Improved Bit-mappings for IEEE 802.11 LDPC Codes with Applications to Amplitude Shaping

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Extended Abstract

Binary low-density parity-check (LDPC) codes are linear block codes which can be represented by their $(n-k)$-by-n parity-check matrix $\boldsymbol{H}$. Here, k is the length of the binary information sequence $\boldsymbol{u} = (u_1, u_2, \ldots, u_k)$, and n is the length of the binary codeword $\boldsymbol{c} = (c_1, c_2, \ldots, c_n)$. Each codeword $\boldsymbol{c}$ in the code $\mathcal{C}$ satisfies $\boldsymbol{c}\boldsymbol{H}^T = \boldsymbol{0}$, i.e., the code $\mathcal{C}$ is the null space of $\boldsymbol{H}$. Equivalently, a valid codeword has to satisfy $n-k$ parity-check equations. In Fig. 1 (left), parity-check matrix of the $[n = 7, k = 4, (n-k) = 3]$ Hamming code is shown as an example [1, p. 15].

Any linear block code can be represented by a Tanner graph. A Tanner graph is a bipartite graph that consists of n variable nodes and $n-k$ check nodes. There is a connection between the i^{th} variable node and the j^{th} check node if $\boldsymbol{H}_{ji} = 1$. The Tanner graph that corresponds to the parity-check matrix in Fig. 1 (left) is shown in Fig. 1 (right). Here, circles represent variable nodes, i.e., coded bits in $\boldsymbol{c}$, squares represent check nodes, i.e., parity-check equations.

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

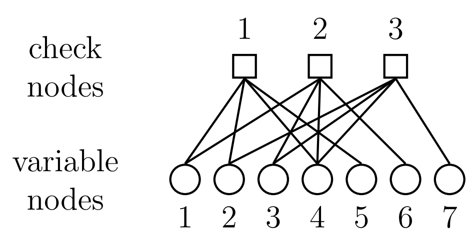

Figure 1: The Tanner graph and the corresponding parity-check matrix.

The degree of a node in a Tanner graph is defined as the cardinality of the set of nodes that it is connected to. Thus, the degree of the i^{th} variable node is given by the sum of all elements of the i^{th} column of $\boldsymbol{H}$. As an example, there are three degree-1, three degree-2, and one degree-3 variable nodes in the Tanner graph in Fig. 1. A coded bit is a part of several parity-check equations, and this number is given by the degree of the corresponding variable node.

In bit-interleaved coded modulation (BICM), the n-bit output of a binary forward error correction (FEC) code is typically mapped to an N-tuple of 2^m-ary amplitude-shift keying (2^m-ASK) symbols. This mapping is achieved using a binary labeling strategy that assigns an m-bit binary label $(B_1, B_2, \ldots, B_m)$ to each symbol, e.g., the binary reflected Gray code (BRGC) [2, Defn. 2.10]. Here, we assumed that $N = mn$.

For BICM systems, the error probability is not identical for different bit levels, and each bit is not "protected" equally against errors. This phenomenon is called unequal error protection (UEP). As an example, we show in Fig. 2 the bit error rate (BER) of different bit levels for uniform and shaped 16-ASK with BRGC. Here, shaping is realized using enumerative sphere shaping [3, 4]. For uniform 16-ASK, the protection is higher for more significant bit levels. However, for shaped 16-ASK, the most protected bit level is B_2. This is because the ratio $\Pr\{B_i = 0\}/\Pr\{B_i = 1\}$ is the largest for $i = 2$, an effect observed earlier in [5, Sec. III-C].

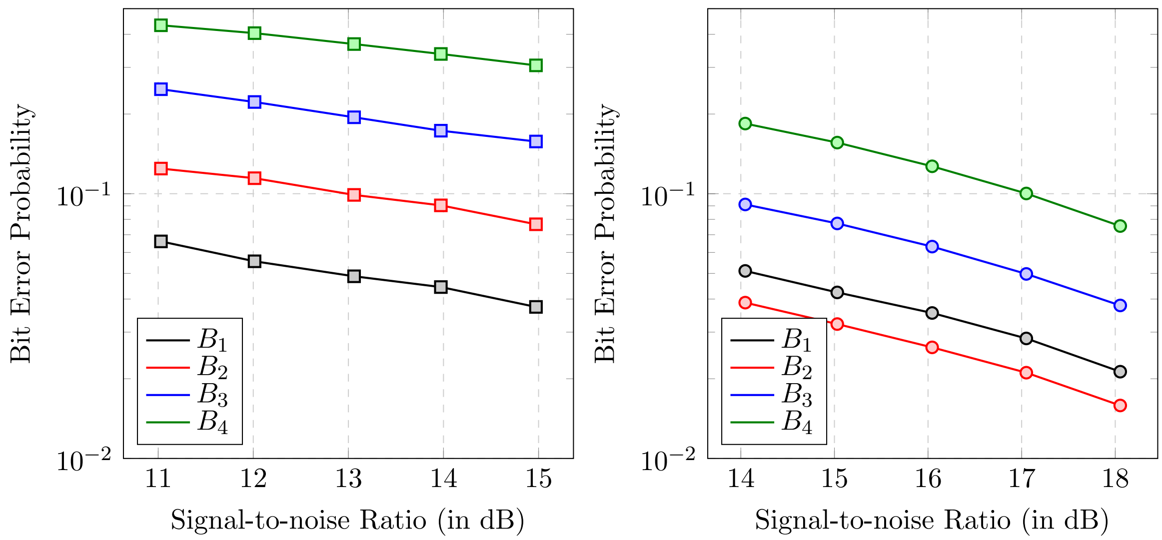

Figure 2: Pre-FEC, hard-detection BER for uniform (left) and shaped (right) 16-ASK with 4 and 3.5 bits of entropy, respectively.

UEP observed in Fig. 2 implies that the performance of an LDPC code depends on how the coded bits, i.e., variable nodes, are "connected" to different bit levels. A coded bit with a corresponding variable node with a higher degree is a part of a higher number of parity checks and can be considered to play a more important role during the decoding procedure. Thus, connecting variable nodes with higher degrees to more protected bit levels can improve the performance.

In Fig. 3, the performance of uniform BICM and probabilistic amplitude shaping (PAS) [6] is shown for 16-ASK and the corresponding BRGC. As the FEC code, the rate-5/6 and 3/4 648-bit systematic LDPC codes from the IEEE 802.11 standard [7] are used for uniform and shaped signaling, respectively. Both the regular bit-mapping and the heuristically modified bit-mapping explained above are simulated. In regular bit-mapping, bits at the output of the FEC encoder are consecutively gathered into groups of m and mapped to 2^m-ASK symbols. In modified bit-mapping, bits at the output of the FEC encoder are re-ordered before ASK mapping such that the ones that correspond to higher degree variable nodes are mapped to a more protected bit level according to Fig. 2. The transmission rates of the uniform and the shaped schemes are 3.33 and 2.5 bit/1D, respectively. We see that at a BER of 10^{-5}, modified bit mapping provides 0.2 and 0.1 dB gains over the regular mapping for uniform and shaped signaling, respectively. We believe the reason why the gain is smaller for shaped signaling is due to a constraint imposed by PAS: all parity bits should be used as the sign bit level B_1 [6, Fig. 5]. Accordingly, we were only able to change the mapping for variable nodes that correspond to systematic bits. A future research direction can be to investigate the effect of this constraint by using less restrictive shaping approaches as in [8, 9].

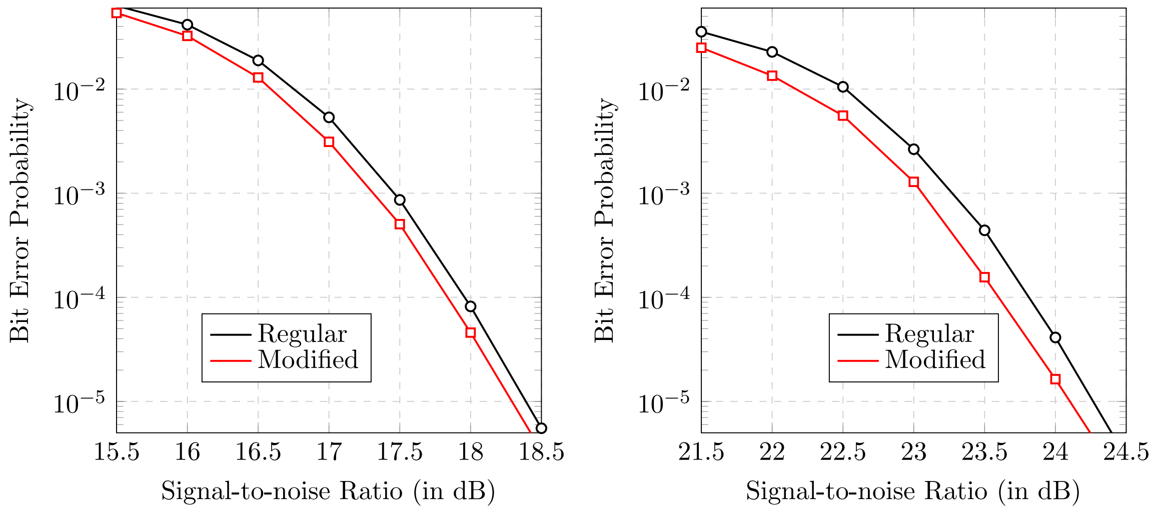

Figure 3: Post-FEC BER for shaped (left) and uniform (right) signaling.

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