

# Multirate Performance Quantification using Time-Lifting and Local Polynomial Modeling

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# Multirate Performance Quantification using Time-Lifting and Local Polynomial Modeling

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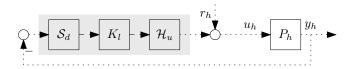
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# 1 Background

Increasing performance requirements for mechatronic systems leads to continuous-time performance evaluation becoming more important. Continuous-time performance is typically evaluated using a significant higher sampling rate for the plant compared to the sampling rate of the controller, resulting in multirate systems. Similarly to single-rate systems, multirate systems require performance quantification.

## 2 Problem Formulation

Consider the multirate control structure in Figure 1, where the controller  $K_l$  is sampled at a low-rate  $\omega_{s,l} = \omega_{s,h}/F$ and  $P_h$  at a high-rate  $\omega_{s,h} = 2\pi/h_h$ . Performance criteria



**Figure 1:** High-rate plant  $P_h$  operating in multirate feedback loop with low-rate controller  $K_l$ . The signals are up- and downsampled using  $\mathcal{H}_u$  and  $\mathcal{S}_d$ .

for Linear Time Invariant (LTI) systems are typically evaluated using Frequency Response Functions (FRFs). However, since multirate systems are Linear Periodically Time-Varying (LPTV) [1], evaluating frequency domain models is not trivial. Several definitions for FRFs for multirate systems are available [2], requiring multiple experiments. Hence, the aim of this paper is to develop a more timeefficient method to identify multirate FRFs.

# **3** Approach

This paper considers the Performance Frequency Gain (PFG) definition of multirate FRFs, given by [2]

$$\mathcal{P}\left(e^{j\omega h_h}\right) = \sup_{w_h \neq 0} \frac{\|\zeta_h\|_{\mathcal{P}}}{\|w_h\|_{\mathcal{P}}},\tag{1}$$

where  $\zeta_h$  and  $w_h$  are chosen by the user, with  $w_h$  containing only one frequency component. The PFG represents the maximum response of a system for a single input frequency, hence requiring a multitude of experiments to determine for a frequency grid. An alternative representation of the PFG uses  $K_l$  and  $P_h$  [2]. To identify  $P_h$ , this paper uses the timelifted representation, translating a SISO LPTV into a MIMO LTI representation [1]. First, define  $\underline{J} : \underline{r} \mapsto \underline{y}$  and  $\underline{S} : \underline{r} \mapsto \underline{u}$ , where  $\underline{r} = \mathcal{L}r_h$ ,  $\underline{y} = \mathcal{L}y_h$ ,  $\underline{u} = \mathcal{L}u_h$  and  $\mathcal{L}$  the time-lifting operator.  $\hat{J}$  and  $\hat{S}$  are estimated with local polynomial modeling, since it identifies MIMO systems in a single experiment [3]. Second, recover the time-lifted original system as  $\hat{P} = \mathcal{L}\hat{P}_h\mathcal{L}^{-1} = \hat{J}\hat{S}^{-1}$ . The high-rate system  $\hat{P}_h$  is found by inverse lifting [1, Section 6.2.1]. Finally, the PFG is calculated by performing the procedure in [2], using  $\hat{P}_h$  and  $K_l$ .

# 4 Initial Results

A high-rate fourth-order system  $P_h$  with controller  $K_l$  is considered with F = 3. The PFG is determined based on an estimate of  $P_h$ , both using an Emperical Transfer Function Estimate (ETFE) and the developed approach. The estimation error of the PFG for these methods is shown in Figure 2.

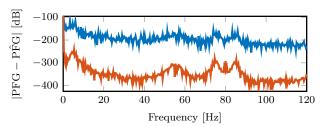


Figure 2: Error for estimating the PFG for ETFE (---) and the developed approach (---).

#### **5** Ongoing Research

Ongoing research is focused at validating the framework in an experimental setting.

### Acknowledgments

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