Abstracted Reduction of Interconnected Structural Models

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1 Introduction

To assess their dynamic behaviour, complex structural systems are usually modeled using finite element methods. The large order of these models necessitates the use of model reduction techniques to enable efficient dynamic analysis. These models, represented by the system of linear differential equations $\mathbf{\Sigma}$, often consist of an interconnection of substructures $\mathbf{\Sigma}_j,\ j = 1,\ldots,k$. In practice, model reduction is often performed on individual substructures $\mathbf{\Sigma}_j$, by, e.g., component mode synthesis methods (CMS), because (i) direct reduction of $\mathbf{\Sigma}$ is not computationally tractable and (ii) substructures are typically developed by independent teams. However, if the reduction of a substructure $\mathbf{\Sigma}_j$ to its reduced representation $\hat{\mathbf{\Sigma}}_j$ does not consider the coupling to the other substructure dynamics, the accuracy of the coupled, reduced-order model (ROM), $\hat{\mathbf{\Sigma}}$, cannot be ensured.

2 Research approach

We introduce the idea of reducing $\mathbf{\Sigma}_j$ in interconnection with a low-order approximation of the other substructures, to improve the accuracy of $\hat{\mathbf{\Sigma}}$. Stated differently, instead of considering (and reducing) $\mathbf{\Sigma}_j$ in isolation, we consider the interconnection of $\mathbf{\Sigma}_j$ with an *abstraction* of its environment. This ensures that the reduced $\hat{\mathbf{\Sigma}}_j$ is relevant in the scope of the overall structure. By using only an abstraction instead of the full substructure models, the computational cost of the reduction problem decreases and only a basic, nominal designs of the other substructures are required.

Reduction of the interconnection of $\mathbf{\Sigma}_j$ and the corresponding abstraction using standard reduction methods would destroy the interconnection structure and results in one unified, reduced model. Therefore, structure-preserving reduction methods, such as structure-preserving balanced truncation (SPBT) of [1], are employed to retain the interconnection structure and thus retain access to the reduced subsystems $\hat{\mathbf{\Sigma}}_j$. The combination of SPBT with the use of abstractions is denoted *abstracted* SPBT (ASPBT).

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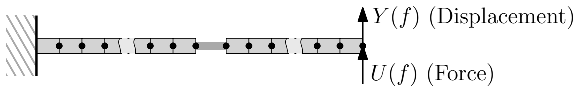

Figure 1: Schematic drawing of the coupled beam model.

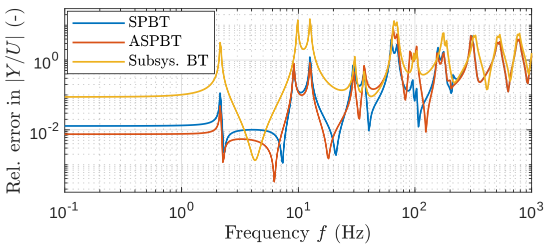

Figure 2: Relative reduction error in $|Y/U|$.

3 Numerical example

Observe the SISO system of two equivalent Euler beams, with 2% modal damping each, in Figure 1. The coupling consists of a translational and rotational spring, with, respectively, 50 and 5 times the left cantilever's translational and rotational stiffness at its end. The discretized cantilever and free beam substructures consist of 60 and 62 degrees of freedom (DoF), respectively, and are reduced to 4 DoF each. Reduction is performed using (i) SPBT, (ii) ASPBT (using CMS-reduced abstractions of 3 DoF) and (iii) subsystem balanced truncation (BT). The resulting relative error magnitudes of the collocated FRF is shown in Figure 2.

4 Conclusion and outlook

Numerical evaluation indicates that ASPBT is superior to subsystem BT in terms of input-output accuracy of the ROM, and is comparable to SPBT. This implies that low-order abstractions are sufficient to capture the relation between $\mathbf{\Sigma}_j$ and $\mathbf{\Sigma}$, while significantly improving computational tractability. In further work, we will formulate error bounds to be able to further compare ASPBT to alternatives.

References

[1] H. Sandberg and R. M. Murray, “Model reduction of interconnected linear systems,” *Optimal Control Applications and Methods*, vol. 30, no. 3, pp. 225–245, May 2009.