Enhancing the computational speed of the modal Green function for the electric-field integral equation for a body of revolution

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Summary. We propose an interpolation technique to reduce the computation time of the integrals involved in the electric field integral equation modal Green function for a perfectly conducting body of revolution in free space. The proposed technique is based on applying an appropriate interpolation to the singular part of the modal Green function, which is computationally expensive. By analyzing the electromagnetic scattering of various objects, it is shown that the proposed interpolation scheme can reduce the corresponding computational time by more than a factor of 100.

1 Introduction

Scattering by perfectly electrically conducting (PEC) bodies of revolutions (BOR) in free space have been widely studied in computational electromagnetics [1, 3, 4]. A popular approach to characterize the scattering of such kind of structures resides in solving the associated electric field integral equations (EFIE), obtained from the equivalence principle.

Many reports exist on solving the scattering by PEC BORs based on the EFIE [2, 5, 6]. Yet, the computation of the modal Green function of the EFIE can be challenging, especially when the observation point approaches the source point. In particular, such a coincidence results in singular behavior for the modal Green function (MGF) of the EFIE. Such a singularity renders the direct computation of the corresponding MGF inaccurate.

To avoid such inaccuracies, one can extract the singular part of the MGF and calculate it individually. This approach, known as the singularity extraction method, has been leveraged in several prior studies for addressing the scattering of PEC BORs with different geometries [2, 6]. In this regard, one of the most efficient and accurate techniques has been provided in [6] when the analysis concerns many modes. In this proposal, the MGF is written as the combination of a regular and a singular parts, which are then computed, respectively, by means of a fast Fourier transform (FFT) and a three-term recurrence relation. Such a decomposition enables the accurate calculation of the MGF, overcoming the aforementioned singularity issue. However, this approach is not yet optimized in terms of computation time, as it implies the repetitive computation of the singular part of the MGF for similar Green-function parameters.

Here, by avoiding the redundant calculation of the singular part of the EFIE MGF using an interpolation scheme, we propose to improve the computational speed of the singularity extraction method proposed in [6]. The performance of the proposed approach is evaluated by analyzing the scattering by BORs of different size, and by comparing the corresponding speed with the one from the original method in [6]. It is shown that the proposed approach can boost the computational speed by a factor of more than 100. It is further shown that, by increasing the size of the body, the enhancement in the computational speed becomes even more significant, thus illustrating the importance of the proposed technique for addressing the scattering of larger-scale structures.

2 EFIE Modal Green function

The EFIE modal Green function is represented as follows [6]

$$g_n(\rho,\rho',z,z') = \int_0^{\pi} \frac{e^{-jkR(\phi)}}{4\pi R(\phi)} \cos(n\phi)\, d\phi, \quad (1)$$

in which $R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(n\phi) + (z-z')^2}$ denotes the distance between source and observation points, n is the mode number, and $k = 2\pi\lambda$ is the wave number, where λ is the wavelength. It can be deduced from Eq. (1) that, when the source point gets close to the observation point ($R \to 0$), g_n becomes singular. To handle this issue, a singularity extraction method has been provided in [6], based on decomposing g_n into a regular ($g_{n,1}$) and a singular parts ($g_{n,2}$) as follows

$$g_n(\rho,\rho',z,z') = g_{n,1} + g_{n,2} = \int_0^{\pi} \frac{e^{-jkR(\phi)} - 1}{4\pi R(\phi)} \cos(n\phi)\, d\phi + \int_0^{\pi} \frac{\cos(n\phi)}{4\pi R(\phi)} d\phi, \quad (2)$$

$g_{n,1}$ is continuous and periodic. Thus, it can be computed accurately for all modes n simultaneously using an FFT. On the other hand, the singular part ($g_{n,2}$) can be rewritten as

$$g_{n,2} = \frac{1}{4\pi}(-1)^n \sqrt{\frac{w}{\rho\rho'}} \int_0^{\pi/2} \frac{\cos(2n\phi)}{\sqrt{(1-w\sin^2\phi)}} d\phi$$
$$= \frac{1}{4\pi}(-1)^n \sqrt{\frac{w}{\rho\rho'}} Q_n(w), \tag{3}$$

in which

$$w = \frac{4\rho\rho'}{(\rho+\rho')^2 + (z-z')^2}. \tag{4}$$

and $0 \le w \le 1$. In [6], a three-term recurrence relation was proposed to compute $Q_n(w)$.

For a cylindrical BOR with radius of 0.25λ and height of 0.5λ (the total perimeter of the generating curve is λ), we compute the corresponding impedance matrix while keeping track of the values of w in the process. The histogram of the parameter w is shown in Fig. 1. From the results of this figure, one realizes that the parameter w is close to one most of the time. This is consistent with the fact that, when the source and observation points are close to each other (which is where we apply the singularity extraction method), w approaches 1 according to Eq. (4) (except for the regions close to the axis of rotation of the BOR). This form of distribution implies many redundant computations of $g_{n,2}$ in the small region of $0.99 \le w \le 1$. To avoid this redundancy, we compute $g_{n,2}$ based on an interpolation scheme. In this scheme, $g_{n,2}$ is computed at specific reference points in the region $0.99 \le w \le 1$, based on which a look-up table is created. This look-up table is then used to interpolate $g_{n,2}$ for all values of w.

To evaluate the performance of the proposed technique, we consider the computation time for the impedance matrix for PEC BOR cylinders with increasing length of their perimeter based on the normal, i.e. without interpolation, and the proposed interpolation approach. The corresponding computation times, as well as the number of segments on the perimeter of the generating curve, are summarized in Table 1. Note that 16 Fourier modes, i.e. $n = 0, \ldots, 15$, were considered in all cases. As can be observed, the optimized approach offers much shorter computation time, i.e. by more than a factor of 100. Interestingly, the reduction in computation time becomes more significant when increasing the perimeter of the BOR by a factor of 2 and 4.

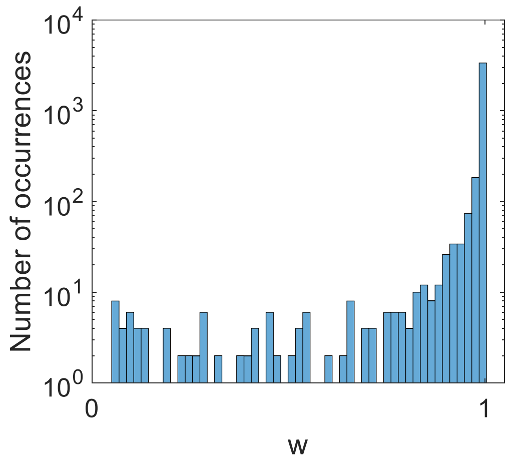

Fig. 1. Histogram for w when computing the impedance matrix of a cylindrical PEC BOR with dimensions specified in the text and perimeter length λ.

Table 1. Computational time of the singular part of EFIE MGF

Object	number of segments	normal (s)	interpolated (s)
Cylinder (λ)	40	159.31	1.40
Cylinder (2λ)	80	319.35	1.90
Cylinder (4λ)	160	641.12	4.55

3 Conclusion

We proposed an interpolation scheme for optimizing the computation of the singular part of the EFIE modal Green function. It is demonstrated that the proposed method can reduce the computation time by a factor of more than 100.

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