Fast model-based control and prediction of the safety factor profile evolution in tokamak plasmas

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Advanced tokamak operation requires control over the q-profile evolution during current ramp-up and flat-top phase, as it determines both stability and performance of the plasma. This can be done by feedback controlling the q-profile evolution. For ITER, simultaneous control of the q-profile and additional control tasks needs to be demonstrated, where a governing supervisory controller (SC) needs to share the same actuators for both tasks. This implies that the q-profile controller should be able to handle real-time varying constraints on actuators and plasma physics, set by the SC. This paper describes an algorithm to control and predict the q-profile evolution using Model Predictive Control (MPC). MPC is a general optimal control technology which uses a predictive model to compute the control action and can deal routinely with real-time varying actuator and state constraints [1]. The prediction can be made available to the SC. Simulation results show the effectiveness of this approach.

Approach

For control purposes, the 1D plasma transport physics inside the tokamak are modeled as a dynamical system, linking actuators to outputs (q at several locations in the plasma) via the state. The state is a complete description of the considered system in terms of plasma parameters at a particular moment in time. The actuators are the inductive and other non-inductive heating and current drives (H&CD). The envisioned implementation of the MPC-controller in a tokamak is shown in Figure 1. A supervisory controller receives real-time predictions and warnings of the MPC-controller. In turn it provides the MPC-controller with real-time constraints. With these constraints the MPC-controller can calculate the optimal feedback signal which is provided in com-

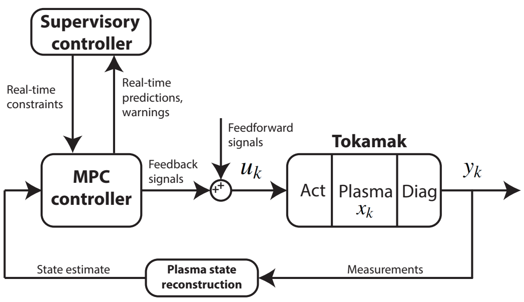

Fig. 1: Envisioned implementation in tokamak.

bination with the feedforward signal (reference actuator trajectory) to the tokamak. From the available measurements the plasma state can be reconstructed and fed to the MPC controller. In this work we replace the tokamak and plasma state reconstruction by the RAPTOR simulation code, where we have direct access to the state. The SC is replaced with artificial time-varying constraints.

Figure 2 illustrates how a reference trajectory can be tracked using prediction with (blue) and without a constraint (red). The prediction of the future output evolutions is based on knowledge of the dynamics, current state and future actuator inputs. The predicted actuator trajectory to get back to the reference trajectory is shown. In case of the constraint, the controller recognizes that in the future a constraint will be violated and computes an actuator sequence to avoid this constraint.

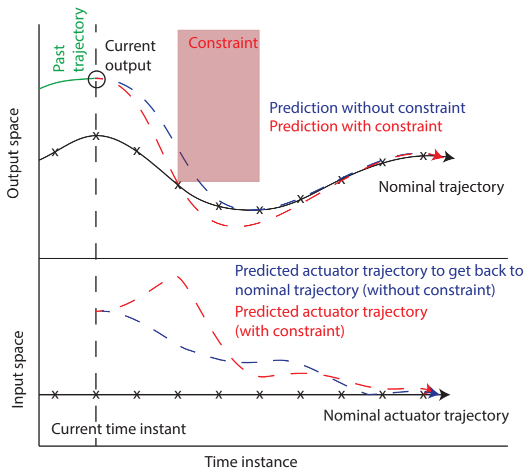

Fig. 2: Cartoon illustration of prediction and constraint handling.

In this work we use RAPTOR as a simulator of the 1D plasma transport physics [2]. RAPTOR is a rapid 1D plasma transport code which solves the nonlinear coupled evolution of poloidal magnetic flux and electron temperature. RAPTOR uses a fixed 2D flux surface geometry, simplified source models and a simplified (Bohm-gyro-Bohm [3]) transport model for χ_e. We use RAPTOR in this paper for: 1) optimization of the reference trajectory [4]; 2) derivation of linearized models around the reference trajectory; 3) nonlinear simulator for testing the controller. RAPTOR can also be used for real-time profile reconstruction and fault detection [5].

It is assumed that the plasma nominally evolves along a pre-calculated trajectory in its operating (state) space. This reference trajectory is derived, using the method in [4], in which the desired q-profile is reached after $t = 100$ and is in stationary state (flat loop-voltage profile). Once the reference nonlinear trajectory is known, local linearizations are derived from RAPTOR at each time instance and used as a model for prediction and control.

The MPC-controller requires a prediction model to predict future plasma behavior, a cost function optimizer subject to constraints for future actuator trajectory generation and knowledge of the current state. The prediction model is constructed by using the linearizations along the trajectory at all time steps till the prediction horizon N. The prediction model predicts the deviation of the actual q-profile evolution from the reference q-profile evolution. The cost function is defined as $J_k = \sum_{k=1}^{P} || \frac{1}{q_{\text{ref}}(k)} - \frac{1}{q(k)} ||^2$, quadratically penalizing the future error in the q-profile evolution and an additional term for smoothing the actuator inputs (not shown). The constraints are composed of actuator amplitude constraints (e.g. $P_{\mathrm{EC}} < c_1$), actuator ramp rate constraints (e.g $\frac{dI_p}{dt} < c_2$), mixed actuator constraints (e.g $\sum P_{\mathrm{EC}} < c_3$) and plasma physics constraints (e.g $q > 1$, $c_4 < \frac{I_{\mathrm{NI}}}{I_p} < 1$, $V_{\mathrm{loop}} > 0$). The cost function and constraints can be reformulated into a well-known Quadratic Programming (QP) problem. QP problems are computationally cheap. While the time-varying dynamics from the 1D transport physics are taken into account, the algorithm is fast enough to be implemented on currently operational tokamaks. MPC controllers for the q profile have been designed in the past, but contained a simplified linear physics model [6] or were computationally more demanding as it solved a nonlinear optimization at each time step [7]. Both designs did not include plasma physics constraints.

Results

The effectiveness of this approach is demonstrated in ITER simulations, presenting an example of real-time varying constraint handling.

The reference trajectory is optimized such that a stationary hybrid-like q-profile is obtained at the beginning of the flat-top phase (100 seconds). The controlled actuators are the plasma current I_p and power to three EC beams at ρ =0.2, 0.4 and 0.55. The following settings are chosen: plasma current I_p<15 MA (8.5 MA nominal), the total EC power ΣP_{EC}<20 MW (16.2 MW nominal). The NBI power is uncontrolled and fixed at 16.5MW starting at 60s. The prediction horizon is set to 40 seconds. This results in 2ms computation time per time step of 1 second.

Figure 3 shows the results for the constraint handling example. A constraint is added after 160 seconds: $q(\rho = 0) > 1.09$ for $t > 200$ (indicated by the red area in column 2). The evolution of $\iota(\rho = 0)$ shows clearly how the constraint is negotiated (-·- in column 2). The constraint is avoided, but $\iota(\rho = 0)$ remains as close as possible to the reference. The controller computes the necessary actuator inputs (blue in column 1) and especially reduces the P_{EC}. It can be noticed that with these actuator locations the controller cannot independently control $\iota(\rho = 0)$ and $\iota(\rho = 0.2)$, but that $\iota(\rho = 0.2)$ is affected by the constraint handling on $\iota(\rho = 0)$. Moreover the evolution of $P_{\mathrm{EC}}(\rho = 0.55)$ (-·- in column 1) and W_{tot} (-·- in column 3) indicates that the constraint cannot be sustained without loss of performance. This reveals that the controller is naturally subjected to the limitations of the 1D plasma transport dynamics, but can anticipate the constraint and optimize the inputs to negotiate this constraint.

Conclusions and outlook

The MPC-controller can track a predefined reference trajectory of the q-profile evolution. Simulation results show that constraints can be handled and that the controller can compensate

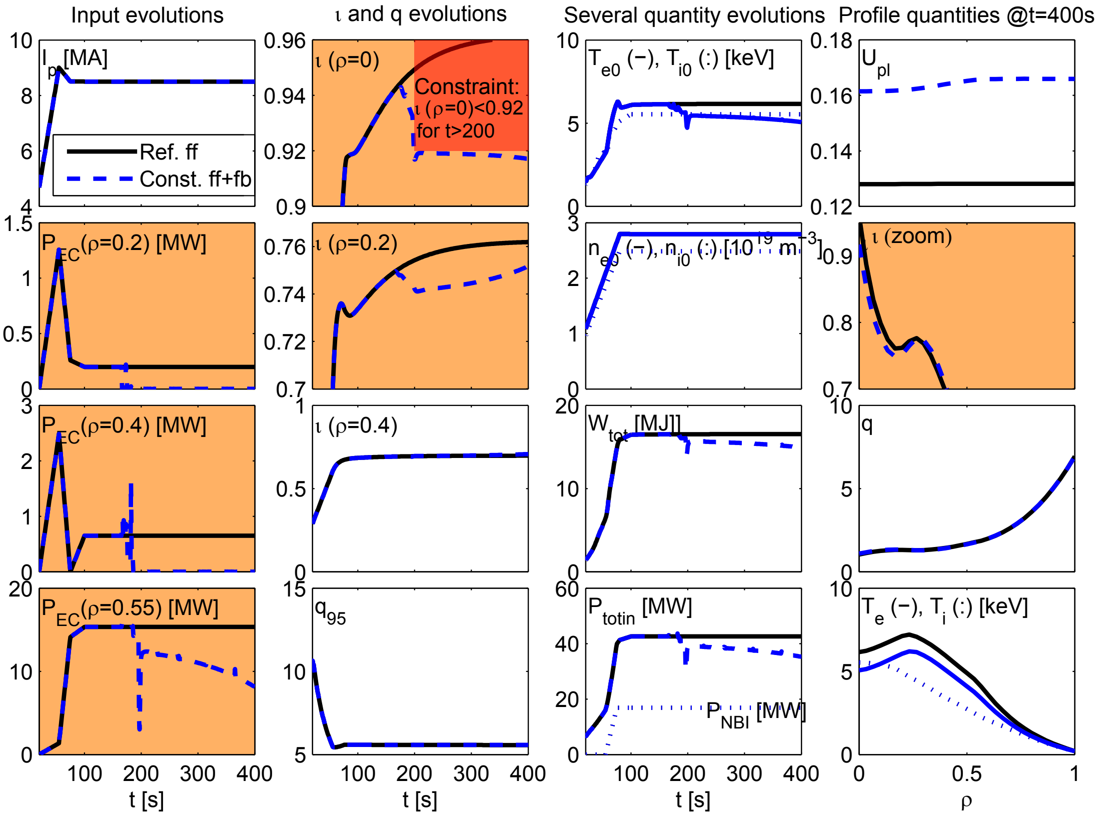

Fig. 3: Results constraint handling. A constraint is added after 160s: $\iota(\rho = 0) < 0.92$, for $t > 200$. Note: $\iota = 1/q$. Reference trajectory (-), constrained trajectory with feedback (-·-).

for model uncertainties and disturbances.

The main advantages over previous work [6, 7, 8, 9, 10] on q-profile control are: 1) Model-based approach, no identification experiments necessary; 2) Based on model that can represent any operating mode and tokamak; 3) Local linearizations accurately represent highly nonlinear dynamics, especially due to time-varying resistivity; 4) Handle time-varying constraints on actuators and plasma physics; 5) Predictions are available to supervisory controller.

Future work entails the application of the controller with RAPTOR to ITER advanced scenarios, the validation of the controller on more complete simulators like CRONOS and in experiments on currently operational tokamaks. Furthermore, we will exploit the advantages of predictions and real-time constraint handling in connection with a supervisory controller.

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