Language in the Mathematics Classroom: An introduction to the papers and presentations within ETC 7

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Research considering the role of language in the teaching and learning of mathematics continues to grow and develop, drawing on a range of theoretical, methodological and pedagogical approaches. In this introduction, we detail the discussions had and issues raised at the 7th ERME Topic Conference as a result of the bringing together of the theoretical perspectives, foci and findings of the papers presented that are included in these proceedings. These rich discussions also raised new challenges for those researching language and mathematics and identified new possibilities for the future work of the Mathematics and Language thematic working group.

Keywords: Language and mathematics, multilingual contexts, meaning-making, interaction.

Introduction

Research into language and mathematics has seen a shift away from distinctions between the language of the teacher, the language of the students, and the language of mathematics, towards a more integrated understanding focusing on language in interaction and the role of language in meaning-making. Much of this research is now situated in classrooms, and the focus is more on the interactions between teachers, students and mathematics than the language of each as exemplified by the focus of ETC 4 (Planas & Schütte, 2018). This attention to interactions also blurs the boundaries and distinctions between the foci of different researchers, theories and the discussions between researchers. Whilst the papers in this conference have been grouped into three themes, there is considerable overlap between the themes, and we learn a great deal from discussions both within and between the themes.

The first theme is *language in multilingual contexts* where the multilingual nature of mathematics learning and teaching is the focus of the discussions. The second theme of *language for meaning-making* focuses more on the language aspects of the conceptual understanding of particular mathematical concepts such as multiplication, angle or proof. The third theme concentrates on *classroom interactions and discursive practices* such as argumentation or explaining. The research presented within these themes often intersects with each other. For example, the issue of learning the discursive practices of proving could equally well have been investigated within the third theme as in the second theme. However, as universities increasingly become multilingual, this issue could also

have been investigated in the first theme. And indeed, a current question is how to make intricate logical relations accessible to students who learn mathematics in a second or third language.

The interaction between themes highlights why continuing work on language in mathematics is necessary. The papers in this conference draw from an increasing diversity of theoretical approaches, many with their origins in other fields such as sociology, linguistics and psychology. This diversity enables researchers to consider a range of contexts, issues and foci but also raises the question of how coherence can be sustained and developed as we learn more about the role of language in the learning and teaching of mathematics. Furthermore, this diversity also emphasises the challenge of communicating the research included in this conference, alongside and building upon other language-sensitive research (Planas, Farrugia, Ingram, & Schütte, 2019). However, as past research has shown, such diversity can often be the starting point for identifying further contexts, issues and foci worth investigating.

Language in multilingual contexts

A total of 6 papers and 1 poster were presented in the theme of *language in multilingual contexts*. The theme *language in multilingual contexts* has a long tradition in research into language and mathematics. The focus of the theme within ERME conferences has been on multilingual students' resources for learning mathematics (e.g. Planas, 2018) since the beginning, and on how learning and teaching practices in mathematics could allow multilingual students to better utilise their resources (e.g. Barwell, 2020; Norén, 2015).

At this second topic study conference on language and mathematics, a new issue emerged in the theme of language in multilingual contexts. This new emerging issue concerns the learning and teaching of mathematics in a university context. In the past, research on language in multilingual contexts has often focused on primary and secondary education. This new emerging issue has become relevant in the recent years because of new emerging contexts for learning and teaching at university: 1. African countries are strengthening their own cultural identities, which results in moves towards utilizing students' first language for learning and teaching of mathematics in schools. For example, Arabic and Berber languages replace French in Algeria, which was the regular academic language since colonial times. However, French continues to be the lingua franca at the university level, so that many students learn mathematics in their second (or third) language (Azrou). Similarly, to strengthen the Irish cultural identity, one Irish University utilises Irish as a language of instruction in the first year (Ní Ríordáin). 2. Many universities in Europe start to switch their education towards English as Medium of Instruction (EMI). This means that all students, international or national students alike, learn mathematics in their second (or third) language, English. Traditionally, a similar language context could be found in the US and Australia / New Zealand, where international students learn mathematics in their second (or third) language English, which was found to be challenging for students (Barton, Chan, King, Neville-Barton, & Sneddon, 2005). This situation is now becoming more common in Europe as well, for a larger group of students. 3. Universities in regions with a multilingual population teach in their respective languages of instruction (e.g. South Africa, Russia), so that many students learn mathematics in a second or third language. Related to this is the need for student teachers to learn to communicate multilingually in their future school teaching.

At the conference, the issue of multilingual mathematics learning and teaching has been investigated from various perspectives, but mainly with the perspective on cultural backgrounds and language resources to *describe* new phenomena of bilingual mathematics learning at university. With respect to the first context outlined above, Ní Ríordáin studies the teachers' use of Irish language and how they connect to representations and finds that English is used as a language to clarify meanings. Framed within the traditional perspective of language differences, Azrou investigates the different ways in which different languages allow to express conditional statements, which can impact logical reasoning. While this analysis focuses on structural aspects of different languages, it highlights potential difficulties that need to be investigated further in students' actual language use in the future. It could be hypothesized that such structural deficiencies of certain languages can be compensated by translingual practices, where multiple languages can be used as a combined resource. In a similar perspective on structural features of languages, Durand-Guerrier argues that the translation of logical statements into formal statements could build bridges to connect multiple languages. However, in the discussion it was questioned whether logical reasoning with formal statements can be independent of the specific language contexts from which they have been translated.

Within the context of university learning with a multilingual student body, Salekhova as well as Meaney and Rangnes investigate student teachers' learning. Salekhova proposes a model to rate the quality of multilingual mathematical communication in school mathematics classrooms. Meaney and Rangnes investigate student teachers' learning in a multilingual university context and find that a multilingual context offers opportunities to make implicit assumptions about mathematics learning explicit. However, if English is used as a shared language in such a context, such implicit assumptions can often go unnoticed, so that these opportunities are often not realized. Within the context of the impact of EMI on university mathematics learning, Schüler-Meyer investigates students' writing practices. As writing becomes a central medium of communication at university, challenges with writing mathematically in a multilingual context could potentially hinder mathematical understanding.

In addition to the focus on university contexts, there is a continuation in investigating multilingual mathematics learning in a secondary school context. For example, Barwell introduces the constructs of "flow of language" and of "scales", where the first is a metaphor for the fact that students, by speaking in a classroom, insert themselves into a flow of language, which has been formed by previous speakers, and will be changed by his or her utterance. Scales describe different levels of sources of meaning, where, for example, a mathematical idea of a student can be acceptable by peers (smaller scale), but not when being considered in the context of nationwide exams (bigger scale). This study illustrates how new theoretical constructs continue to highlight new and relevant phenomena in multilingual mathematics learning.

Language for meaning-making

A total of nine papers and one poster were presented in the theme of *language for meaning-making*. As mentioned in the introduction, there is a certain overlap between the three groups, so several of the papers also in this group could equally well have been presented in the other groups. A characteristic of most of the papers in this group is that the object of study is closely linked to a particular mathematical topic. In the papers and presentations, one can find examples of studies

connected to counting and early number understanding (Farrugia), measurement (Chesnais & Constantin), decimal numbers and fractions (Coulange & Train), early algebra (Dohle & Prediger), multiplicative structures (Rønning), geometry (Akdoğan, Güçler, & Argün; Bolondi, Branchetti, & Giberti; Mithalal) and probability (Post & Prediger), sometimes also combinations of these topics. Contexts for the papers span from kindergarten to secondary school. Most of the papers are based on observational (classroom) studies but examples of quantitative studies are also present.

The role of language for meaning-making and for conceptual understanding has been acknowledged for a long time and a central issue is the development of a mathematical discourse necessary for competent participation in mathematical practices (Moschkovich, 2015). One dimension of competent participation is facilitating learners to transition from an everyday to a mathematical discourse. This dimension is indeed addressed by several of the papers. Mithalal's study is situated in early learning of geometry in France and is connected to the curricular requirement that children at a very early stage should use '*specific vocabulary'*. Coulange and Train, in their work with decimal numbers and fractions, are interested in what they, with reference to Bakthin, denote as a transition from first discourses to second discourses. Chesnais and Constantin discuss *implicitness* in the mathematics classroom, meaning that some elements of the mathematical practice are not made clear to the students and therefore may hinder conceptual understanding and the development of the mathematical discourse.

Mathematical topics covered in the papers in the group are predominantly topics that are central in the middle grades of compulsory school. Dohle and Prediger look into early algebra by investigating fifth graders' meaning-making when transforming expressions like $8 \times 12 + 2 \times 4$, using a variety of representations, and justifying why the transformations are valid. Representations, and connecting representations, are also important in the work by Coulange and Train on decimal numbers and fractions. They refer to the connecting of representations as constructing coherence between voices. In the paper by Rønning, the central theme is proportionality, and the challenges involved in expressing shape-preserving enlargement as a multiplicative structure.

Bolondi et al. take as a starting point Fischbein's (1993) theory of figural concepts. Using a geometrical figure from Fischbein's work they analyse students' judgement of the truth value of certain assertions connected to this figure. Their work is with students in Grade 9, and also Post and Prediger, in their work with conditional probability, frame their study among students of similar age. Their main interest is to explore what academic language demands students meet when developing conceptual understanding for conditional probability. The same age group is also addressed in Elçin et al.'s work when they look at one 16-year-old student's discursive development on the topic of reflection in relation to the teacher's discourse. The authors look at this development using Sfard's (2008) theory of commognition. Farrugia also refers to Sfard in her work on pre-school children's development of a mathematical discourse during play.

In addition to the nine papers, one poster was presented, by Hache, Dias, Millon-Fauré and Azaoui. The setting in the poster is among multilingual immigrant learners and their development of a mathematical discourse in the French language.

In a summary of the work in the group, one of the big questions for further work was formulated as follows: How do students develop from emerging (everyday, informal) discourses to new discourses

(new words and new usages) and get deeper understanding? Other topics to address were phrased as to look at the discourse and meaning transformation arising from the task compared to the external discourse appropriation and to study links between students' discourse and their activity in general.

Language in interaction.

The ten papers discussed in the group fill the common theme *language in interaction* differently. However, all seek to better understand students' mathematical learning in collaborative settings or to better facilitate student communication and learning mathematics at different levels. Two themes occurred in the discussions of the presented research that highlight possibilities for future collaboration and interaction between the researchers that presented their work in the group.

First, the group expressed a need for further coordination of theories that particularly consider theories (and the corresponding tools) from outside mathematics education. Already within established theories in the context of language and interaction, different theoretical notions and their implications for analysing and interpreting student data were discussed. For instance, mediation and Habermas' (1988) construct of communicative rationality (e.g. in Boero & Turiano on the new construct of rational mathematical templates) as well as negotiation of meaning connected to a more interactionist perspective on learning mathematics (Krummheuer, 2011) were examined. The latter is elucidated in numerous studies. For example, in Bitterlich's analysis of real-world contexts and their impact on language and learning, Ludes-Adamy and Schütte's identification of dissent and consensus situational structures, or Friesen and Schütte's observation of interactional obligations for participating in deeper collective argumentations. With great interest, the group also discussed new ideas on theorising subject choice and consideration of time as suggested by Smith in the context of analysing students' accounts for choosing advanced mathematical pathways. Also Ingram and Andrews' insights from a study on working with teachers on improving students' communication skills based on the Discipline of Noticing (Mason, 2012) amplified the range of presented frameworks. Furthermore, the question of how to approach the problem of balancing theory and empirical insights (transcripts) when talking or writing about research on language in interaction was discussed (as they were in Planas et al., 2019). This first theme of networking theories is part of the ongoing discussion not only in the CERME TWG on language and mathematics but also in other working groups and sometimes even by means of a dedicated working group on networking theories in CERMEs. However, the second theme of researching language in interaction in the context of digital learning environments arose for the first time.

The second theme of discussion was initiated by a presentation on developing digital learning environments for mathematics classrooms which aim at supporting students' collaboration and language production (Albano, Coppola, Dello Iacono, & Pierri), and a presentation on a digital tool facilitating teachers to use students' written answers to organize meaningful whole class discussions (Zöchbauer & Hohenwarter). The group extensively discussed the issues of collaboration, interaction and language against the background of chances and obstacles of digital tools and environments for learning mathematics. Furthermore, the question arose as to which theories, already implemented in mathematics education research, and with a focus on language in interaction, are functional for researching classroom interaction connected to digital learning environments. Of course, the group only started this discussion and did not reach any definitive answers and solutions. However, in

particular with the Corona Virus forcing teachers and students around the world to engage in various kinds of E-Learning, home-schooling etc., the question of how we can transfer our existing insights on language in interaction into teaching and learning with digital tools and environments are important areas for collaborative research in the near future. For example, it would be interesting to discuss to what extent Tewes' work on support systems for participation in collective argumentations or Zindel's study on teacher moves for promoting students' participation for language learners can contribute to developing and refining digital learning environments. Likewise, to what extent can digital tools and environments help teachers to promote and support language in mathematics classrooms? And to what extent becomes "language of the tool" another language learning goal in mathematics classrooms?

Linking the existing work on language, interaction and learning mathematics to digital learning environments and tools for teaching might be one of the challenges in the coming years not only for our subgroup but for all researchers in the area of language in mathematics education. For example, it could be fascinating to see which opportunities of relating registers and representations (as indicated by Albano et al.) arise by digital means as well as what opportunities and obstacles online group work involves compared to face-to-face interaction. Thus, a focus on online interaction as a form of interaction in mathematics education seems to be a new field for future collaboration, which could also be perfectly combined with research on meaning-making and multilingual contexts, as was in focus for the other subgroups.

Plenary sessions

Plenary talks allowed the two speakers, Candia Morgan and David Pimm to expose and present some of their consequent and inspiring research work. This gave the opportunity for people who were not as familiar with the nuance of their work to discover it further and for others to deepen their comprehension of it and to better identify the richness and significance of their work.

Candia Morgan, in her talk about "Conceptualising and researching mathematics classrooms as sites of communication" defended the potentialities of switching, in research, from looking at language and communication (including not only written and spoken language but also gestures, bodily movements or visual representations) in the classroom to considering mathematics teaching and learning as a communicative situation. David Pimm's talk about "Speaking, writing and mathematics registers: Denying "the dream of a common language" for mathematics", on his part, led him to pledge for a strict recognition of Halliday's notion of registers to avoid risking a too broad understanding of what counts as communication in mathematics. For example, he warned against conceptualizing symbolic language of mathematics as a means of communication that can be independent from spoken language.

Both talks also allowed the presenters to share ideas which could provide support for designing new research directions and support each of us in reflecting on the landscape of research on the topic of language and mathematics education, as well as on the theoretical assumptions which underpin the research field. These two themes continued throughout the discussions within each of the groups as they considered the future direction for research in multilingual contexts, language for meaning-making, and language in interaction within mathematics education.

Conclusion

The contexts in which we are researching continue to evolve. Mathematics classrooms today do not look like those of 20 years ago, and it is likely that in 20 years' time we will be saying the same thing about classrooms today. Multilingualism is now a common feature of many classrooms, lecture theatres and other educational settings. Choices or decisions (unconscious or deliberate) about which languages to use are politically and socially complex, as are the choices and decisions around the use of mathematical language, mathematical discursive practices and representations. These choices or decisions are becoming even more complex as we embrace and utilise digital technologies.

There are many challenges that we continue to face as the contexts in which we research evolve, and as research on language in mathematics education evolves and develops. As Candia Morgan reminded us, we draw on a range of theories from different disciplines that we have found useful in helping us to understand the relationships between mathematics and language, and in particular the role of language in the learning of mathematics. Yet we need to continue to revisit and evolve these theories to specifically address the concerns of mathematics education.

The focus of this ERME Topic Conference on classroom interactions also points to the growing need for teacher education to take account of language in the teaching and learning of mathematics. Teachers are increasingly working in multilingual and linguistically diverse contexts and there is now significant research that points to the role of language in meaning-making in mathematics, and in interaction. One challenge here is to not only to make the research accessible to teachers, and teacher educators, but also to make it relevant and of interest.

Research on language in the mathematics classroom has come a long way in the last few decades, and the journey continues. Opportunities like those in the ERME conferences offer us insights from different perspectives, disciplines and foci, as well as providing an opportunity to discuss and examine the issues and challenges we encounter as we move the research of the working group forward. These proceedings can only offer a glimpse of the richness of the discussions that were had, but we hope they will enable you to become as intrigued and enthused by the theoretical and empirical insights the different authors offer.

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