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Revenue Maximization in an Optical Router Node Using Multiple Wavelengths

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***Abstract*—In this paper an optical router node with multiple wavelengths is considered. We introduce revenue for successful transmission and study the ensuing revenue maximization problem. We present an efficient and accurate heuristic procedure for solving the NP-hard revenue maximization problem and investigate the advantage offered by having multiple wavelengths.**

***Index Terms*—optical routing, optical node, revenue, optimization, multiple wavelengths**

I. Introduction

In the last decades, optical fibers have emerged as the dominant transport medium in communication networks, because they offer major advantages over copper cables: huge bandwidth, extremely low losses and an extra dimension, viz., a choice of wavelengths (wavelength division multiplexing). Multiple wavelengths are to be used in order to enable the packet routing at various planes in the network (each at a specific wavelength); by including wavelength conversion, packets can be transferred between these planes, and thus congestion points can be circumvented. To handle packets at the IP layer would imply lots of packet conversions from optical to electronic, do the IP processing in the electrical domain, and then convert back to optical. The O/E-IP processing-E/O conversions introduce relatively significant time delays, which means extra latency. Such extra latency can seriously reduce the network throughput for interactive high-speed communication between users. All-optical routing in the nodes, as proposed and studied in this paper, is valuable to minimize the latencies.

Optical routing also offers substantial challenges [6], [8]. Photons cannot be stored easily, and hence buffering of optical packets is different from buffering in conventional communication systems. When photons need to be buffered, they are sent into a local fiber loop, which thus provides a small discrete delay to the photons without displacing or losing them. Packets can be inserted into and extracted from the fiber loop by means of a cross/bar switch. If after a loop completion the photon still cannot be transmitted, then it could again be sent into the fiber loop, or be considered as lost. Such optical nodes are to be used in an all-optical packet-routing network, having multiple hops.

In [1] we modeled a single-wavelength optical routing node as a queueing system with a single server (the wavelength) and N stations – the N ports of the routing node. We assumed that each successful transmission of a packet ("customer") brings a certain profit. Our aim in [1] was to maximize the router performance by maximizing that profit. As a communication system typically works in frame time, we demanded that the time it takes the server to complete one cycle of the N stations is a given constant C. We then wanted to assign fixed amounts of time $V_1, \dots, V_N$ to the visit periods (also called service windows) of the stations, such that $\sum_{i=1}^N V_i = C - \sum_{i=1}^N S_i$, where S_i is the time to switch to station $i \in \{1, 2, \dots, N\}$. We introduced the probability $p_i(V_i)$ that a packet in a retrial loop of station i retries during visit period V_i, and the probability $q_i(V_i)$ that a packet is dropped when it fails to retry during V_i. Under reasonable assumptions on those retry and drop probabilities, the revenue optimization problem in [1] was shown to be a separable concave optimization problem – a well-studied type of optimization problem that allows for an efficient and insightful algorithm (RANK; cf. [5]) that yields the optimal solution.

Our goals in the present paper are (i) to investigate the advantage offered by having *multiple wavelengths*, and (ii) to formulate and solve the revenue optimization problem for an optical routing node with multiple wavelengths. We shall show that the advantage, in terms of revenues, is very significant (in particular, to go from one to two wavelengths). While solving the revenue optimization problem for multiple wavelengths is an NP-hard problem, we develop a heuristic that works very well. Our numerical results give insight into the sensitivity of various parameters and modeling assumptions.

The paper is organized as follows. Section II presents a detailed description of the optical routing node model. The revenue maximization problem, that amounts to a resource allocation problem (assigning stations to wavelengths and assigning visit times to stations), is discussed

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in Section III. Numerical examples are shown in Section IV. Section V contains conclusions and suggestions for further research.

II. Optical routing node model

Consider a K-wavelength optical routing node with N ports (stations) to route packets and with retrial loops to store packets. We represent it by a queueing model with K servers which visit N queues. We shall assume that there is a fixed assignment of stations to servers (how to do that assignment is part of our optimization problem), in which each server always serves a fixed set of stations.

The customers: Packets (also called customers in queueing terminology) of type j, $j = 1, \cdots, M$, arrive at station i, $i = 1, \cdots, N$, according to independent Poisson processes with rate λ_{ij}, for all i, j. We allow several customer types because there can be several types of data at each port. If at the time of packet arrival the station is being served, then the packet is instantaneously transmitted; else it enters a retrial loop. We assume the retrial time to be random, because delay loops of various lengths may be used. If, at the time of retrial, the station is not in service then the packet again goes into a retrial loop and this process continues.

The servers: The servers go through cycles of fixed length C (the frame time). In each cycle a server visits each of its assigned stations once, for a fixed period of time V_i for station i. A visit to i is preceded by a deterministic switchover (setup) time S_i of the server. During V_i, there may be two types of arrivals: (i) newly arriving packets, and (ii) packets which were in a retrial loop; we assume the latter retry during V_i with probability $p_i(V_i)$. In view of the huge available bandwidth, we assume the server serves all these packets (new arrivals + retrials) instantaneously, i.e., whenever a station is being served, any packet which arrives at it or retries, is transmitted immediately. Hence for practical purposes the service times are negligible. At the end of the visit of station i each packet which still resides in a retrial loop of i is dropped with probability $q_i(V_i)$. Hence the probability that a packet in a retrial loop of station i leaves the system, either served during a visit at station i or dropped after a visit of station i, is $r_i(V_i) := p_i(V_i) + q_i(V_i) - p_i(V_i)q_i(V_i)$.

Revenue: Every served customer generates a profit and every lost customer incurs a loss to the system. Our goal is to assign stations to servers, and subsequently visit times within a frame time C to stations, such that the revenue of the system is maximized. Assume that:

- a customer of type j served at station i gives a profit γ_{ij} (depending both on the type of packet and the type of source).
- a customer of type j dropped at station i causes a penalty θ_{ij}. Indeed, the server has an obligation to meet the contract it has with each source. If the server fails to meet this contract it incurs a penalty: loss of packets/reputation/further contracts. One could also view $\Theta_i := \sum_j \lambda_{ij}\theta_{ij}$ as contract costs of the service provider per time unit, and $\Gamma_i := \sum_j \lambda_{ij}(\gamma_{ij} + \theta_{ij})$ as the maximum revenue that can subsequently be earned back by successfully serving customers.

For $K = 1$ wavelength (cf. also [1] where that case was studied), the mean earnings per cycle are

$$\sum_j \lambda_{ij}\gamma_{ij}\left[(C - V_i)\frac{p_i(V_i)}{r_i(V_i)} + V_i\right],$$

and the mean costs per cycle are

$$\sum_j \lambda_{ij}\theta_{ij}\left[(C - V_i)(1 - \frac{p_i(V_i)}{r_i(V_i)})\right],$$

yielding the following net revenue for station i per cycle:

$$R_i(V_i) = M_i(V_i) - C\Theta_i,$$

where for all $i = 1, \ldots, N$,

$$M_i(V_i) := \Gamma_i\left[(C - V_i)\frac{p_i(V_i)}{r_i(V_i)} + V_i\right]. \qquad (1)$$

In the next section we present an algorithm to allocate the stations to different wavelengths such that each wavelength has a set of stations to serve; subsequently the visit periods are chosen such that the revenue for each wavelength is maximized.

III. Resource allocation

In this section we propose a procedure for solving the revenue maximization problem that was globally described in Section II. For each wavelength k, we have $C = \sum_{i \in \mathbf{P}_k}(S_i + V_i)$ where $\mathbf{P}_k$ represents the set of all stations served by wavelength k. Note that if there is only one station being served by a wavelength, then there is no switchover involved. In that case, $V_i = C$ where i is the only element of $\mathbf{P}_k$. Further we denote the set of stations which are each served by one complete wavelength as $\mathbf{P}$ and the set of stations which are not served by any wavelength as $\mathbf{Q}$.

We now define the optimization problem REVENUE which produces maximum revenue via an optimal allocation of stations to wavelengths and visit periods to stations. **REVENUE**

$$\max \sum_{i=1}^{N} M_i(V_i)$$

subject to

$$\begin{aligned}\sum_{i=1}^{N}[(S_i+V_i)x_{ik}+V_i y_{ik}] &= C, \quad \forall\ k=1,2,\cdots,K,\\ \sum_{k=1}^{K}[x_{ik}+y_{ik}] &\le 1, \quad \forall\ i=1,2,\cdots,N,\\ \sum_{i=1}^{N}x_{ik}+N\sum_{i=1}^{N}y_{ik} &\le N, \quad \forall\ k,\\ x_{ik},\ y_{ik}\in\{0,1\} \quad \text{and} \quad 0\le V_i\le C, &\quad \forall\ i,\ k.\end{aligned}$$

Here $M_i(V_i)$ is given in Eq. (1). $x_{ik}=1$ if station i is served by wavelength k, but station i is not the only one being served by it, and is 0 otherwise, $y_{ik}=1$ if station i is the only station being served by wavelength k, and is 0 otherwise. This is stated in the third condition where if for a wavelength k some $y_{ik}=1$ then no other station can be served on it. The second condition states that each station i can only be served by at most one wavelength. The first and last conditions are system properties, and they state that the allocation per wavelength should be equal to its capacity C and the visit period cannot be negative or more than the total cycle time of one wavelength. This problem is a non-linear mixed integer programming problem. Under certain realistic assumptions regarding the system parameters (see also [1]), we can reduce the objective function of this maximization problem to separable concave terms; however, the occurrence of the integers $x_{ik},\ y_{ik}$ prevents us from using the RANK algorithm [5] that was used in [1]. The so-called BALANCE problem, which is NP-complete [4], is a special case of REVENUE. Hence REVENUE is an NP-hard problem; below we propose a heuristic to solve REVENUE. We argue that this heuristic should produce results which are close to optimal, and we provide numerical results in Section IV to support that claim.

The idea behind our approach is the following. In *Step 1* we do as if there is only one wavelength, but a frame time of length KC. We use the RANK algorithm to get an optimal choice of the visit periods $\tilde{V}_i$ for such a situation. That should already give a quite good first estimate of the visit periods. In *Step 2* we use those $\tilde{V}_i$ values to assign stations to wavelengths. This is done such that each of the K wavelengths gets roughly the same $\sum(S_i+\tilde{V}_i)$ – which hence should be close to C. Finally, in *Step 3*, with those K allocations we use RANK again, but now for K separate single-wavelength problems. Below we provide the details of these three steps.

Step 1: We first define the following optimization problem.

ONE

$$\begin{aligned}&\max \sum_i M_i(\tilde{V}_i)\\ \text{subject to } \sum_i \tilde{V}_i &= KC-\sum_i S_i,\\ \text{and} \quad 0\le \tilde{V}_i &\le C-S_i, \quad \forall i.\end{aligned}$$

The solution of this optimization problem gives us the values of $\tilde{V}_i$ required by each station to give the maximum revenue, subject to the condition that the maximum amount of resource available is KC. The upper bound on $\tilde{V}_i$ is included because a station cannot be served by more than one wavelength. Note that $M_i(\tilde{V}_i)$ is the same as given in Eq. (1).

We solve the (separable, concave) optimization problem ONE using RANK, and we thus obtain values of $\tilde{V}_i$. Every station i which has $S_i+\tilde{V}_i=C$, is allocated to a single wavelength. These stations belong to the set $\mathbf{P}$ and as described at the start of this section, all stations belonging to this set have their visit periods equal to the cycle time C. Further, all the stations with $\tilde{V}_i=0$ belong to the set $\mathbf{Q}$. These stations will not be allocated to any wavelength, and as mentioned earlier they will have zero visit period. By renumbering, we may assume that the stations in $\mathbf{Q}$ are the highest numbered stations, immediately preceded by the stations in $\mathbf{P}$. Also assume that the latter $N(\mathbf{P})$ stations are assigned to the $N(\mathbf{P})$ highest numbered wavelengths.

We now turn to our procedure for assigning stations to wavelengths (Step 2) and subsequently determining the exact visit periods (Step 3).

Step 2: Take the values of $S_i+\tilde{V}_i$ for the first $N-N(\mathbf{P}+\mathbf{Q})$ stations (i.e., those not in $\mathbf{P}$ or $\mathbf{Q}$). Sort these values in descending order, say $S_1+\tilde{V}_1\ge S_2+\tilde{V}_2\ge\cdots\ge S_{N-N(\mathbf{P}+\mathbf{Q})}+\tilde{V}_{N-N(\mathbf{P}+\mathbf{Q})}$. Then allocate those stations to the first $K-N(\mathbf{P})$ wavelengths following the so-called *Longest Processing Time first* (LPT) rule. This amounts to first assigning stations $1,\ldots,K-N(\mathbf{P})$ to wavelengths $1,\ldots,K-N(\mathbf{P})$; and subsequently assigning each of the remaining stations, one by one in descending order of their values, to that wavelength for which the sum of the already assigned values is the smallest. This procedure is continued until all stations have been assigned.

Remark. The idea to use LPT comes from multiprocessor scheduling. Consider a set of N tasks which have to be served on K parallel servers. The service of a task on a server, once started, cannot be interrupted. In multiprocessor scheduling the goal often is to minimize the *makespan*, i.e., the time until all tasks are completed. This is an NP-hard problem. The makespan minimization problem can be reformulated in the terminology of bin-packing, where it amounts to finding the smallest common capacity of the bins, sufficient to pack all N pieces. Many heuristics have been developed for solving the bin-packing or makespan minimization problem; see, e.g., [3]. LPT is a simple and accurate heuristic procedure. It is intuitively clear that assigning tasks in decreasing order of size should work well when K and N are not too small: because the smallest tasks are assigned last, it is likely that all makespans are close to each other. See [7] for a probabilistic analysis of various bin-packing heuristics, and [2] for a probabilistic analysis of LPT list scheduling.

Step 3: Now that we have assigned all stations to a wavelength, we still need to determine the visit periods for those stations that use wavelengths $1,\dots,K-N(\mathbf{P})$, because the extended visit periods $S_i+\tilde{V}_i$ of the stations that are assigned to a particular wavelength do not exactly sum up to C. For this we solve optimization problem TWO, for $k=1,\dots,K-N(\mathbf{P})$:

TWO

$$\begin{aligned} &\max \sum_{i\in\mathbf{P}_k} M_i(V_i) \\ &\text{subject to} \sum_{i\in\mathbf{P}_k} V_i \;=\; C-\sum_{i\in\mathbf{P}_k} S_i, \\ &\text{and} \qquad V_i \;\geq\; 0, \quad \forall i\in\mathbf{P}_k. \end{aligned}$$

The solution of this optimization problem gives us the values of V_i required by each station allocated to wavelength k, subject to the maximum amount of resource available at that wavelength. We thus obtain new extended visit periods S_i+V_i for stations $1,\dots,N-N(\mathbf{P}+\mathbf{Q})$.

Remark. If, in *Step 2*, a station i^* is the only one being assigned to a wavelength, then we do not run TWO for it but take $V_{i^*}=C$.

This concludes the description of the heuristic procedure. In the next section we shall investigate its accuracy. Its computational complexity is low. The optimization problems ONE and TWO are concave separable with linear constraints and can be solved in polynomial time; and we use ONE once, TWO at most K times. We also use LPT once. Further, we need to sort the extended visit periods in Step 2 once.

IV. Numerical examples

In this section we present a few numerical examples to illustrate various properties of our system. For all the examples in this section we assume that the probability of retrial and drop probability for a station i are given by $p_i(V_i)=1-e^{-\nu_i V_i}$ and $q_i(V_i)=e^{-\mu_i V_i}$. Further, the revenue of a station i is equal to $M_i(V_i)$ as given in Eq. (1).

Example 1:

We first consider a toy example with $K=2$ wavelengths and either $N=3$ or $N=4$ stations, for which all possible assignments allocating all stations to a wavelength are listed. For each station i, the parameters ν_i and μ_i are equal to 0.5. The switchover times $S_i=0.2$ for each station i and cycle time $C=2$. Finally, $\Gamma_i=i$, for each station i. The allocation of stations to different wavelengths is shown, along with the corresponding visit period (obtained by using TWO) and the revenue obtained by the system. Note that an allocation 0 implies that the station was not allocated to any wavelength.

TABLE I: 3 station system

Allocation	Visit Period	Revenue
[1 1 2]	**[0.48 1.12 2.00]**	**10.11**
[1 2 1]	[0.28 2.00 1.32]	9.81
[2 1 1]	[2.00 0.61 0.99]	8.65

TABLE II: 4 station system

Allocation	Visit Period	Revenue
[0 1 1 2]	**[0.00 0.61 0.99 2.00]**	**14.65**
[1 2 2 1]	[0.14 0.61 0.99 1.46]	14.25
[1 2 1 2]	[0.28 0.48 1.32 1.12]	14.03
[1 1 2 2]	[0.48 1.12 0.67 0.93]	13.34
[1 1 1 2]	[0.00 0.61 0.99 2.00]	14.65
[1 1 2 1]	[0.00 0.48 2.00 1.12]	14.22
[1 2 1 1]	[0.00 2.00 0.67 0.93]	13.23
[2 1 1 1]	[2.00 0.00 0.67 0.93]	11.23

In Tables I and II the values given by our procedure described in the previous section are printed boldface. We observe that in both cases our procedure gives the best allocation.

Example 2:

In this example we compare the results obtained using our procedure with the results obtained by randomly allocating wavelengths to different stations and then optimizing the visit periods at each wavelength. We show numerical results for five different cases for a system with $N=16$ stations, $K=4$ wavelengths and frame time $C=8$. In each of the first four cases, we vary one parameter while keeping all the other constant and in the last case we use random system parameters; the Γ_i are uniformly distributed on $(0,8)$; the ν_i and μ_i on $(0,1)$, and the S_i on $(0,0.4)$.

We take 10000 independent allocations of wavelengths in two different ways, (i) and (ii). In (i) we allocate stations in such a way that each wavelength gets at most 4 stations, whereas in (ii) there is no restriction on the number of stations allocated to a wavelength. In both cases we subsequently use TWO. For both (i) and (ii) we show the maximum, the average and the minimum obtained revenue among the 10000 cases and the percentage of allocations which generated a revenue above the value generated using our algorithm.

TABLE III: Varying Γ_i

	Maximum	Average	Minimum	Percent
(i)	475.72	468.89	454.24	1.46
(ii)	475.50	441.36	300.33	0.24
Algorithm		474.51		

$\Gamma_i = 0.5 * i$, $\nu_i = 0.5$, $\mu_i = 0.5$ and $S = 0.2$.

TABLE IV: Varying ν_i

	Maximum	Average	Minimum	Percent
(i)	387.29	384.58	381.94	9.89
(ii)	387.14	358.36	224.93	0.87
Algorithm		385.65		

$\Gamma_i = 4$, $\nu_i = 0.05 * i$, $\mu_i = 0.5$ and $S = 0.2$.

TABLE V: Varying μ_i

	Maximum	Average	Minimum	Percent
(i)	413.19	413.15	412.98	0.00
(ii)	413.19	377.54	231.52	0.00
Algorithm		413.19		

$\Gamma_i = 4$, $\nu_i = 0.5$, $\mu_i = 0.05 * i$ and $S = 0.2$.

TABLE VI: Varying S_i

	Maximum	Average	Minimum	Percent
(i)	398.81	398.06	395.60	0.05
(ii)	398.79	351.53	181.94	0.00
Algorithm		398.81		

$\Gamma_i = 4$, $\nu_i = 0.5$, $\mu_i = 0.5$ and $S = 0.05 * i$.

TABLE VII: Completely Random

	Maximum	Average	Minimum	Percent
(i)	360.85	355.23	338.07	4.56
(ii)	360.83	338.14	231.45	0.62
Algorithm		359.93		

$\Gamma_i \sim U(0,8)$, $\nu_i \sim U(0,1)$, $\mu_i \sim U(0,1)$, and $S_i \sim U(0,0.4)$

Tables III-VII suggest that a random assignment of stations to wavelengths, but still using TWO to subsequently choose V_i, is much worse than the assignment of our algorithm. However, the symmetric assignment, in which each of the four wavelengths serves (at most) four out of the 16 stations, and for which the visit times are calculated using TWO, yields results that are typically quite close to the values obtained using our algorithm (and in a few cases even better).

Example 3:

In this example we study which effect increasing the number K of wavelengths has on the revenue of the system. We take the allocation obtained using the procedure of Section III. For each K we take $N = 16$ stations, $S_i = \mu_i = \nu_i = 0.05 * i$, $\Gamma_i = 0.5 * i$ and $C = 8$.

We observe that increasing the number of wavelengths increases the revenue obtained and also the number of stations served. However, the marginal increment decreases with an addition of each wavelength. In this example the change from $K = 1$ to $K = 2$ almost doubles the revenue and more than doubles the number of stations served, whereas the change from $K = 7$ to $K = 8$ increases the revenue by less than two percent (and the number of stations served does not change). In the case of $K = 16$, the revenue equals $C * \sum_{i=1}^{16} \Gamma_i = 544$. The system operator can choose an optimal number of wavelengths so as to maximize its utility. This observation may be of interest in networks where traffic is highly variable and the cost of running extra resources is high.

TABLE VIII: Varying the number of wavelengths

K	Revenue	# of stations served
1	170.54	3
2	322.62	8
3	400.97	11
4	452.88	13
5	480.40	14
6	499.60	14
7	517.23	15
8	525.21	15
16	544.00	16

Example 4:

In this example we consider a system with $N = 16$ stations, $K = 4$ wavelengths, frame time $C = 8$ and switchover period from each station $S_i = 0.2$, for all $i = 1, \ldots, N$. We show three different cases, each of which has one of Γ_i, ν_i, and μ_i different for all stations, the other two parameters being equal for all stations. In these numerical experiments we study how the procedure described in Section III allocates resources depending on each factor, and develop insight into the influence of these factors on the system performance. In Table IX, we mention the wavelength to which each station is assigned, the visit period each station receives and the revenue each station gives, for the three cases.

From Table IX(a) we see that in general $\Gamma_i > \Gamma_j$ does not imply $V_i > V_j$, but when i and j are allocated to the same wavelength this implication appears to be true. Also, if the value of Γ_i is very low, then – even though our procedure allocates that station to a wavelength – it may not receive any service (equivalent to not being allocated).

In Table IX(b) we see that in general, within a wavelength, stations with lower ν_i receive higher V_i. This happens because the system tries to allocate longer visit periods to stations with low retrial rates so as to maximize the number of customers it can serve. However, if ν_i is very low (see station 1), then the system, subject to limited resources, might not allocate any resource to that station.

From Table IX(c) one can generally observe that the stations with higher drop probability, i.e., lower μ_i, receive higher visit periods to have fewer losses. Also, like in the previous case the difference in revenue generated from each station is not big.

TABLE IX: Visit period allocation and corresponding revenue obtained

Station	(a) Allocation	(a) Visit	(a) Revenue	(b) Allocation	(b) Visit	(b) Revenue	(c) Allocation	(c) Visit	(c) Revenue
1	0	0.00	0.00	0	0.00	0.00	3	1.85	22.76
2	0	0.00	0.00	1	3.35	26.05	4	1.86	23.36
3	3	0.93	6.54	2	2.33	22.33	2	1.87	23.94
4	4	1.22	10.68	3	2.18	23.09	1	1.87	24.48
5	4	1.45	14.89	4	2.07	23.83	3	1.86	24.90
6	3	1.67	19.27	4	1.97	24.37	4	1.85	25.29
7	2	2.16	24.96	3	1.88	24.80	2	1.84	25.66
8	1	2.25	28.90	2	1.83	25.30	1	1.83	26.01
9	1	2.34	32.89	1	2.16	28.02	1	1.82	26.32
10	2	2.46	37.00	4	1.69	25.85	3	1.80	26.56
11	3	2.20	39.45	3	1.64	26.09	2	1.78	26.81
12	4	2.23	43.23	2	1.60	26.42	4	1.76	27.03
13	4	2.30	47.24	1	1.89	28.59	4	1.73	27.23
14	3	2.40	51.49	3	1.50	26.76	2	1.71	27.43
15	2	2.78	57.03	4	1.47	26.96	3	1.69	27.62
16	1	2.81	60.94	2	1.44	27.19	1	1.68	27.79
Total		29.20	474.51		29.00	385.65		28.80	413.19

(a) $\Gamma_i = 0.5 * i$, $\nu_i = 0.5$ and $\mu_i = 0.5$.

(b) $\Gamma_i = 4$, $\nu_i = 0.05 * i$ and $\mu_i = 0.5$.

(c) $\Gamma_i = 4$, $\nu_i = 0.5$ and $\mu_i = 0.05 * i$.

Three final observations: 1. The spread in visit periods is small in IX(c) compared to those in IX(a) and IX(b). This suggests that the factor μ_i is less important than the factors ν_i and Γ_i in the solution of this problem. 2. Our procedure often results in a more or less even spread of revenues among stations if Γ_i are equal. This suggests that the procedure makes the system reasonably fair, i.e., tries to provide the best service to each station. 3. Even though the revenues obtained from stations with different retrial rates and drop probabilities are similar, the resources required by these stations are different. For a lower retrial rate and/or higher drop probability, a longer visit period is required to give similar revenue. This is a techno-economic trade-off to consider while designing the router.

V. Conclusions and suggestions for further research

To understand the behaviour and study the performance of future optical networks, we have considered a revenue optimization problem for a multiple wavelength optical routing node. This is a mixed integer non-linear programming problem and hence extremely time-consuming to solve even for a small number of wavelengths. Since one would like to solve this revenue optimization problem quite frequently, we have developed an efficient and near-optimal heuristic procedure for (i) assigning stations to wavelengths and subsequently (ii) assigning visit times to stations within a fixed frame time.

Several topics for further research suggest themselves. Firstly, one might consider variants of the proposed heuristic procedure. For example, a consequence of the use of LPT is that, for each wavelength, one has a sum of assigned $S_i + V_i$ that is not exactly equal to C. We subsequently used TWO to make final choices for the visit periods V_i. Instead, one could simply scale all V_i, that belong to one and the same wavelength, by the same factor α such that $\sum(S_i + \alpha V_i) = C$. Secondly, it could be interesting to relax two modeling assumptions, viz., to take the finiteness of the fiber loops into account more explicitly, and to remove the assumption of negligible service times. Thirdly, one could consider completely different ways of assigning stations to wavelengths, allowing for example that the same station uses more than one wavelength. Finally, it would be worthwhile to study the trade-off between using more wavelengths and investing in a higher number of fiber loop buffers - which can be translated into a lower drop probability, in terms of node throughput and economics.

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