Film thickness formulas for circular EHL contacts

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Film thickness formulas for circular EHL contacts

The quest for a more accurate central film thickness formula

15th Arnold Tross Colloquium
Hamburg, May 17th, 2019

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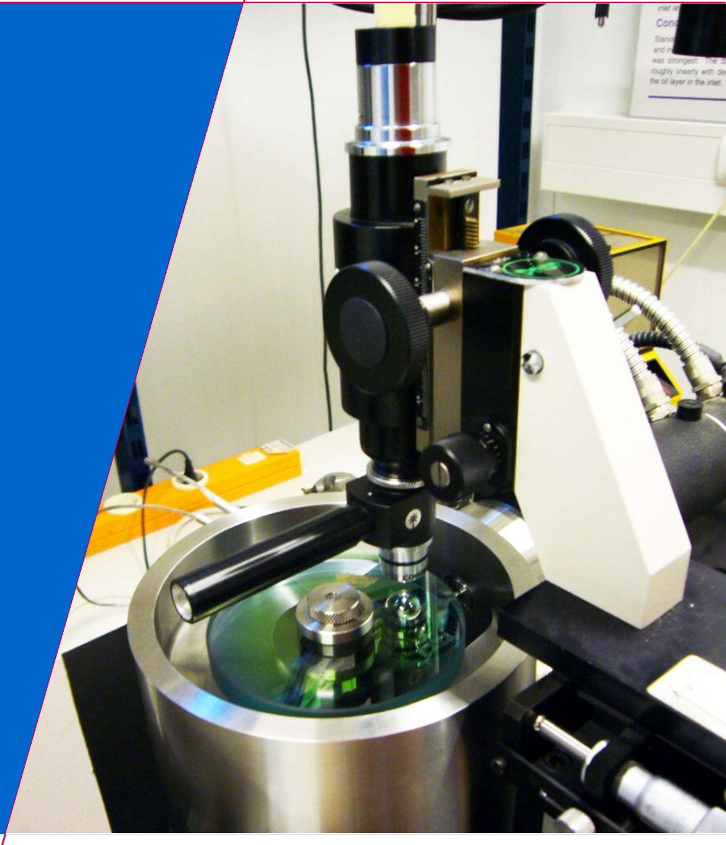

Technische Universiteit Eindhoven University of Technology

Where innovation starts

Presentation Outline

1. **Introduction and aim**
2. Analysis and method
3. Current EHL regime asymptotes
4. A critical review of asymptotic behaviour
5. The quest for a comprehensive film thickness formula
6. First results
7. Discussion
8. Conclusions

Motivation

- The ***pressure viscosity coefficient*** of a lubricant, alpha (α), is of utmost importance in elastohydrodynamic lubrication (EHL).
- A ***considerable problem*** is that it is barely known for real lubricants.
- The determination of α is possible by an indirect method based on ***accurate measurements*** of the central film thickness and on an ***accurate approximation formula*** for it, with an inaccuracy of about ± 12% for the best formulas, which were proposed by ***Hamrock and Dowson*** (1977) and by ***Chittenden et.al.*** (1985).

The early attempts and fundamentals of this subject have been treated at

- 3rd Arnold Tross Colloquium, Hamburg, 8th of June, 2007,
- 5th Arnold Tross Colloquium, Hamburg, 19th of June, 2009,
- 13th Arnold Tross Colloquium, Hamburg, 15th of May, 2017,
- 14th Arnold Tross Colloquium, Hamburg, 25th of May, 2018

1. Introduction and aim - 2

A considerable ***improvement of the accuracy of the*** α ***value*** is only to be expected from a approximation formula, not yet available, much closer to the numerical simulations than any existing one.

This seems to be unnoticed!

We reported at the last ATK that

- the complete **Moes model** (a curve fit) almost always has *too high values* for the film thickness, resulting in a too small value for alpha in the indirect assessment method, because:
- the **Moes VE formula** (for the severe EHL regime), part of the Moes model, contains an *erroneous sign* in the exponent of the elasticity (stiffness), which results in physically unacceptable film behaviour with stiffness

Knowing this setback, the quest for an improved approximation can continue. Can the Moes model be improved? Can the asymptotes for the four regimes be trusted?

Themes:

1. ***Are the current asymptotes reliable?***
2. ***Is there any advantage of an comprehensive formula?***
3. ***Is it possible to find an improved approximation formula?***

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2. Analysis and method

Available

- central film thickness measurement results
- film thickness approximation formulas (a lot!)
- software for film thickness calculations (through Venner's MG programme)

Methodology

- a critical review of formulas
- proper choice of nondimensional groups in a survey
- improvement of asymptotic behaviour based on numerical calculations only
- construction of improved formulas
- test of these formulas for known alpha values

Limitations and possibilities

- circular contacts sufficient
- classical EHL
- wide operational range

Presentation Outline

The importance of using sets of nondimensional groups in EHL was shown at the 13th Arnold Tross Colloquium, May 15th, 2017.

For elliptical EHL contact film thickness can be described by:

The Delft/Blok/Moes EHL groups

$$\hat{H} = \hat{H}\left(N, L, \omega\right)$$

Note that

$$\hat{H} = \left(\frac{h}{R_e}\right)\left(\frac{E_r R_e}{2\,\eta_0 \bar{u}}\right)^{\frac{1}{2}}$$

$$\hat{H} = \hat{H}\left(\eta_0, \bar{u}, R_e, E_r, h\right)$$

$$N = \omega^{-\frac{1}{2}} M = \omega^{-\frac{1}{2}} \left(\frac{F}{E_r R_e^{\,2}}\right)\left(\frac{E_r R_e}{2\eta_0 \bar{u}}\right)^{\frac{3}{4}}$$

$$M = M\left(\eta_0, \bar{u}, R_e, E_r, F\right)$$

$$L = \left(\alpha E_r\right)\left(\frac{2\eta_0 \bar{u}}{E_r R_e}\right)^{\frac{1}{4}}$$

$$L = L\left(\eta_0, \bar{u}, R_e, E_r, \alpha\right)$$

$$\omega = \left(\frac{R_t}{R_e}\right)$$

radii of curvature (crowning) ratio

3. Current EHL regime asymptotes - 2

In the nondimensional groups proposed by Johnson:

The Johnson EHL groups

$$h' = h'\left(g'_E, g'_v, \omega\right)$$

$$h' = \left(\frac{F}{\eta_0 \bar{u} R_e}\right)^2 \left(\frac{h}{R_e}\right)$$

$$g'_E = \left(\frac{F^8}{E_r^{\,2} R_e^{\,10} \eta_0^{\,6} \bar{u}^6}\right)^{\frac{1}{3}}$$

$$g'_v = \alpha \left(\frac{F^3}{\eta_0^{\,2} \bar{u}^2 R_e^{\,4}}\right)$$

$$\omega = \left(\frac{R_t}{R_e}\right)$$

Note that

$$h' = h'\left(\eta_0, \bar{u}, R_e, F, h\right)$$

$$g'_E = g'_E\left(\eta_0, \bar{u}, R_e, F, E_r\right)$$

$$g'_v = g'_v\left(\eta_0, \bar{u}, R_e, F, \alpha\right)$$

3. Current EHL regime asymptotes - 3

Their relation is given in the conversion table

Table 1: Coversion table for nondimensional groups in elliptical EHL contacts

	Johnson (1970) h', g_V', g_E', $\omega = 1$	**Moes (2000)** $\hat{H}$, N, L, λ $N = M\sqrt{\lambda}$	**Hamrock and Dowson (1974 - 2004)** H, U, W, G, $k = k(\omega)$
Johnson (1970) h', g_V', g_E', $\omega=1$	---	$h' = 4\hat{H}M^2$ $g_v' = 4LM^3$ $g_E' = 4M^{\frac{8}{3}}$ $\omega = \lambda^{-1}$	$h' = HW^2U^{-2}$ $g_v' = GW^3U^{-2}$ $g_E' = W^{\frac{8}{3}}U^{-2}$ $k = k(\omega)$
Moes (2000) $\hat{H}$, N, L, λ $N = M\sqrt{\lambda}$	$\hat{H} = 2^{-\frac{1}{2}} h' g_E'^{-\frac{3}{4}}$ $M = 2^{-\frac{3}{4}} g_E'^{\frac{3}{8}}$ $L = 2^{\frac{1}{4}} g_E'^{-\frac{9}{8}} g_v'$ $\lambda = \omega^{-1}$	---	$\hat{H} = 2^{-\frac{1}{2}} HU^{-\frac{1}{2}}$ $M = 2^{-\frac{3}{4}} WU^{-\frac{3}{4}}$ $L = 2^{\frac{1}{4}} GU^{\frac{1}{4}}$ $\lambda = \omega^{-1}$

…. and allows a transformation of these sets into eachother.

3. Current EHL regime asymptotes - 4

To disclose the effect of solid elasticity (E_r) and pressure sensitivity of the fluid viscosity (α) Johnson groups have to be employed.

Generally 4 regimes in EHL have been accepted – see Johnson (1970) and Hamrock et al. (2004):

- IR = isoviscous rigid => classical HD
- IE = isoviscous elastic => soft contacts
- VR = piezoviscous rigid => highly loaded rigid contacts ("Blok regime")
- VE = piezoviscous elastic => highly loaded elastic contacts

Johnson suggested film thickness asymptotes for these 4 regimes. Since then there is a consensus on the existence of the asymptotes.

3. Current EHL regime asymptotes - 5

To disclose the effect of elasticity (E_r) and pressure-viscosity coefficient (α) the Johnson groups have to be employed.

Moes asymptotes in Delft notation:

$$\hat{H}_{IR,c} \approx 41.35\, M^{-2}$$

$$\hat{H}_{IE,c} \approx 2.418\, N^{-\frac{2}{15}}$$

$$\hat{H}_{VR,c} \approx 0.909\, L^{+\frac{2}{3}}$$

$$\tilde{H}_{VE,c} \approx 1.247\, N^{-\frac{1}{12}}\, L^{\frac{3}{4}}$$

Moes asymptotes in Johnson notation:

$$h'_{IR,c\,\odot} \approx 165.38$$

$$h'_{IE,c\,\odot} \approx 3.666\, g_E'^{\;0.7000}$$

$$h'_{VR,c\,\odot} \approx 1.443\, g_V'^{\;0.6666}$$

$$h'_{VE,c\,\odot} \approx 2.098\, g_V'^{\;0.75}\, g_E'^{\;-0.1250}$$

3. Current EHL regime asymptotes - 6

The full Moes curve fit after the transformation to Johnson groups reads:

The Moes (2000) magic formula:

$$h'_c = \left[\left\{ h'^{\frac{3}{2}}_{IR} + \left(h'^{-4}_{IE} + 0.025\, g'^{-3}_E \right)^{-\frac{3}{8}} \right\}^{\frac{2}{3}\hat{s}} + \left(h'^{-8}_{VR} + h'^{-8}_{VE} \right)^{-\frac{1}{8}\hat{s}} \right]^{\frac{1}{\hat{s}}}$$

where

$$\hat{s} = \frac{3}{2}\left\{ 1 + \exp\left(-1.2\frac{h'_{IE}}{h'_{IR}} \right) \right\}$$

Presentation Outline

Asymptotic EHL calculations (Johnson groups)

Numerical calculations were preformed for circular contact, employing a code from C.H. Venner, 2008, Software Code ehl2d0 version 1.00 in a range of conditions.

A large number of calculations of the central film thickness h'_c was performed for fixed values of $10^2 \leq g_E \leq 10^9$ and $10^0 \leq g_V \leq 10^{15}$

Note:
The disadvantage of the Venner software is the calculation grid, which is related to the dimensions of the Hertz contact. This is too small for IR regimes. Results having $h'_c < 250$ will therefore be discarded.

Therefore, of the four regimes, the IR area cannot be investigated. What remains to investigate are regimes IE, VR and VE.

4.1 The IE asymptote in EHL

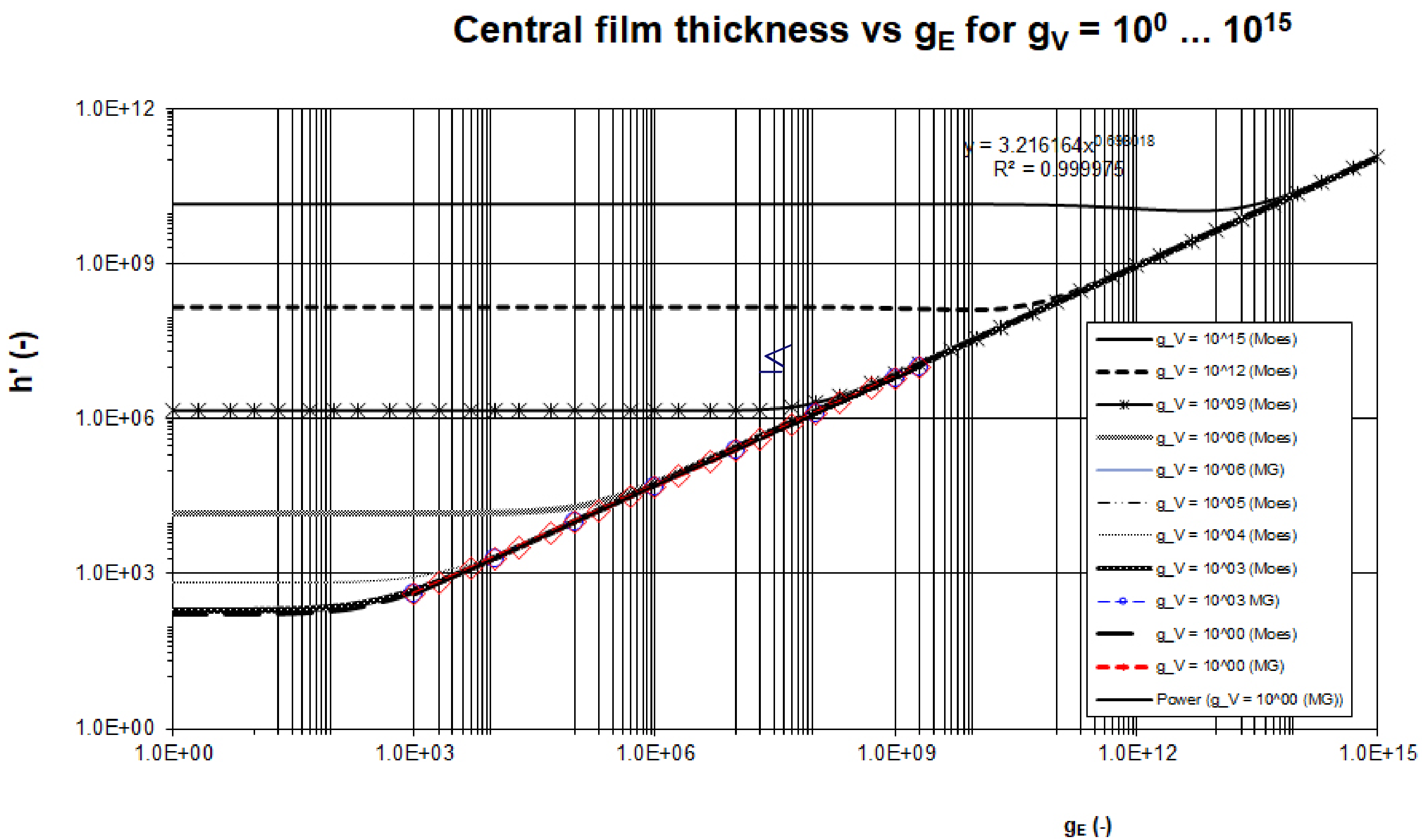

Figure 1: Nondimensional film thickness h' as a function of elasticity group g_E (for $10^0 \leq g_V \leq 10^{15}$)

4. A critical review of asymptotic behavior - 3

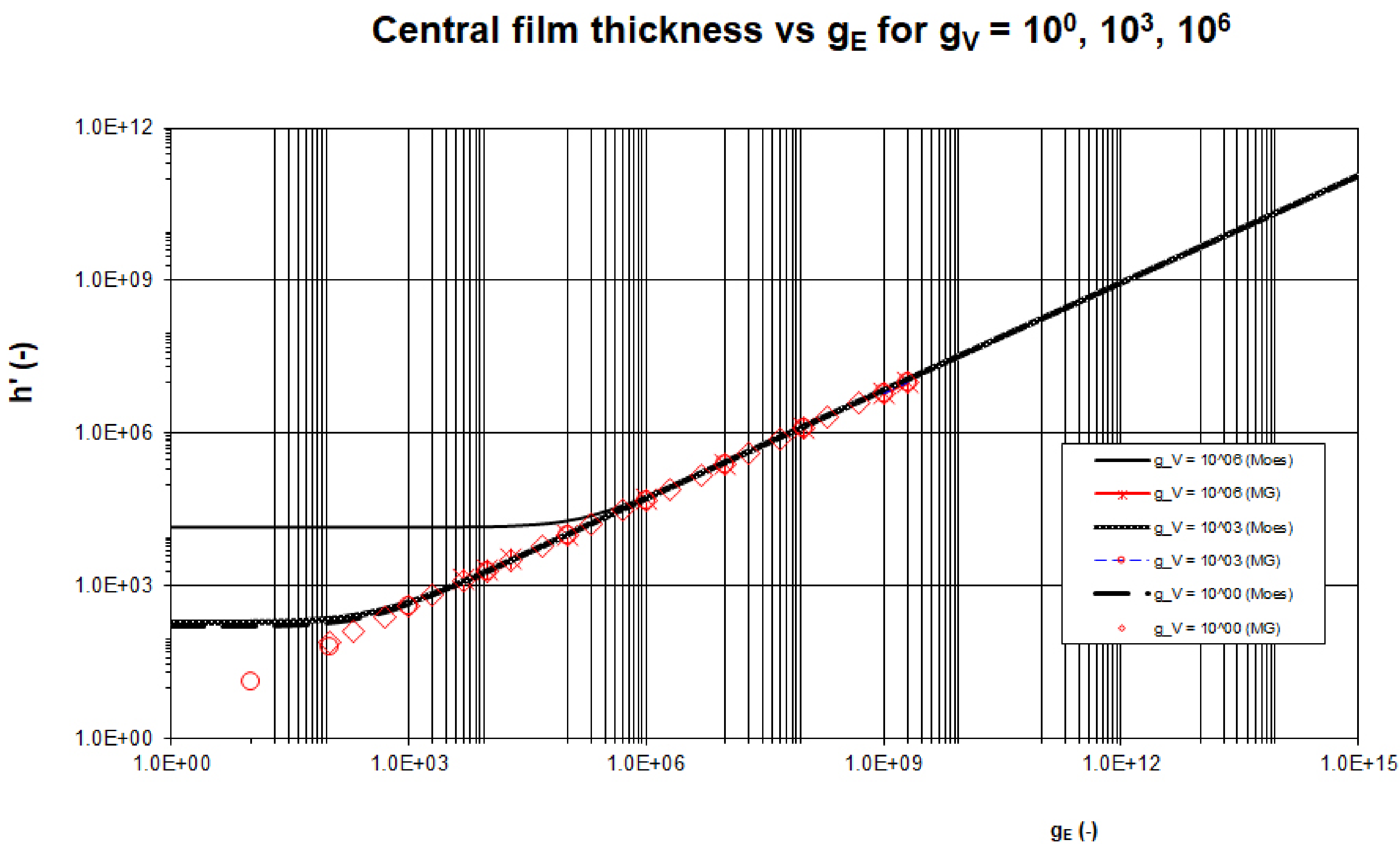

Figure 2: Nondimensional film thickness h' as a function of elasticity group g_E (for $10^0 \leq g_V \leq 10^6$)

4. A critical review of asymptotic behavior - 4

Observations:

The behaviour of h'_{IE} with g_E at high values of g_E show:

- a slope of the Moes IE asymptote of 0.700
- a slope of for Hamrock and Dowson's IE formula of 0.668
- a slope with Venner's code of 0.697

$$h'_{IE,c\ new\odot} \approx 3.292\, g_E'^{\,0.696675}$$

$$h'_{IE,c\ Moes\odot} \approx 3.666\, g_E'^{\,0.7000}$$

$$h'_{IE,c\ H\&D\odot} \approx 4.365\, g_E'^{\,0.6675}$$

4. A critical review of asymptotic behavior - 5

4.2 The VR asymptote in EHL

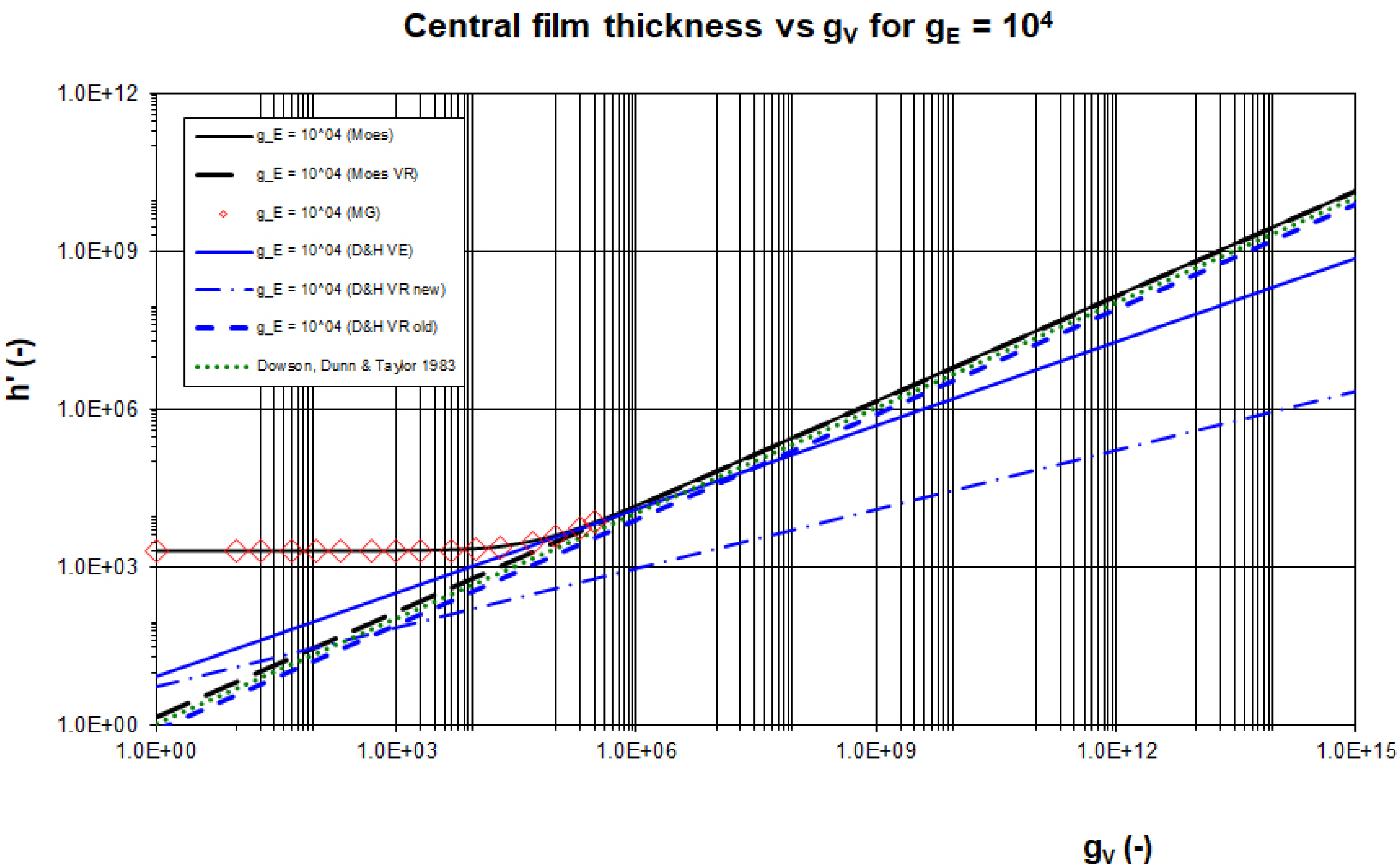

Figure 3: Nondimensional film thickness h_c' as a function of viscosity group g_V (for $g_E = 10^4$)

4. A critical review of asymptotic behavior - 6

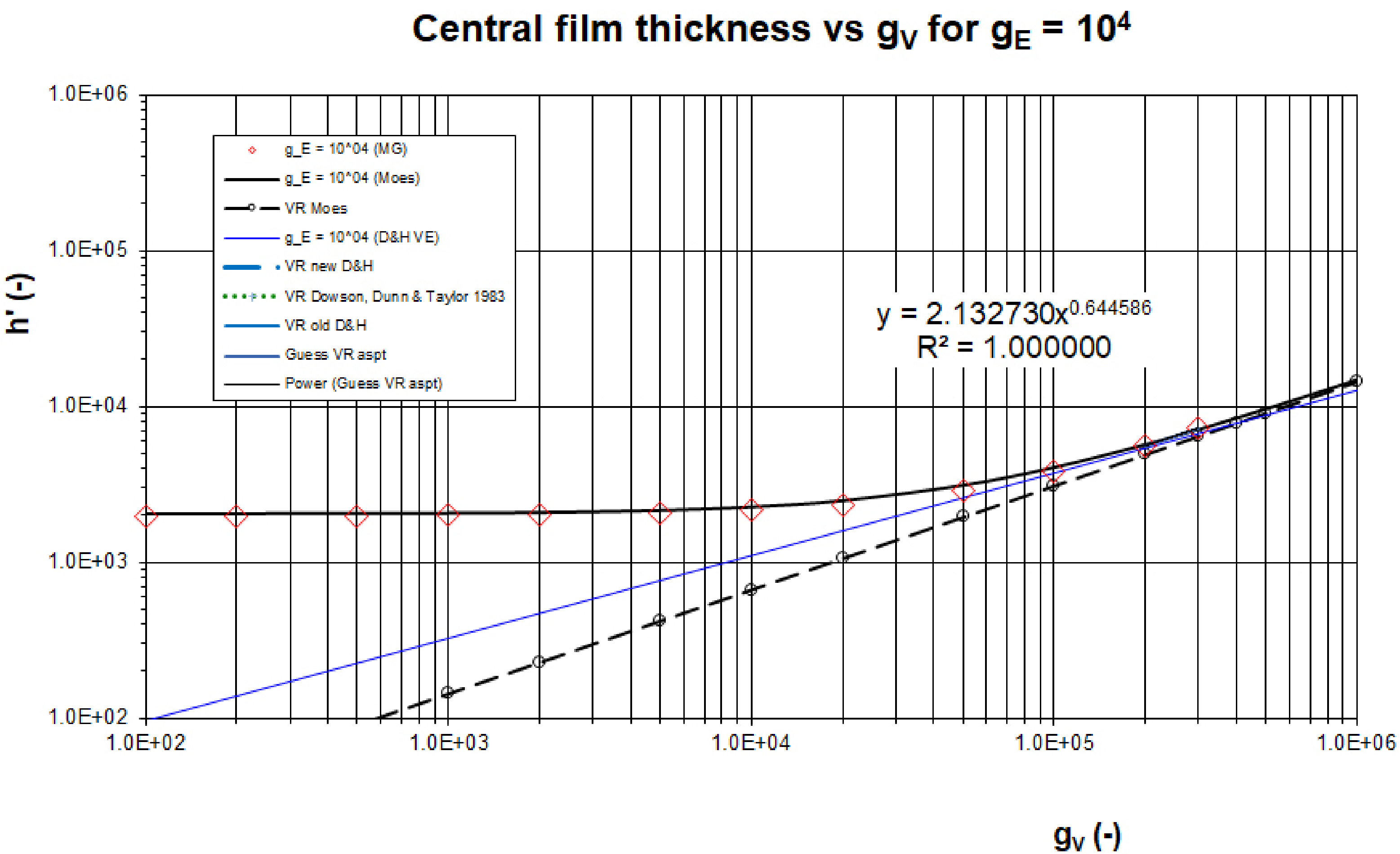

Figure 4: Nondimensional film thickness h_c' as a function of viscosity group g_V (for $g_E = 10^4$)

4. A critical review of asymptotic behavior - 7

Observations:

The behaviour of h'_{VR} with g_E at high values of g_E show:

- a slope of the Moes VR asymptote of 2/3
- a slope for the old Hamrock and Dowson's VR formula (1978) of 2/3
- a slope with Venner's code of 0.645

$$h'_{VR,c\ new\odot} \approx 2.1327\ g_V'^{\,0.6446}$$

$$h'_{VR,c\ Moes\odot} \approx 1.443\, g_V'^{\,2/3}$$

$$h'_{VR,c\ H\&D\,(old)\odot} \approx 0.819\ g_V'^{\,2/3}$$

4.3 The VE asymptote in EHL

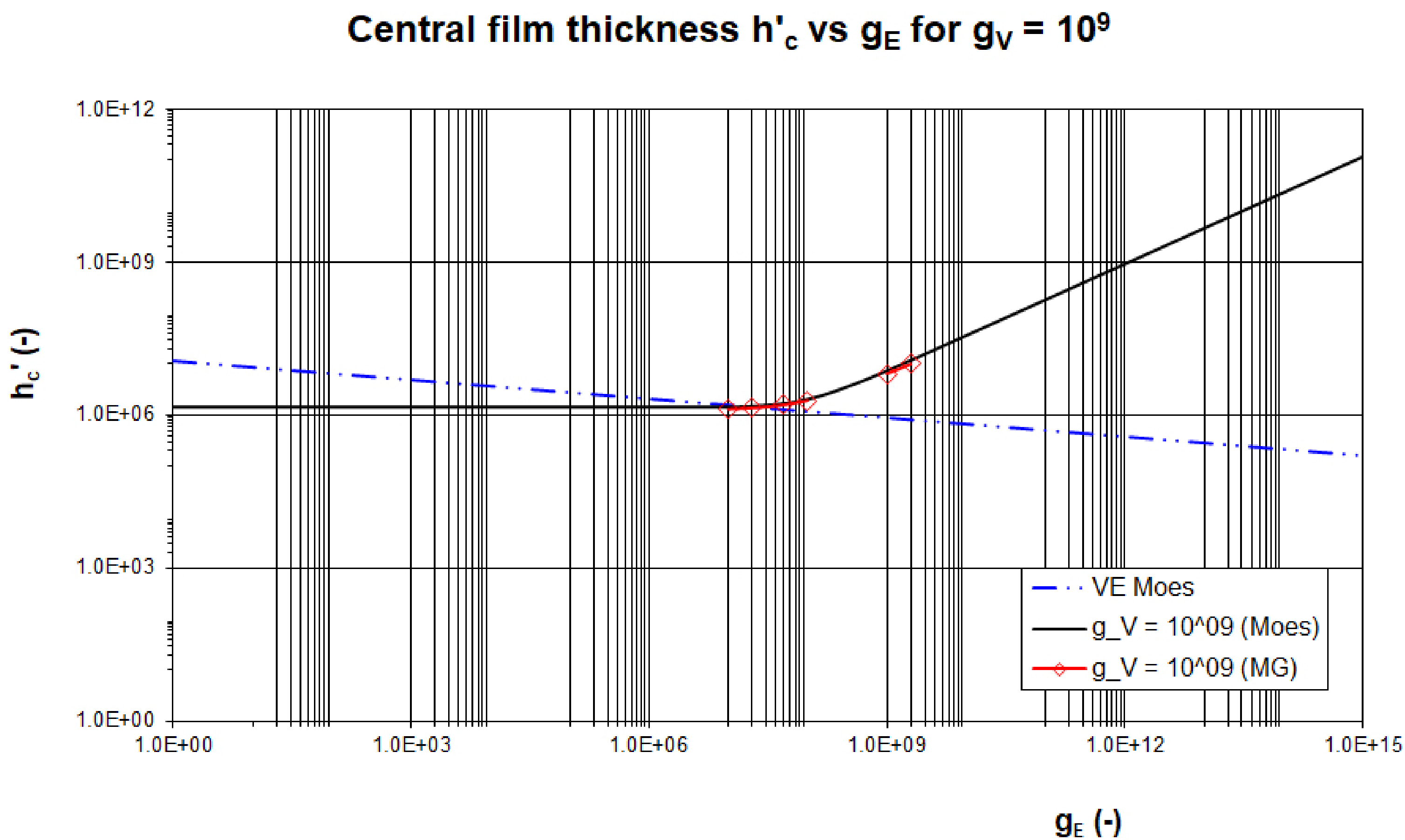

Figure 5: Nondimensional film thickness h_c' as a function of elasticity group g_E (for $g_V = 10^9$)

4. A critical review of asymptotic behavior - 9

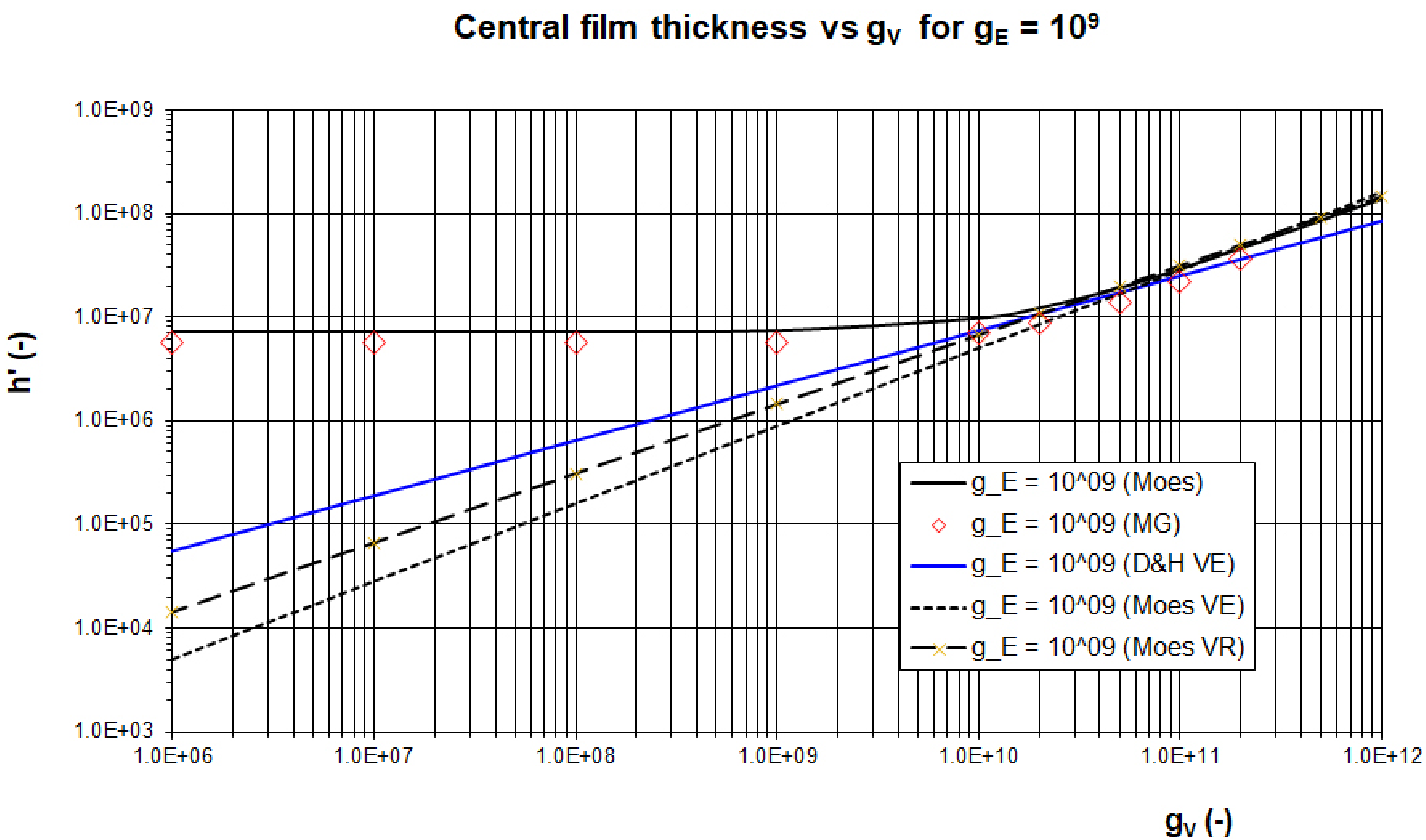

Figure 6: Nondimensional film thickness h_c' as a function of viscosity group g_V (for $g_E = 10^9$)

4. A critical review of asymptotic behavior - 10

Observations:

The behaviour of h'_c with both g_E as well as g_V at very high values show:

- a slope of the Moes VE asymptote of -0.125 is in gross error
- for $h'_c(g_E)$ at very high g_V the slope with g_E shows IE behaviour (0.700)
- for $h'_c(g_V)$ at very high g_E the slope with g_V shows VR behaviour (0.666)

So … there apparently is no such a thing as asymptotic behaviour!

Also, the ***magic in the comprehensive Moes formula*** is, that one part (its VE asymptote) is completely wrong, but its total result is ***not so bad after all***. And there is no alternative for it yet!

Presentation Outline

The Johnson diagram based on the full Moes solution

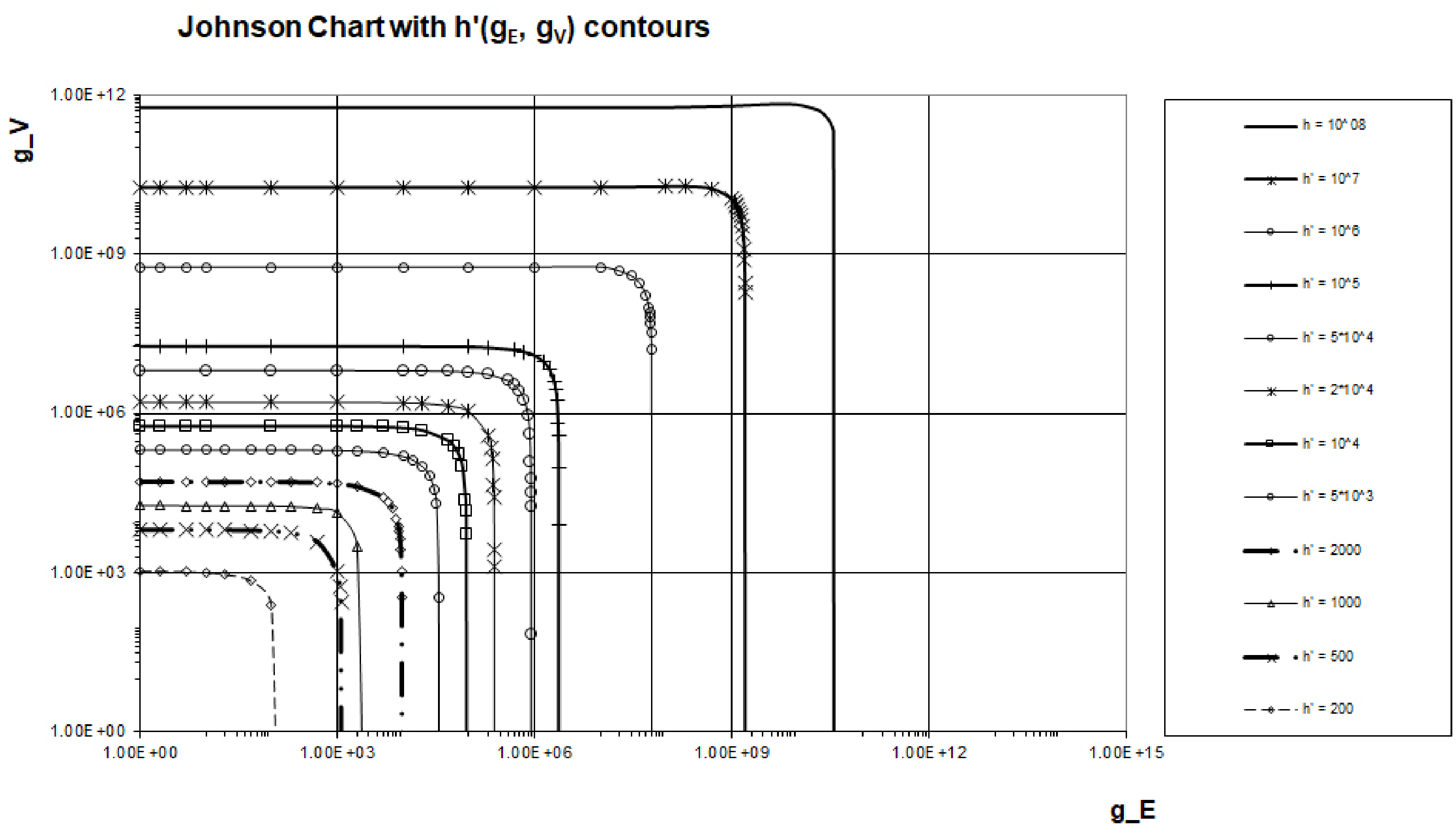

Figure 7A: Contour plot for nondimensional film thickness h_c according to Moes' comprehensive formula

The Johnson diagram based on the full Moes solution

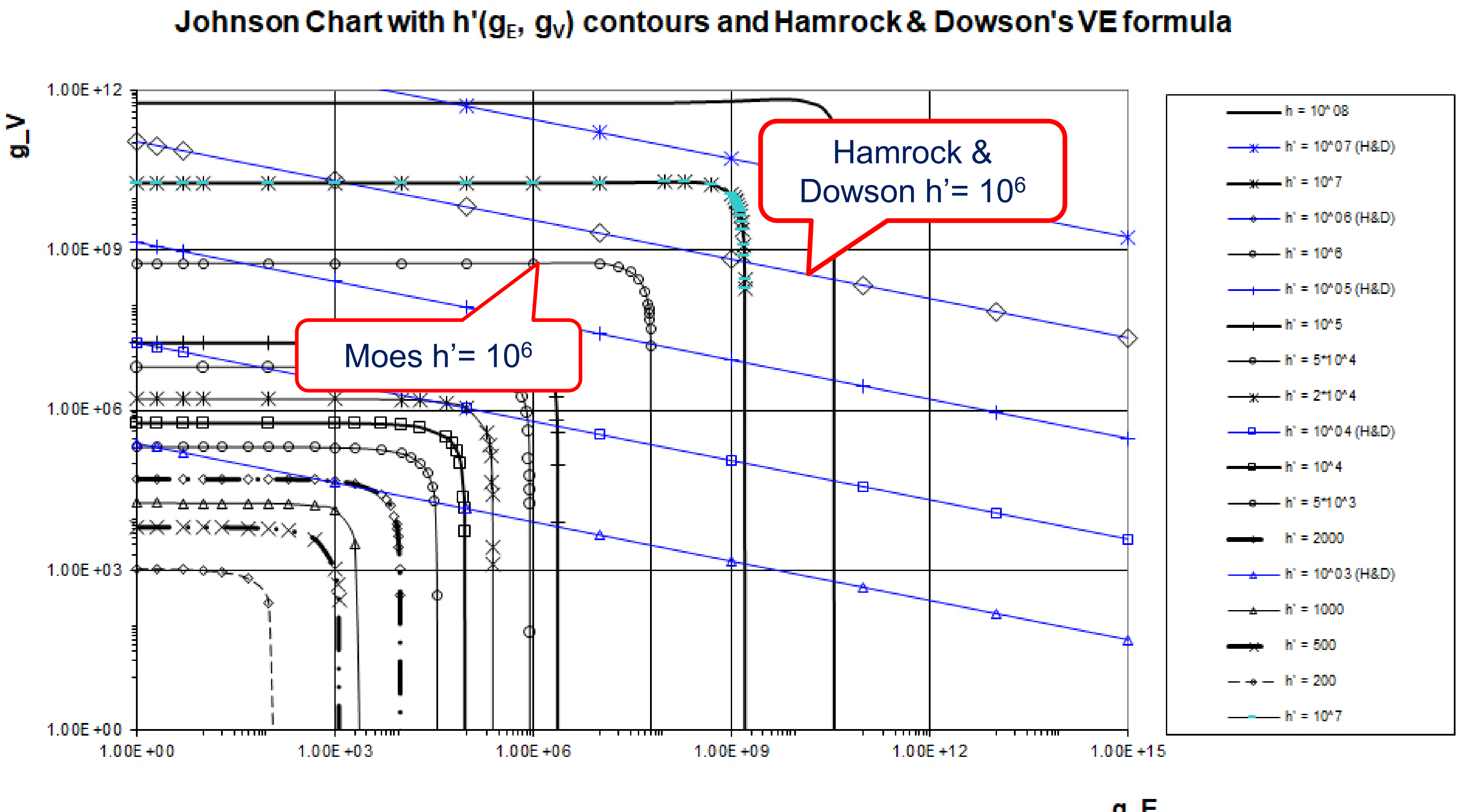

Figure 7B: Contour plot for nondimensional film thickness h_c according to Moes' comprehensive formula

The Johnson diagram based on the full Moes solution

Johnson Chart with h'(g_E, g_V) contours, own experimental data and tresholds

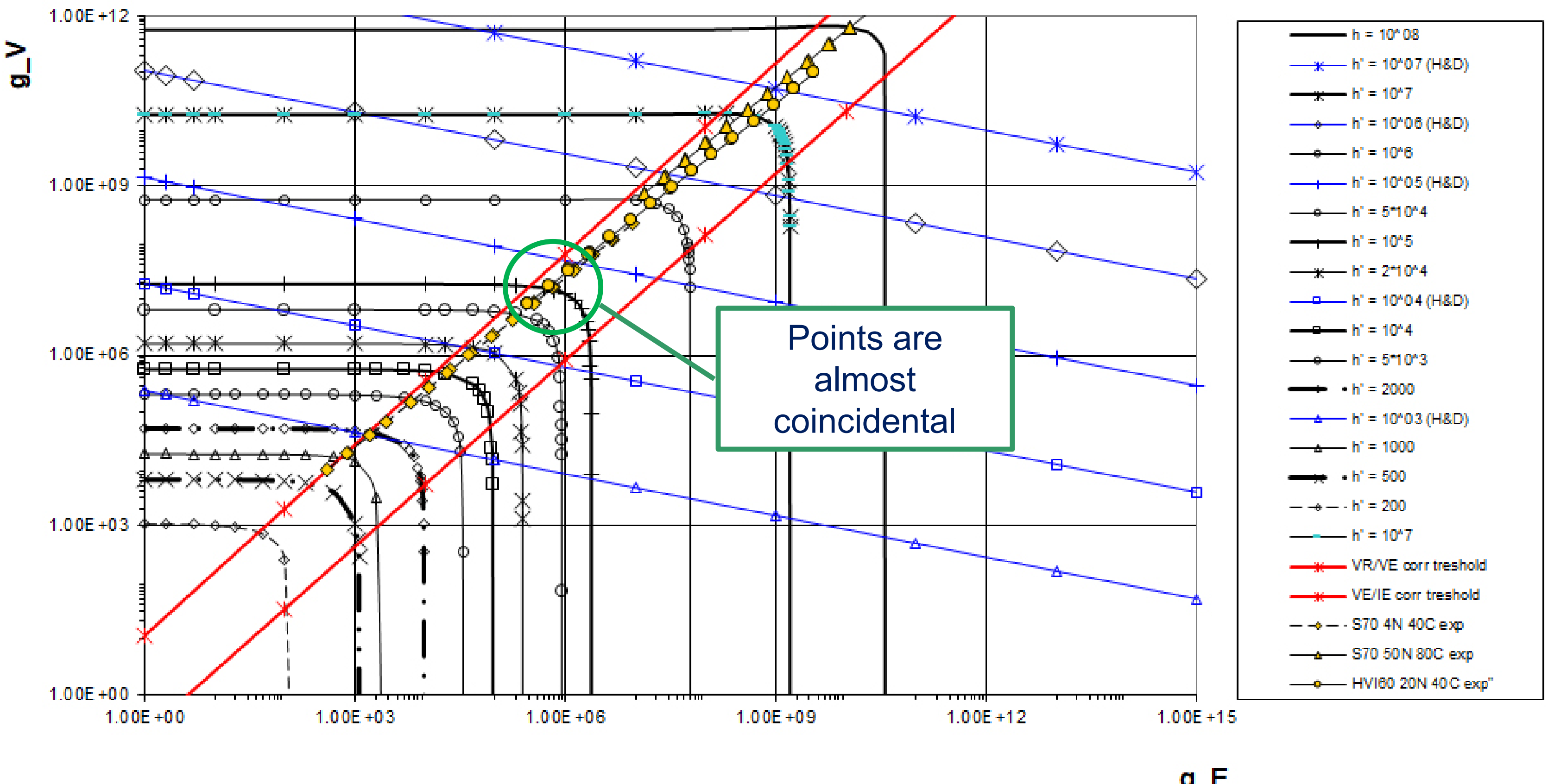

Figure 7C: Contour plot for nondimensional film thickness h_c according to Moes' comprehensive formula

The Johnson diagram based on the full Moes solution

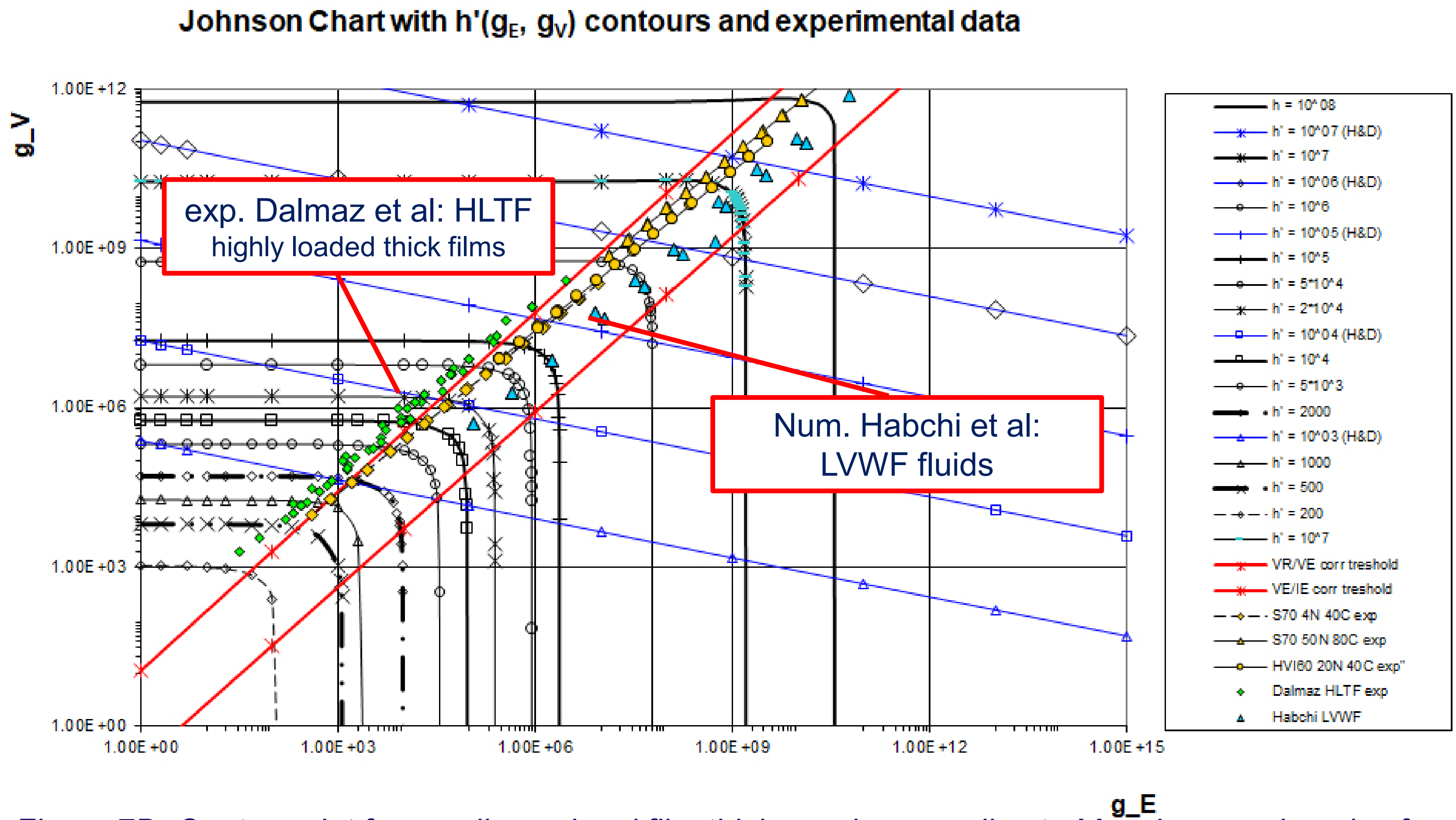

Figure 7D: Contour plot for nondimensional film thickness h_c according to Moes' comprehensive formula

The Johnson diagram based on the full Moes solution

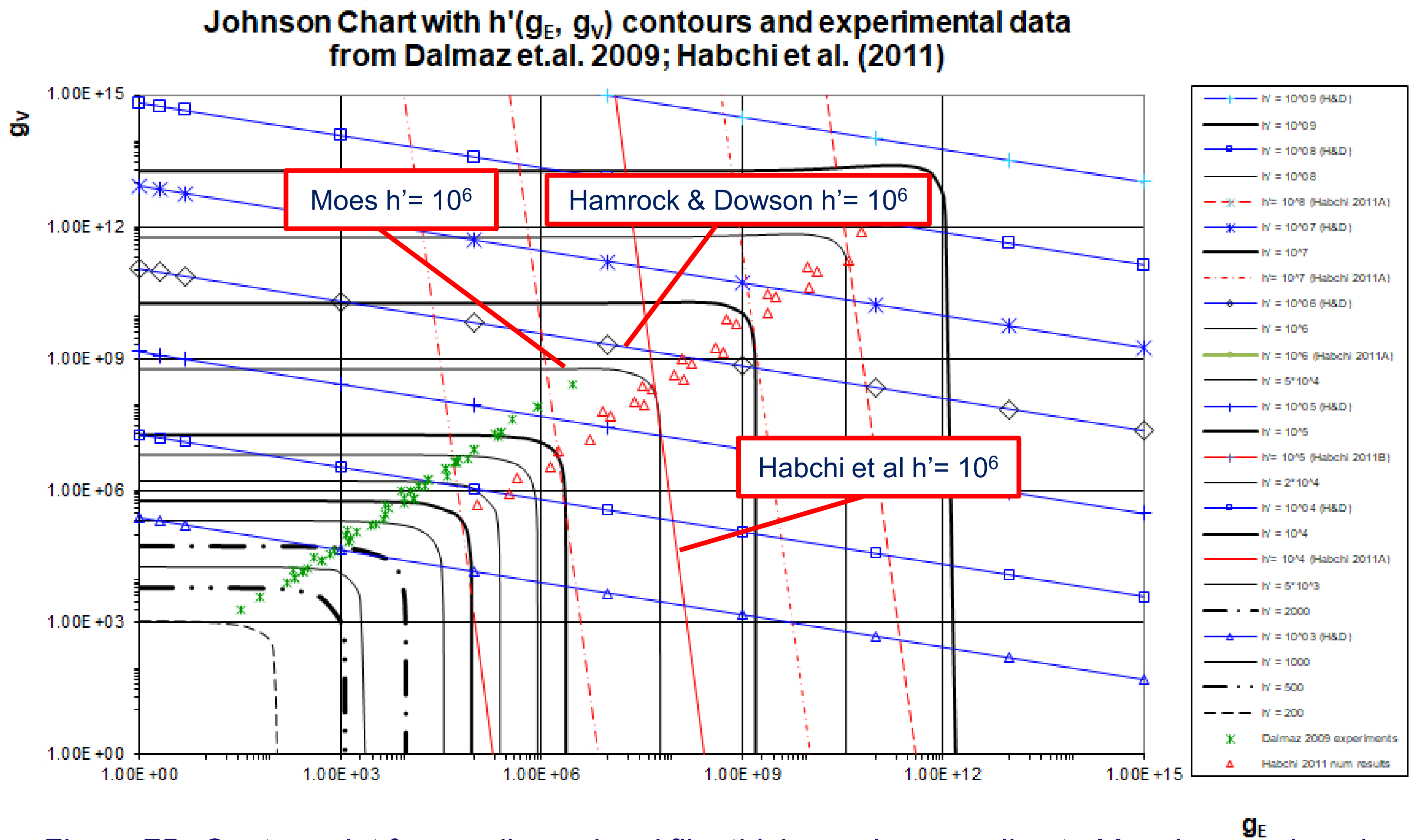

Figure 7D: Contour plot for nondimensional film thickness h_c according to Moes'comprehensive formula

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5. The quest for a comprehensive film thickness formula - 6

Observations:

- The comprehensive ***Moes fit yields higher film thickness*** than Hamrock & Dowson's. This holds true for the entire Moes formula families and the Dowson formula families.
- The better part of all interferometric film thickness experiments have been performed where all approximative formulas almost coincide.
  This is a ***lucky coincidence*** and the reason why the assessment procedure to find the pressure viscosity coefficient has been so succesful. A fortunate coincidence, with the retrospective reasoning.
- Further away from the standard experimental conditions all exponential fits are in error.
- Many authors put forward many (own) curve fits for their limited conditions. A unifying model covering all circumstances is needed.
- The comprehensive ***Moes model is the only one*** which could meet these needs, but has too high film thickness values.
- Habchi et al.'s (2011) numerical study on low viscosity working fluids (LVWF) supports this viewpoint.

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6. First results

6.1 In Delft/Blok notation H = H(L,M)

Depending on the choice of the experimental or numerical conditions, different exponential fits will be obtained. E.g., see Habchi et.al.'s fit vs. Hamrock & Dowson's fit.

About 125 MG results were obtained in a range $10 \leq M \leq 10^4$, $0.1 \leq L \leq 50$
The Moes magic formula yields average + 27.78%, deviation 13.08%, whereas the Hamrock and Dowson VE fit returns average -6.78%, deviation 26.74%.
See table.

Corrections in the comprehensive Moes model by

- replacing the original Moes VE fit by a much better VE fit
- adding sensitivity multiplying factors to the 3 or 4 asymptotes

	ΔH&D VE-MG	*ΔMoes orig-MG*	*ΔMoes adptd-MG*	*Δnew-MG*
average L 0.1-50	-6.78%	27.78%	-20.66%	-21.16%
st.dev. L0.1-50	26.74%	13.08%	29.19%	28.95%
average L1-40	5.64%	30.66%	-7.32%	-7.61%
st.dev. L1-40	12.65%	13.13%	15.76%	14.80%
average L10-30	8.26%	35.56%	3.80%	2.54%
st.dev. L10-30	9.73%	13.90%	3.66%	1.97%

The improvement is not impressive enough to recommend its use.

6.2 In Johnson notation $h' = h'(g_E, g_V)$

The next thought is to transform the results into Johnson groups and then try to find a curve fit based on 3 asymptotes (IR, IE and VR):

$$h'_{new,c\odot} \approx h'_{IR} + \left\{ \left(\frac{h'_{IE} - h'_{IE}*}{c_{IE}} \right)^m + \left(\frac{h'_{VR} - h'_{VR}*}{c_{VR}} \right)^m \right\}^{\frac{1}{m}}$$

where

$$h'_{IR,c\odot} \approx 140.11$$

$$h'_{IE,c\odot} \approx 3.2923\, g_E'^{\;0.7000}$$

$$h'_{VR,c\odot} \approx 2.1327\, g_V'^{\;0.645}$$

and $m, h*_{IE}, c_{IE}, h*_{VR}, c_{VR}$

are constants which are fitted on the results

However, this was not successful. Another proposition has been tried, without success yet. What next?

Presentation Outline

The quest for an improved magic formula continues

Over 400 MG results were obtained in a wide range of circumstances.

For fitting purposes it seems advantageous to use the Blok notation.
An alternative to Blok's and Johnson's representation is a nondimensional grouping as suggested by Venner and Lubrecht (2000):

$$\tilde{h} = \left(\frac{h}{R_e}\right)\left(\frac{R_e}{a_{Hz}}\right)^2 = \frac{3^{-\frac{2}{3}}}{\mu_\omega^{\;2}}\left(\frac{h}{R_e}\right)\left(\frac{F}{E_r R_e^{\;2}}\right)^{-\frac{2}{3}}$$

$$\tilde{\alpha} = \alpha\sigma_{Hz} = \frac{3^{\frac{1}{3}}}{2\pi}\frac{1}{\mu_\omega \nu_\omega}\left(\frac{F\alpha^3 E_r^{\;2}}{R_e^{\;2}}\right)^{\frac{1}{3}}$$

$$\tilde{\lambda} = \frac{12\eta_0 \bar{u} R^2}{a_{Hz}^{\;3}\sigma_{Hz}} = \frac{8\pi}{3^{\frac{1}{3}}}\frac{\nu_\omega}{\mu_\omega^{\;2}}\left(\frac{\eta_0 \bar{u}}{E_r R_e}\right)\left(\frac{F}{E_r R_e^{\;2}}\right)^{-\frac{4}{3}}$$

$$\omega = \frac{R_t}{R_e}$$

So:

$$\tilde{h} = \tilde{h}\left(F, E_r, R_e, h\right)$$

$$\tilde{\alpha} = \tilde{\alpha}\left(F, E_r, R_e, \alpha\right)$$

$$\tilde{\lambda} = \tilde{\lambda}\left(F, E_r, R, \eta_0 \bullet \bar{u}\right)$$

$$\omega = \frac{R_t}{R_e}$$

6. Discussion - 2

In many experiments where film thickness is used to assess the alpha value, rolling velocity is varied, while geometry, temperature, load, viscosity and hence alpha is not varied during one measurement series.

It may be advantageous to use a nondimensional parameter group like α* above:

$$\tilde{\alpha} = \alpha \sigma_{Hz} = \frac{3^{\frac{1}{3}}}{2\pi} \frac{1}{\mu_\omega \nu_\omega} \left(\frac{F \alpha^3 E_r^2}{R_e^2} \right)^{\frac{1}{3}}$$

$$\tilde{\lambda} = \frac{12 \eta_0 \bar{u} R^2}{a_{Hz}^3 \sigma_{Hz}} = \frac{8\pi}{3^{\frac{1}{3}}} \frac{\nu_\omega}{\mu_\omega^2} \left(\frac{\eta_0 \bar{u}}{E_r R_e} \right) \left(\frac{F}{E_r R_e^2} \right)^{-\frac{4}{3}}$$

The existing data can be mapped into these nondimensional groups and represented in another map.

The nondimensional film thickness h as a function of the hydrodynamics group Λ and a given α*

6. Discussion - 3

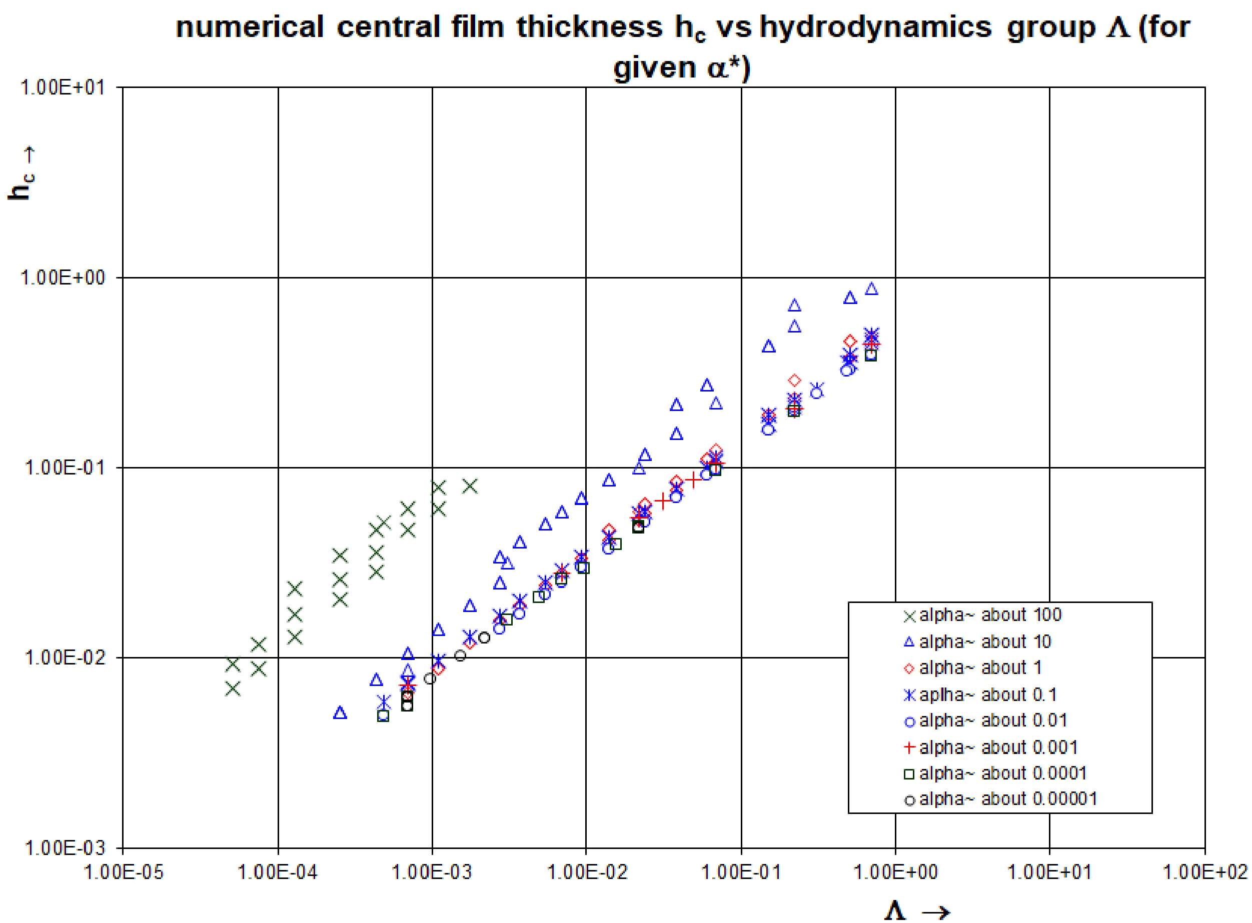

Figure 8A: Nondimensional film thickness h_c as a function of Λ and α**

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Conclusions

- The Moes VE asymptote is erroneous. The comprehensive formula surprisingly performs much better than expected.
- All 4 asymptotes have been analysed. There is no VE asymptote. The IE and VR asymptote found by MG have somewhat higher values than Moes.
- An alternative to the magic Moes formula is not available yet, but is very much needed
- Possibly a notation in Venner & Lubrecht nondimensional groups gives an opening to realise this

Acknowledgements

Hans Moes (retired from Twente University, Enschede, Netherlands) for his teaching and **Kees Venner** (Twente University, Enschede, Netherlands) for making available the software code.

Questions:

Questions?

I would like to address them!

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