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Suppression of axial-torsional vibrations in drilling system described by neutral-type delay differential equations

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Abstract. Vibrations in deep drilling systems lead to efficiency deterioration and may even cause the system failure. In this paper, a controller is designed aiming at mitigation of these vibrations, which is based on a neutral-type time delay model that represents distributed axial and torsional dynamics. First, the stability of the associated linearized dynamics is analyzed using a spectral approach. Furthermore, the open-loop system is shown to be stabilizable by state feedback which supports subsequent controller design. An optimization-based continuous pole placement technique has been employed to design a stabilizing controller, which mitigates steady-state drill-string vibrations. The effectiveness of the controller is shown in a representative case study.

Introduction

Drilling systems are used for exploration and harvesting of oil, gas and geo-thermal energy and suffer from complex coupled dynamics that involve axial, torsional, and lateral vibrations. Different approaches have been employed to model these vibrational phenomena: lumped parameter models [1], distributed parameter models [2, 3] and neutral-type time delay (NTD) dynamic model [4, 5]. The NTD model is obtained directly from the distributed parameter model by neglecting the damping along the drill string and was first introduced in [4] and modified in [5]. In the current paper, the NTD model is employed to study coupled axial-torsional vibrations in drilling string with a drag bit. Motivated by the work in [6], both the cutting process and frictional contact effects are taken into consideration in the bit-rock interaction model. The resulting equations of motion are neutral-type delay differential equations (NDDEs) with several constant state delays (related to the wave propagation speed along the drill string), state-dependent state delays (induced by the bit-rock interaction) and constant input delays. The stability of vibrational models for drilling system have been investigated in [1] for finite-dimensional models and in [3, 7] for distributed models. In this paper, a spectral approach is employed to analyze the stability of the associated linearized dynamics, to study the root causes of the steady-state vibrations and as a basis for controller design. Using the optimization-based continuous pole placement method [8], the controller is designed to eliminate the undesired axial-torsional vibrations. Since the objective function of the underlying optimization problem is non-smooth and to avoid convergence to a local minima, a combination of particle swarm optimization (PSO) algorithm and hybrid algorithm for nonsmooth optimization (HANSO) is used to design the controller gains.

Distributed Drill string model

The NTD model for the coupled axial-torsional dynamics of the drill string is expressed as follows:

$$M_B\ddot{U}_b(t) - C_1\ddot{U}_b(t-2\tau_a) = -C_2\dot{U}_b(t) - C_1C_2\dot{U}_b(t-2\tau_a) - W(t) - C_1W(t-2\tau_a) + C_3u_H(t-\tau_a), \quad (1)$$

$$I_B\ddot{\theta}_b(t) - C_4\ddot{\theta}_b(t-2\tau_t) = C_5\dot{\theta}_b(t) - C_4C_5B\dot{\theta}_b(t-2\tau_t) - T(t) - C_4T(t-2\tau_t) + C_6u_T(t-\tau_t), \quad (2)$$

where the axial and angular position of the bit are denoted by U_b and θ_b, respectively, $C_i, i = 1, ..., 6$, are constants related to drill string characteristics and (M_B, I_B) are the bottom hole assembly (BHA) inertias. The time delays in (1) and (2), τ_a and τ_t, are the time required for the axial and torsional waves to travel from one extremity of the drill string to the other which can be assumed constants since the drill-string length is quasi-constant on the relevant vibrational time scale. The system has two delayed control inputs, the hook force and the top-drive torque, both exerted at the rig which are specified by u_H and u_T, respectively. On the other extremity, i.e. at the bit, W and T are the resistive force and torque applied from the formation, called weight-on-bit (WOB) and torque-on-bit (TOB). For the case of normal cutting, as presented in [6], these are modelled by the following (linearized) bit-rock interaction law:

$$W(t) = \epsilon a\zeta d(t) + \sigma al, \quad T(t) = (1/2)(\epsilon a^2 d(t) + \mu\gamma a^2\sigma l), \quad (3)$$

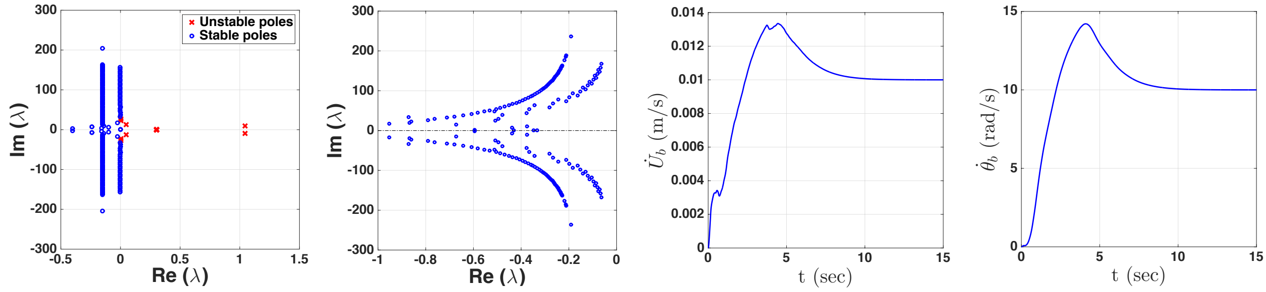
where the parameters $\epsilon, a, \zeta, \sigma, l, \mu, \gamma$ characterize bit and rock properties. The depth of cut $d(t)$ in (3) is defined by $d(t) = n\big(U_b(t) - U_b(t-\tau_n)\big)$ for a n-blade bit, where the state-dependent delay τ_n is the time required for the bit to rotate by the angle of $2\pi/n$, obtained by solving $\theta_b(t) - \theta_b(t-\tau_n) = 2\pi/n$.

Stability analysis and stabilizability

The linearized dimensionless perturbation dynamics with respect to the nominal steady-state solution, associated to constant velocity drilling conditions, is obtained along the lines of [1, 9]. The TDS-STABIL Matlab package is used to find the stability-relevant characteristic roots of this infinite-dimensional model. If this linearized system is exponentially stable, then the nominal solution of the associated non-linear system is locally exponentially stable [10]. The open-loop spectrum is shown in Fig. 1(a), which illustrates that the system is intrinsically unstable. Moreover, the system is proved to be "formally stable" and "spectrally stabilizable", so the stabilizability of the system is assured by state feedback [11], which provides a good starting point for controller design.

Controller design

The state-feedback control law, established by continuous pole-placement method, is finite-dimensional but it can be used for the stabilization of the infinite-dimensional model because the system is proved to be "formally stable" which means that the number of unstable characteristic roots is finite, see Fig. 1(a). The resulting closed-loop spectrum is demonstrated in Fig. 1(b) that confirms closed-loop stability. Note that although the damping along the drill string has been neglected in NTD model, this does not blemish the designed controller since stabilizing the un-damped system guarantees the stability of the associated damped system. To avoid big overshoots and high frequencies in the control inputs, a feed-forward and a low-pass filter have been also designed to complement the state feedback controller. As illustrated in Fig. 1(c) and Fig. 1(d), the closed-loop bit velocities show a satisfactory performance regarding the stability and response time.



(a) Open-loop spectrum (b) Closed-loop spectrum (c) Bit axial velocity (d) Bit angular velocity

Figure 1: Simulation results.

Conclusions

The distributed dynamics of the drill string is analyzed by using a spectral approach. Employing optimization-based continuous pole-placement method, a state-feedback controller is proposed to mitigate axial-torsional vibrations of the drilling system. The simulation results shows the feasibility of the proposed controller.

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