Hybrid evolutionary algorithms and Lagrangian relaxation for multi-period star hub median problem considering financial and service quality issues

Engineering Applications of Artificial Intelligence 97 (2021) 104056

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Hybrid evolutionary algorithms and Lagrangian relaxation for multi-period star hub median problem considering financial and service quality issues

Hamid Tikani [a], Reza Ramezanian [a,*], Mostafa Setak [a], Tom Van Woensel [b]

[a] Department of Industrial Engineering, K. N. Toosi University of Technology, Tehran, Iran
[b] School of Industrial Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

ARTICLE INFO

Keywords:
Multi-period planning
Star hub median problem
Budget constraints
Meta-heuristics
Lagrangian relaxation

ABSTRACT

Hub facilities are centralized locations that consolidate and distribute the commodities in transportation networks. In many real world applications, transport service providers may prefer to lease hub facilities for a time horizon rather than being owned or constructed. In this paper, a modeling framework is proposed for the multi-period hub location problem that arises in the design of the star–star network with two types of hubs and links. It includes a designated static central hub, some movable hub facilities and a set of nodes with pairwise demands. A periodic growth in the amount of budget is considered at each period to expand the transportation network and an interest rate is also applied to the unused budget available during each period. Since the overall quality of services in the hub and spoke systems rely on the length of the paths, upper bound constraints are considered for the paths between nodes. Numerical experiments are carried out to show the applicability of the proposed model. Due to the computational complexity of the model, an improved genetic algorithm (GA) and a hybrid particle swarm optimization (HPSO) are utilized to find near optimal solutions. Both algorithms employ caching strategy to improve the computation times. Moreover, the HPSO benefits from genetic operators and local search methods to update the particles. In order to assess the effectiveness of the proposed methods, the results are compared with a pure GA and a proper lower bound achieved by a Lagrangian relaxation method.

1. Introduction

The hub location problem (HLP) plays a crucial role in many transportation and telecommunications systems and can be considered as one of the fundamental models in the classical facility location problems. It concerns the optimization of shipping of some commodities or flows by determining the potential locations for hub facilities and assigning nodes to these facilities with the aim of routing the flow among origin–destination pairs. More precisely, the flow in a hub-and-spoke network involves three phases, consisting of collecting, routing and distributing. Overall, the flow goes through the located hubs. After consolidating the flow in the hub facilities, they traverse to the inter-hub links (if necessary) in the routing phase. Finally, the flow departs from the hubs to reach their destinations (Karimi and Setak, 2014). Hub-and-spoke networks are often designed to model the problems that require the transfer of large number of commodities (Randall, 2008). By explaining these points, hub and spoke structure helps to decrease the transportation costs in comparing with the point-to-point networks and leads to benefit the economies of scale. General purposes in many HLPs are related to the design of an efficient network for minimizing the total routing costs in addition to the installation costs for the hubs. We usually see three basic assumptions for the classical hub location problems (Kratica et al., 2011; Amin-Naseri et al., 2018): (1) the inter hub network is a complete graph, in which every hub pair is directly connected to each other (Fig. 1(a)); (2) Using the inter hub links, exploiting a discounting factor (α) which reflects the economies of scale ($0 \leq \alpha \leq 1$); (3) It is not possible to tranship the flow between two non-hub nodes, directly.

In this study, we relax the first assumption (1) of the classical hub location problem. Specifically, instead of using a complete graph between each hub node, a star network configuration is employed between hub nodes. We applied a two level star/star network in which a central hub (CH) is established and some of the periodic hubs (PH) should be selected among the user nodes. Then, each hub is connected using direct links to the central hub and each of the remaining nodes is directly connected to exactly one hub. As such, the backbone network that connects the PHs to CH has a star shape, and the network connecting demand nodes to PHs, is also a star (Yaman and Elloumi, 2012). Fig. 1(b) depicts an incomplete hierarchical network with a star/star

* Corresponding author.
E-mail addresses: Hamid.tikani@email.kntu.ac.ir (H. Tikani), Ramezanian@kntu.ac.ir (R. Ramezanian), Setak@kntu.ac.ir (M. Setak), T.v.Woensel@tue.nl (T. Van Woensel).

https://doi.org/10.1016/j.engappai.2020.104056
Received 14 August 2019; Received in revised form 21 October 2020; Accepted 25 October 2020
Available online 11 November 2020
0952-1976/

form. This type of structure is managed easier and possible problems can be distinguished and resolved quickly. A practical example of such network is a kind of three-layer internetworking model, which is applicable in designing a scalable and cost-effective internetwork. In this regard, a typical hierarchical network model is broken up into three various layers including access, distribution, and core. The access layer provides users to connect to the network. The distribution layer (non-central hub) manages the local traffic and also connects the access layer to the core (Graziani and Vachon, 2014). Furthermore, investigations of Elmastaş (2006) on different cargo delivery firms in Turkey shows that one of the major companies has a star–star network configuration where the main transfer center (CH) is located in Ankara. The customers deliver their cargos to non-central transfer center and these points are connected to the CH with direct links (Elmastaş, 2006). All previous works in the literature (e.g., Yaman and Elloumi, 2012; Yaman, 2008) modeled cargo delivery in the star–star network through a static environment. In the mentioned examples, the core layer and the main transfer center referred to as the network backbone which contains special equipment (e.g., data center) to provide fast transportation between the elements. Clearly, non-central (local) hubs contain some moveable facilities (e.g., electronic equipment) that can be relocated according to situations but the CH is not functionally movable.

In this study, the service quality is incorporated into the design process problem. The purpose is to design a transportation network in which the length of the paths that connect each pair does not exceed a pre-specified threshold value (Yaman and Elloumi, 2012). According to Yaman and Elloumi (2012), the distance between each pair of nodes can be considered as a measure of service quality. This problem is known as the HLP with bounded path lengths.

In many real applications, transportation companies have several hub facilities (usually combinations of company-owned hubs and/or leased hubs). Leasing hubs facilitates relocating the facilities based on the demand fluctuations or transportation cost variations. It also helps to expand their networks to increase their market share in the future (Gelareh et al., 2015). Clearly, a multi-period facility location problem makes it possible to open new facilities or closing current facilities proportionating to the variation in demands, costs, governments imposes, sale strategies, tax regulations, and other parameters involved in the decision-making processes.

In the current work, we study a multi-period star hub median problem with budget constraints and bounded path lengths, denoted as the MSHMP-BC-BP. We have three main contributions:

1. This study focuses on a multi-period HLP in the design of the star/star network with regard to service quality considerations. Two types of static and dynamic hub facilities are considered in the model. The network also exploits one static central hub that is owned by the company. The non-portable crucial equipment should be located in this hub.
2. We consider the Time Value of Money (TVM) as an important factor for managerial decision-making. In practice, in each period, the budget that is available to invest in opening (lease) or closing hubs is limited and an interest rate is accounted for the unused budget.
3. We build two meta-heuristic algorithms, including a caching genetic algorithm (CGA) and a hybrid particle swarm optimization (HPSO) to solve the MSHMP-BC-BP. The HPSO utilizes crossover and mutation operators to update the particles. The proposed algorithms apply an immigration operator to avoid local optima in searching process and benefit from a caching strategy to avoid unnecessary calculation of objective values for repetitive individuals during the executions (Kratica et al., 2007). The solution algorithms are further discussed in Section 4. In addition to the mentioned methods, a Lagrangian relaxation (LR) approach is adopted to find a tight lower bound for the proposed model.

This paper is structured as follows. Section 2 is dedicated to the relevant literature. We describe the problem and formulate the model in Section 3. In Section 4, the procedures of the proposed CGA, HPSO and the LR are described in detail. In Section 5, the results of various computational experiments for the model are reported. Additionally, the performance of the proposed algorithms is analyzed in this section. Section 6 provides a sensitivity analysis on some parameters of the model and presents some managerial insights and guidance for decision makers. Finally, in Section 7, conclusions are summarized and some directions for further researches are suggested.

2. Literature review

In this section, we provide a brief literature review of the hub location problem and related solution algorithms by concentrating on the multi period network design problems. Firstly, the idea of using hubs in a network is presented by Goldman (1971). Then, O'kelly (1987) formulates a mathematical model for the single hub location problem. Thereafter, many of researchers developed numerous hub location models and various methods for solving them. For example, we refer to Najy and Diabat (2020), Ghaffarinasab and Kara (2019) and Yang et al. (2019). To sum up the trends of researches on HLPs in the last three decades, the studies in the late 1980s focused on modeling frameworks, in the 1990s on both modeling and optimizing, and eventually in recent years on advanced formulations and method solutions. Among various types of hub location models with different objective functions such as median, covering and center, the most frequently researches focused on hub median location problems (Farahani et al., 2013). A detailed review on HLPs and the categorizations are provided in Contreras and O'Kelly (2019) and Farahani et al. (2013). Concerning reviewing the studies in the field of HLP, it comes out that a large variety of mathematic formulations and solution algorithms have been proposed during the past decade. Due to the NP-Hardness of the HLP, many researchers have employed meta-heuristics to solve the hub location problems (Özgün-Kibiroğlu et al., 2019; Aboytes-Ojeda et al., 2020; Lüer-Villagra et al., 2019). Among various methods, GAs and their modifications can be considered as effective and practical solution methods that utilized to find acceptable solutions for HLP. Moreover, hybridization of genetic operators and local searches with other meta-heuristics such as particle swarm optimization (PSO) helps to prevent premature convergence and increase the performance of proposed algorithms (see Yang et al., 2013; Gao and Qin, 2016).

Some recent papers that employed the principals of GA in solving the HLPs are Kratica et al. (2011), Bashiri et al. (2013), Mohammadi et al. (2013), Kratica et al. (2007), Bashiri et al. (2013), Yang et al. (2013), Damgacioglu et al. (2015), Ebrahimi-Zade et al. (2016), Gao and Qin (2016), Qin and Gao (2017), Hasanzadeh et al. (2016), Bashiri et al. (2017), and Lüer-Villagra et al. (2019). In the current paper, we aim to study two improved meta-heuristics by taking the following features into account:

- So far, most of the studies investigated the HLP in a static environment; however, according to the Farahani et al. (2013), formulations of single period HLP and their solution approaches are not pertinent to real-world applications. Because after some years, the location of selected hubs is no longer optimum due to the significant changes in the initial data. This fact motivated us to provide efficient algorithms for dynamic HLPs.
- Recalculation of fitness values for repetitive individuals that appear during the optimization process can significantly affect the quality of the algorithms (Kratica et al., 2007). In order to remedy this drawback, we utilized caching technique to speed up the searching process. The provided cache table is also utilized to fill the population list as varied as possible and enhance the searching scheme.

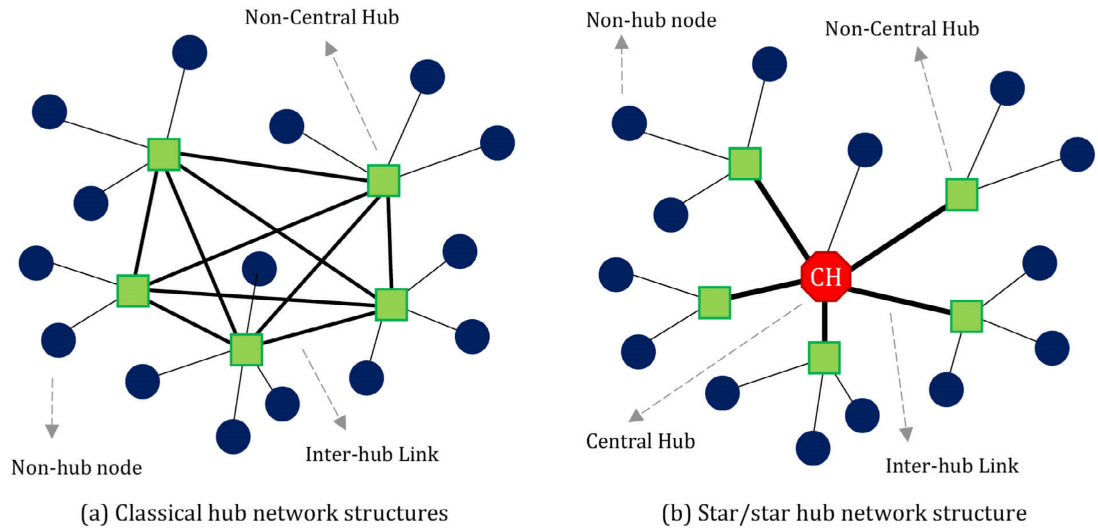


Fig. 1. Examples of hub networks.

- Some of the existing methods cannot bring enough diversification to escape from the poor local optima. In the current work, we embedded several diversification strategies (e.g. immigration operator and LS strategies) through our proposed algorithms to prevent premature convergence.

Since the proposed model in the current study is a kind of incomplete hub network, the rest of the literature is dedicated to incomplete hub location models and after that, we review the multi-period hub location problems.

In the field of hub location problems there exist various studies with incomplete structure among inter-hub links. This incomplete configuration of hub networks can be classified into different categories such as tree shape, ring network, star shape and some special forms. For example, Wang et al. (2006), Contreras et al. (2009, 2010), Karimi and Setak (2014) and Karimi and Setak (2016), developed hub location models with incomplete structures. Some other researches that relaxed the assumption of complete hub and spoke network include, Nickel et al. (2001), Yaman et al. (2007), Yoon and Current (2008), Yaman (2009), Alumur et al. (2009), Calık et al. (2009), and Khodemani-Yazdi et al. (2019). Star topology network in hub structures is employed in different studies. Gavish (1982) utilized a star form network for a telecommunications system that flows are only to and from a central site. Yaman (2008) developed a hub location model in the design of a star–star configuration considering modular arc capacities. Yaman and Elloumi (2012) presented two star p-hub location problems, including star p-hub center problem and the star p-hub median problem. In the first model, the aim is to minimize the longest length of paths in the network while the second model strives to minimize the total transportation routing cost considering an upper bound for path lengths, which reflects the service quality in the network. Some other studies are Wasner and Zäpfel (2004), Labbé and Yaman (2008) and Tikani et al. (2016, 2018).

The next part of our review focuses on multi period hub location models.

Nowadays, it is very scarce that the amount of supplies, demands or generally the market sizes do not change as time goes on. More precisely, each service provider may start with some initial facilities and over the course of a planning horizon based on various conditions decide about expanding the network by opening some new hub facilities or contracting it with closing them. Campbell (1990) developed the first model that considered a hub location problem in a dynamic situation. He introduced a continuous approximation model for the general freighting carrier that operates in a fixed region with an increasing density of demands. Thereafter, Gelareh (2008) studied a hub location problem in which a transportation company begins its operation by an initial hub network. Then, facility location model strives to minimize the total transportation and facility costs over the specific planning horizon. Contreras et al. (2011) considered a dynamic uncapacitated HLP with single assignments. They proposed quadratic model that minimizes the cost of locating, routing and closing the hubs throughout the planning horizon. In another work, Taghipourian et al. (2012) introduced a dynamic virtual hub location problem considering the fuzziness in the planning. Their model decides on location of virtual hubs and the paths to connect the origin–destination pairs over the planning periods. Recently, Gelareh et al. (2015) presented a multi-period hub location problem. In their proposed model, the hub facilities can be changed during the time. In detail, they assume that facilities are leased instead of being owned or constructed; therefore, they can be easily relocated over the course of a planning period. Ghodratnama et al. (2015) presented hub median problem with opening and re-opening modes in an uncertain environment. The objective function of their models includes the costs associated with covering, transporting, activating, opening and reopening operations. Ebrahimi-Zade et al. (2016) presented a multi-period hub set covering problem with single assignments considering flexible covering radius. The covering radius is assumed to be an exogenous parameter. Their model strives to find the optimal network proportional to the various involving parameters by establishing or closing the facilities, periodically. Alumur et al. (2016) formulated a mixed-integer linear program for multi-period hub location problems in a situation that network structure can be progressively constructed and the capacity of hubs gradually expanded during the time. Correia et al. (2017) studied multi-period hub location problems with the multiple allocations using two-stage stochastic modeling framework. The first stage of the model determines the location of hub facilities and sets their initial capacity for the entire planning horizon. While the second-stage involves tactical/operational decisions including allocation of the non-hub nodes to the selected hubs, routing between origin–destination pairs, and the amount of capacity expansion for the hubs. Bashiri et al. (2017) presented a mathematical model for the dynamic *p*-mobile hub location problem. They considered facilities with mobility feature that can be transferred to other nodes to meet the demands. They declared that using such facilities helps to save extra hub establishment and closing costs in the hub networks. Ghodratnama et al. (2018) addressed a bi-objective HLP model in manufacturing supply chains. In this model various transportation modes, congestion effects and production scheduling are taken into account over the planning horizon. Saadati and Hosseininezhad (2019) developed a multi-period hub-and-spoke model for a bagasse-based Bioethanol in a five echelons supply chain. The proposed model seeks to minimize the total costs and co2 emission, simultaneously. They tested the model using real-world data associated with the Iran sugar industry.

Table 1 summarizes the studies about multi-period hub location problems and shows the innovations of the current paper in comparison with the previous works.

From the literature review, we conclude that the multi-period hub location problem has not received too much attention in the past decades and only few studies are published in recent years. The third column of Table 1 indicates that the current paper is the first one addressing star/star hub median problem in a multi-period setting. In particular, we consider that a company established a central hub facility with some special traits while several periodic hubs can be leased for a specific time horizon. We additionally incorporate service quality considerations and budget constraints with TVM concept to reflect more actual factors in the model, which again, is considered for the first time within a multi-period HLP. Another main contribution of the study is provision of efficient solution algorithms to cope with real size problems including modified genetic algorithm, hybrid PSO and Lagrangian relaxation method.

3. Problem definition

In this section, we formally describe the problem and formulate it as a mixed integer programming problem. Based on the star/star hub network design (proposed by Yaman, 2008), this paper deals with a two-level star structure, including a designated central hub to consolidate the flow among the hubs (Yaman, 2008). Consider an origin–destination (OD) pair that both of them are non-hub nodes. In this configuration, the flow originates from a node, goes through its assigned hub with a direct connection. We have three cases as follows:

- If both origin and destination (O–D) are assigned to the same hub, then the flow goes from the hub to its destination. The corresponding path is *Origin→hub→ Destination.*
- If an (O–D) pair is dedicated to different non central hubs, then the flow traverses a path in the form of *Origin→Hub→ Central-hub → Hub → Destination.*
- If one of the (O–D) pairs is connected to the central hub (for example the origin is connected to the CH), then the flow traverses a path in the form of *Origin→ Central-hub → Hub → Destination*.

When one of the OD pairs is a hub or central hub itself, the flow in the network is a subset of the mentioned incidents. The hub facilities in the star/star network are categorized into two types:

Central hub (CH): There exists only one CH in the communication network. It is the fundamental hub to be established and can be considered as a centralized point in the backbone network that connects the periodic hubs (PH) to each other by a star structure. All basic equipment that is not removable is in the CH. Due to the importance of CH, the potential locations for CH are predetermined with regards to different considerations.

Periodic hub (PH): The number of PHs in the network is a model decision. These hubs are constituents of accessible networks and not owned by a company. They can be leased for a specific time horizon. PHs do not need installation costs but opening or closing them is costly. Each user node is a potential location for opening a hub. The facilities in such hubs are classified into static and movable facilities (Ebrahimi-Zade et al., 2016). Static facilities in PH refer to the facilities and staffs, which are not possible to be relocated, and when a hub is closed they remain useless. Moveable facilities refer to the facilities and staffs that can be transferred to the newly opened hubs. We consider that the moving facilities of a closed hub can only be used in one hub. This process eventuates some savings in the total costs. The related savings are subtracted from the total costs on each period.

Other assumptions and concepts of the model are described. (*i*) Direct transshipment between nodes and PHs is not allowable; (*ii*) uncapacitated single assignment version of HLP is considered for the problem. In fact, a demanded node should be connected to only one hub with a direct link and no limitation is considered on the capacity of the hubs; (*iii*) routing cost using inter-hub links is discounted with the classical discount factor $0 < \alpha \leq 1$, which represents the economies of scale; *(iv)* the length of a route between two pairs of nodes is considered as a measure for quality of services. The model controls the lengths such that the connection between each two pair of nodes does not exceed a pre-specified threshold value. This service quality has been improved over the time by decreasing the threshold; *(v)* the planning horizon is finite and the time horizon is divided into various periods; *(vi)* the total investment for opening hubs is limited to available budget on each period. This amount is periodically increasing to expand hub and spoke network. The incremental budget is determined based on the demanding fluctuations and using cost predictions. Unused budget on each period is counted as saving in the objective function; *(vii)* objective function computed by present worth value and all costs are converted to the present form by knowing the interest rate and using formulas in the factor conversion table.

We first present the notation:

Sets and Parameters

N	The number of nodes (spokes) in the hub–spoke network.
i, j	Indices for nodes $i, j = 1 \dots N$.
k, l	Indices for hubs.
n	Indices for central hubs.
t	Indices for planning horizon length, $t = 1, 2, \dots, T$
d_{ij}	Distance from node i to node j.
C_t	Transfer cost (per unit distance) between nodes in period t.
D_{ij}^t	Flow to be sent from origin node i to destination j in period t.
F_n^{Fix}	Fix cost for establishing central hub n.
F_{kt}^{rent}	Related cost for opening (renting) a non-central hub k in period t.
F_{kt}^{excrete}	Related cost for closing (excreting) a non-central hub k in period t.
S_t	The benefits from movable facilities in a closed PH in period t.
U_t	The longest possible path between nodes in period t.
ARC	Basic allowable sleep capital related to open hubs periodically.
EI_t	Allowable marginal capital investment related to open hubs in period t.

Decision variables

x_{tij}^{lnk}	A binary decision variable, which is 1 if the flow from node i to node j goes through hubs located at node k and l and central hub n in period t, it takes 0 otherwise.
O_n	A binary decision variable, which is 1 if node n established as a central hub, it takes 0 otherwise.
w_{ik}^t	A binary decision variable, which is 1 if node i is assigned to be hub k in period t, it takes 0 otherwise.
p_{tk}	A binary decision variable, which is 1 if a new hub is opened (rented) at node k in period t and otherwise equals 0.
q_{tk}	A binary decision variable, which is 1 if the existing hub k is closed in period t and otherwise equals 0.
z_t	Equals to minimum value of $\sum_k p_{tk}$ and $\sum_k q_{tk}$.
e_t	Unused budget in the period t.

3.1. Financial process of expanding the network

Time value of money is a basic principle in financial management. The HLP can be considered as an important investment project, as establishing (and even leasing) the hub facilities is costly and needs a large amount of capital. Studying multi-period HLPs by ignoring the

Table 1
A summary of related work in the literature (Complete: complete graph in spoke-level network).

Authors	Year	Multi period planning	Inter hub network structure	Types of facility		Relocation is allowed	Budget constraints	Service quality considerations	Time value of money	Solution algorithm
				Static facilities	Moveable facilities					
Gelareh (2008)	2008	✔	Complete	✔	–	✔	✔	–	–	Benders Algorithm/Lagrangian Relaxation/Greedy Algorithms
Contreras et al. (2011)	2011	✔	Complete	✔	–	✔	–	–	–	Lagrangian Relaxation/ Branch and bound algorithm
Taghipourian et al. (2012)	2012	✔	Complete	✔	–	✔	✔	–	–	CPLEX commercial solver
Gelareh et al. (2015)	2015	✔	Complete	✔	–	✔	✔	–	–	Meta-heuristic method/Benders Algorithm
Ghodratnama et al. (2015)	2015	✔	Complete	–	✔	✔	–	–	–	commercial solver (Gams software)
Ebrahimi-Zade et al. (2016)	2016	✔	Complete	–	✔	✔	–	–	–	Genetic algorithm
Alumur et al. (2016)	2016	✔	Incomplete	✔	–	✔	–	–	–	CPLEX commercial solver using valid inequalities
Correia et al. (2017)	2017	✔	Complete	✔	–	✔	–	–	–	commercial solver using valid inequalities
Bashiri et al. (2017)	2017	✔	Complete	✔	✔	✔	–	–	–	Genetic algorithm/ simulated annealing algorithm
Ghodratnama et al. (2018)	2018	✔	Complete	✔	–	✔	–	–	–	Goal attainment and LP metric method
de Sá et al. (2018)	2018	–	Incomplete	✔	–	–	✔	–	–	Benders decomposition
Pearce and Forbes (2018)	2018	✔	Complete	–	✔	✔	✔	–	–	Benders decomposition
Saadati and Hosseininezhad (2019)	2019	✔	Complete	✔	–	✔	–	–	–	ε-constraint method
Ghaffarinasab (2020)	2020	–	Star–star network	✔	–	–	–	✔	–	Tabu search heuristic
This paper		✔	Star–star network	✔	✔	✔	✔	✔	✔	CPLEX commercial solver/Modified caching genetic algorithm/Hybrid particle swarm optimization/Lagrangian Relaxation

concept of TVM brings suboptimal decisions. As described earlier, the amount of investing in opening hubs is limited in each period and is affected by demand fluctuation. Determining these parameters is out of this paper's scope, but a comprehensive business plan includes the business goals and the associated plans can be applied to decide on the available budget on each period. Many companies usually start their business with a special share of the market with less investment. To this end, we exert a minimum sleep capital (ARC) for network structure on each period to support the initial market share. Moreover, a marginal capital investment EI_t is added to ARC for expanding the network structure in each period t. The amount of EI_t relies on the marketing strategies and business model of the company and utilized to gradually expand the foundations. We employed $\left(\frac{\mathrm{F}}{\mathrm{P}}, \%\mathrm{i}, \mathrm{t}\right)$ factor to convert the financial amounts to the present form. For example, consider that the ARC is set to 200 unit and we aim to invest a uniform gradient amount 100 unit on expanding the network during 5 periods. The cash flow diagram of this process is depicted in Fig. 2. The arrows in the figure represent the cash.

Accordingly, the allowable investment on opening hubs in the period 2 is calculated by $ARC + EI_2$, that equals to 300 units.

3.2. The MSHMP-BC-BP modeling framework

The mathematic model for MSHMP-BC-BP problem can be formulated as follows:

$$Min \sum_i \sum_{j\neq i} \sum_k \sum_{l\neq k} \sum_n \sum_t [C_t(d_{ik} + d_{lj}) + \alpha C_t(d_{kn} + d_{nl})] x^{knl}_{tij} D^t_{ij} (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) + \sum_i \sum_{j\neq i} \sum_k \sum_n \sum_t [C_t(d_{ik} + d_{kj})] x^{knk}_{tij} D^t_{ij} (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) + \sum_t \sum_k F^{\mathrm{rent}}_{kt} p_{tk} (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) \quad (1)$$

$$+ \sum_t \sum_k F^{\mathrm{excrete}}_{kt} q_{tk} (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) - \sum_t S_t z_t (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) - \sum_t e_t (\mathrm{P/F}, \%\mathrm{i}, \mathrm{t}) + \sum_n F^{\mathrm{Fix}}_n O_n$$

$$\sum_n O_n = 1 \quad (2)$$

$$\sum_t \sum_i \sum_j \sum_k \sum_l x^{knl}_{tij} \leq M.O_n \quad \forall n \quad (3)$$

$$\sum_n \sum_l \sum_k x^{knl}_{tij} = 1 \quad \forall i, j, t \quad (4)$$

$$O_n \leq \sum_t w^t_{nn} \quad \forall n \quad (5)$$

$$2 \times \sum_n x^{knl}_{tij} \leq w^t_{jl} + w^t_{ik} \quad \forall i, j, t, k, l \quad (6)$$

$$w^t_{ik} \leq w^t_{kk} \quad \forall i, k, t \quad (7)$$

$$\sum_k w^t_{ik} = 1 \quad \forall i, t \quad (8)$$

$$\sum_k F^{\mathrm{rent}}_{kt} w^t_{kk} + e_t = ARC + EI_t \quad \forall t \quad (9)$$

$$p_{tk} - q_{tk} = w_{tkk} - w_{t-1kk} \quad \forall k, t \quad (10)$$

$$p_{tk} + q_{tk} \leq 1 \quad \forall k, t \quad (11)$$

$$z_t = \min \left\{ \sum_k p_{tk}, \sum_k q_{tk} \right\} \quad \forall t \quad (12)$$

$$x^{knl}_{tij} \left(d_{ik} + d_{kn} + d_{nl} + d_{lj}\right) \leq U_t \quad \forall i, j, t, k, n, l \quad (13)$$

$$e_t \geq 0 \quad \forall t \quad (14)$$

$$O_n, p_{tk}, q_{tk}, x^{knl}_{tij}, w^t_{ik} \in \{0, 1\} \quad (15)$$

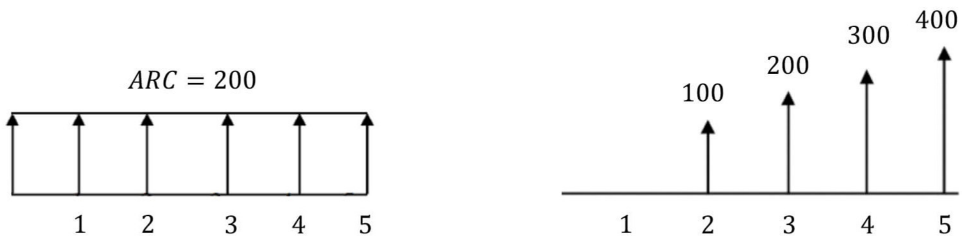


Fig. 2. An example of cash flow diagram for available investments.

The objective function (1) minimizes the total costs by considering savings in different periods. All the financial flows for each period are discounted to the beginning of the planning horizon by a financial factor $(P/F, \%i, t)$. The objective function includes:

- Total transportation cost from various origins to destinations. We consider a discount factor α to reflect the economies of scale associated with the use of links between non-central hubs and the central hub (part 1).
- The second part is dedicated to the opening (renting) or closing (excreting) periodic non-central hubs on each period (part 2).
- The third part considers the benefits of movable facilities in periodic non-central hubs (part 3).
- Interest rates related to the unused budget available (for expanding the network) during different periods are calculated in the fourth part (part 4).
- Finally, the cost of installing the central hub is considered in the last part (part 5).

Constraint (2) ensures that one hub is selected to be installed as a central hub. Constraint (3) forces that the flow from node i to j are zero unless hub n is assigned to the central hub. Constraint (4) forces the flow to go through one or two hubs and a central hub. Constraint (5) ensures that a central hub should be chosen among the hubs. Constraint (6) ensures that the path from i to j, using hubs k and l can be utilized, if both i and j are connected to hubs k and l, respectively. Constraint (7) assures that in each period, ordinary node i may be connected to k, if it is set as a hub. Constraint (8) ensures that on each period every node is exactly assigned to one hub (single allocation constraint). Constraint (9) assures that the amount of budget, invested in non-central hubs should not exceed the available budget at each period. According to constraint (10), for a given node k in period t, when a hub is newly rented, a binary variable p_{tk} equals 1,a binary variable q_{tk} equals 0 and when the existing hub in a node is closed, q_{tk} equals 1 and p_{tk} equals 0. Otherwise, both variables will be zero. Constraint (11) enforces that a non-central hub node can be opened or closed in each period. Constraint (12) specifies the number of possible movements in each period. It equates to the minimum number of rented and closed hubs. These constraints can be expressed as a lemma as follows:

Lemma: As a rule, in each period, three possible cases can be considered (Ebrahimi-Zade et al., 2016):

- $\sum_k p_{tk} > \sum_k q_{tk}$: In this case, the total number of established (leased) PHs is more than the total number of closed PHs. It states that all movable facilities associated to close PHs can be reused. Consequently, the total numbers of movements equal to $\sum_k q_{tk}$.
- $\sum_k p_{tk} = \sum_k q_{tk}$: In this case, total numbers of opened and closed PHs are equal to each other. Consequently, we can dedicate all movable facilities from closed PHs to established PHs and the total numbers of movements equal to $\sum_k p_{tk}$ or $\sum_k q_{tk}$.
- $\sum_k p_{tk} < \sum_k q_{tk}$: In this case, the total numbers of opened (leased)PHs are less than the total number of closed PHs (for example due to the decline in demands or an increase in the cost of opening PHs). Thereupon, only $\sum_k p_{tk}$ number of moving facilities of closed PHs can be used in newly opened PHs.

Constraint (13) controls that U_t can be the length of the longest path between origin–destination pairs in the resulting star/star network in period t. Constraint (14) shows that the unused budget on each period should take positive value. Finally, constraint (15) defines the types of decision variables.

Solving of the MSHMP-BC-BP problem formulation is computationally intractable. In particular, even when the MSHMP-BC-BP is studied under a single period and the set of CH and PHs are given, the achieved sub-problem that concerns the optimal assignment of non-hub nodes to the selected hubs is proven to be NP-hard (see Kratica et al., 2007). The proposed MSHMP-BC-BP belongs to the class of NP-hard problems. Therefore, in order to solve the problem in a reasonable computation time, efficient solution meta-heuristic algorithms and Lagrangian relaxation method need to be developed. We discuss our proposed methods in the next section.

4. Solution approaches

Over the past decade, the genetic operators are hybridized with various evolutionary algorithms and yield to achieve high-quality outcomes for combinatorial optimization problems. In addition, variants of PSO algorithm are employed in the literature to efficiently solve different complex problems (e.g. De et al., 2016, 2017; Maiyar and Thakkar, 2019). In the current study, we apply GA's operators couple with some innovative techniques in two improved evolutionary algorithms including genetic algorithm and particle swarm optimization to handle MSHMP-BC-BP in an efficient way. In what follows, our proposed optimization algorithms are discussed.

4.1. Modified caching genetic algorithm

The genetic algorithm, as proposed by Holland (1975), imitates the mechanics of genetic evolution and natural selection. An evolution process in a typical GA consists of a selection procedure, crossover operator, and mutation operator. The cycle of reproduction of the populations and elitism is re-iterated until a well-defined stopping criterion is met. The proposed genetic algorithm exploits an immigration operator and an internal data storage that guides the search process to obtain better solutions. The next sub-sections are dedicated to details of the proposed genetic algorithm.

4.1.1. Representation of chromosome

In addition to simplicity, the structure of chromosome contains the necessary information for defining a solution to the problem. In the hub location problem a chromosome represents the network configuration by locating the hub facilities and the assignments of simple nodes to the located hubs. In the present context, direct use of location–allocation matrix as a chromosome is too complicated due to the genetic operators. To this end, we applied a continuous solution representation (CSR) form of the chromosomes to prevent generation of infeasible solutions by the operators. It makes the searching process smoother and easier. In detail, we designed a $[\boldsymbol{K} \times (\boldsymbol{t} \times \boldsymbol{K})]$ dimension matrix containing the numbers between [0,1] to represent the given network in which $\boldsymbol{K}$ denotes the number of nodes and $\boldsymbol{t}$ represents the number of periods. The procedure of finding number of PHs on each period t in the network ($\boldsymbol{NH_t}$) is described in Algorithm 1. This algorithm benefits the caching strategy, which is described in more details in Section 4.1.6. In order to complete the network structure, we proposed a decoding process that transfers the chromosome to a network with one central hub and a $\boldsymbol{NH_t}$ number of hubs on each period (correspond to Step 4 and 5 in Algorithm 1) as follows:

Step1: Sorting the numbers in the diagonal matrix of the first period.
Step2: The largest number (among potential nodes) denotes the central hub for all periods.
Step3: Assigning the PHs according to the sorted list until the numbers of hubs are completed (NH_t number of hubs should be chosen in period t).
Step4: Assigning the non-hub nodes to the hubs by comparing the values at the intersection of non-hub node's column and the rows, which are assigned as hubs (the highest number determines the assignment).

Step 3 and Step 4 in the above process are repeated for all periods. This approach guarantees that each non-hub node is connected to only one hub. In order to clarify the method, we illustrate a sample chromosome for the network with six nodes in Fig. 3. Assume that Algorithm 1 proposes one PH for the first period and two PHs for the second and third periods based on the budget constraints. In the decoded structure, the one value at the main diagonal denotes the hubs, while other elements with value 1 represent the allocated user nodes. The corresponding network configurations after decoding process are depicted in Fig. 3.

The chromosome representation and the associated decoding process are proposed in such a way to handle most of the constraints. In detail, the suggested method aims to satisfy all constraints except Constraints (9) and (13). In the following, a method is provided to select a set of hubs regarding the budget constraint. Although the proposed method does not necessarily satisfy the maximum path length constraint, it steers the explorations toward achieving feasible solutions.

In order to specify the numbers of PHs (Step 3 in Algorithm 1), we apply an approach based on a roulette wheel selection for selecting the numbers of PHs. In this method, the probability of selecting each set of hubs is estimated based on the deviation in corresponded path length constraints, in particular: (Maximum deviation in path length constraints $\downarrow \propto$ the probability $\uparrow$). This problem can be considered as a star p-hub location problem with limited path length, which is abbreviated with (SpHP-lPl). The SpHP-lPl aims at minimizing the deviation between the largest path in the network and predetermined upper bound. Before presenting the mathematical model of SpHP-lPl, we describe the decision variables and parameters:

y_{ij}^{knl} A binary decision variable, which is 1 if flow from node i to node j goes through hubs located at node k and l and central hub n, it takes 0 otherwise.
v_{ik} A binary decision variable, which is 1 if node i is assigned to hub k, it takes 0 otherwise.
UP The longest possible path between nodes.
β_{min} Deviation of largest path.
P Number of hubs in the network.

The mathematical formulation of SpHP-lP is written as follows:

$$\text{Min} \quad \beta_{min} \tag{16}$$

$$\sum_n O_n = 1 \tag{17}$$

$$\sum_i \sum_j \sum_k \sum_l y_{ij}^{knl} \le M.O_n \quad \forall n \tag{18}$$

$$\sum_n \sum_k \sum_l y_{ij}^{knl} = 1 \quad \forall i,j \tag{19}$$

$$O_n \le v_{nn} \quad \forall n \tag{20}$$

$$2 \times \sum_n y_{ij}^{knl} \le v_{jl} + v_{ik} \quad \forall i,j,k,l \tag{21}$$

$$v_{ik} \le v_{kk} \quad \forall i,k \tag{22}$$

$$\sum_k v_{kk} = P \tag{23}$$

$$\sum_k v_{ik} = 1 \quad \forall i \tag{24}$$

$$y_{ij}^{knl}\left(d_{ik} + d_{kn} + d_{nl} + d_{lj}\right) - UP \le \beta_{min} \quad \forall i,j,k,n,l \tag{25}$$

$$O_n, y_{ij}^{knl}, v_{kk} \in \{0,1\}, \beta_{min} \ge 0 \tag{26}$$

Eq. (16) represents the objective function of SpHP-lP. Eq. (17) exerts that one hub should be selected as a central hub. Eq. (18) states that the flow from node i to j is to be zero unless hub n is assigned to be a central hub. Eq. (19) forces the flow to go through one or two hubs and a central hub. Eq. (20) guarantees that central hub should be chosen among the set of hubs. Eq. (21) imposes that a path from i to j by using the hubs k and l is usable if both i and j are linked to hubs k and l, respectively. Eq. (22) assures that it is possible for an ordinary node i to be connected to hub k, if it is set as a hub. Eq. (23) enforces that exactly, P hubs should be selected. Eq. (24) applies the single allocation strategies in hub network. Eq. (25) implies that β_{min} equals to the largest deviation between predetermined upper bound and the longest existed path. Finally, Eq. (26) propagates the types of decision variables.

4.1.2. Initial population

The initial population of the proposed genetic algorithm consists of $[\boldsymbol{K}\times(\boldsymbol{t}\times\boldsymbol{K})]$-dimensional matrices where the number of matrices equals the number of individuals.

4.1.3. Fitness evaluation

One of the usual methods to overcome the constraints in meta-heuristic algorithms is considering intermediate infeasible solutions during the searching process by relaxing some of the constraints. In the proposed meta-heuristic, we relaxed the constraints, which related to the path lengths in order to enhance the method exploration capabilities. Let $\bar{s}$ represents an infeasible solution for the proposed model. Consequently, the penalized objective function with constraint relaxation can be achieved by Eq. (27). In Eq. (27), φ is a non-negative coefficient, which is initialized by the user.

$$C\left(\bar{s}\right) = C\left(\bar{s}\right) + \varphi \times w(\bar{s}) \tag{27}$$

where $C(\bar{s})$ represents the standard objective function and $w(\bar{s})$ measures the maximum path length violation over all connections.

4.1.4. Crossover operator

The crossover operator in the genetic algorithm exchanges the information between two selected chromosomes (parents) in order to generate a new offspring with better features according to the evolution theory. Here, roulette wheel selection is utilized to choose the individuals, which undergo crossover. Moreover, we applied convex crossover with a one-cut point. In detail, two new offsprings, p_{1new} and p_{2new} can be achieved by two parents p_{fn} and p_{sn} using Eqs. (28) and (29). In these equations b is a random matrix with size of the parents on the interval [0, 1].

$$p_{1new} = b.p_{fn} + (1-b).p_{sn} \tag{28}$$

$$p_{2new} = (1-b).p_{fn} + b.p_{sn} \tag{29}$$

4.1.5. Mutation operator

The mutation process is an important building block of the GA aiming to escape local optima moving to a global solution. Hence, we devised some random changes for the chromosomes to steer the explorations in the solution space. For each mutation, one of the prepared operators is randomly chosen to produce an offspring with new characteristics. The process of the designated mutations are expressed as follows:

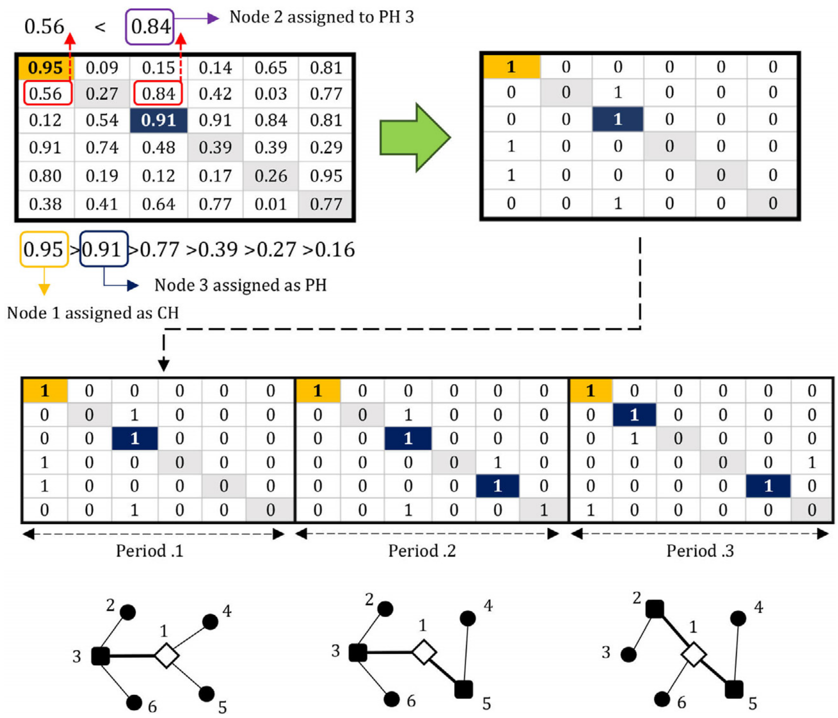


Fig. 3. Decoding process for sample network.

Algorithm 1. Decoding the chromosome with regard to the budget constraints

Initialization: PE←number of periods, RB (Remained budget) ← 0
Step1: Determine the central hub
→ it corresponds to the largest number of potential nodes in the diagonal of the matrix in period 1
For each $t \in PE$
  Step2: Determine the maximum number of hubs that can be opened with regard to budget constraint on period t
    $RB = ARC + EI_t$
    $Count = 1$
    $S \leftarrow$ { The set of potential hubs according to the sorted numbers in diagonal of matrix in period t}
    while RB ≥ 0 & | S | > 0
      $\mathrm{RB} \leftarrow \mathrm{RB} - F^{\mathrm{rent}}_{S\{\mathrm{Count}\}\,t}$
      Delete $S\{Count\}$ from the set S ($S := S \backslash \{S\{Count\}\}$)
      $Count \leftarrow Count + 1$
    end while
  Step3: Determine the number of PHs by roulette wheel rule
    For $i = 1\colon Count$
      $\mathrm{fitness}_i$ = Calculate the fitness function of SpHP-lPl with i number hubs (or use cache table)
    End for
    $SUM = \sum_{i=1}^{Count} \mathrm{fitness}_i$
    Sum of probabilities = 0
    For $i = 1\colon Count$
      $\mathrm{Probability}_i$ = Sum of probabilities + ($\mathrm{fitness}_i$ / SUM)
      Sum of probabilities ← Sum of probabilities + $\mathrm{Probability}_i$
    End for
    RAND= a random number between 0 and 1
    Find the first i such that $\mathrm{Probability}_i \geq$ RAND
    $NH_t = i$
  Step4: Determine the set of hubs in period t // *based on* NH_t *and* S
  Step5: Determine the assignments of nodes in period t
End for
Step 6: Return the decoded structure for the network configuration

- *CH Exchange Mutation:* This operator exchanges the central hub. In particular, the related gene ρ that causes the node to be central hub is selected and then the value of the related gene (value_ρ) is replaced by ($\text{value}_\rho^{new} = 1 - \text{value}_\rho$), so the next priority is selected to be the central hub.
- *PH Exchange Mutation:* This operator removes a node from the set of PHs, randomly. In fact, one of PHs (for example θ) is elected by chance and then the value of the related gene (value_θ) is replaced by ($\text{value}_\theta^{new} = 1 - \text{value}_\theta$) such that the hub node (θ) converses to an ordinary node and the next priority can be selected as a PH.
- *Transposition Mutation:* In this operator, the associated matrix of each period is transposed separately before the decoding process. Thus, the set of hubs (include CH and PHs) remains unchanged but the assignments of the ordinary nodes to hubs entirely rearrange.
- *Swap Mutation:* This mutation operator selects two columns on the chromosome with a chance, and then interchange the allele values among them to generate new offspring.

Fig. 4 visualizes the performance of proposed mutation operators on a part of the chromosome that relates to the first period.

4.1.6. Caching strategy

The caching genetic algorithm prevents excess calculation of fitness values relates to repetitive chromosomes during the searching operations (Kratica et al., 2011, 2007). In this method, a caching table is prepared to store the fitness values of the chromosomes. If we recognize a genetic code that exists in cache table, we use the cache information instead of re-computing the objective function. Same as Kratica et al. (2007), we employed simple but practical caching strategy named Least Recently Used (LRU) to limit the data storage with storage size $N_{cache} = 8000$.

4.1.7. Migration operator

In this study, we employ an immigration operator in addition to the crossover and mutation operators. The immigration operator helps to search the solution space more widely and prevents converging to a (non-acceptable) local optimum. The basic idea of immigration relates to societies in which some new individuals (i.e. immigrants) join the current population, perpetually. The immigrant's chromosomes are randomly created by some distribution. Thus, no genetic material of the present generation is brought in. On the whole, in the proposed genetic algorithm, some new offsprings and several immigrants attach to the main population. After sorting the individuals based on their finesses, better individuals go through to the next generation. This process is repeated until a stopping criterion is met.

The complete flowchart of proposed CGA is given in Fig. 5. The algorithm exploits a feature which seeks to fill the population list as varied as possible. To this end, the objective value of repetitive individuals in each population is set to the upper bound. It eventuates the non-existence of members with the identical genetic code in the next generation. It eventuates the non-existence of members with the identical genetic code in the next generation and reduce the size of population list without any loss of quality.

4.2. Proposed hybrid particle swarm optimization (HPSO)

Over the past decade, the genetic operators have been hybridized with various evolutionary algorithms to efficiently solve different complex problems. Another useful trend, which has been utilized in the literature to achieve higher quality solutions, is embedding local search strategies in the evolution process. In this study, we applied these features to devise an improved algorithm based on particle swarm optimization. In what follows, the process of proposed optimization algorithm is discussed.

Particle swarm optimization (PSO) is an evolutionary computation algorithm based on the foraging of birds and fish schooling. The procedure of the algorithm is initially proposed by Kennedy and Eberhart in 1995 (Kennedy and Eberhart, 1995). Same as many meta-heuristic algorithms PSO is a population-based method and contains various particles, which represent the potential solutions to a certain problem. PSO employs a simple mechanism that imitates swarm behavior of birds. In details, it starts with a group of random particles as an initial population and then iteratively seeks for the globally optimal solution. Here, we combine PSO with proposed genetic operators in Section 4.2.6. Moreover, some local search (LS) strategies are added to the proposed algorithm in order to make better particles in Section 4.2.7. The features of the proposed hybrid approach are the following:

- We employed a continuous solution representation (CSR) form of the particles to keep the feasibility of solutions after updating the positions.
- In many variants of PSO algorithms, the $Pbest$-$Gbest$ based formula is utilized to create new solutions and explore the solution space. The $Pbest$ represents the best fitness discovered by each particle and $Gbest$ denotes the best position observed by the whole population. We proposed a new update strategy for the particles by incorporating the genetic operators (crossover and mutation). Proposed probabilistic-based method strives to keep the diversity of the solutions during the searching process.
- In order to prevent premature convergence through the local optima, we applied immigration operator. In this approach, we devised a strategy to exchange a poor particle with a new immigrant particle.
- We employed the LS heuristic to a sub-swarm of the whole population on each iteration. This approach helps to explore the solution space with more precision and keep the particles far away from a local optimum.

The mentioned improvements of the proposed hybrid method solution are as follows:

4.2.1. Participles representation

The solution representation in the modified HPSO is similar to proposed CGA and employs real values. The mentioned decoding process (Algorithm 1) can be implemented to transfer the particle to a hub configuration.

4.2.2. Initial population

The initial population in the proposed method is a group of random particles represented by some matrices. The entries in the matrices have a uniform distribution in [0, 1].

4.2.3. Fitness evaluation

We employed a penalized objective function same as proposed CGA in Section 4.1.3. Clearly, the particles with lower objective values are evaluated with higher fitness.

4.2.4. Migration operator

In the proposed hybrid method, we incorporate immigration operator to escape from local optima and explore the new areas of the solution space. To do this, after M iterations (M is set by user); one-tenth of population that has the lowest $x^k_{Pbest,i}$ in the swarm are replaced by immigrant individuals.

4.2.5. Caching strategy

Same as proposed CGA, we incorporated caching techniques into the presented HPSO to improve the computational times in the searching procedure. The complete information about the caching method is presented in Section 4.1.6.

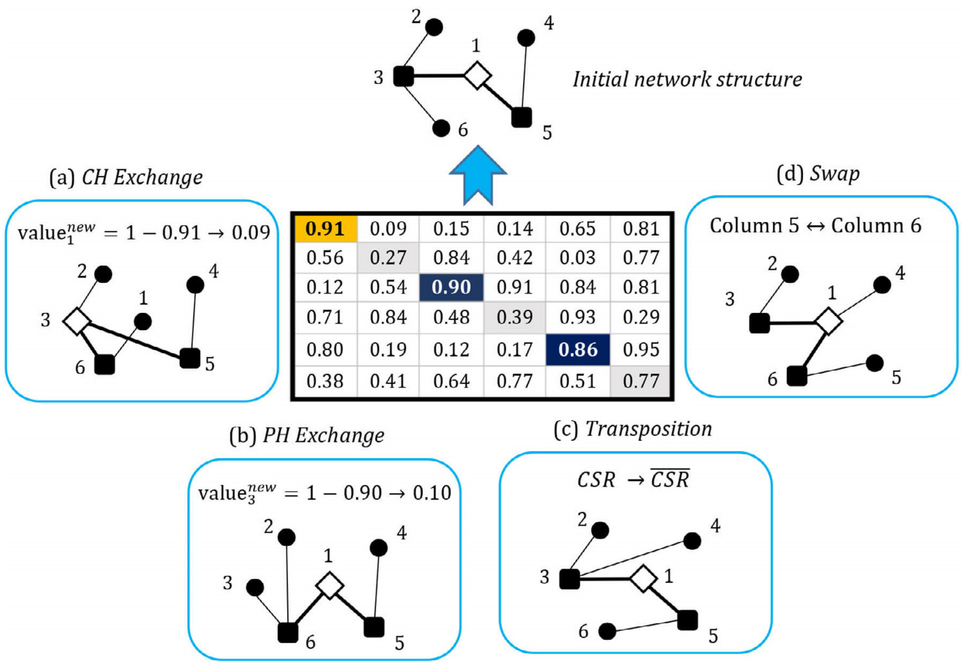


Fig. 4. The effect of different mutations on chromosomes and the resulting network created by (a) CH-Exchange, (b) PH-Exchange, (c) Transposition, (d) Swap.

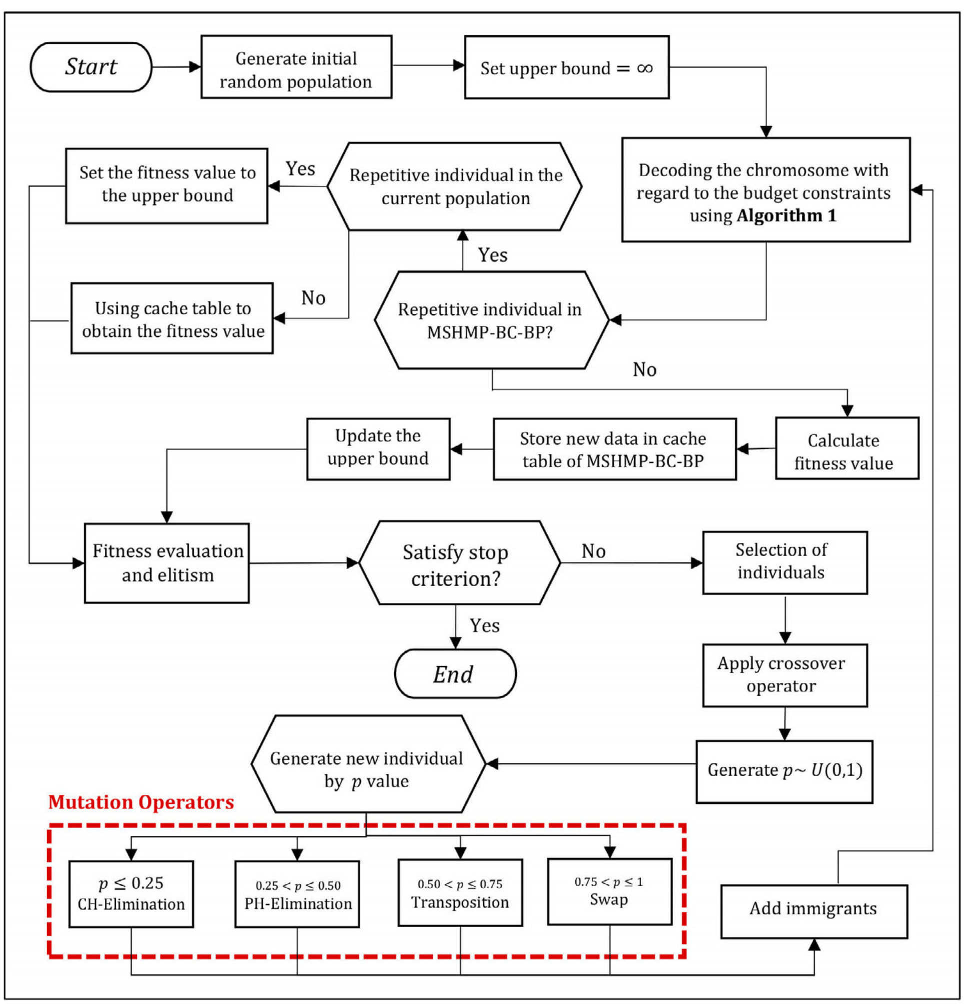


Fig. 5. Flowchart process of modified caching genetic algorithm.

4.2.6. Updating process

This process aims at updating the particle to achieve new solution x_i^{k+1} form x_i^k. In the proposed hybrid algorithm, we define an updating formula like Lüer-Villagra et al. (2019) by utilizing genetic operators. The formula is presented in the below:

$$x_i^{k+1} = \left(x_{Pbest,i}^k \boxtimes x_i^k\right) \odot \left(x_{Gbest,i}^k \boxtimes x_i^k\right) \odot \hat{x}_i^k \quad (30)$$

In Eq. (30), $x_{Pbest,i}^k$ denotes the best fitness value discovered by ith particle. $x_{Gbest,i}^k$ presents the best fitness value observed by the whole swarm and x_i^k expresses the position of the ith particle at the kth iteration. The mark $\boxtimes$ in Eq. (30) denotes the crossover operator of two particles. The convex crossover with one-cut point is utilized to create offsprings (see Section 4.1.4). Moreover $\hat{x}_i^k$ Specify the particle obtained by mutation operator on x_i^k (see Section 4.1.5). The mark $\odot$ also expresses the selection strategy among the archived individuals.

The following method is proposed to choose the new particle among the individuals created by $x_{Pbest,i}^k \boxtimes x_i^k$ and $x_{Gbest,i}^k \boxtimes x_i^k$ and $\hat{x}_i^k$. Fig. 6 illustrates the procedure of $\odot$ operator. In this process, first, we choose a use number E to specify that which operator should be applied. Let C be a constant number in the interval of [0,1] which should be initialized by the user. Moreover, consider a real random number R_i in the interval of [0,1] is produced for particle i. If $R_i \geq C$ then offspring 1 or offspring 3 should be chosen to replace with the particle, otherwise offspring 2 or offspring 4 is selected to update the particle. The offspring 5, which is created by mutation operator is sent, directly.

Premature convergence is the main deficiency of the PSO method. In detail, it is plausible for the algorithm to be stuck in a poor local optima when all of the members are unable to find a better feasible solution. To prevent this state we utilized immigration operator and employ some LS strategies to enhance the particles. The scheme of proposed hybrid PSO is depicted in Fig. 7.

4.2.7. LS improvements

Local search is a classical optimization method, which generates solutions from an initial solution by employing local changes (neighborhoods). Implementing appropriate movements to generate the neighborhood structures helps to achieve acceptable solutions. Furthermore, designing various neighborhood structures slower leads to a local optimum.

In the proposed method, we enhance a sub-swarm implementing LS heuristics. In each iteration, one-tenth of the swarm is randomly selected and improved by LS strategies. Each of the proposed methods is a simple rule that reassigns some of the nodes to hubs or changes the hubs. For this purpose, we adopt five various neighborhood structures for the hub network design. Fig. 8 illustrates the proposed neighborhood structures, which are defined as follows:

a. Shift node: it switches the assignment of a single non-hub node to a different hub (includes central hub).
b. Swap node: it exchanges allocations of two non-hub nodes with each other.
c. Exchange hub: it exchanges a hub node with a non-hub. Accordingly, some of the assignments will be changed.
d. Decrease number of hubs by one: delete one of the non-central hubs and update the assignments.
e. Increase the number of hubs by one: add additional non-central hubs and update the assignments. This operator is employed with regard to the budget constraint.

Base on above descriptions, the procedure of the proposed HPSO algorithm is summarized in the Algorithm 2.

4.3. MSHMP-BC-BP: Lagrangian lower bound

As mentioned earlier, the MSHMP-BC-BP is an NP-hard problem. Consequently, we may face excessively large resultant problem even for average sized instances. Indeed, the computational time increases non-polynomially with the size growth of the problem using commercial solvers (i.e. CPLEX). This motivates us to develop a Lagrangian relaxation-based algorithm to achieve proper lower bounds for the problem.

To this end, we relaxed the set of constraints (5) and (6) using Lagrange multipliers λ_{ijkn} and u_{tn} yields the following problem:
(MSHMP-BC-BP_LG):

$$\begin{aligned} L(\lambda,u) = Min \sum_i \sum_{j\neq i} \sum_k \sum_{l\neq k} \sum_n \sum_t [C_t(d_{ik}+d_{lj}) + \alpha C_t(d_{kn}+d_{nl})] x_{tij}^{knl} \\ \times D_{ij}^t(\mathrm{P/F},\%\mathrm{i},\mathrm{t}) \\ + \sum_i \sum_{j\neq i} \sum_k \sum_n \sum_t [C_t(d_{ik}+d_{kj})] x_{tij}^{knk} D_{ij}^t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ + \sum_t \sum_k F_{kt}^{\mathrm{rent}} p_{tk}\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ + \sum_t \sum_k F_{kt}^{\mathrm{excrete}} q_{tk}\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) - \sum_t S_t z_t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ - \sum_t e_t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) + \sum_n F_n^{\mathrm{Fix}} O_n \\ + \sum_i \sum_j \sum_k \sum_n \lambda_{ijkn}\left(2\times \sum_n x_{tij}^{knl} - (w_{jl}^t + w_{ik}^t)\right) \\ + \sum_t \sum_n u_{tn}(O_n - w_{nn}^t) \end{aligned} \quad (31)$$

Subject to constraints (2)–(4) and (7)–(15).

Note that the obtained $L(\lambda,u)$ can be decomposed into two sub-problems: the first problem (1) is in the solution space of two binary variables O_n, x_{tij}^{knl}, and the second problem (2) should be solved in the space of the binary variables p_{tk}, q_{tk}, w_{ik}^t and positive variable e_t.

The first sub-problem is:
Sub problem 1:

$$\begin{aligned} LR_{xo} = Min \sum_i \sum_{j\neq i} \sum_k \sum_{l\neq k} \sum_n \sum_t [C_t(d_{ik}+d_{lj}) + \alpha C_t(d_{kn}+d_{nl})] x_{tij}^{knl} \\ D_{ij}^t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ + \sum_i \sum_{j\neq i} \sum_k \sum_n \sum_t [C_t(d_{ik}+d_{kj})] x_{tij}^{knk} D_{ij}^t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ + \sum_n F_n^{\mathrm{Fix}} O_n + 2 \\ \times \sum_i \sum_j \sum_k \sum_n \lambda_{ijkn} \sum_n x_{tij}^{knl} + \sum_t \sum_n u_{tn} O_n \end{aligned} \quad (32)$$

Subject to constraints (2)–(4) and (13) where $O_n, x_{tij}^{knl} \in \{0,1\}$.

The solution to LR_{xo} is given by choosing one central hub (constraint 2) and presents one path between two various nodes (constraint 4) such that the path should pass through the central hub (constraint 3) and finally controls the maximum allowable distance between two nodes constraint (13). It is notable that the feasible solutions of LR_{xo} do not consider the non-central hubs. Moreover, in the achieved solutions, nodes are not necessarily connected to a single open hub on each period. Furthermore, the second sub-problem can be rewritten as follows:
Sub problem 2:

$$\begin{aligned} LR_{pqwe} = Min \sum_t \sum_k F_{kt}^{\mathrm{rent}} p_{tk}(\mathrm{P/F},\%\mathrm{i},\mathrm{t}) + \sum_t \sum_k F_{kt}^{\mathrm{excrete}} q_{tk}(\mathrm{P/F},\%\mathrm{i},\mathrm{t}) \\ - \sum_t S_t z_t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) \\ - \sum_t e_t\left(\frac{\mathrm{P}}{\mathrm{F}},\%\mathrm{i},\mathrm{t}\right) - \sum_i \sum_j \sum_k \sum_n \lambda_{ijkn}\left(w_{jl}^t + w_{ik}^t\right) \\ - \sum_t \sum_n u_{tn} w_{nn}^t \end{aligned} \quad (33)$$

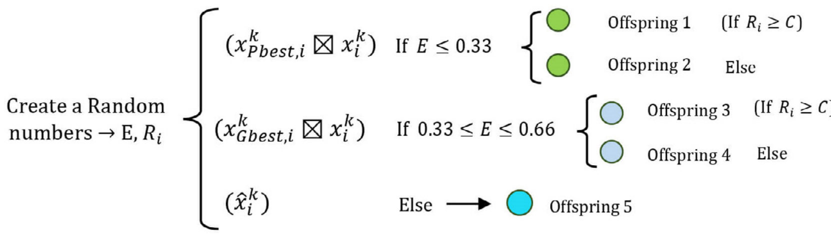


Fig. 6. The selection procedure in the proposed hybrid PSO.

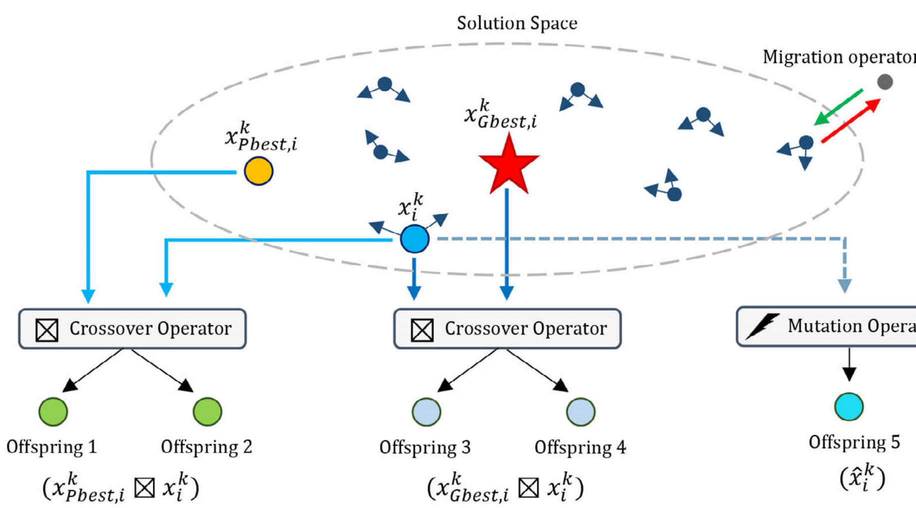


Fig. 7. The sketch of proposed hybrid PSO.

Algorithm 2. Algorithm of HPSO with local search method

Input: Parameters of MSHMP-BC-BP, Number of iterations, M (relates to immigration operator), C (relates to the selection process), $m \leftarrow 0$, UP (upper bound of fitness values)
While termination condition has not met ***do***
  Evaluate the fitness for each particle i specify the local best $x_{Pbest,i}$, and the global best $x_{Gbest,i}$
  Update the positions of particles in the swarm using formula (30) and selection process using C value
  Improve a sub-swarm of particles using local search strategies
  // fitness function evaluation for each particle
  If (the particles are new then store the related data in the cache table) **then**
    If (one particle is repetitive in the current population) **then**
      Set the fitness value of the particle to UP
    Else
    Use the cache table to achieve the fitness function
    End if
  Else
    Calculate the fitness function of the particle
    Update the UP and the cache table according to the evaluated particle
  End if
  If $(m \ge M)$ **then**
    <u>*Apply migration operator*</u> and relocate the poorest particle with a new immigrant
    Set $m \leftarrow 0$
  Else
    Set $m \leftarrow m + 1$;
  End if
End while
Return the best position found by all of particles x_{Gbest} and the corresponding objective value

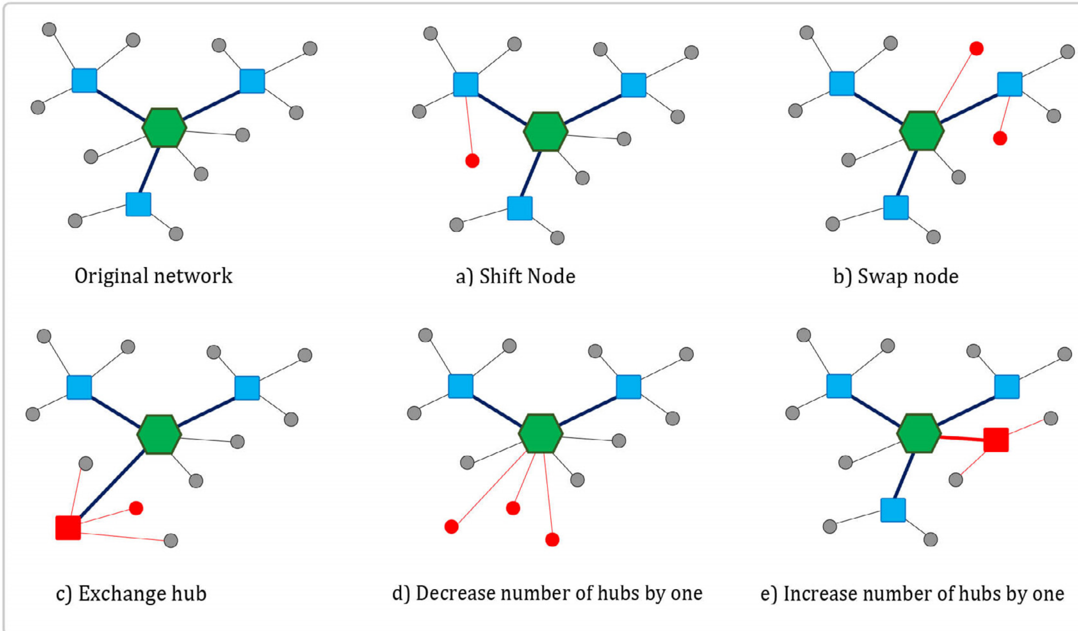


Fig. 8. Neighborhood structures.

Subject to constraints (5) and (7)–(12) where $p_{tk}, q_{tk}, w^t_{ik} \in \{0, 1\}$ and $e_t \geq 0$.

The solution to LR_{pqwe} is given by choosing non-central hubs according to the budget constraints (constraint 9) and determines the variables p_{tk} and q_{tk}, which relate to the opening or closing the hubs.

The process of the sub-gradient algorithm is provided in Algorithm 3. The algorithm gives a lower bound $\varkappa_D$ and $\mathcal{U}\mathcal{P}$ represents a known upper bound obtained from meta-heuristic methods. The parameter ρ_k is initially set to 2 and halved after 30 consecutive iterations without any improvement in the lower bound. The optimization algorithm will be terminated by three conditions include: (1) all the components of the subgradients (λ_{ijkn} and u_{tn}) are equal to zero. In this case, the algorithm reaches an optimal solution; (2) the maximum computation time t_{max} is reached; (3) the convergence criteria with optimality tolerance $\frac{\mathcal{U}\mathcal{P}-\varkappa_D}{\varkappa_D} \leq \mathcal{E}$ is met.

5. Experimental results

In this section, we validate the performance and report the results of several computational experiments. The instances are generated based on a real case of passenger transportation in Iran. The real transportation case is introduced by Karimi and Bashiri (2011) and includes 37 cities in Iran. This data set is created based on two criteria, containing tourism and industry and can be found in http://www.shahed.ac.ir/bashiri/Lists/List13/Attachments/1/IAD(dataset).rar. In this study, the studied instances with different sizes exploit some subsets of mentioned data set.

In order to find an optimum solution for the problem, the instances are coded in GAMS 24.1.3 with ILOG CPLEX 12.5 64-Bit optimization routines. Moreover, the meta-heuristic algorithms are coded in MATLAB 2014a software. All programs are executed on a computer with an Intel Core i5-3337U (1.8 GHz) with 6 GB of RAM. In all instances, we consider three courses of the planning horizon. In order to create different scales of problems; the proposed model was implemented on some subsets of the mentioned data set by taking the first n number of cities participating in the network with two discounting factors $\alpha = 0.2 \text{ and } 04$. Additional required parameters are generated according to Table 2.

In Table 2, $U[a, b]$ denotes a continuous uniform of distributional function on the interval of a and b, and DU(a, b) represents a discretional uniform distribution on the interval of a and b, where $a < b$. Parameter C is a transferring cost per unit distance between OD pairs. It is discounted by parameter α in inter-hub links. The potential cities for the establishment of central hub include Tehran, Esfahan, Shiraz, and Ahvaz.

Table 2
Generated input parameters for the experiments.

C_t	$t = 1 \to 5/t = 2 \to 6/t = 3 \to 7$	$\forall t$
α	$\alpha = 0.2, 0.4$	
D^t_{ij}	$t = 1 \to DU(100, 500)/t = 2 \to DU(200, 600)/t = 3 \to DU(300, 700)$	$\forall i, j, t$
F^{Fix}_n	$U(7500, 25000)$	$\forall n$
F^{rent}_{kt}	$U(500, 1000)$	$\forall k, t$
F^{excrete}_{kt}	U(300, 900)	
S_t	$U[250, 850]$	$\forall t$
U_t (in thousand kilometers)	$5 - t/3$	$\forall t$
ARC	1500	
EI_T	$t = 1 \to 0/t = 2 \to 600/t = 3 \to 1200$	$\forall t$
Interest rate	0.25	

5.1. Algorithms parameter tuning

In this section, we calibrate the parameters involved meta-heuristic algorithms by Taguchi design method. This method finds the optimal level of the signal factors by minimizing the variances of quality characteristics. In particular, it uses a measure of variation called signal-to-noise (S/N) ratio to ascertain the best level for each factor. In order to change objective values to non-scale data, the relative percentage deviation (RPD) is utilized. The RPD is computed according to the following equation:

$$RPD = \frac{|Alg_{sol} - Min_{sol}|}{|Min_{sol}|} \times 100 \tag{34}$$

In Eq. (34), Alg_{sol} represents the value of objective function that obtained by the meta-heuristic algorithms while Min_{sol} presents one optimal solution obtained by the GAMS software. The results of algorithms are comparable when their parameters are tuned with caution and uniformly. In this regard, the parameters of employed evolutionary algorithms are divided into three different levels and then adjusted

Algorithm 3. Subgradient Method

Iteration $k \leftarrow 0$
Initialize: $z_D \leftarrow -\infty$; $\lambda^0 \leftarrow 0$; $u^0 \leftarrow 0$; $\rho_k \leftarrow 2$; set the $t_{max} = 3600$s
Let $\mathcal{up}$ be a known upper bound achieved by meta-heuristics.
While *(Stopping criterion := false)* ***do***
  If (*One of the defined stopping criterion is satisfied*) **then**
    Stopping criterion :=true;
  End if
  Solve the Lagrangian function $L(\lambda^k, u^k)$
  If $L(\lambda^k, u^k) > z_D$ **then**
    $z_D \leftarrow L(\lambda^k, u^k)$
  End if
  Evaluate the subgradient $\gamma(\lambda^k, u^k)$
  Calculate the step length $T^k \leftarrow \rho_k \frac{(\mathcal{up} - L(\lambda^k, u^k))}{\|\gamma(\lambda^k, u^k)\|^2}$
  $(\lambda^{k+1}, u^{k+1}) \leftarrow \max\{0, (\lambda^k, u^k) + T^k \times \gamma(\lambda^k, u^k)\}$
  $k \leftarrow k + 1$
End while

for a medium-sized problem. Here we selected the instance 20 and determined the most appropriate level by analysis of experiments in Minitab 16.2 software. The levels of each parameter are presented in Fig. 9.

According to the Taguchi's plan, we employed the L9 design. The results of the S/N ratio for the meta-heuristic algorithms are shown in Fig. 9. The largest value of the S/N ratio on each graph and for each parameter shows a higher performance level.

5.2. Numerical instances

In this section, we solve various problems with different features such as numbers of nodes and discounting factors. The results of computational experiments are reported to assess the effectiveness of the proposed algorithms in comparison with optimal solutions obtained by CPLEX and LR. Table 3 contains the sample features and experimental outputs of GAMS using CPLEX solver. In the first and second columns, instance's numbers and the features are given, respectively. The instances are considered with two discounting factors $\alpha = 0.2, 0.4$. The optimum results of the mathematical model and associated computational times are given in the third and fourth column of Table 3. By paying attention to Table 3, it is obvious that by increasing the size of the instances the computation times increase exponentially. From sample 15 onwards, it is not possible anymore to obtain an optimal solution using GAMS/CPLEX in less than one hour. As a result, we use our algorithms to tackle the problem for larger instances. The total number of iterations is set to ($MAX\_IT = 150$) for the GAs and HPSO. From instance 13, we set population size to 90 individuals to achieve high quality solutions.

The results of solving the problems with GAs are tabulated in Tables 3 and 4. We run the GA algorithms 25 times for each of the instances. The best-achieved solution among all runs is reported in the CGA and GA columns. If the algorithm obtains an optimum solution (by comparison with mathematical model) it is marked with opt. The averaged required time that GA consumes to reach the best value during the execution is shown in the column $t(s)$ (in seconds), t_{tot} represents the average of total time that GA needed to complete the searching process. Moreover, the average number of iterations in which GA reaches the best answer is given in *Gen.* As described earlier, caching GA prevents computations for repetitive individuals. Thus, the algorithm does not calculate the fitness function for all the members of generations. In this regard, column $eval$ in Table 4 presents the average amount of evaluations. On each execution, the CGA uses the cache-table instead of calculation of the objective function, many times. We applied the criterion *cache* to indicate the average percentage of using internal storing data to achieve the fitness function. In practice, this criterion can be applied to denote the run-time savings. Note that the computational results are indeed a fair comparison since the number of calls to the evaluation function in all algorithms is similar. The existing difference in the values of $eval$ for CGA and HPSO is mainly due to the use of caching techniques and the performance of operators in diversifying the members and generating high-quality individuals during the searching process.

We calculate the relative gap to evaluate the quality of the solutions. For this purpose, we used the following equation for each run:

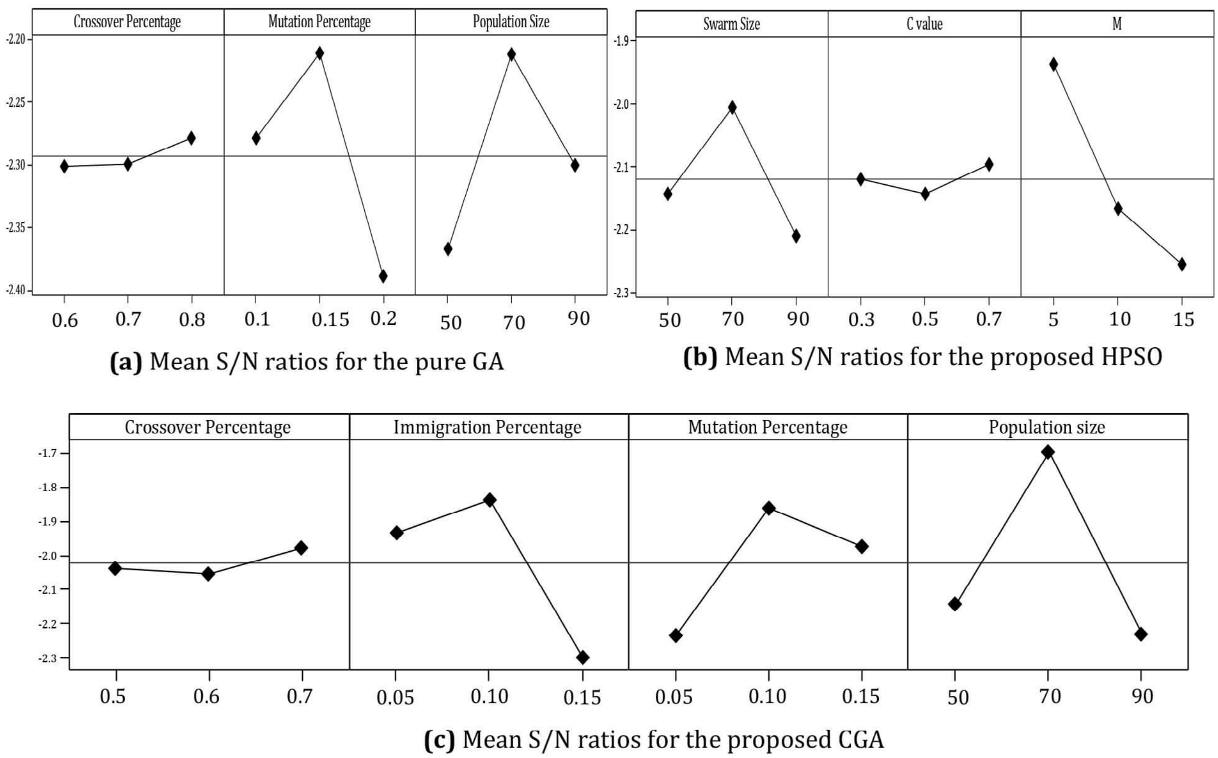
$$Gap = 100 \times \frac{Alg_{sol} - Best_{sol}}{Best_{sol}} \tag{35}$$

In Eq. (35) Alg_{sol} expresses the solution found by the algorithms (the best objective function among all the executions) and $Best_{sol}$ shows one optimal solution obtained by mathematic model and otherwise, it is the tight lower bound found by the Lagrangian relaxation method.

The results of solving the problems with HPSO are presented in Table 5. Again, we run HPSO algorithm 25 times for each of the instances. Most of the columns in Table 5 are same as Table 4. Column $iter$ represents the average number of iterations in which HPSO reaches the best answer. In addition, column LR shows the tight lower bounds obtained by the proposed Lagrangian relaxation method.

By referring to the Tables 3–5, variations of discounting factor α do not provide a meaningful effect on the total computational times for both exact and meta-heuristic algorithms. However, the objective function value increases by decreasing the discounts. As can be seen from Tables 3–5, in all experiments, except the problem with only five nodes, HPSO and CGA solve faster than CPLEX. The exponential growth in the computational time of the CPLEX solver in comparing with the required time of meta-heuristics is depicted in Fig. 10(a). In addition, Fig. 10(b) shows the benefit of using the caching technique in the CGA by comparing the computation times with the pure GA. The figure confirms that generally increasing the instance size yields a lower cache usage. All in all, this feature significantly improves the computation times in all instances.

The superiority of the CGA and HPSO to pure GA can be concluded from the Fig. 10(b) and Tables 3–5. The average of total times and Gens in Table 4 (or $Iters$ in Table 5) indicates that proposed meta-heuristics can reach the solutions relatively short computation times ($t_{tot} \leq 156$ s) with fewer iterations. From Tables 4 and 5, it is evident that the provided meta-heuristics are capable to obtain acceptable solutions for MSHMP-BC-BP in relatively short computation times ($t_{tot} \leq 156$ s).



(a) Mean S/N ratios for the pure GA

(b) Mean S/N ratios for the proposed HPSO

(c) Mean S/N ratios for the proposed CGA

Fig. 9. Mean S/N ratios for the meta-heuristics.

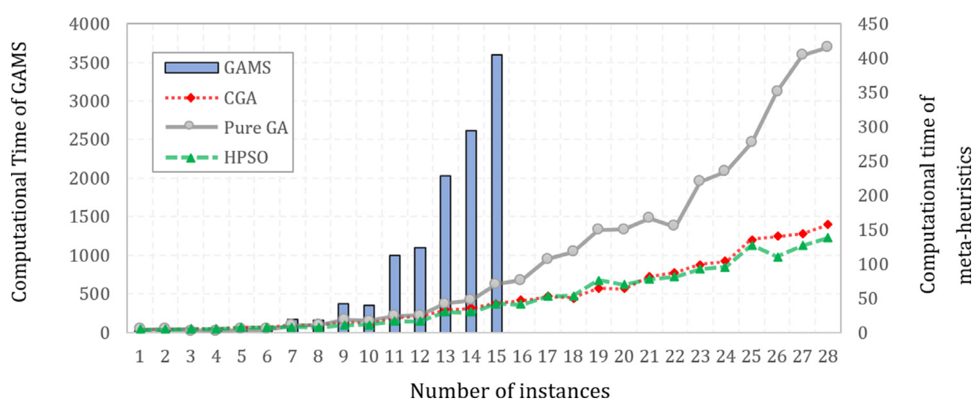


Fig. 10(a). Comparison the computational time of various method solutions.

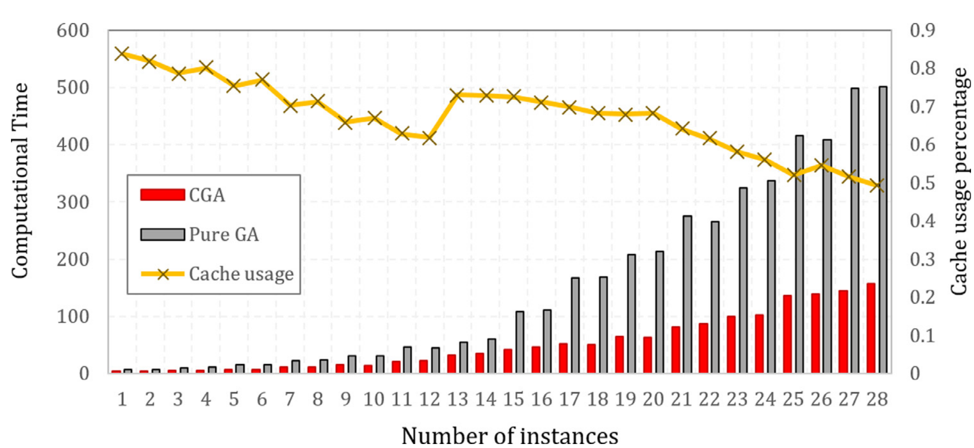


Fig. 10(b). Benefit of caching process in the CGA comparing to the pure GA.

Table 3
Features of the sample problems solved by GAMS and standard GA.

NO.	Sample feature	Number of variables	Number of constraints	GAMS	t (s)	GA	t (s)	t_{tot} (s)				Gen	$Gap\%$			
								Ave	Best	Worst	σ		Ave	Best	Worst	σ
1	5-0.4	9426	7718	101 935 241.87	2.273	Opt	4.111	7.491	6.876	8.149	0.214	27.1	0	0	0	0
2	5-0.2	9426	7718	89 328 316.85	2.205	Opt	4.042	7.392	6.323	8.397	0.353	26.3	0	0	0	0
3	6-0.4	23 394	15 847	193 168 663.85	11.055	Opt	3.653	11.19	10.24	12.12	0.351	29.5	0	0	0	0
4	6-0.2	23 394	15 847	174 466 358.34	11.019	Opt	3.484	11.63	10.54	12.84	0.408	27.6	0	0	0	0
5	7-0.4	50 504	29 196	315 373 505.34	52.033	Opt	8.190	17.41	16.68	18.17	0.265	37.6	0	0	0	0
6	7-0.2	50 504	29 196	286 143 475.92	58.351	Opt	9.332	18.01	17.25	18.75	0.233	38.4	0	0	0	0
7	8-0.4	98 406	49 637	331 132 832.28	165.19	Opt	10.44	25.15	23.70	26.42	0.465	48.4	0	0	0	0
8	8-0.2	98 406	49 637	268 473 263.93	158.33	Opt	12.07	24.78	24.27	25.34	0.181	50.0	0	0	0	0
9	9-0.4	177 270	79 330	462 618 576.08	365.92	Opt	20.11	35.87	34.94	36.87	0.346	60.4	0	0	0	0
10	9-0.2	177 270	79 330	394 089 945.76	354.18	Opt	19.40	33.46	31.64	35.24	0.630	59.1	0	0	0	0
11	10-0.4	300 146	120 723	568 803 119.88	992.67	Opt	26.44	48.11	44.41	51.15	1.142	64.1	0	0	0	0
12	10-0.2	300 146	120 723	497 643 399.19	1101.3	Opt	27.09	49.82	45.63	53.95	1.469	65.3	0	0	0	0
13	11-0.4	483 324	176 552	712 653 516.78	2029.18	714 230 785.19	44.06	58.61	53.73	63.48	1.728	70.1	0.2213	0	0.4610	0.0411
14	11-0.2	483 324	176 552	620 984 784.41	2618.29	Opt	47.50	67.18	59.84	75.79	2.695	77.3	0	0	0	0
15	12-0.4	746 694	249 841	838 453 018.15	>3600	848 563 032.45	78.91	115.2	108.6	120. 6	2.050	89.1	1.2058	0	1.3503	0.0513
16	12-0.2	746 694	249 841	716 576 090.62	>3600	726 216 108.31	82.42	117.1	106.2	127. 2	3.726	91.5	1.3453	0	1.5830	0.0830
17	13-0.4	1114 106	343 902	–	–	1 087 089 732.25	104.4	162.7	149.9	174. 4	3.943	97.8	1.2315	0.1601	1.5413	0.0701
18	13-0.2	1114 106	343 902	–	–	957 697 579.26	110.1	165.0	152.3	178.7	4.272	99.2	0	0	0	0
19	14-0.4	1613 730	462 335	–	–	1 359 280 369.17	148.9	206.6	193.1	220.6	4.804	105.0	2.0885	0.7501	2.7710	0.0774
20	14-0.2	1613 730	462 335	–	–	1 192 977 046.96	157.2	221.2	204.5	239.4	5.868	114.6	1.1015	0.8320	1.9510	0.8310
21	15-0.4	2278 416	609 028	–	–	1 596 211 292.10	163.5	283.1	265.2	300.6	6.143	122.0	2.9004	1.0161	3.1035	0.9016
22	15-0.2	2278 416	609 028	–	–	1 385 569 409.64	158.3	270.9	249.2	292.8	7.421	119.2	1.5018	1.0018	1.8430	0.0832
23	16-0.4	3146 054	788 157	–	–	1 808 418 821.09	223.9	329.2	306.9	352.3	8.350	123.8	0.4539	0	1.0694	0.0840
24	16-0.2	3146 054	788 157	–	–	1 604 837 705.27	229.7	334.0	306.1	365.7	10.02	125.1	1.4332	0.9713	1.7101	0.1012
25	17-0.4	4 259 934	1 004 186	–	–	2 060 608 548.72	313.4	421.8	391.5	449.5	9.863	127.4	2.3097	1.3521	2.7310	0.1004
26	17-0.2	4 259 934	1 004 186	–	–	1 884 144 900.35	329.1	430.4	398.2	466. 9	11.01	131.1	0.9798	0.5716	1.2910	0.0943
27	18-0.4	5 669 106	1 261 867	–	–	2 297 636 676.19	412.0	509.3	475.1	542. 5	10.69	141.3	1.4505	1.0813	2.8127	0.1051
28	18-0.2	5 669 106	1 261 867	–	–	2 089 676 851.45	418.3	514.2	485.7	548. 2	10.39	145.7	3.1514	1.8613	3.5615	0.1362
Total average :		–	–	–	–	–	113.2	160.60	149.23	172.35	3.893	82.64	0.7633	0.3427	0.9921	0.0985

Table 4
Features of the sample problems solved by modified CGA.

NO.	Sample feature	CGA	t (s)	t_{tot} (s)				Gen	$Gap\%$				$eval$	$cache$
				Ave	Best	Worse	σ		Ave	Best	Worse	$\sigma\%$		
1	5-0.4	Opt	2.112	4.421	3.963	4.879	0.164	24.4	0	0	0	0	1162.0	0.827
2	5-0.2	Opt	2.061	4.393	3.850	4.936	0.194	23.8	0	0	0	0	1298.3	0.814
3	6-0.4	Opt	2.823	5.860	4.974	6.745	0.269	27.0	0	0	0	0	1503.8	0.783
4	6-0.2	Opt	2.682	5.724	5.084	6.363	0.201	25.5	0	0	0	0	1476.1	0.770
5	7-0.4	Opt	4.081	7.847	7.173	8.520	0.212	32.8	0	0	0	0	1712.1	0.743
6	7-0.2	Opt	4.576	8.030	7.523	8.537	0.191	34.6	0	0	0	0	1771.4	0.739
7	8-0.4	Opt	8.441	12.14	11.09	13.18	0.341	44.1	0	0	0	0	2123.1	0.730
8	8-0.2	Opt	8.912	11.52	10.82	12.21	0.251	46.5	0	0	0	0	2101.0	0.744
9	9-0.4	Opt	12.02	16.40	15.51	17.28	0.289	57.6	0	0	0	0	2481.6	0.663
10	9-0.2	Opt	11.33	15.80	14.49	17.10	0.462	55.3	0	0	0	0	2404.2	0.652
11	10-0.4	Opt	15.07	23.05	20.53	25.56	0.914	60.7	0	0	0	0	2590.1	0.608
12	10-0.2	Opt	16.92	25.10	22.03	28.16	1.077	63.1	0	0	0	0	2641.6	0.620
13	11-0.4	Opt	25.07	36.18	30.16	42.19	1.825	54.4	0	0	0	0	3011.7	0.704
14	11-0.2	Opt	27.15	37.02	31.34	42.69	2.046	56.7	0	0	0	0	3185.5	0.691
15	12-0.4	Opt	32.92	44.18	37.69	50.66	2.440	63.4	0	0	0	0	2946.0	0.701
16	12-0.2	Opt	37.09	48.33	40.73	55.92	2.656	68.9	0	0	0	0	3192.7	0.693
17	13-0.4	1 073 864 858.01	46.19	59.08	50.40	67.75	3.154	76.0	0	0	0	0	3385.0	0.670
18	13-0.2	957 697 579.26	45.20	63.12	52.53	73.70	3.560	79.8	0	0	0	0	3506.4	0.680
19	14-0.4	1 331 472 836.43	56.44	75.11	63.86	86.35	3.843	83.4	0	0	0	0	3464.0	0.659
20	14-0.2	1 179 979 429.40	54.17	74.19	59.35	89.02	4.695	81.2	0	0	0	0	3427.9	0.662
21	15-0.4	1 551 246 188.61	59.45	89.57	74.93	104.2	4.914	82.5	0.0017	0.0011	0.0231	0.0087	3582.4	0.637
22	15-0.2	1 365 569 409.64	64.12	88.22	71.27	105.1	5.442	85.0	0.0167	0.0096	0.0342	0.0128	3654.8	0.619
23	16-0.4	1 800 336 608.80	84.57	101.0	80.36	121.6	6.923	92.1	0.0150	0.0115	0.0377	0.0108	3708.5	0.570
24	16-0.2	1 582 244 247.28	87.20	106.1	84.47	127.7	7.349	94.2	0.0202	0.0173	0.0398	0.0114	3749.0	0.553
25	17-0.4	2 014 810 426.10	109.1	133.0	110.1	155.8	7.891	105.8	0.0358	0.0231	0.0501	0.0149	3896.1	0.531
26	17-0.2	1 866 162 654.46	114.7	137.2	110.7	163.6	8.075	111.3	0.0260	0.0156	0.0492	0.0141	3915.7	0.547
27	18-0.4	2 266 162 654.46	133.2	149.0	125.9	172.0	8.199	119.5	0.0608	0.0425	0.1014	0.0224	4001.0	0.507
28	18-0.2	2 037 663 162.54	135.0	155.4	130.6	180.1	8.319	124.9	0.0333	0.0210	0.1120	0.0276	4022.2	0.491
Total average :		–	42.95	54.89	45.77	63.99	3.067	66.94	0. 0074	0.0050	0.01598	0.0043	2854.0	0.664

Moreover, the average of gaps shows that the HPSO can achieve higher quality solutions with better computational times.

The convergence curves related to the executions of meta-heuristic algorithms for the instance 11 are presented in Fig. 11. The initial population is generated randomly and used for all proposed algorithms. As seen from Fig. 11, the proposed HPSO and CGA are able to reach one optimal solution with less iterations, because these methods are able to diverge the population and explore the solution space more properly.

As described earlier, we implemented a LR method to obtain a tight lower bound for all instances. For better illustration, the proposed algorithm is conducted for instance 11, and changes in the objective function during various iterations are presented in Fig. 12. The figure

Table 5
Features of the sample problems solved HPSO and Lagrangian relaxation.

NO.	Sample feature	*HPSO*	*t* (s)	t_{tot} (s)				*eval*	*cache*	*Iter*	*Gap*%				*LR*
				Ave	Best	Worst	σ				Ave	Best	Worst	σ%	
1	5-0.4	Opt	1.220	3.716	3.286	4.180	0.143	989.20	0.814	21.7	0	0	0	0	Opt
2	5-0.2	Opt	1.416	3.810	3.177	4.551	0.236	1049.4	0.804	20.1	0	0	0	0	Opt
3	6-0.4	Opt	2.216	5.001	4.281	5.780	0.246	1374.6	0.776	24.1	0	0	0	0	Opt
4	6-0.2	Opt	2.340	5.044	4.274	5.841	0.286	1401.0	0.755	23.2	0	0	0	0	Opt
5	7-0.4	Opt	3.822	6.942	6.349	7.460	0.186	1612.1	0.713	31.2	0	0	0	0	Opt
6	7-0.2	Opt	4.019	7.045	6.521	7.543	0.164	1627.3	0.722	32.4	0	0	0	0	Opt
7	8-0.4	Opt	7.291	9.048	8.027	10.06	0.326	1933.0	0.680	42.1	0	0	0	0	Opt
8	8-0.2	Opt	7.582	9.175	8.777	9.531	0.127	1976.5	0.696	44.3	0	0	0	0	Opt
9	9-0.4	Opt	9.284	14.02	13.35	14.67	0.231	2209.2	0.602	54.1	0	0	0	0	Opt
10	9-0.2	Opt	8.723	13.80	12.49	15.20	0.462	2151.7	0.629	50.6	0	0	0	0	Opt
11	10-0.4	Opt	12.05	18.73	15.96	21.47	0.838	2380.1	0.524	53.6	0	0	0	0	Opt
12	10-0.2	Opt	14.72	19.45	16.63	22.51	0.980	2490.4	0.550	58.2	0	0	0	0	Opt
13	11-0.4	Opt	22.17	34.06	30.97	37.73	1.152	2909.5	0.652	52.4	0	0	0	0	Opt
14	11-0.2	Opt	24.73	34.65	29.12	40.30	1.887	2916.0	0.674	55.5	0	0	0	0	Opt
15	12-0.4	Opt	30.03	46.30	41.69	50.87	1.504	3071.9	0.640	59.7	0	0	0	0	Opt
16	12-0.2	Opt	27.23	42.07	35.01	52.04	2.484	2916.3	0.622	56.0	0	0	0	0	Opt
17	13-0.4	1 073 864 858.01	40.54	57.11	49.39	65.09	2.760	3163.5	0.631	68.3	0	0	0	0	1 073 864 858.01
18	13-0.2	957 697 579.26	43.12	60.28	52.04	69.27	2.991	3220.2	0.622	70.4	0	0	0	0	957 697 579.26
19	14-0.4	1 331 472 836.43	53.10	73.20	62.80	83.20	3.203	3316.7	0.601	72.3	0	0	0	0	1 331 472 836.43
20	14-0.2	1 179 979 429.40	52.78	78.14	66.14	90.69	3.913	3410.8	0.579	75.8	0	0	0	0	1 179 979 429.40
21	15-0.4	1 551 231 046.23	57.44	80.16	67.10	93.51	4.505	3590.1	0.540	77.1	0.0007	0.0005	0.0009	0.0064	1 551 220 286.13
22	15-0.2	1 365 274 302.01	60.83	83.20	66.67	99.75	5.442	3644.3	0.534	80.5	0.0151	0.0090	0.0158	0.0093	1 365 068 652.29
23	16-0.4	1 800 336 608.80	75.06	87.60	71.25	104.2	5.845	3720.0	0.539	88.2	0.0050	0.0038	0.0068	0.0071	1 800 247 328.91
24	16-0.2	1 582 231 369.56	79.33	90.88	68.10	112.0	7.349	3806.7	0.515	90.9	0.0043	0.0031	0.0049	0.0083	1 582 162 627.06
25	17-0.4	2 014 563 287.85	101.2	118.7	100.6	139.0	6.576	3797.1	0.529	97.3	0.0235	0.0130	0.0280	0.0114	2 014 089 443.75
26	17-0.2	1 866 162 654.46	110.6	126.6	101.5	152.7	8.075	3862.8	0.502	103.7	0.0160	0.0106	0.0183	0.0116	1 865 863 888.56
27	18-0.4	2 266 036 654.12	120.7	139.4	118.1	160.6	7.843	4014.2	0.471	100.3	0.0553	0.0349	0.0579	0.0173	2 264 784 961.49
28	18-0.2	2 037 403 621.96	126.2	141.2	127.6	162.9	7.626	3970.6	0.452	107.4	0.0206	0.0117	0.0223	0.0189	2 036 983 990.53
Total average :		–	39.27	50.33	42.54	58.66	2.763	2733.0	0.620	61.12	0.0050	0.0031	0.0055	0.0032	–

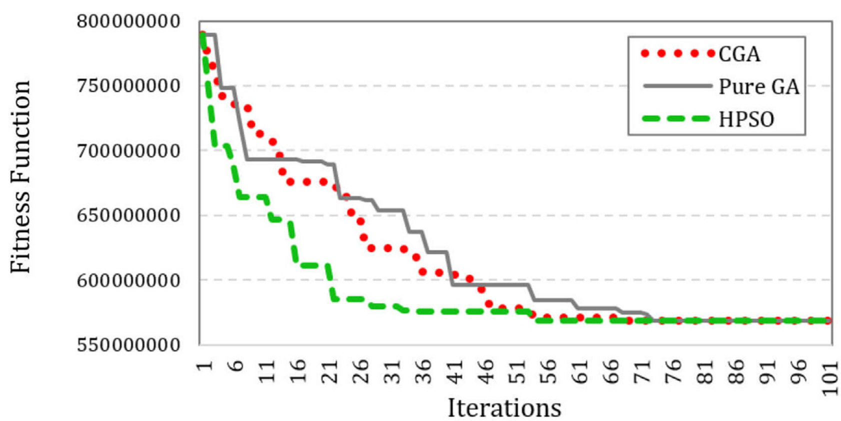


Fig. 11. Comparison of convergence curves of meta-heuristics.

indicates that the difference between the lower bound of the problem and the cost of one optimal solution decreases for a higher number of iterations. In particular, this approach succeeded to provide a good lower bound for medium and large instance sizes.

5.3. Pairwise statistical analysis of the solution approaches

In order to compare the solution methods, statistical tests are conducted. The aim is to justify whether there are significant differences between the solutions of the proposed meta-heuristics. Applying the Kolmogorov–Smirnov test with a significance level of 0.05 shows that the obtained results are not normally distributed. Therefore, the non-parametric Wilcoxon signed rank test is employed with a significance level of 0.05. The values of asymptotic significance (A.S.) levels and the Z statistic are presented in Table 6. Based on the outcomes of the statistical tests, the differences between the solutions of the algorithms are statistically significant. In detail, both HPSO and Modified CGA outperform the pure GA. Moreover, the proposed HPSO outperforms the modified CGA in a statistical sense.

Table 6
Wilcoxon signed rank test for the solution approaches.

Solution methods	CGA-Pure GA	HPSO-CGA	HPSO-Pure GA
Z	−3.296	−2.201	−3.296
A.S.	0.001	0.028	0.001

5.4. Evolution of diversity in the genetic algorithms

The diversity of population greatly impacts the efficiency of genetic algorithms. A too rapid decrease in the diversity of generations leads to a premature convergence in the search process. In this study, we employed a measure proposed by Topcuoglu et al. (2005) to evaluate the diversity. They proposed the hamming distance between Assign arrays as a measure of diversity. The hamming distance between two assigned matrices A_1 and A_2 equates to the numbers of rearrangements in the assignments for converting A_1 into A_2. The diversity of each population is computed by average hamming distance among each member of the population with the best individual and all other members of that

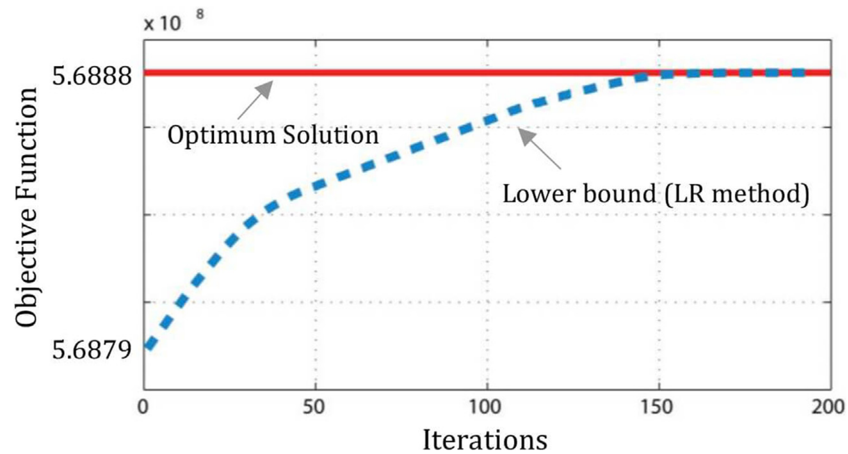


Fig. 12. Convergence of Lagrangian relaxation method to one optimal solution.

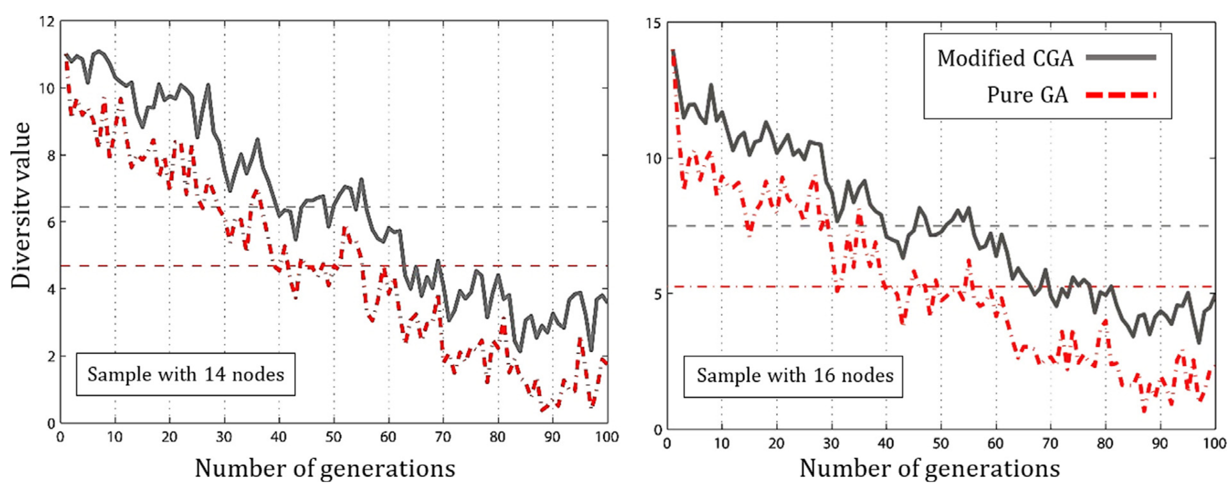


Fig. 13. Diversity values for the algorithms with two different problem sizes.

population. Fig. 13 shows the diversity of population for two sample problems with 16 and 14 nodes. For a more accurate comparison, we applied an identical initial population (which is randomly generated) with 100 numbers of iterations for both algorithms.

As seen from Fig. 13, instances with 14 and 16 nodes have an initial population with diversity values of 11 and 14, respectively. These values imply that the initialization phase of the algorithm is completely able to create the distinct initial individuals to avoid premature convergence. It is clear that diversity values decrease by increasing the numbers of generations, gradually. As can be seen from Fig. 13, in both instances, modified GA can provide a better diversification in the searching procedure rather than pure GA.

6. Sensitivity analysis and managerial insights

In what follows, a sensitivity analysis is provided for some specific parameters of MSHMP-BC-BP to identify their influences on the system's performance. Then, some managerial insights have been extracted for the decision makers.

6.1. Sensitivity analysis on discount factor and interest rate

In the majority of hub-and-spoke transportation systems, a discount factor is incorporated to reflect the savings due to economies of scale compared to hub-to-hub arcs. Fig. 14 presents the effects of the discount factor variation on the total cost for the instance with 10 number of nodes. It is clear that decreasing the discount rate increases the total costs.

To evaluate the effect of interest rate on the objective function, we consider an instance with 10 nodes and a discount factor of 0.2 for different interest rates. The results are depicted in Fig. 15. Increasing the interest rate leads to a lower objective function value. In fact, when the periodic costs are converted by the higher interest rate the present expense becomes lower. We also prepared a detailed breakdown on the components of objective function for the aforementioned instance in Fig. 16. The first point that can be drawn from the figure is that in MSHMP-BC-BP the transportation cost has a significant share of the total cost. The present value of renting/excreting costs for non-central hubs is the second major role. The other determinant cost among the defined components is the establishment of CH. In overall, we conclude that transportation cost and the related costs for opening or closing hubs have a large share in the overall costs and the unused budget or benefits from movable facilities only helps to partially decrease the operational cost in each period.

6.2. Evaluating significance of a multi-period network with leased hubs

To demonstrate the necessity of multi-period hub location problem, we ignore the possibility of opening or closing during the planning horizon. In fact, we consider a hub location problem with the star/star design in which some numbers of hubs should be established in the beginning of the horizon and it is not acceptable to change the location of hubs during the planning horizon. In this case, the installation

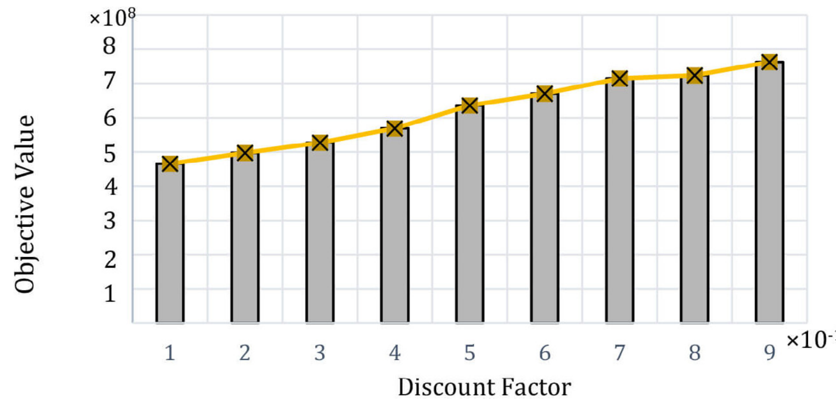


Fig. 14. Total costs for various discount factors.

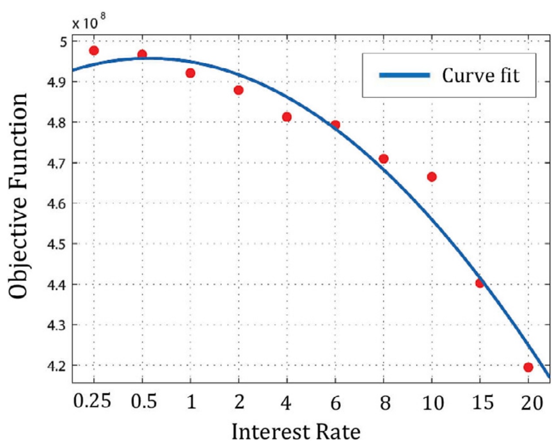


Fig. 15. Total costs for various interest rates.

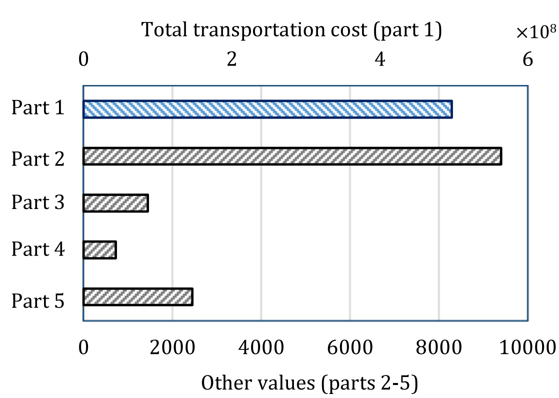


Fig. 16. The results of objective function and its components.

costs for non-central hub are decuple of opening (renting) a hub in MSHMP-BC-BP. We also consider an initial budget of 40 000, which is available for establishing the network. This traditional HLP and proposed MSHMP-BC-BP are solved for an instance with 37 numbers of nodes and the results are shown in Fig. 17. As expected the obtained objective function, in the case of the multi period HLP, is much better than the case of the traditional hub location model (see Table 7). In both problems, the model chooses Isfahan (one of the central provinces of Iran) as a central hub. Other employed hubs are listed in Table 7.

Table 7
Comparing MSHMP-BC-BP with traditional HLP.

Model	Central hub	Employed Non-central hubs	Total objective function
MSHMP-BC-BP	Isfahan	Tehran, Kerman, Ahvaz, Shiraz, Hamedan	3 658 675 440.74
Traditional HLP	Isfahan	Sanandaj, Shahrud, Kerman	5 426 430 837.08

6.3. Managerial insights

This paper provides some important suggestions and effective principles for scientific decision making especially in newly emerged companies.

1. Establishing a hub-and-spoke network is very costly as the initial investment is large. While by using MSHMP-BC-BP, a company can start its own business by less initial investment and gradually expands the network according to the various conditions using leased hubs.
2. The analysis shows that, according to the budget constraints, MSHMP-BC-BP chooses better hubs with strategic locations. Conversely, traditional HLP should operate with a non-flexible initial set of facilities and cannot employ appropriate hubs due to the limited budget and service quality considerations. Moreover, the proposed model makes it possible to cover a part of costs relating to expand the network with the incomes earned by the company during the periods.
3. In MSHMP-BC-BP the service quality can be improved over the time by adjusting the threshold parameter U_l. In this regard, MSHMP-BC-BP considers the defined threshold and might relocated some of the leased hubs due to fluctuations in demand if it is cost effective.
4. We add this point that the proposed MSHMP-BC-BP model can be extended to provide the initial funds and capital using loans. In this situation, some further analysis is required to determine the business' capability to repay loans and incorporate the financial process in the objective function.

7. Conclusions and future remarks

In this study, we presented a multi-period hub location problem with the possibility of opening (leasing) new facilities or closing current facilities proportionate to variation in demands, costs, sales strategies, and other parameters that involved in the decisions making processes. The proposed hub-and-spoke structure arises in the form of a star/star network. It exploits a strategic central hub that should be established by a company and several periodic hubs that can be opened (leased) for a

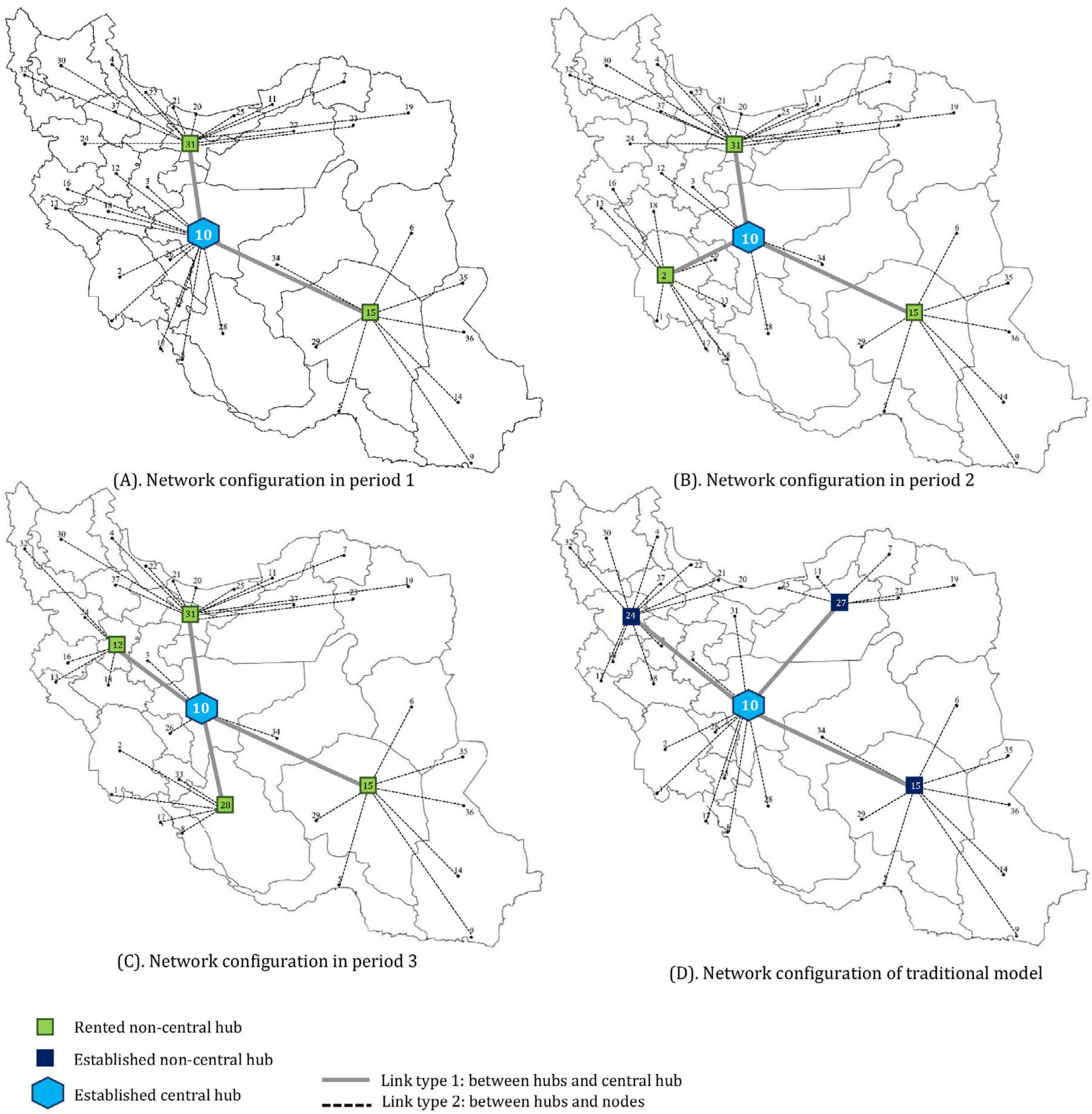


Fig. 17. Comparison between network structures for MSHMP-BC-BP and traditional HLP.

specific time horizon. Central hub applied as a centralized point in the backbone network that connects to the periodic hubs with a star shape. A pre-specified threshold value is defined to control the path lengths among each pair as a measure of service quality constraint. The aim of the proposed model is to minimize the total financial costs, which is converted to present value by knowing the interest rate and using the formula in the factor of the conversion table. The amount of the budget that is available to invest in opening new hubs is limited and an interest rate is accounted for the unused budget during the periods. To this end, a mixed integer mathematical model is presented to model the problem.

A computational study with various numerical instances is carried out. According to the studied instances, the proposed model substantially helps to decrease the total network costs in comparing with the traditional hub location models. Due to the computational complexity of proposed model, the CPLEX solver is only able to solve small instances. Therefore, we developed and tested efficient algorithms including a modified caching genetic algorithm and a hybrid particle swarm optimization to solve the large-scale problems in a reasonable time. The proposed evolutionary algorithms include an immigration operator in order to keep the diversity and increase the explorations. A caching technique is also embedded in the algorithms to improve the computational time of the searching process. The performance of the algorithms is compared with a pure GA and the mathematical model. Moreover, the Lagrangian relaxation method is employed to achieve tight lower bounds for the samples. The results demonstrate the value and quality of proposed algorithms. The methods are able to reach high-quality solutions with less computational time.

Further research directions include incorporating other actual factors such as maintenance costs or government regulations, applying robust optimization, finding some sets of valid inequalities to enhance the model and solving the problem with other exact method solutions. Moreover, the uncertainty of the parameters can be taken into consideration by formulating the problem using a two-stage stochastic programming approach.

CRediT authorship contribution statement

Hamid Tikani: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Writing - original draft. **Reza Ramezanian:** Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Supervision, Validation, Visualization, Writing - review & editing. **Mostafa Setak:** Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Supervision, Validation, Visualization, Writing - review & editing. **Tom Van Woensel:** Conceptualization, Formal analysis, Methodology, Validation, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Aboytes-Ojeda, Mario, Castillo-Villar, Krystel K., Roni, Mohammad S., 2020. A decomposition approach based on meta-heuristics and exact methods for solving a two-stage stochastic biofuel hub-and-spoke network problem. J. Cleaner Prod. 247, 119176.

Alumur, Sibel A., Kara, Bahar Y., Karasan, Oya E., 2009. The design of single allocation incomplete hub networks. Transp. Res. B 43 (10), 936–951.

Alumur, Sibel A., et al., 2016. Multi-period hub network design problems with modular capacities. Ann. Oper. Res. 246 (1–2), 289–312.

Amin-Naseri, Mohammad Reza, Yazdekhasti, Amin, Salmasnia, Ali, 2018. Robust bi-objective optimization of uncapacitated single allocation p-hub median problem using a hybrid heuristic algorithm. Neural Comput. Appl. 29 (9), 511–532.

Bashiri, Mahdi, Mirzaei, Masoud, Randall, Marcus, 2013. Modeling fuzzy capacitated p-hub center problem and a genetic algorithm solution. Appl. Math. Model. 37 (5), 3513–3525.

Bashiri, Mahdi, et al., 2017. Mathematical modeling for a p-mobile hub location problem in a dynamic environment with a genetic algorithm solution approach. Appl. Math. Model..

Calık, Hatice, et al., 2009. A tabu-search based heuristic for the hub covering problem over incomplete hub networks. Comput. Oper. Res. 36 (12), 3088–3096.

Campbell, James F., 1990. Locating transportation terminals to serve an expanding demand. Transp. Res. B 24 (3), 173–192.

Contreras, Ivan, Cordeau, Jean-François, Laporte, Gilbert, 2011. The dynamic uncapacitated hub location problem. Transp. Sci. 45 (1), 18–32.

Contreras, Ivan, Fernández, Elena, Marín, Alfredo, 2009. Tight bounds from a path based formulation for the tree of hub location problem. Comput. Oper. Res. 36 (12), 3117–3127.

Contreras, Ivan, Fernández, Elena, Marín, Alfredo, 2010. The tree of hubs location problem. European J. Oper. Res. 202 (2), 390–400.

Contreras, Ivan, O'Kelly, Morton, 2019. Hub location problems. In: Location Science. Springer, Cham, pp. 327–363.

Correia, Isabel, Nickel, Stefan, Saldanha-da-Gama, Francisco, 2017. A stochastic multi-period capacitated multiple allocation hub location problem: Formulation and inequalities. Omega.

Damgacioglu, Haluk, et al., 2015. A genetic algorithm for the uncapacitated single allocation planar hub location problem. Comput. Oper. Res. 62, 224–236.

De, A., Kumar, S.K., Gunasekaran, A., Tiwari, M.K., 2017. Sustainable maritime inventory routing problem with time window constraints. Eng. Appl. Artif. Intell. 61, 77–95.

De, Arijit, et al., 2016. Composite particle algorithm for sustainable integrated dynamic ship routing and scheduling optimization. Comput. Ind. Eng. 96, 201–215.

de Sá, Elisangela Martins, Morabito, Reinaldo, de Camargo, Ricardo Saraiva, 2018. Benders decomposition applied to a robust multiple allocation incomplete hub location problem. Comput. Oper. Res. 89, 31–50.

Ebrahimi-Zade, Amir, Hosseini-Nasab, Hasan, Zahmatkesh, Alireza, 2016. Multi-period hub set covering problems with flexible radius: A modified genetic solution. Appl. Math. Model. 40 (4), 2968–2982.

Elmastaş, S., 2006. Hub Location Problem for Air-Ground Transportation Sistems with Time Restrictions (Doctoral dissertation). Bilkent University.

Farahani, Reza Zanjirani, et al., 2013. Hub location problems: A review of models, classification, solution techniques, and applications. Comput. Ind. Eng. 64 (4), 1096–1109.

Gao, Yuan, Qin, Zhongfeng, 2016. A chance constrained programming approach for uncertain p-hub center location problem. Comput. Ind. Eng. 102, 10–20.

Gavish, Bezalel, 1982. Topological design of centralized computer networks—formulations and algorithms. Networks 12 (4), 355–377.

Gelareh, S., 2008. Hub Location Models in Public Transport Planning (Ph.D. thesis). Tecnical University of Kaiserslautern, Germany.

Gelareh, Shahin, Monemi, Rahimeh Neamatian, Nickel, Stefan, 2015. Multi-period hub location problems in transportation. Transp. Res. E 75, 67–94.

Ghaffarinasab, Nader, 2020. A tabu search heuristic for the bi-objective star hub location problem. Int. J. Manage. Sci. Eng. Manage. 1–13.

Ghaffarinasab, Nader, Kara, Bahar Y., 2019. Benders decomposition algorithms for two variants of the single allocation hub location problem. Netw. Spat. Econ. 19 (1), 83–108.

Ghodratnama, A., Arbabi, H.R., Azaron, Amir, 2018. A bi-objective hub location-allocation model considering congestion. Oper. Res. 1–40.

Ghodratnama, Ali, Tavakkoli-Moghaddam, Reza, Azaron, Amir, 2015. A stochastic approach for a novel p-hub location-allocation problem with opening and reopening modes. Int. J. Bus. Perform. Supply Chain Model. 7 (4), 305–337.

Goldman, A.J., 1971. Optimal center location in simple networks. Transp. Sci. 5 (2), 212–221.

Graziani, R., Vachon, B., 2014. Cisco Networking Academy: Connecting Networks Companion Guide. Cisco Press.

Hasanzadeh, Hamid, Bashiri, Mahdi, Amiri, Amirhossein, 2016. A new approach to optimize a hub covering location problem with a queue estimation component using genetic programming. Soft Comput. 1–13.

Holland, John H., 1975. Adaptation in Natural and Artificial Systems. An Introductory Analysis with Application to Biology, Control, and Artificial Intelligence. University of Michigan Press, Ann Arbor, MI.

Karimi, H., Bashiri, M., 2011. Hub covering location problems with different coverage types. Sci. Iran. 18 (6), 1571–1578.

Karimi, Hossein, Setak, Mostafa, 2014. Proprietor and customer costs in the incomplete hub location-routing network topology. Appl. Math. Model. 38 (3), 1011–1023.

Karimi, Hossein, Setak, Mostafa, 2016. Flow shipment scheduling in an incomplete hub location-routing network design problem. Comput. Appl. Math. 1–33.

Kennedy, J., Eberhart, R., 1995. Particle swarm optimization. In: IEEE International Conference on Neural Networks, Vol. 4. pp. 1942–1948.

Khodemani-Yazdi, Melahat, et al., 2019. Solving a new bi-objective hierarchical hub location problem with an M/ M/ c queuing framework. Eng. Appl. Artif. Intell. 78, 53–70.

Kratica, Jozef, et al., 2007. Two genetic algorithms for solving the uncapacitated single allocation p-hub median problem. European J. Oper. Res. 182 (1), 15–28.

Kratica, Jozef, et al., 2011. An evolutionary-based approach for solving a capacitated hub location problem. Appl. Soft Comput. 11 (2), 1858–1866.

Labbé, Martine, Yaman, Hande, 2008. Solving the hub location problem in a star–star network. Networks 51 (1), 19–33.

Lüer-Villagra, Armin, Eiselt, Horst A., Marianov, Vladimir, 2019. A single allocation p-hub median problem with general piecewise-linear costs in arcs. Comput. Ind. Eng. 128, 477–491.

Maiyar, L.M., Thakkar, J.J., 2019. Environmentally conscious logistics planning for food grain industry considering wastages employing multi objective hybrid particle swarm optimization. Transp. Res. E 127, 220–248.

Mohammadi, Mehrdad, Jolai, Fariborz, Tavakkoli-Moghaddam, Reza, 2013. Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm. Appl. Math. Model. 37 (24), 10053–10073.

Najy, Waleed, Diabat, Ali, 2020. Benders decomposition for multiple-allocation hub-and-spoke network design with economies of scale and node congestion. Transp. Res. B 133, 62–84.

Nickel, Stefan, Schöbel, Anita, Sonneborn, Tim, 2001. Hub location problems in urban traffic networks. Math. Methods Optim. Transp. Syst. 48, 95.

O'kelly, Morton E., 1987. A quadratic integer program for the location of interacting hub facilities. European J. Oper. Res. 32 (3), 393–404.

Özgün-Kibiroğlu, Ç., Serarslan, M.N., Topcu, Y.İ., 2019. Particle swarm optimization for uncapacitated multiple allocation hub location problem under congestion. Expert Syst. Appl. 119, 1–19.

Pearce, Robin H., Forbes, Michael, 2018. Disaggregated Benders decomposition and branch-and-cut for solving the budget-constrained dynamic uncapacitated facility location and network design problem. European J. Oper. Res. 270 (1), 78–88.

Qin, Zhongfeng, Gao, Yuan, 2017. Uncapacitated p-hub location problem with fixed costs and uncertain flows. J. Intell. Manuf. 28 (3), 705–716.

Randall, Marcus, 2008. Solution approaches for the capacitated single allocation hub location problem using ant colony optimisation. Comput. Optim. Appl. 39 (2), 239–261.

Saadati, Mahdi, Hosseininezhad, Seyed Javad, 2019. Designing a hub location model in a bagasse-based bioethanol supply chain network in Iran (case study: Iran sugar industry). Biomass Bioenergy 122, 238–256.

Taghipourian, Farzin, et al., 2012. A fuzzy programming approach for dynamic virtual hub location problem. Appl. Math. Model. 36 (7), 3257–3270.

Tikani, Hamid, Honarvar, Mahboobeh, Mehrjerdi, Yahia Zare, 2016. Joint optimization of star p-hub median problem and seat inventory control decisions considering a hybrid routing transportation system. Int. J. Supply Oper. Manage. 3 (3), 1.

Tikani, H., Honarvar, M., Zare Mehrjerdi, Y., 2018. Developing an integrated hub location and revenue management model considering multi-classes of customers in the airline industry. Comput. Appl. Math. 37 (3), 3334–3364.

Topcuoglu, Haluk, et al., 2005. Solving the uncapacitated hub location problem using genetic algorithms. Comput. Oper. Res. 32 (4), 967–984.

Wang, Zhaoxin, Lin, Chinlon, Chan, Chun-Kit, 2006. Demonstration of a single-fiber self-healing CWDM metro access ring network with unidirectional OADM. IEEE Photonics Technol. Lett. 18 (1), 163–165.
Wasner, Michael, Zäpfel, Günther, 2004. An integrated multi-depot hub-location vehicle routing model for network planning of parcel service. Int. J. Prod. Econ. 90 (3), 403–419.
Yaman, Hande, 2008. Star p-hub median problem with modular arc capacities. Comput. Oper. Res. 35 (9), 3009–3019.
Yaman, Hande, 2009. The hierarchical hub median problem with single assignment. Transp. Res. B 43 (6), 643–658.
Yaman, Hande, Elloumi, Sourour, 2012. Star p-hub center problem and star p-hub median problem with bounded path lengths. Comput. Oper. Res. 39 (11), 2725–2732.
Yaman, Hande, Kara, Bahar Y., Tansel, Barbaros Ç., 2007. The latest arrival hub location problem for cargo delivery systems with stopovers. Transp. Res. B 41 (8), 906–919.
Yang, Xiao, Bostel, Nathalie, Dejax, Pierre, 2019. A MILP model and memetic algorithm for the hub location and routing problem with distinct collection and delivery tours. Comput. Ind. Eng. 135, 105–119.
Yang, Kai, Liu, Yan-Kui, Yang, Guo-Qing, 2013. Solving fuzzy p-hub center problem by genetic algorithm incorporating local search. Appl. Soft Comput. 13 (5), 2624–2632.
Yoon, Moon-Gil, Current, John, 2008. The hub location and network design problem with fixed and variable arc costs: formulation and dual-based solution heuristic. J. Oper. Res. Soc. 59 (1), 80–89.