Vector Control of Induction Machines

Vector Control of Induction Machines

J.L. Duarte

Abstract: Various vector control methods, which were introduced in the past as being quite different approaches, are present as particular cases that fit a more general treatment. An analytical description is derived to facilitate the evaluation of the common back-ground among then.

Introduction

In the so-called *scalar* approaches for induction motor control, the machine model is considered just for steady state. As expected, the control flexibility available with these methods is very limited and their application is, in the main, restricted to general-purpose ac drives where transient response and low speed-performance are not critical. Until recently, high performance applications have almost exclusively been the domain of dc motors. With *vector control*, however, induction motor drives are more than a match for dc drives.

The Field-Oriented Control - FOC - proposed by Blaschke (1972) was the first and the most characteristic vector control method. It can be understood from the *Reference-Frame Theory* (Krause, 1986). The reference frame used in FOC is one whose direct axis coincides with the cage-flux vector. This frame is not static and does not have constant speed during transients. Actually, it was not a commonly used reference frame for the analysis of electric machines. The great advantage of this *non-inertial* frame is that the induction machine is seen as a separately-excited DC machine. As a result, for impressed stator currents, field orientation allows for independent flux and torque control.

A drawback of FOC is that this method is sensitive to parameter variations, principally the rotor time constant. Moreover, the determination of the cage-flux vector, to be used as referential for field orientation, and the implementation of the current control loop, necessary to impose the stator currents, can be performed in different ways. Therefore, alternative vector control strategies have been proposed since the 80's, trying to achieve robustness and/or ease-of-implementation.

Among the new vector control methods, the following can be mentioned: Field Acceleration Method - FAM (Yamamura, 1986), Improved Field Acceleration Method - IFAM (Takahashi & Noguchi, 1986), Direct Self Control - DSC (Depenbrock, 1988), Universal Field Orientation - UFO (De Doncker & Novotny, 1994), Direct Torque Control - DTC (Tiitinen *et al.*, 1995), and others. As different mathematical notations have been used in these studies (e.g. spatial vectors, spatial phasors, spiral vectors, matrices, ...) it is not straightforward to evaluate the correlation among then.

For the purpose of attaining a basic understanding of the joint concepts involved, a general description of the control approaches is given in this paper, following the classification proposed by Santisteban & Stephan (1995). As it will be seen in the next sections, the proposals can be grouped in two main categories: *quadrature control* methods and *slip control* methods.

Basic Relationships

The vector representation to be used in this paper allows for the analysis of induction machines in arbitrary reference frames (e.g. frames oriented with stator, rotor, stator flux, cage flux, air-gap flux, etc..) with relative simplicity. The motivation for the chosen notation is given in Appendix, being a particular case of the classical vector theory.

In terms of vector quantities, the fundamental voltage/current/flux relationships of an induction machine are summarized by

$$\mathbf{u}_s^s = r_s \mathbf{i}_s^s + D\{\mathbf{\Psi}_s^s\}, \quad (1)$$
$$\mathbf{u}_k^r = r_k \mathbf{i}_k^r + D\{\mathbf{\Psi}_k^r\} \equiv 0, \quad (2)$$

where

$\mathbf{u}_s^s, \mathbf{u}_k^r$ are the stator and cage voltage vectors,

$\mathbf{i}_s^s, \mathbf{i}_k^r$ are the stator and cage current vectors,

$\mathbf{\Psi}_s^s$, $\mathbf{\Psi}_k^r$ denote the flux vectors linking the windings at stator and the cage windings at the rotor, respectively,

r_s, r_k represent the stator and cage resistances,

$D\{\,\}$ is an operator that performs differentiation with respect to time ($\equiv \mathrm{d}\{\,\}/\mathrm{d}t$).

In Eq.(1) the vectors are projected onto a stationary reference frame oriented with the stator (superscript s), while in Eq.(2) the reference frame is rotating synchronously with the rotor (superscript r).

The projection of the vectors in Eqs.(1-2) onto an *arbitrary* reference frame leads to

$$\mathbf{u}_s^a = r_s \mathbf{i}_s^a + D\{\mathbf{\Psi}_s^a\} + \dot{\gamma}_a^s \mathbf{R}(\pi/2)\mathbf{\Psi}_s^a, \quad (3)$$
$$0 = r_k \mathbf{i}_k^a + D\{\mathbf{\Psi}_k^a\} + \dot{\gamma}_a^r \mathbf{R}(\pi/2)\mathbf{\Psi}_k^a, \quad (4)$$

with $\dot{\gamma}_a^s \equiv D\{\gamma_a^s\}$, $\dot{\gamma}_a^r \equiv D\{\gamma_a^r\}$ denoting the angular velocity of the arbitrary frame with respect to the stator and the rotor, respectively; and

$$\begin{aligned} \mathbf{\Psi}_s^a &= \mathbf{\Psi}_l^a + \ell_{\sigma s}\mathbf{\imath}_s^a \\ \mathbf{\Psi}_k^a &= \mathbf{\Psi}_l^a + \ell_{\sigma k}\mathbf{\imath}_k^a \\ \mathbf{\Psi}_l^a &= \ell(\mathbf{\imath}_s^a + \mathbf{\imath}_k^a), \end{aligned} \tag{5}$$

where $\mathbf{\Psi}_l^a$ is the vector related to the total flux crossing the air-gap, ℓ is de machine main inductance, $\ell_{\sigma s}$ and $\ell_{\sigma k}$ are the leakage inductances of the stator and cage windings.

In view of Eqs.(3-4), the electric torque developed by the induction machine is determined by conservation of energy to be

$$m_{el} = [\mathbf{R}(\pi/2)\mathbf{\Psi}_s^a]^{\top}\mathbf{\imath}_s^a. \tag{6}$$

where $^{\top}$ means vector transposition.

It is convenient to define a *generalized-flux* vector (De Donker & Novotny, 1994) as follows

$$\mathbf{\Psi}_x^a = \ell(\chi\,\mathbf{\imath}_s^a + \mathbf{\imath}_k^a), \tag{7}$$

where χ can be seen as an unspecified turn-ratio between stator windings and (equivalent) cage windings. According to the value of χ, $\mathbf{\Psi}_x^a$ represents different fluxes:

$$\begin{array}{lll} \chi = (1+\sigma_s) & : \mathbf{\Psi}_x^a \equiv \mathbf{\Psi}_s^a & \text{(stator flux)}, \\ \chi = 1 & : \mathbf{\Psi}_x^a \equiv \mathbf{\Psi}_l^a & \text{(air-gap flux)}, \\ \chi = 1/(1+\sigma_k) & : \mathbf{\Psi}_x^a \equiv \mathbf{\Psi}_k'^{\,a} & \text{(cage flux)}, \end{array}$$

where $\sigma_s = \ell_{\sigma s}/\ell$ and $\sigma_k = \ell_{\sigma k}/\ell$ are leakage factors, and

$$\mathbf{\Psi}_k'^{\,a} = \frac{1}{(1+\sigma_k)}\,\mathbf{\Psi}_k^a$$

represents, in fact, a 'reduced' cage-flux vector.

By assuming impressed stator currents, it can be shown that the projection of Eqs.(4-6) onto a reference frame oriented with the generalized-flux vector $\mathbf{\Psi}_x$ yields

$$\dot{\varphi}_x^r = \frac{\imath_s^{\psi x2} + \sigma_\chi \frac{\ell'}{r_k'} D\{\imath_s^{\psi x2}\}}{(1-\sigma_\chi)\frac{1}{r_k'}\psi_x - \sigma_\chi \frac{\ell'}{r_k'}\imath_s^{\psi x1}}, \tag{8}$$

$$\begin{aligned} \psi_x + \frac{\ell'}{r_k'} D\{\psi_x\} &= \frac{1}{(1-\sigma_\chi)}\ell' \times \\ &\times \left[\imath_s^{\psi x1} + \sigma_\chi \frac{\ell'}{r_k'} D\{\imath_s^{\psi x1}\} - \sigma_\chi \frac{\ell'}{r_k'} \dot{\varphi}_x^r \imath_s^{\psi x2}\right], \end{aligned} \tag{9}$$

$$m_{el} = \psi_x\, \imath_s^{\psi x2}, \tag{10}$$

where

$\dot{\varphi}_x^r \equiv D\{\varphi_x^r\}$ is the *slip* (angular velocity) of the generalized-flux vector with respect to the rotor;

$\psi_x \equiv \psi_x^{\psi x1}$ is the direct component of the generalized flux projected onto the reference frame linked to the generalized-flux vector (of course, $\psi_x^{\psi x2} \equiv 0$);

$\imath_s^{\psi x1}, \imath_s^{\psi x2}$ are the stator current direct and quadrature components, respectively;

$\ell' = \ell/(1+\sigma_k)$ and $r_k' = r_k/(1+\sigma_k)^2$ are 'reduced' machine parameters;

and

$$\sigma_\chi = \frac{1}{\chi}\left[\chi - \frac{1}{(1+\sigma_k)}\right]. \tag{11}$$

Eqs.(8-10) are shown as a block diagram in Fig. 1. To emphasize the cross-coupling effect of the current components over the flux and the electric torque, the term $\imath_c$ has been introduced in Fig. 1 following

$$\imath_c + \sigma_\chi \frac{\ell'}{r_k'} D\{\imath_c\} = \sigma_\chi \frac{\ell'}{r_k'} \dot{\varphi}_x^r \imath_s^{\psi x2}. \tag{12}$$

In the case where the cage-flux vector is chosen as reference (hence $\sigma_\chi = 0$), it can be observed from Fig. 1 that the influence of $\imath_s^{\psi k2} (\equiv \imath_s^{\psi x2})$ over the flux is eliminated. This approach corresponds to the field-oriented model proposed by Blaschke. As it can be also seen from the block diagram with $\sigma_\chi = 0$, the flux can be controlled independently by $\imath_s^{\psi k1} (\equiv \imath_s^{\psi x1})$, and the torque by $\imath_s^{\psi k2}$ (while maintaining constant flux level).

Torque Equations

Torque is the quantity that makes the interface between the mechanical and electrical parts of a drive system, being therefore an essential variable to be considered in the control of electrical machines.

The mechanical part can be summarized by

$$D\{\dot{\rho}^s\} = \frac{1}{\Theta}(m_{el} - m_{load},) \tag{13}$$

with $\dot{\rho}^s \equiv D\{\rho^s\}$ representing the (electric) angular rotor velocity and

$$\begin{aligned} \rho^s &= \mathbf{p}\,\rho_{shaft}, \\ m_{load} &= (1/\mathbf{p})\,M_{load}, \\ \Theta &= (1/\mathbf{p}^2)\,\Theta_{shaft}, \end{aligned} \tag{14}$$

where $\mathbf{p}$ is the number of pole-pairs, ρ_{shaft} the physical shaft angle with respect to the stator, Θ_{shaft} the total inertia of all rotating parts on the shaft, and M_{load} all other instantaneous load torques.

Eq.(10) presents the electrical torque as the product of a generalized flux and the stator current component orthogonal to it. This equation is similar to the one for a separately-excited dc machine. In this paper the control methods based on Eq.(10) will be called *quadrature control* methods.

On the other hand, by using Eqs.(8-9), it is possible to write $\imath_s^{\psi x2}$ as a function of $\dot{\varphi}_x^r$. The substitution of this expression for $\imath_s^{\psi x2}$ into Eq.(10) yields a non-liner function between electric torque and slip $\dot{\varphi}_x^r$ as follows

$$\begin{aligned} \left[1 - \sigma_\chi \frac{\ell'}{r_k'}\frac{D\{\psi_x\}}{\psi_x}\right] m_{el} + \sigma_\chi \frac{\ell'}{r_k'} D\{m_{el}\} &= \\ = (1-\sigma_\chi)\frac{1}{r_k'}\left[\psi_k'^{\,\psi x1}\psi_x\right]\dot{\varphi}_x^r, \end{aligned} \tag{15}$$

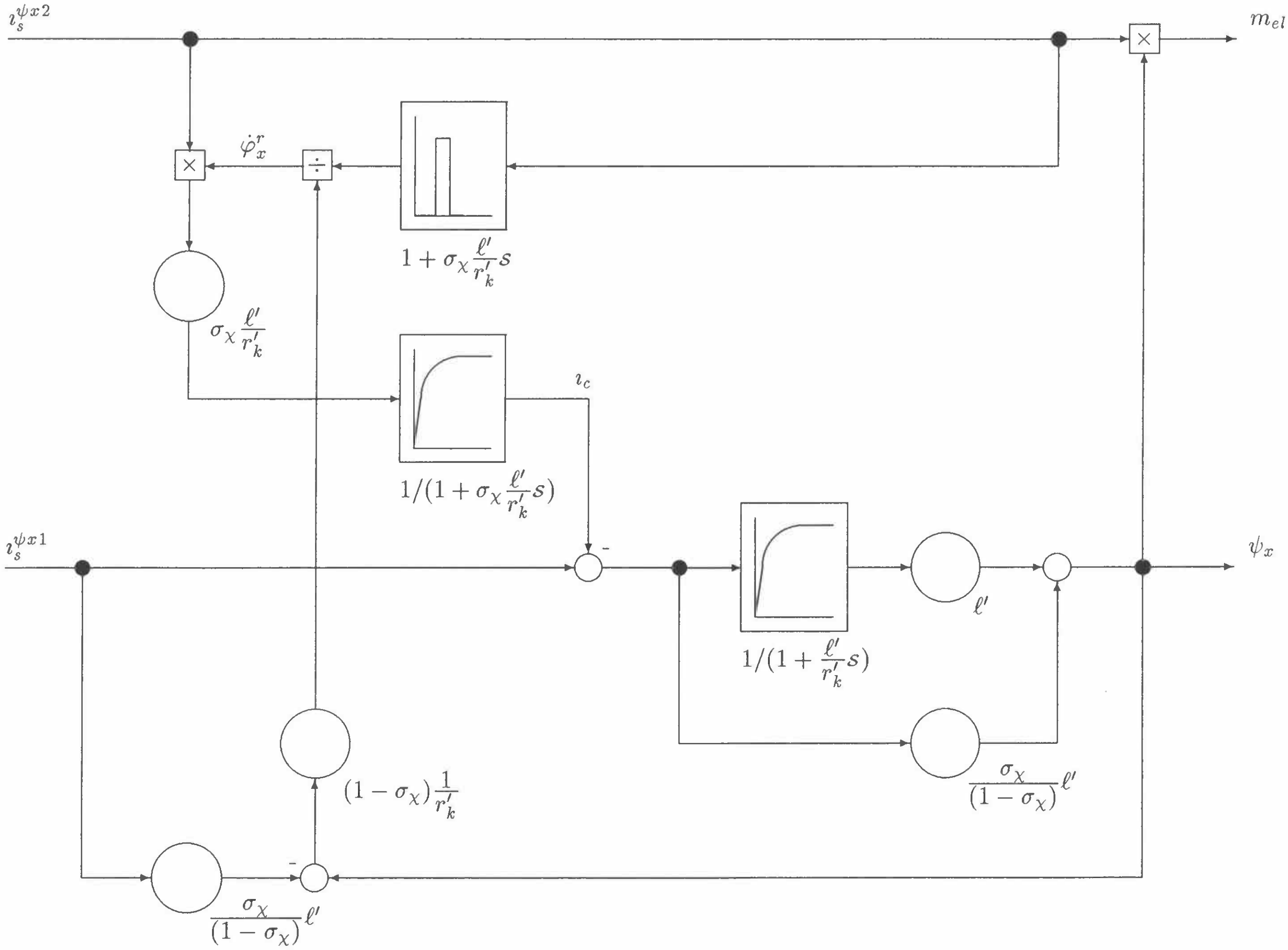

Figure 1: Induction machine model with impressed currents, projected onto a reference frame oriented with the generalized-flux vector. The linear blocks are given as transfer functions in the complex Laplace-variable s.

where $\psi_k'^{\psi x1}$ is the 'reduced' cage-flux direct component, which, in light of Eqs.(5)&(7), follows from

$$\mathbf{\Psi}_k'^{\,a} = \mathbf{\Psi}_x^a - \frac{\sigma_\chi}{(1-\sigma_\chi)}\ell'\,\mathbf{i}_s^a. \tag{16}$$

The control methods based on Eq.(15) will be called *slip control* methods.

If nearly-constant flux operation ($D\{\psi_x\} \approx 0$, $\psi_k'^{\psi x1} \approx \psi_x$) is assumed in Eq.(15), an *approximately-linear* first-order differential equation between torque and slip is found :

$$m_{el} + \sigma_\chi \frac{\ell'}{r_k'} D\{m_{el}\} = \left[(1-\sigma_\chi)\frac{1}{r_k'}(\psi_x)^2\right] \dot{\varphi}_x^r. \tag{17}$$

However, when the cage-flux vector is chosen as reference ($\sigma_\chi = 0$), it results from Eq.(15) that

$$m_{el} = \left[\frac{1}{r_k'}(\psi_k')^2\right] \dot{\varphi}_k^r, \tag{18}$$

which leads to a direct relationship between torque and slip under constant flux operation. It is worthwhile to notice that Eq.(18) is always *exact*, also under transient conditions.

Quadrature Control Methods

As already pointed out, these control methods are based on Eq.(10). They can be further subdivided into *direct* and *indirect* methods. Direct ones have a control loop for the flux, a flux estimator or flux sensors being therefore necessary. Indirect ones assume that the flux amplitude is constant, the spatial position of which being obtained by means of a feedforward block that has the rotor speed as input.

Direct Quadrature Control Methods

The general control scheme is presented in Fig. 2, where the superscript $\tilde{}$ labels values that are provided by observers (Verguese & Sanders, 1988). The superscript $\hat{}$ denotes *estimated* (therefore non-exact) values for the machine parameters, while the superscript * represents *desired* (or commanded) quantities.

The PI-regulators shown in Fig. 2 may be substituted by other classical or modern controllers. Also, to improve the dynamic response, the desired stator current quadrature component may be given by

$$\imath_s^{\psi x2*} = \frac{{m_{el}}^*}{\tilde{\psi}_x}.$$

To obtain decoupled torque and flux control loops, it is necessary to compensate for the cross-coupling current $\imath_c$ (cf. Eq.(12)). In the general case, this can be achieved by adding the term $\hat{\imath}_c$ to the output of the flux controller, as shown in Fig. 2, being calculated from observed values . Decoupling occurs naturally with cage-flux orientation ($\imath_c \equiv 0$).

Various examples of direct quadrature control methods are given by Jansen & Lorenz (1993), showing that it is also possible to perform regulation in a different reference frame than the one used for field orientation.

Indirect Quadrature Control Methods

In this case, there is no flux control loop and $\imath_s^{\psi x1*}$ in Fig. 2 is imposed constant. Some examples of this control method are given with the indirect UFO scheme (De Doncker & Novotny, 1994), which is based on the inverse model of the induction motor. Another one is the well-known indirect FOC (Murphy & Turnbull, 1988).

Slip Control Methods

These methods are based on Eq.(15).

Cage-flux Slip Control

As already mentioned, in this case $\sigma_\chi = 0$ and Eq.(15) can be written exactly as Eq.(18). Moreover it can be seen from Eq.(8) with $\sigma_\chi = 0$ that there exists a direct relationship between $\imath_s^{\psi k2}$ and the slip. Therefore, when $\sigma_\chi = 0$, Eq.(8) is the equivalent quadrature form of Eq.(15). This means that the cage-flux based slip control and the cage-flux based quadrature control are equivalent.

An example of cage-flux slip control is the FAM applied to the T-I model (Yamamura, 1986). The equivalence of this control scheme and the indirect FOC was shown by Stephan (1991).

Air-gap and Stator-flux Slip Control

FAM applied to T and T-II models (Yamamura, 1986) are examples of air-gap and stator-flux slip control, respectively, where the derivative terms in Eq.(15) are negleted, yielding

$$m_{el} = \frac{(1-\sigma_\chi)^2 \frac{(\psi_x)^2}{r_k'}\,\dot{\varphi}_x^r}{1 + (\sigma_\chi \frac{\ell'}{r_k'}\,\dot{\varphi}_x^r)^2}. \tag{19}$$

As it can be expected, by using Eq.(19), a linear control can not produce the best dynamic behaviour during transients.

IFAM, DSC and DTC are other examples of stator-flux slip control methods, where non-linear controllers are applied. In contrast to field orientation, the non-linear approach in these cases does not require an accurate instantaneous flux angular position, being based on hysteresis control of the electromagnetic state of the motor.

Concluding Remarks

An overview about vector control of induction machines has been presented. Analytical comparisons were derived and a general classification given.

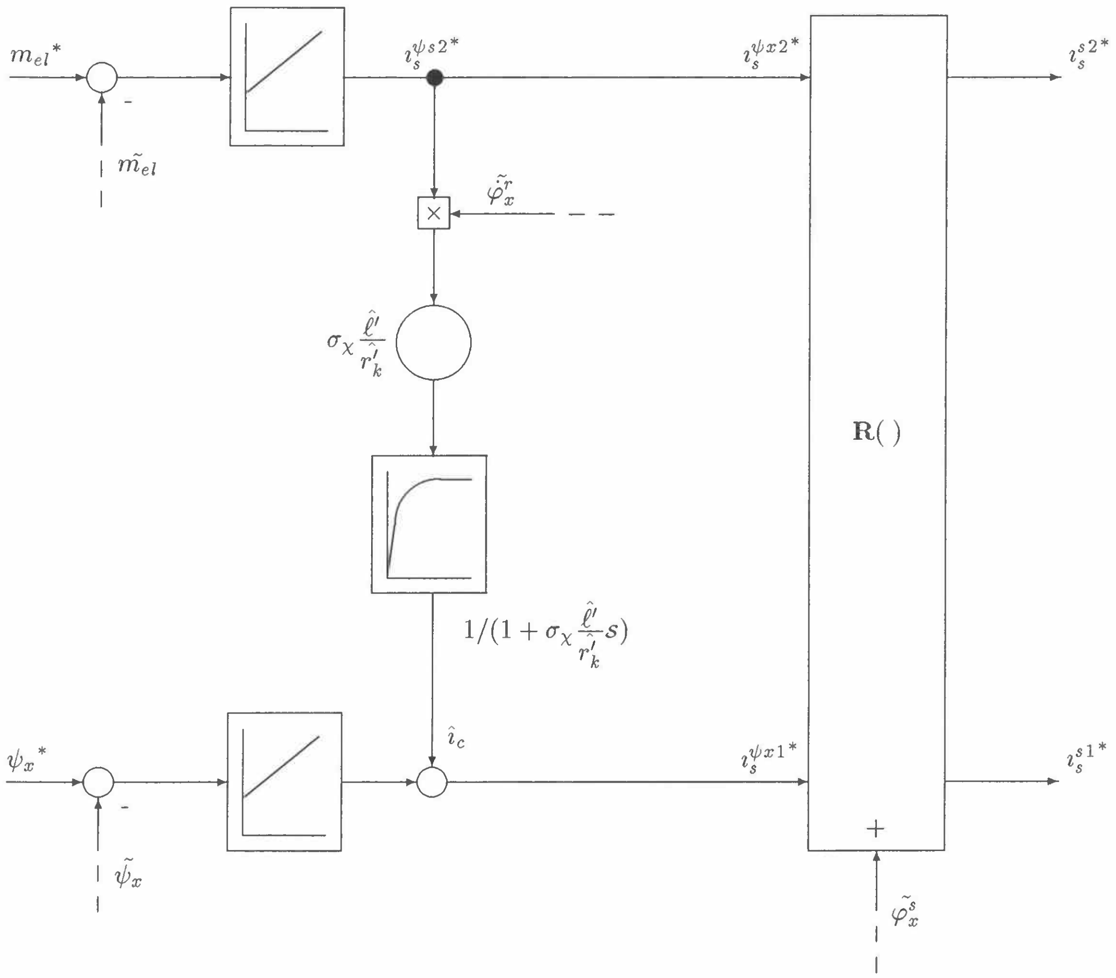

Figure 2: Direct quadrature control.

From the analysis, the following conclusions can be deduced:

- The quadrature control methods are based on exact torque equations. Therefore, if the motor parameters necessary for the control algorithm are known, the transient performance can be optimal.

- The slip control methods which are based on approximate torque equations can not produce optimal transient responses for all operating conditions.

- The cage-flux based slip control is equivalent to the cage-flux based quadrature control.

- All direct quadrature control methods can present equivalent optimal transient responses.

REFERENCES

Blaschke F.; 1972; *The principle of field orientation as applied to the new Transvektor closed-loop control system for rotating-field machines*; Siemens Review; vol. 39, no. 5; pp. 217-220.

Krause P; 1986; *Analysis of Electric Machinery*; Mc Graw-Hill, New York.

Yamamura S.; 1986; *AC motors for high performance applications*; Mercel Dekker, New York.

Takahashi I. & T. Noguchi; 1986; *A new quick-response and high-efficiency control strategy of an induction motor* ; IEEE Trans. on Ind. Applic.; vol. 22, no. 5; pp. 820-827.

Depenbrock M.; 1988; *Direct Self-Control of inverter-fed induction machine*; IEEE Trans. on Power Elect.; vol. 3, no. 4; pp. 420-429.

Murphy J. & F. Turnbull; 1988; *Power Electronic Control of AC motors*; Pergamon Press, Oxford.

Verguese G.C. & S.R. Sanders; 1988; *Observers for flux estimation in induction machines*; IEEE Trans. Ind. Electron., vol. 35, no.1; pp. 85-94.

Stephan R.; 1991; *Field oriented and Field acceleration control for induction motors: Is there a difference?*; IECON Conf. Proc., pp. 567-572.

Serrano-Iribarnegaray L.; 1993; *The modern Space-phasor Theory, Part II: Comparison with the Generalized Machine Theory and the Space-Vector Theory*; ETEP Trans.; vol. 3, no. 3; pp. 213-219.

De Donker R. & D. Novotny; 1994; *The Universal Field Oriented Controller*; IEEE Trans. on Ind. Applic.; vol. 30, no. 1; pp. 92-100.

Jansen R. & R. Lorenz; 1994; *A Physically insightful approach to the design and accuracy assessment of flux observers for field oriented induction machine drives*; IEEE Trans. on Ind. Applic.; vol. 30, no. 1; pp. 101-110.

Tiitinen, P., P. Pohjalainen & J. Lalu; 1995; *The next generation motor control method: Direct Torque Control*; EPE Journal; vol. 5, no. 5; pp. 14-18.

Santisteban J.A. & R.M. Stephan; 1995; *Vector control methods for induction machines: an overview*; IFAC-CETTI (Workshop on Control Education and Technology Transfer Issues); Curitiba-BR.

APPENDIX : *Vector notation*

Accurate lumped-parameter models for electric machines can be readily derived from vector principles (see Serrano-Iribarnegaray, 1993, for more details). In this paper, bold symbols such as $\mathbf{u}_s$ and $\mathbf{\imath}_s$ represent space vectors related to machine quantities (in case, $\mathbf{u}_s$ denotes the stator voltage and $\mathbf{\imath}_s$ is the stator current, respectively).

Specific vector component *values* have always to be given with reference to a coordinate system. For instance, the components of $\mathbf{u}_s$ and $\mathbf{\imath}_s$ that are related to a coordinate frame of two orthogonal axis, one of which being oriented with a stator winding axis (usually phase a), may be noted as

$$\mathbf{u}_s^s = \begin{bmatrix} u_s^{s1} \\ u_s^{s2} \end{bmatrix}, \ \mathbf{\imath}_s^s = \begin{bmatrix} \imath_s^{s1} \\ \imath_s^{s2} \end{bmatrix},$$

where u_s^{s1} is the projection of $\mathbf{u}_s$ onto the stator winding axis (direct component), u_s^{s2} the projection of $\mathbf{u}_s$ onto the stator winding orthogonal axis (quadrature component), etc..

The (power-invariant) transformation of the machine terminal quantities to this stationary stator reference frame can be obtained by means of measurements of the motor phase-to-phase voltages u_{ab}, u_{bc} and the line currents $\imath_{sa}, \imath_{sb}$ following

$$\begin{aligned} \begin{bmatrix} u_s^{s1} \\ u_s^{s2} \end{bmatrix} &= \sqrt{2/3} \begin{bmatrix} 1 & \sqrt{1/2} \\ 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} u_{ab} \\ u_{bc} \end{bmatrix}, \\ \begin{bmatrix} \imath_s^{s1} \\ \imath_s^{s2} \end{bmatrix} &= \sqrt{2/3} \begin{bmatrix} 3/2 & 0 \\ \sqrt{3}/2 & \sqrt{3} \end{bmatrix} \begin{bmatrix} \imath_{sa} \\ \imath_{sb} \end{bmatrix}, \end{aligned}$$

assuming that $\imath_{sa} + \imath_{sb} + \imath_{sc} = 0$ (no neutral wiring).

With reference to the stationary stator frame, the stator-flux vector is found to be

$$\mathbf{\Psi}_s^s = \begin{bmatrix} \psi_s^{s1} \\ \psi_s^{s2} \end{bmatrix} = \int (\mathbf{u}_s^s - r_s \mathbf{\imath}_s^s)\, dt.$$

which may also be described as

$$\mathbf{\Psi}_s^s = \begin{bmatrix} \psi_s \cos(\varphi_s^s) \\ \psi_s \sin(\varphi_s^s) \end{bmatrix},$$

with

$$\begin{aligned} \psi_s &= \sqrt{(\psi_s^{s1})^2 + (\psi_s^{s2})^2}, \\ \varphi_s^s &= \arctan\left(\psi_s^{s2}/\psi_s^{s1}\right). \end{aligned}$$

In fact, φ_s^s is the angular position of $\mathbf{\Psi}_s$ with respect to the direct axis of the stator reference frame, being therefore the basis quantity for transforming vector coordinates from the stationary stator frame to a reference frame rotating synchronously with $\mathbf{\Psi}_s$ (or vice-versa). For instance,

$$\mathbf{u}_s^{\psi s} = \mathbf{R}(-\varphi_s^s)\mathbf{u}_s^s \quad \text{or} \quad \mathbf{u}_s^s = \mathbf{R}(\varphi_s^s)\mathbf{u}_s^{\psi s},$$

where, in general,

$$\mathbf{R}(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{bmatrix},$$

and

$$\mathbf{u}_s^{\psi s} = \begin{bmatrix} u_s^{\psi s1} \\ u_s^{\psi s2} \end{bmatrix},$$

with the direct component $u_s^{\psi s1}$ denoting now the projection value of $\mathbf{u}_s$ onto the direction of $\mathbf{\Psi}_s$, and the quadrature component $u_s^{\psi s2}$ the projection value onto the orthogonal direction. It is obvious that

$$\mathbf{\Psi}_s^{\psi s} = \begin{bmatrix} \psi_s^{\psi s1} \\ \psi_s^{\psi s2} \end{bmatrix} = \begin{bmatrix} \psi_s \\ 0 \end{bmatrix}.$$