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Short communication

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A B S T R A C T

Intensity Modulated (IM) optical communication over an LED channel requires the use of a non-negative signal that can also cope with the low-pass nature of LEDs. For this purpose, dedicated schemes such as Flip-OFDM and Assymetrically Clipped Optical (ACO)-OFDM have been proposed. We derive a common mathematical description on which both schemes rely. Exploiting this insight, we propose Continuous Phase Flip-OFDM (CP-Flip-OFDM) as an enhancement to Flip-OFDM. It ensures phase continuity at the transition between the two successive copies of the OFDM blocks, thereby it obviates the Cyclic Midfix between the first OFDM block and its flipped copy. Simultaneously, we derive a less compute-intensive way to generate and detect ACO-OFDM. Instead of creating an Hermitian-symmetry at the transmit Inverse FFT input, which is common in optical IM OFDM, we use zero padding. After the transmit IFFT, a phase ramp-up, i.e., a multiplication with a complex-valued exponential, is applied before truncating to a real and non-negative signal.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is popular for Intensity-Modulated (IM) Optical Wireless Communications (OWC) [1,2]. In fact, the LED channel, in particular with a phosphor-coating to generate white light, but also monochromatic LEDs in the visible or IR spectrum, heavily attenuates high modulation frequencies. A key advantage of OFDM is that individual symbols are transmitted in parallel as a narrow-band signal, free of InterSymbol Interference (ISI). Different subcarriers see different attenuation, while a simple scaling can be used as equalizer. Moreover, every subcarrier can carry its own, optimized signal constellation. Although OFDM is sometimes seen as a versatile modulation method that solves many channel-related problems, the probability density function of the signal closely resembles a Gaussian distribution.

The intensity-modulated optical channel requires the signal to be neither complex nor negative, which makes OFDM as-is unattractive. Besides adding a DC bias, a variety of solutions have been proposed to create a uni-polar signal, e.g. [3–9]. Flip-OFDM [4,5] and Asymmetrically Clipped Optical OFDM (ACO-OFDM) [6], which have been compared extensively, but mainly via simulations. However, as far as we see, their inherent mathematical similarity has not been identified before. Our analysis here allows a harmonization into a common description, which can also explain previously reported (simulation and experimental) results that observe virtually identical performance on many criteria. Nonetheless, we identify spectral differences and differences in handling channel dispersion. Moreover, based on new insights, we discover a novel implementation framework. Hitherto, ACO and Flip OFDM have been described as resulting from different recipes. However, mathematical commonalities allow an alternative, computationally-efficient processing of ACO-OFDM and allow an improvement of Flip-OFDM by making the transition Continuous Phase (CP-Flip-OFDM). Further, we identify a broader class of inserting an additional DC-biased stream to repair the 50% throughput loss of ACO-OFDM. As a further result, in contrast to common belief, there is no need to explicitly impose an Hermitian symmetry at the IFFT input in OWC. Zero-padding of the higher subcarriers and removal of the (non-zero) imaginary-valued signal part suffices. In fact, we even use the non-zero, redundant imaginary part to our advantage.

We investigate the time-frequency signal footprint of every input data symbol. For DCO-OFDM, this is known to be confined to the vicinity of the subcarrier frequency. It appears that, particularly for Flip-OFDM, but also for ACO-OFDM this footprint is much wider. Every symbol spreads half of its signal over the entire signal band, but this 50% of the energy-per-bit is simply discarded during

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reception. Only for the effectively used 50% of bit energy, the common belief is correct that despite the highly non-linear clipping, the received signal per subcarrier only depends on the channel transfer function at a frequency that corresponds to that particular subcarrier or that subcarrier streams would remain orthogonal.

2. Background

Both Flip- and ACO-OFDM systems convert N PAM or, equivalently, $N/2$ QAM data signals $d_0 \ldots d_{N-1}$ into $2N$ non-negative transmit samples, where mostly N is a power of 2. The process to reach these transmit signals is different, but the outcome has similarities. As a first step, $N/2$ complex valued QAM signals $X_0, \ldots, X_{N/2-1}$ are generated with $X_n = d_n + jd_{N/2+n}$ where $n = 0, 1, \ldots N/2-1$. Other symbol mappings may be used, but these are equivalent if we allow a renumbering of the symbols, without loss of generality. To create a real-valued output signal, it is common practice that these are extended into $X_{N/2}$,..., X_{N-1} to create an Hermitian-symmetric signal $X_{N-k} = \overline{X_k}$, so $X_n = d_{N-n} - jd_{N+N/2-n}$ for $n > N/2$.

2.1. Flip-OFDM formulation

In OWC, an N-sized IFFT is performed on vector $\boldsymbol{X} = X_1, \ldots, X_{N-1}$, so for $k = 0, 1, ..N-1$, the real output is

$$z_k = \sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi nk}{N}} + \left(d_n - jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi(N-n)k}{N}} + \ldots \quad (1)$$

where the "$+\ldots$" accounts for the subcarriers 0 and $N/2$, which are often not used. We ignore these details of modulation of d_0 and $d_{N/2}$ on the DC subcarrier and the maximum-frequency subcarrier, to avoid that these complicate the notation unnecessarily, while it does not give a deeper insight. The upper half of the subcarriers at the IFFT input are the complex conjugate of the lower half of the subcarriers, to satisfy an Hermitian symmetry. This yields a real signal at the IFFT output. Then, Flip-OFDM uses an explicit copy-and-flip operation, where $z_{N+k} = -z_k$. Thus, before clipping,

$$z_k = \begin{cases} \sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi nk}{N}} + \left(d_n - jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi(N-n)k}{N}}, \text{ for } k \in (0,\ N-1) \\ -\sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi nk}{N}} - \left(d_n - jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi(N-n)k}{N}}, \text{ for } k \in (N,\ 2N-1) \end{cases}, \quad (2)$$

thus where z_k is extended beyond $k = N-1$ with the copy-and-flipped block. Due to the minus sign in the signal (2) for $k = N$, a crude, undesirable swap of polarity occurs after $k = N-1$. Consequently, Flip-OFDM not only uses a Cyclic Prefix (CP) preceding the first block but also has to insert a second CP before the second, flipped block [4]. We will refer to the latter as the *Cyclic Midfix* (CM) which is located between $k = N-1$ and $k = N$. It is necessary to ensure clean reception of the second (flipped) block. Formally, for a CM of length N_{MF}, samples $N, ..2N-1$ are shifted to $N + N_{MF}, ..2N + N_{MF} - 1$ and the CM is defined as $z_{N+i} = z_{2N-1-N_{MF}+i}$ for $i = 0, ..N_{MF}$. This reduces the effective bit rate.

In fact, if the channel exhibits a delay spread, signals from the first block spill into the second block and cause crosstalk between (parts of) symbols. The optical multipath delay spread may be moderate in indoor systems, where excess path lengths due to reflections are a few meters at most, but other effects can also be significant. For instance, the limited bandwidth of the emitting LED, often rolling off below 10 MHz disperses the signal over hundreds of ns and imposes a mimimum duration for the Mid-Fix. Also, filtering in the LED modulator and impedance matching contribute to group delay and dispersion. Moreover, the foreseen use of distributed MIMO in which mutual delays from spatially separated emitters connected in the ceiling via different cabling lengths impose a minimum duration on any cyclic pre, mid or postfix.

Reduction of the length of cyclic fixes has been a topic of OFDM research, to avoid throughput losses. A mechanism to eliminate the need for a mid-fix altogether can be attractive. In fact, we propose CP-Flip-OFDM, such that the mid-fix can be omitted without jeopardizing performance. From $e^{\frac{j2\pi nk}{N}} = 1$ at $k = N$, it is known that a cyclic continuation (no minus sign; no flip) of the OFDM block would yield continuous phases at all subcarriers. In contract to this, Flip-OFDM, inverts polarity at the transition. We propose to overcome discontinuities by gradually rotating the phase in the first block (thus also in the copy-flipped second block), to linearly build up a phase rotation from 0 to π over the first N samples $k = 0, 1, ..N-1$, thus anticipating a polarity flip. The next sample $k = N$ adopts the phase of sample zero, but with a minus sign. Thereby, it becomes continuous between $N-1$ and N, thus eliminates the need for a mid-fix. As this obviates the midfix, we will not unnecessarily complicate our notation and just take $N_{MF} = 0$.

2.2. ACO-OFDM formulation

Before we express CP-Flip-OFDM and the phase rotations mathematically, we first evaluate ACO-OFDM. We can show the mathematical similarity of CP-Flip and ACO-OFDM if we compare both with the same number of symbols in running parallel and at the same symbol rate. That is, we compare (CP-)Flip-OFDM using two concatenated IFFT data blocks each of size N and each generated by an IFFT of size N, to ACO-OFDM using an IFFT of size $2N$. Both cases carry $2N$ real-valued time samples at the same bandwidth. Following our earlier choice of data mapping, ACO-OFDM maps the nth plus j times the $(N/2+n)$th real data symbol to (odd) subcarrier $2n+1$ of the $2N$-IFFT, where the input meets the Hermitian symmetry. In fact, we see that for the $2N$ IFFT, ACO-OFDM creates

$$z_k = \sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi(2n+1)k}{2N}} + \left(d_n - jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi(2N-2n-1)k}{2N}}, \quad (3)$$

where the first part already describes the transmission data while the second term is created by the Hermitian-symmetric input to cancel the imaginary output. ACO-OFDM, then transmits z_k^+, thus clips away any negative signal parts, with $z_k^+ = \max(0, z_k)$. ACO-OFDM is known to satisfy the symmetry property (before clipping) $z_{N+k} = -z_k$. This can be verified by inserting

$$e^{\frac{j2\pi(2n+1)(N+k)}{2N}} = (-1)^{2n+1} e^{\frac{j2\pi(2n+1)k}{2N}} = -e^{\frac{j2\pi(2n+1)k}{2N}} \quad (4)$$

into (3) for z_{N+k}. This suggests that one may alternatively create an ACO-OFDM signal by initially generating only N samples and then copy, shift over N and flip polarity, then clip. In this paper, we explore whether one can reduce the size of the IFFT from $2N$ to N. Forcing (3) into an N-sized IFFT-like notation, we see

$$z_k = \sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi\left(n+\frac{1}{2}\right)k}{N}} + \left(d_n - jd_{\frac{N}{2}+n}\right) e^{\frac{-j2\pi\left(n+\frac{1}{2}\right)k}{N}}, \quad (5)$$

for $k = 0, 1, \ldots N-1$, and an anti-cyclic extension (flipped polarity) for $k = N, N+1, ..2N-1$. The denominator in the complex exponential is N, not $2N$. However, (5) is not exactly of the form of

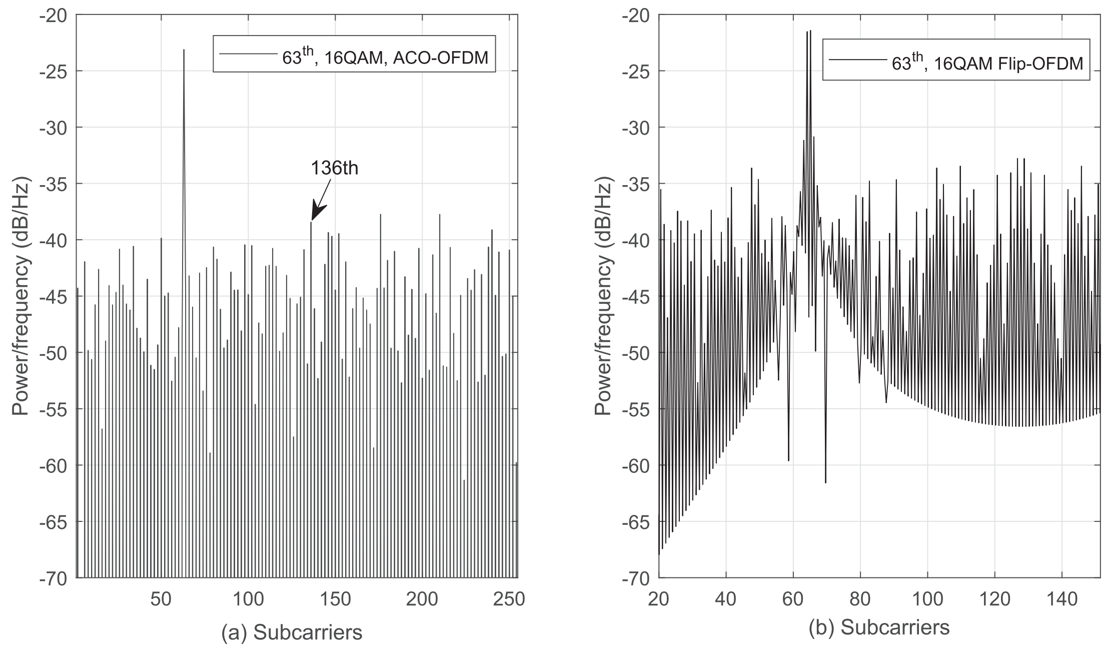

Fig. 1. Subcarrier power spreading due to clipping in ACO-OFDM (a), and flip and clipping in Flip-OFDM (b), setting the 63th subcarrier as zero, $N = 256$.

a standard IFFT, as $n + 1/2$ appears in the twiddle factors. We will show that the standard IFFT, with its efficient butterfly implementation, can nonetheless be used. We interpret from (5) that the difference between Flip-OFDM and ACO-OFDM is just a shift by half a subcarrier $n \to n + 1/2$ before clipping. Intuitively speaking, modulating the odd subcarriers on a $2N$ sized IFFT is equivalent to loading subcarrier $(n + 1/2)$ on an N-sized IFFT. We can perform this after the transmit IFFT by multiplying the output samples by $e^{\pm \frac{j\pi k}{N}}$. That is, we exploit that in (5) this factor 1/2 is independent of the running index n, thus can be taken in front of the summing.

3. Spectral consequences of clipping

OFDM is often used to ensure that any particular symbol is mapped to a specific frequency. Then, the constellation and power used for this symbol can be optimized for the channel transfer at its frequency. However, severe clipping does not preserve such strict frequency mapping. While BER simulations in previous papers suggest that clipped-OFDM can recover orthogonal subcarrier channels, we found no earlier investigations in literature of how the footprint of an individual Flip- or ACO-OFDM symbol is mapped to the spectrum of the modulation signal. To answer the question of whether in clipped OFDM, a single symbol on each subcarrier still travels over a narrowband well–contained frequency channel, without spreading far outside the subcarrier frequency slot, we simulated this effect. In particular, for a single tone in isolation, the flip-clip operation in Flip-OFDM causes strong periodic harmonics. However, this gives little insight in what happens in the ensemble of all subcarriers being clipped in OFDM. To study the latter, we compared the spectrum of an OFDM signal with one that carries the same data, but with one subcarrier symbol set to zero.

Simulations in Fig. 1(a) shows that for ACO-OFDM, clipping artefacts on even subcarriers are around 24 dB below the main signal. Since 50% of the samples are clipped, half of the power is omitted. Intuitively it is appealing to argue that, half of the remaining power actively supports the original target subcarrier frequency (No. 63 in Fig. 1), while the other half is spectrally spread over the band. If we interpret an FFT as just being a unitary matrix operation, (non-clipped) OFDM can be seen as a sequence of independent identically distributed (iid) Gaussian random variables. Clipping any such time sample gives a flat error spectrum. In such case, the clipping noise would fall equally strong on all subcarriers. However, a clip of sample k_c occurs either at k_c or at $N + k_c$ but not at both instants. Thus, (odd) frequencies with an anticyclic symmetry always catch the signal at either position, while cyclic (even) subcarriers see every clip artefact with random polarity, depending on where the clip occurs. N such randomly–swapped samples add up as noise. Therefore, odd ACO subcarriers can be proven to experience zero clipping noise, while the even subcarriers expect to see a noise variance of $1/N$ (-24 dB, per loaded subcarrier) below the main signal, also as simulated. Similarly, Flip-OFDM creates clipping artefacts that do not interfere with signals carried on other subcarriers in an N-sized FFT grid, after folding back. However, if one performs a $2N$ receive-FFT, because of the antipolar flip, the signal seen at frequency $2n$ is zero. Moreover two sidebands occur, and clipping noise spreads to odd frequencies in the $2N$ grid, thus in between subcarriers in an N sized grid.

To refine the resolution, we evaluate the spectral mapping beyond the scope of a limited time window that only sees discrete samples of the spectrum. To be more specific, we use an $N = 256$ sized IFFT for Flip-OFDM and we evaluate its spectrum by performing a $16N = 4096$ sized FFT on the $2N = 512$ samples of a copy-flipped time signal with zero padding outside the signal block. Fig. 2 shows that before clipping, the CP-Flip-OFDM (also ACO-OFDM) signals are much better confined in spectrum as Flip-OFDM signals, because the OFDM block window of $2N$ samples with phase continuity is essentially two times as wide in time domain compared to Flip-OFDM swapping phase after N samples. From Fig. 2(b), we see more clearly that the Flip-OFDM peak is 3 dB below the main CP-Flip-OFDM peak.

4. New CP-flip-OFDM description

We can adapt Flip-OFDM to enforce phase continuity and simultaneously make it mathematically equivalent to ACO-OFDM, i.e., harmonize Flip-OFDM in (2) and ACO-OFDM in (5). This allows us to simplify the signal generation of ACO-OFDM, but also brings (efficiency) advantages to Flip-OFDM. In this paper, we will use the

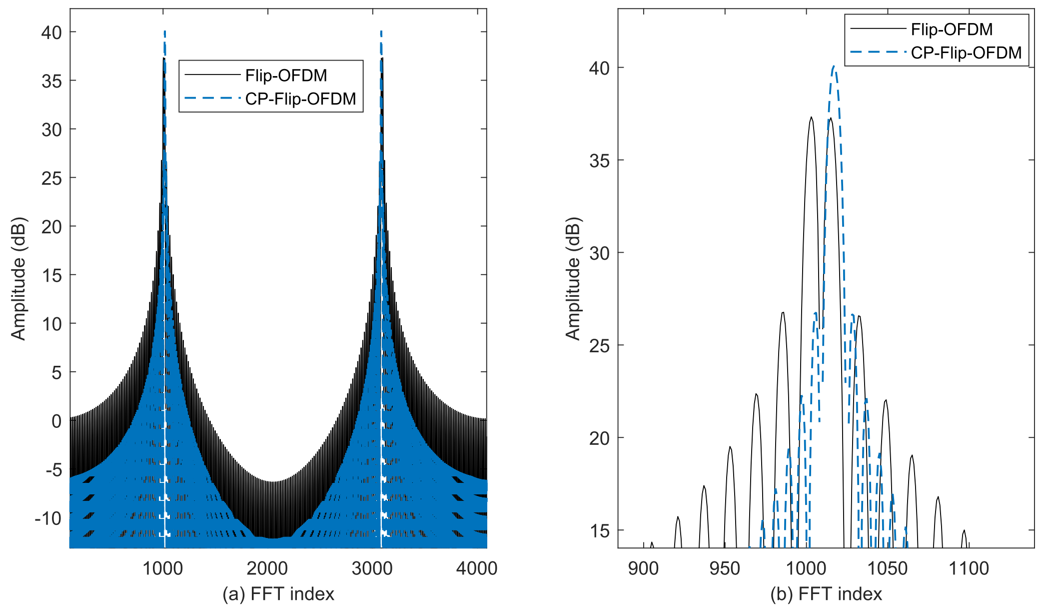

Fig. 2. Envelope of the subcarrier power spectrum caused by copy-flipping (no clipping) the 63th subcarrier and $N = 256$, for Flip-OFDM (black) and CP-Flip-OFDM (blue), using rectangular window to increase the frequency resolution 16 fold. This is also the spectrum that a receiver observes by its unflip-merge operation. (b) zoom-in on main peak(s) from (a). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

name CP-Flip-OFDM whenever we specifically refer to the new algorithms using IFFTs of size-N, and use the name ACO-OFDM when we refer to the generated signal waveform itself.

4.1. CP-Flip-OFDM generation

Both in (2) and (5), the second term is just the complex conjugate of the first term. To facilitate an efficient implementation, and since $\overline{ab} = \overline{a}\overline{b}$ and $\mathrm{Re}[2a] = a + \overline{a}$, we argue that the transmit samples z_k to be transmitted may as well be written as deleting the imaginary part in a signal that only loads the lower subcarriers (thus without the second terms). In fact, we have we can re-write ACO-OFDM in (5) as

$$\begin{aligned} z_k &= \mathrm{Re}\left[2e^{\frac{j\pi k}{N}} \sum_{n=1}^{N/2-1} \left(d_n + jd_{\frac{N}{2}+n}\right) e^{\frac{j2\pi nk}{N}}\right] \\ &= \mathrm{Re}\left[2e^{\frac{j\pi k}{N}} \mathrm{FFT_N}\left(d_n + jd_{\frac{N}{2}+n}\right)\right], \end{aligned} \tag{6}$$

for $k = 0, 1, \ldots, N-1$ and zero-padded inputs. In other words, if we anyhow truncate the imaginary part, we may feed into the upper subcarriers $X_{N-n} = 0$, instead of the usual Hermitian-copy $X_{N-n} = \overline{X_n}$. This alternative zero-padding in combination with only transmitting the real-part of the time-domain samples, instead of using an Hermetian symmetry, can be used not only for CP-FLIP-OFDM but also for DCO-OFDM, ACO-OFDM, etc. In fact, many papers suggest that an Hermetian input symmetry shall be applied in IM-OWC using any form of OFDM. However, alternatives, such as zero-padding, appear to exist.

Because of the periodic frequency spectrum of the FFT, zero-padding the top-half of the subcarriers creates a complex-valued output, called an *analytic* signal as it has no negative frequency components. The real and imaginary parts of an analytic signal are known to be real-valued functions related to each other by the Hilbert transform [10]. Thus, they do not contain any independent information. To create Flip-OFDM, ACO-OFDM or even DCO-OFDM, one can just harmlessly remove this imaginary output. However, we argue that we can even use the imaginary part to our advantage, to create CP-Flip-OFDM.

As can be seen from (6), at the output of the IFFT, every complex-valued output value is phase-rotated by $e^{\frac{j\pi k}{N}}$, thus a linear ramp up to phase π at $k = N$. Here, we thus need to use the Hilbert-transformed imaginary part. Only after phase rotation, the real part is taken by truncating the imaginary part, followed by similar copy-flip-clip operations for Flip-OFDM as in Fig. 3.

Evidently, since we are only interested in the real part of the outcome of this phase rotation, the complex post-IFFT multiplication by $e^{\frac{j\pi k}{N}}$, which typically consists of four real multiplications, can be reduced to two real multiplications, i.e., the product of the real parts minus the product of both imaginary parts. This may also be interpreted as generating Single Side Band (SSB), see Eq. (10), explicitly from inphase and quadrature (I and Q) components. A more detailed comparison on the computational complexity between the proposed CP-Flip-OFDM and ACO-OFDM is listed in Table 1. In fact, one complex-valued multiplication involves four real-valued multiplication and two real-valued addition, and one complex-valued addition needs two real-valued additions. Hence, we see that CP-Flip-OFDM requires less computational resources than the ACO-OFDM, although there is an additional phase rotation which corresponds to a complex-valued multiplication. An alternative approach to reduce the complexity is the use of so-called real-valued FFTs [11], which restructures and punctures the regular butterfly structure and limits the complexity from $4N \log N$ to $2N \log N$. However, this deviates from standard FFT semiconductor IP blocks and adds control complexity in the flow of operations, so the latter appears to be rarely used in practice.

4.2. CP-Flip-OFDM demodulation

4.2.1. Mathematical evaluation

Since in CP-Flip-OFDM and ACO-OFDM, $z_k = z_k^+ - z_{N+k}^-$ with $z_{N+k}^- = \min(0, z_k)$, when received over a dispersion-free channel, received samples can be used as input to an N-sized time-to-frequency transform. We implement the detector by starting off with an operation that copies and adds the second half of the received time samples ξ_k $(k = N, N+1, \ldots, 2N-1)$ to the first block. Thus, it does a flip back-and-merge operation to create the series of variables $\widehat{\xi} = \xi_k - \xi_{N+k}$ for $k = 0, 1, \ldots, N-1$. In litera-

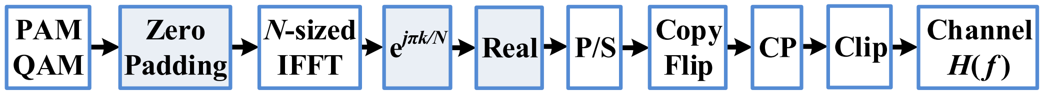

Fig. 3. Block diagram for CP-Flip-OFDM modulator and channel, where the filled-boxes indicate the unique operations compared with Flip-OFDM modulator.

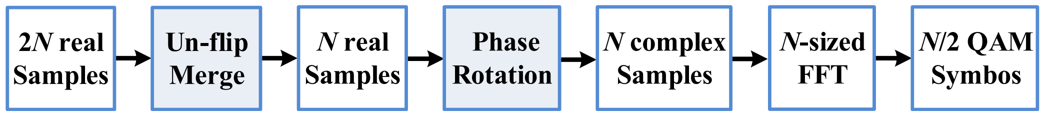

Fig. 4. Block diagram for CP-Flip-OFDM demodulator that can also detect ACO-OFDM, with un-flip merge operation, followed by a complex-valued phase rotation before the FFT.

Table 1
Complexity comparison between CP-Flip-OFDM and ACO-OFDM.

OFDM	Items	Complex multiplication	Complex addition	Real multiplication	Real addition
CP-FLip-OFDM	$e^{\frac{j\pi k}{N}}$	N	0	$2N$	N
CP-FLip-OFDM	N-IFFT	$\frac{N}{2}\log_2(N)$	$N\log_2(N)$	$2N\log_2(N)$	$3N\log_2(N)$
ACO-OFDM	$2N$-IFFT	$N\log_2(2N)$	$2N\log_2(2N)$	$4N\log_2(2N)$	$6N\log_2(2N)$

ture, the reliable working of ACO-OFDM is often explained for a non-dispersive channel, thus with $\xi_k = z_k^+$ and $\xi_{N+k} = z_{N+k}^-$ ($k = 0, 1, \ldots, N-1$). Then, the merged-back signal $\hat{\xi} = z_k^+ - z_{N+k}^-$ becomes an input to the N-sized time-to-frequency transform, to recover the subcarrier symbols. However, as the prime justification for OFDM lies in its ability to handle a dispersive channel, for our verification we explicitly insert an impulse response h_l with L taps, i.e., $l \in \{0, 1, \ldots, L-1\}$ that generate $h_l z_{k-l}^+ - h_l z_{N+k-l}^-$ at the receiver. L needs to be less than the size of CM or CP to avoid the ISI. Although, the preserving of subcarrier orthogonality is mostly implicitly assumed for all schemes, evaluation of the FFT expressions only confirms this for copy-flip-clip schemes, but we see that in copy-flip-timereverse-clip schemes, channel dispersion spoils subcarrier orthogonality. In fact, by writing ACO-OFDM as CP-Flip-OFDM after removing the CP, we can show orthogonality concisely via

$$\varsigma_m = \sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\left(h_l z_{k-l}^+ - h_l z_{N+k-l}^-\right)e^{\frac{-j2\pi\left(m+\frac{1}{2}\right)k}{N}}. \quad (7)$$

We renumber the running indexes of the sums $k - l \to k$ and assume an appropriate cyclic prefix of periodicity $2N$ (not N), such that $x_k = x_{2N+k}$ for the preceeding samples $k = -1, -2, ..L-1$. According to the circular shift property of the IFFT and swapping the sums, (7) can be written as

$$\varsigma_m = \sum_{l=0}^{L-1} h_l e^{-\frac{j2\pi l\left(m+\frac{1}{2}\right)}{N}} \sum_{k=0}^{N-1}\left(z_k^+ - z_{N+k}^-\right)e^{\frac{-j\pi k}{N}} e^{\frac{-j2\pi mk}{N}}. \quad (8)$$

Writing $H_{m+1/2} = \sum_l h_l \exp[-j2\pi(m+1/2)l/N]$, we see that a factor $H_{m+1/2}$ can be taken out from the sum over k such that

$$\varsigma_m = H_{m+\frac{1}{2}}\mathrm{FFT}\left[\left(z_k^+ - z_{N+k}^-\right)e^{\frac{-j\pi k}{N}}, m\right]. \quad (9)$$

This receive FFT also has a size of N instead of $2N$, which reduces computational complexity in the receiver. We see that the signal is attenuated by the transfer of the channel at a frequency shifted upwards by half a subcarrier. Despite clipping and despite the observation in Fig. 1 that ACO and Flip OFDM disperse 50% of the bit energy over the entire band, in a dispersive channel, each energy per received symbol is only determined by the channel response at one particular frequency. One may argue that the other 50% of the energy per symbol (at other frequencies) is simply not captured by the standard receiving recipe, which makes ACO-OFDM 3 dB more sensitive to noise.

Interpreting (9) as a method to build a receiver in Fig. 4, we can use the real samples, merge these by flip-back and add, execute a linearly-increasing complex-valued phase rotation by $e^{\frac{-j\pi k}{N}}$, and use a normal N-sized FFT. So, in contrast to most OWC OFDM receivers previously covered in literature, we feed complex-valued samples into the receive FFT, derived by pre-processing the detected real-valued signal samples in the complex domain.

4.2.2. Shifted Hermitian symmetry

Hermitian Symmetry is known to occur for real-valued signals. However, we feed complex values into the receive FFT. A symmetry similar to an Hermitian one is preserved, but the conjugate symmetry is shifted by one subcarrier position, compared to what a conventional FFT of a real-only signal would exhibit. For $m > N/2$, we can replace $m \to N - m$ in (11), apply $H_{m+1/2} = \overline{H_{N-m-1/2}}$ and take the conjugate. That gives a new type of (shifted) symmetry $\overline{\varsigma_{N-m}} = \varsigma_{m-1}$ with $m \in 1, 2, \ldots N/2 - 1$ for CP-Flip-OFDM.

4.3. Signal spectrum intuition for CP-Flip-OFDM

This section gives a frequency-shifting interpretation of the new recipe for generating and detecting ACO-OFDM as if it is a form of FLIP-OFDM with an upmixing operation. Frequency shifting of OFDM signals for the baseband domain suitable for OWC is currently topic of debate in IEEE 802.11bb standardization to reuse WiFi-like signal over optical intensity modulated channels.

Frequency-shifting of real-valued signals by multiplying with a carrier would create a Double Side Band (DSB) signal of double bandwidth. In contrast to this, baseband CP-Flip-OFDM requires SSB frequency shift without negative frequency components, which is known to give practical difficulties. For the transmitter and the receiver, we use a different method to shift the spectrum of real-valued signals without creating a lower–side band.

- At the *transmitter*, a frequency shift is needed that resembles the creation of an SSB signal $z_{SSB}(t)$. Although this could be done by AM modulation and filtering one side band, particularly for our uplift with $1/(2NT)$, that falls inside the signal bandwidth itself $1/(2T)$ (with T the sampling rate which time-domain signals are emitted), it requires a double conversion to avoid aliasing around DC. Moreover, as the shift is only by half a subcarrier, very steep filters would be needed to avoid aliasing. A more attractive signal processing approach would be to

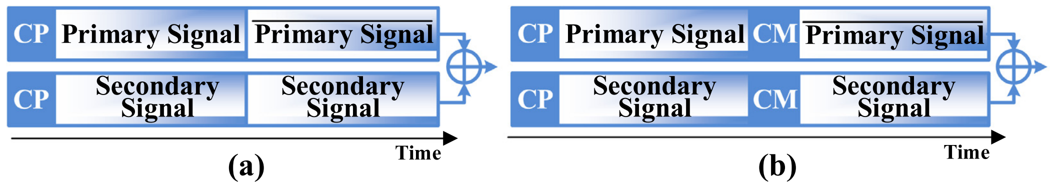

Fig. 5. Dual transmit signal format using (a) CP-Flip-OFDM and DCO-OFDM, and (b) Flip-OFDM and DCO-OFDM with cyclic midfixes. The primary signal is concatenated by a copied and flipped block while the secondary signal also has a copied block. Overline means polarity flipping. Signals in the two rows are added together sample by sample in time domain.

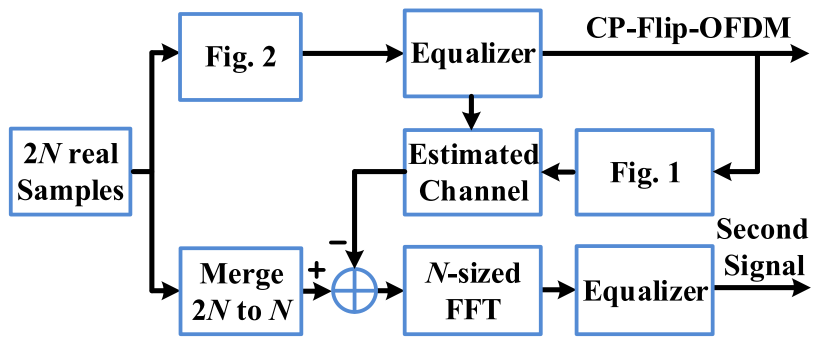

Fig. 6. Block diagram for dual system demodulator.

use inphase-quadrature (I-Q) modulation, based on

$$z_{SSB}(t) = z(t)\cos\left(\frac{2\pi t}{2NT}\right) - \tilde{z}(t)\sin\left(\frac{2\pi t}{2NT}\right), \tag{10}$$

where the Hilbert transform $\tilde{z}(t)$ of $z(t)$ is needed to suppress the lower side band. Creating $\tilde{z}(t)$ involves a phase shift of exactly 90 degrees over a wide frequency band. This Hilbert transformed signal comes for free as the imaginary part of the transmit FFT output, as we propose to replace the commonly-used Hermitian symmetry by zero padding, to create a so-called *analytic* signal (rather than a real signal).

- At the *receiver*, mixing down by half a subcarrier is needed. We avoid Hilbert transforms by feeding a complex-valued signal into the FFT. That is, we invoke the FFT property $z_k e^{-j2\pi\alpha k/N} \leftarrow \text{FFT} \rightarrow Z_{n+\alpha}$. A time-domain multiplication of any signal (thus also of a purely real signal) by $e^{\frac{-j2\pi\beta k}{N}}$ is equivalent to a shift in the frequency domain by β (here $\beta = 1/2$). It creates a complex-valued signal, but this is no problem as an FFT input.

5. Dual systems

Clipped OFDM have the generic disadvantage that two consecutive OFDM blocks are needed to transmit the data. This halves the throughput. This can be repaired by adding a second, DC-biased signal that is periodic with the period of a single OFDM block, as in Fig. 5(a). In fact, dual OFDM systems use a copy-flip-clip for the primary signal and use a copy (no-flip; no-clip) for the secondary signal. In literature, dual methods such as Hybrid ACO-OFDM (HACO-OFDM) and Asymmetrically clipped DC-biased Optical OFDM (ADO-OFDM) were described, while the composite signals were created in different ways. In HACO-OFDM, ACO-OFDM is transmitted on the odd subcarriers and PAM-OFDM is transmitted on the even subcarriers [12]. In ADO-OFDM, ACO-OFDM is run by only loading odd subcarriers, while in a second parallel operation, only the even subcarriers are loaded with a DC-biased DCO-OFDM [3].

A common aspect is that a second data signal s_k is added to z_k, where s_k cyclically extended, thus with $s_{N+k} = s_k$, while z_k is anti-cyclic. Because of this cyclic extension, a midfix is not needed for s_k. In fact, while frequency uplifting by half a subcarrier is favorable for z_k, it would be counterproductive for s_k. In fact, integer subcarriers can cyclically be extended, while half subcarriers can be anti-cyclically extended. One would refer to these as *even* subcarriers in ADO-OFDM. With CP-Flip-OFDM as primary signal, there is no need for a CM between the two copied-flipped-clipped blocks for the clipped part. In fact, the primary signal is made to have phase continuity by means of an uplift of half a subcarrier. Alternatively, if standard (non-CP) Flip-OFDM is used, a CM and windowing transition is used between the two blocks to handle dispersion, as in Fig. 5(b). The secondary signal must then also adhere to a periodic property over a period of one block plus the duration of the CM.

When unflip-merge is used as detection algorithm, any secondary signal s with cyclic extension $s_{N+k} = s_k$ cancels out in the unflip-merge, even if the signal is subject to dispersion, provided that the delay spread fits within the cyclic prefix. To show this, introducing a cyclic s in (7), gives

$$\varsigma_m = \sum_{k=0}^{N-1}\sum_{l=0}^{L-1}\left(h_l z^{+}_{k-l} - h_l z^{-}_{N+k-l} + h_l s_{k-l} - h_l s_{N+k-l}\right) e^{\frac{-j2\pi\left(m+\frac{1}{2}\right)k}{N}}. \tag{11}$$

Thus, without deteriorating the reception of z, this secondary signal can be a DCO-OFDM signal that uses the same subcarrier spacing as z (similar to ADO-OFDM), but we see that s and z may have very different subcarrier grids. A demodulator for a dual system can be separated into two branches as in Fig. 6. Since CP-Flip-OFDM is insensitive to s, the primary signal can be detected exactly as described before. The secondary signal sees interference from the clipped primary signal. Hence, a locally estimated copy for the CP-Flip-OFDM signal including its estimated dispersion needs to be subtracted from this secondary path. It would double the noise and imperfect channel estimation leads to artifacts in the signal. These effects lead to a reduced Signal Noise Ratio (SNR) compared to a situation without the primary signal. Note that, in contrast to ADO-OFDM, we only use FFTs of size N.

6. Conclusions

Both Flip-OFDM and ACO-OFDM create an OFDM signal in which the second half is a polarity-flipped replica of the first part. Flip-OFDM realizes this by repeating and polarity-flipping an OFDM block of length N, while, according to the conventional description, ACO-OFDM employs an IFFT of length $2N$ and only allows signal dimensions that have the required anti-cyclic repetition, thus, only odd subcarriers. An advantage of ACO-OFDM is that all (odd) subcarriers are by design continuous at the split between the two halves. So, the cyclic prefix and windowing are only needed at the beginning of the $2N$ frame, while Flip-OFDM needs cyclic besides prefixes also midfixes are needed between both halves.

We show that ACO-OFDM and Flip-OFDM have more similarities than hitherto reported. In fact, the only difference appears to be that in ACO-OFDM all subcarriers are implicitly shifted up by half a subcarrier and can be created by a copy-flip operation. It appears that this up-shift ensures that all subcarriers have continuous phase. Thereby, ACO-OFDM contains the signal spectrum better, and ACO-OFDM receivers are less sensitive to interference at other frequencies.

We propose a subtle modification to Flip-OFDM, namely CP-Flip-OFDM, that yields these advantages. The idea is to multiply the time sequence by a complex exponential, which mimics a frequency lift of half a subcarrier. We argue that, in contrast to statements repeatedly made in literature, creating an Hermitian symmetry, traditionally used at the transmitter to enforce a real-valued FFT output, is not mandatory. It suffices to only feed the IFFT with complex QAM data for the first half of all subcarriers and leave the rest as zeros. This creates an imaginary part being the Hilbert transform of the real signal. We use this to our advantage to up-shift by half a subcarrier spacing. At the output of the IFFT, we truncate the imaginary part after a complex-valued linear phase ramp-up. So, this leads to a new, versatile signal processing recipe that can create Flip-OFDM, CP Flip-OFDM, ACO-OFDM, or ADO-OFDM. It uses only N-sized IFFTs, but with phase shifting before the copy-(flip)-clip operation.

We believe that our analysis explains why many previously reported comparisons by simulating Flip-OFDM against ACO-OFDM showed identical performance.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Jean-Paul M.G. Linnartz: Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **Xiong Deng:** Investigation, Validation, Writing - original draft, Writing - review & editing.

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