

A bulge test based methodology for characterizing ultra-thin buckled membranes

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1 **A bulge test based methodology**
2 **for characterizing ultra-thin buckled membranes**

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9

10 **Abstract**

11 Buckled membranes become ever more important with further miniaturization and development
12 of ultra-thin film based systems. It is well established that the bulge test method, generally
13 considered the gold standard for characterizing freestanding thin films, is unsuited to characterize
14 buckled membranes, because of compressive residual stresses and a negligible out-of-plane
15 bending stiffness. When pressurized, buckled membranes immediately start entering the ripple
16 regime, but they typically plastically deform or fracture before reaching the cylindrical regime. In
17 this paper the bulge test method is extended to enable characterization of buckled freestanding
18 ultra-thin membranes in the ripple regime. In a combined experimental-numerical approach, the
19 advanced technique of digital height correlation was first extended towards the sub-micron scale,

20 to enable measurement of the highly varying local 3D strain and curvature fields on top of a single
21 ripple in a total region of interest as small as $\sim 25 \mu\text{m}$. Subsequently, a finite element (FE) model
22 was set up to analyze the post-buckled membrane under pressure loading. In the seemingly
23 complex ripple configuration, a suitable combination of local region of interest and pressure
24 range was identified for which the stress-strain state can be extracted from the local strain and
25 curvature fields. This enables the extraction of both the Young's modulus and Poisson's ratio from
26 a single bulge sample, contrary to the conventional bulge test method. Virtual experiments
27 demonstrate the feasibility of the approach, while real proof of principle of the method was
28 demonstrated for fragile specimens with rather narrow ($\sim 25 \mu\text{m}$) ripples.

29 **Keywords:** Bulge Test Method; Buckled Membrane; Wrinkled Membrane, Ultra-thin Films,
30 Freestanding Films; Digital Image Correlation (DIC); Digital Height Correlation (DHC); Global Digital
31 Image Correlation (GDIC)

32 **1. Introduction**

33 The bulge test methodology has become the standard technique for mechanical characterization
34 of thin films [1] especially for freestanding membranes. This is due to (1) the possibility of precise
35 sample processing facilitated by recent developments in micro-fabrication technology; (2) the
36 need for minimal sample handling, which is especially challenging at small scales; (3) the relatively
37 simple data processing for determining the membrane stress and strain values, needed to extract
38 the mechanical properties. The method essentially involves fixing a freestanding membrane over
39 a small window opening and applying a known pressure to it, while measuring the resulting
40 membrane deflection (or curvature). Various models have been developed, based on the sample

41 geometry, to convert the pressure-deflection data to the a (elasto-plastic) stress-strain curve,
 42 which is used to determine mechanical properties such as Young's modulus, Poisson's ratio,
 43 residual stresses, and plasticity parameters [1–3]. During the last 30 years ample research has
 44 been devoted to improve the accuracy of the bulge test by studying the underlying assumptions
 45 such as the influence of bending stiffness [4] and initial conditions e.g. initial film thickness and
 46 residual stress [1,5,6]. Among the different varieties of the bulge test method, the plane-strain
 47 bulge test is most popular [7], where it was shown that for rectangular membranes with in-plane
 48 aspect ratio larger than 4, the stress state in the center of the membrane reduces to a plane strain
 49 condition. This means that the stress and the strain are given by [4]:

$$\kappa_{tt} = \frac{2\delta}{a^2 + \delta^2}, \quad (1)$$

$$\sigma_{tt} = \frac{P}{h \kappa_{tt}}, \quad (2)$$

$$\epsilon_{tt} = \frac{1}{a\kappa_{tt}} \sin^{-1}(a\kappa_{tt}) - 1, \quad (3)$$

50 where κ_{tt} is the curvature in the transverse direction, a is half of the width of the membrane, δ
 51 is the deflection of the apex of the membrane, P is the applied pressure and h is the membrane
 52 thickness. σ_{tt} and ϵ_{tt} denote the normal stress and strain in the transverse direction, respectively.
 53 As a consequence of the plane strain condition in the center of the rectangular membrane, the
 54 transverse stress and the transverse strain can be related by the following constitutive equation:

$$\sigma_{tt} = \left(\frac{E}{1-\nu^2} \right) \epsilon_{tt}, \quad (4)$$

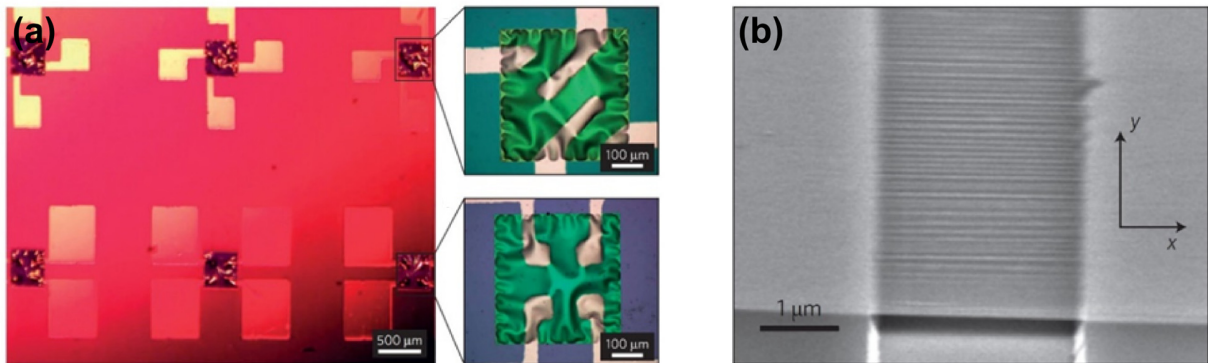
55 where $\left(\frac{E}{1-\nu^2} \right)$ is the plane strain modulus. To extract the Young's modulus (E) and the Poisson's
 56 ratio (ν) separately, an additional test needs to be performed, e.g., a bulge test on a circular or

57 square membrane for which a different stress-strain equation holds, resulting in the biaxial
58 modulus $\left(\frac{E}{1-\nu}\right)$ [2,8].

59 However, such freestanding membranes often buckle as a result of processing induced
60 (compressive) residual stresses in combination with their small out-of-plane bending stiffness,
61 particularly for ultra-thin membranes. In some cases, the buckling is exploited as a functional part
62 in devices, e.g., in bi-stable micro actuators [9–11]. Moreover, the buckling phenomenon in
63 freestanding thin membranes has gained a lot of attention in Micro Solid Oxide Fuel Cells (μ SOFC),
64 where stacks of freestanding membranes serving as electrodes or solid electrolytes, are often
65 buckled as a result of the processing. While initially buckling was considered an issue [12], recent
66 literature suggests that it can actually be beneficial to have these membranes in a buckled state
67 to enhance their functional properties. It has been shown that buckled membranes are
68 mechanically more stable at elevated temperatures, i.e. lower thermomechanical tensile stresses
69 develop compared to a ‘flat’ membrane, often having significant tensile stresses already at room
70 temperature [13], [14]. Mechanical models have been developed to exploit the behavior of such
71 μ SOFC membranes in the post-buckling regime and consequently expand the design space into
72 the low-stress post-buckling regime [12,13]. Recently, controlled buckling patterns in μ SOFC solid
73 electrolyte membranes (Figure 1a) using ‘strain engineering’ have been employed to demonstrate
74 local tuning of ionic conductivity of the electrolyte as an alternative of solid solution doping [15].
75 Furthermore, in the exciting field of graphene, where buckles and ripples are intrinsically present
76 in the suspended configuration (Figure 1b), these phenomena are receiving considerable
77 attention [16] to be exploited in various applications [17], such as improved hydrogen absorption

78 on a rippled graphene surface (due to local curvature), for future efficient hydrogen-based fuel
79 cells [18].

80



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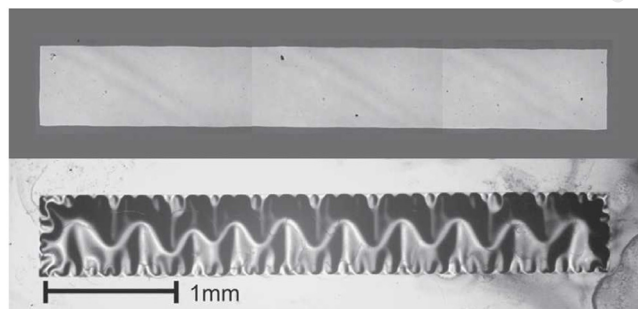
82 *Figure 1: Examples of applications of buckled membranes: (a) design and placement of electrodes to locally*
83 *strain-engineer ionic transport of the solid electrolyte freestanding membrane for μ SOFC application [15]*
84 *(reproduced with permission), (b) SEM micrograph showing ripples in a bilayer suspended graphene*
85 *membrane [16] (reproduced with permission).*

86

87 The presence of such buckling patterns in test samples, as shown in Figure 2, prevents the
88 application of the conventional bulge test. All available literature to date confirms that the
89 conventional bulge test methodology cannot be used to characterize buckled membranes since
90 even at high pressures, buckling patterns typically do not disappear in the membranes and some
91 stress components near the edge of the membrane stay compressive [2,19]. Only in very few
92 cases, the samples can be pressurized, beyond the point where the buckling pattern completely
93 disappears, where the bulge equations (1)-(4) might apply [2,19]. Alternatively, the sample may
94 deform plastically before entering the cylindrical regime. Therefore, such buckled samples are

95 typically discarded and the processing needs to be modified to prevent the buckles to occur, in
96 order to mechanically characterize the membranes accurately. Such processing modifications can
97 be time consuming, costly and sometimes even infeasible or undesired. Moreover, any processing
98 change could influence the actual properties to be determined.

99



100

101 *Figure 2: (Top) Typical 'flat' freestanding bulge test membrane. (Bottom) Buckled membrane*
102 *meandering/telephone cord type pattern [19] (reproduced with permission).*

103

104 Clearly there is a need for a convenient characterization methodology to determine the material
105 properties of the buckled samples in their original (buckled) state.

106 This paper introduces a characterization methodology for testing buckled samples, which builds
107 on the bulge test theory and thus exploits its aforementioned advantages. The approach adopted
108 here is to numerically model the bulge test-like pressure loading of the buckled membrane in the
109 rippled regime, to which the meandering pattern (see Figure 2 bottom) starts transitioning as
110 soon as even minute pressure is applied [20], to understand the mechanics and provide relations
111 for the relevant membrane stress and strain components. To relate the stresses and strains using

112 simple constitutive equations for extracting the material properties, regions of interest (ROI) with
113 simplified stress states are identified and explored. Furthermore, to accurately measure the
114 complex non-uniform three-dimensional displacement field of the buckled membranes, recent
115 advances in bulge test methodology involving integration of Global Digital Image Correlation
116 (GDIC) with conventional bulge test theory to [8] are exploited.

117 **2. Methodology**

118 **2.1 Digital height correlation based bulge test**

119 In conventional bulge test theory, the stresses and the strains obtained using equation (2) and (3)
120 are based on the assumption that they are homogeneous over the membrane, i.e. an infinitely
121 long cylinder or full sphere is assumed. This assumption does not hold anymore when the bending
122 effects at the boundaries play a significant role for films with a relatively large thickness [8].
123 Inhomogeneous fields are also expected in the case of buckled membranes, even for a very small
124 thickness, and in the pressure loaded case. This challenge is addressed by adopting a recently
125 developed Digital Height Correlation (DHC) based bulge test technique [8], to capture the non-
126 uniform 3D displacement fields.

127 The fundamental concept underlying this extension is to apply digital image correlation on the
128 topographical (height) maps of subsequent load increments (pressure increments), resulting in
129 corresponding displacement fields. Curvature fields can subsequently be computed from the
130 displacement fields. Based on the sample geometry, relations such as equation (2) can be used
131 locally to obtain stress from curvature data, while the local strain can be obtained directly from
132 the displacement fields. Therefore, the key assumption of a uniform cylindrical shape and uniform

133 deformation adopted in the conventional bulge test methodology does not need to be fulfilled.

134 Consequently, more accurate, local stress and strain fields can be obtained.

135 Digital Height Correlation is a variant of Global Digital Image Correlation (GDIC) and it is based on

136 the conservation of height, instead of brightness, between a (topographical) reference

137 image $f(\vec{x})$ and a corresponding deformed image $g(\vec{x})$, where \vec{x} is the in-plane position vector,

138 i.e. $\vec{x} = x\vec{e}_x + y\vec{e}_y$, with \vec{e}_x and \vec{e}_y denoting the Cartesian unit vectors. The conservation of height

139 is written as:

$$f(\vec{x}) = g(\vec{x} + u_x(\vec{x})\vec{e}_x + u_y(\vec{x})\vec{e}_y) - u_z(\vec{x}), \quad (5)$$

140 where u_x and u_y are the in-plane displacement components in x and y direction respectively,

141 while u_z is the out-of-plane displacement component. The image residual $r(\vec{x})$ is defined as:

$$r(\vec{x}) = f(\vec{x}) - g(\vec{x} + u_x(\vec{x})\vec{e}_x + u_y(\vec{x})\vec{e}_y) - u_z(\vec{x}) + n_0(\vec{x}), \quad (6)$$

$$\approx g(\vec{x}) + (\vec{\nabla}g \cdot \vec{e}_x)u_x(\vec{x}) + u_y(\vec{x}) + (\vec{\nabla}g \cdot \vec{e}_y)u_y(\vec{x}) - u_z(\vec{x}) + n_0(\vec{x}), \quad (7)$$

142 where n_0 is the image noise and $\vec{\nabla}g$ is the gradient of the deformed image g . The square of the

143 image residual is minimized over the region of interest in the GDIC algorithm,

$$\gamma^2 = \int_{ROI} r(\vec{x})^2 d\vec{x}, \quad (8)$$

144 where γ is the global residual. The 3D displacement vector is given by:

$$\vec{u}(\vec{x}) = u_x(\vec{x})\vec{e}_x + u_y(\vec{x})\vec{e}_y + u_z(\vec{x})\vec{e}_z. \quad (9)$$

145 To make this a well-posed optimization problem, the displacement field is parameterized as a

146 sum of basis functions $\varphi_i(\vec{x})$ that act over the ROI and weighted by a discrete set of degrees of

147 freedom λ_i . The choice of the basis and shape functions is based on the expected deformation
 148 complexity. It should be noted that different shape functions may be required in the x, y and z
 149 directions to capture the deformation. It has been shown that the shape of square and
 150 rectangular bulged membranes are well described by polynomial functions [21].

$$\vec{u}(\vec{x}) = \sum_i^l \lambda_i \varphi_i(\vec{x}) \vec{e}_x + \sum_i^m \lambda_i \varphi_i(\vec{x}) \vec{e}_y + \sum_i^n \lambda_i \varphi_i(\vec{x}) \vec{e}_z, \quad (10)$$

151 where the basis functions φ_i are chosen here to be polynomial functions, given by:

$$\varphi_i = x^\alpha y^\beta \quad (11)$$

152 Subsequently, the strain fields can be computed from the extracted displacement fields using an
 153 appropriate strain definition. To determine the relevant stress components, the curvature field is
 154 obtained with the same DHC measurement. The curvature tensor κ is determined by taking the
 155 spatial gradient of the outward normal vector \vec{n} field as:

$$\kappa(\vec{x}) = \vec{\nabla} \otimes \vec{n}(\vec{x}), \quad (12)$$

156 where the outward normal vector is the normalized gradient of the position field $z(\vec{x})$, as given
 157 by:

$$\vec{n}(\vec{x}) = \frac{\vec{\nabla} z(\vec{x})}{\|\vec{\nabla} z(\vec{x})\|}. \quad (13)$$

158 The curvature fields in transverse direction is given by:

$$\kappa_{tt}(\vec{x}) = \vec{t}_x(\vec{x}) \cdot \kappa(\vec{x}) \cdot \vec{t}_x(\vec{x}), \quad (14)$$

159 where \vec{t}_x is a vector tangent to the membrane surface along the x direction. These curvature
160 fields will be used to determine the local stress components. For instance, for a local region with
161 a plane strain state (in the middle of a non-buckled rectangular membrane), the curvature can be
162 related to the hoop stress by:

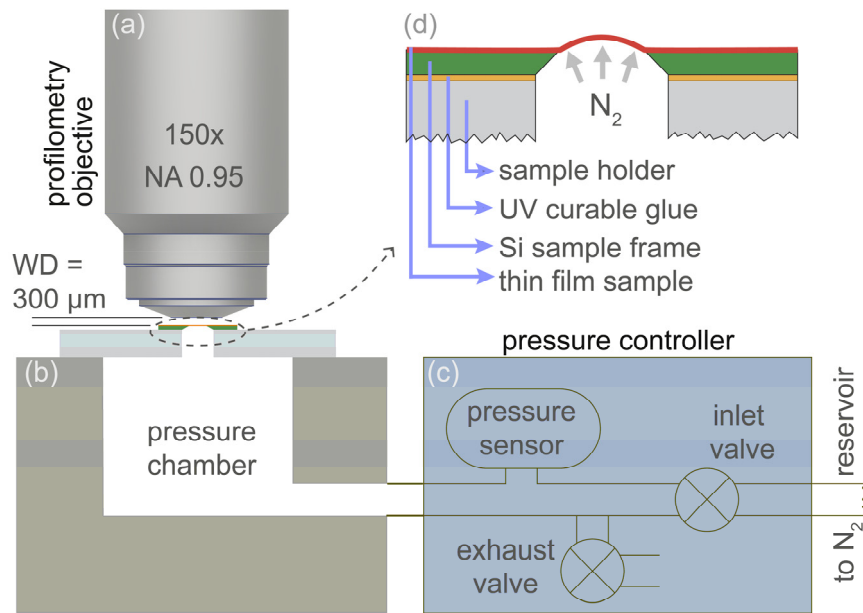
$$\sigma_{tt} = \frac{P}{t\kappa_{tt}}, \quad (15)$$

163 where P is the applied (uniform) pressure and t is the thickness of the membrane.

164 **2.2 Experimental setup and procedure**

165 A custom-made, gaseous pressure medium based bulge test setup (similar to the setup reported
166 in Ref. [22]) is used (see Figure 3b) for the experiments. The setup consists of a sample holder
167 block attached to a pressurized N_2 reservoir through a pressure regulator. A commercially
168 available pressure regulator (MFCS-EZ by Fluigent) was used with a range of 0 – 200 kPa and a
169 resolution of 6 Pa (0.03% of full range). The pressure regulator and consequently the bulge test
170 setup has a fast response and settling time, which is an advantage over liquid based setups. While
171 liquid pressure-medium setups reach higher pressures, as needed for testing stiff (thick or
172 narrow) films or plates, they can suffer adversely from pressure buildup problems in case of a
173 minute leakage or the presence of gas bubbles in the pressure medium. This is usually not a
174 problem in gaseous pressure-medium setups, since any drop in pressure due to leakages is
175 directly compensated through the connected large reservoir. Furthermore, the setup is less
176 sensitive to pressure changes due to ambient temperature variations compared to liquid based
177 setups.

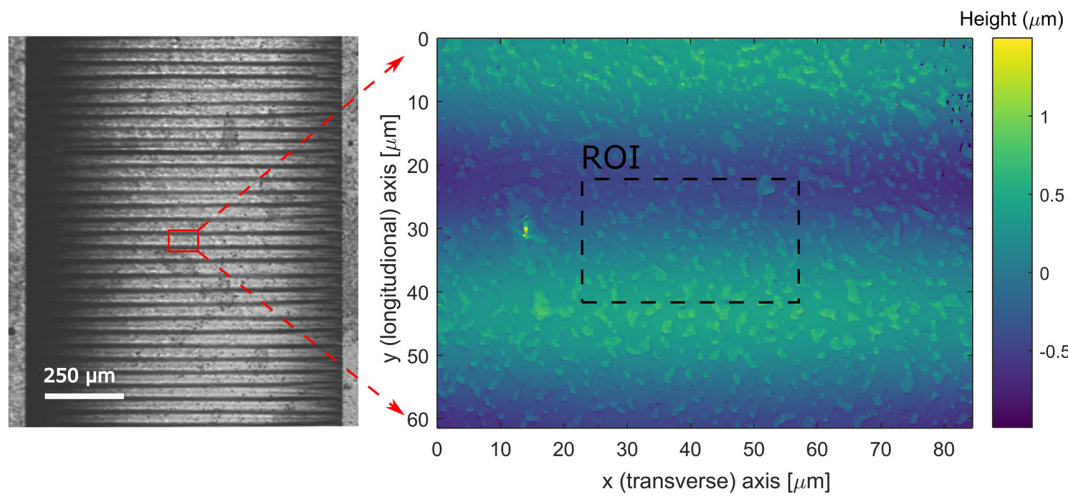
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179
 180 *Figure 3: Schematic of experimental setup for performing bulge test experiments: (a) high-resolution*
 181 *optical profilometry object, (b) bulge test setup, (c) pressure regulator to control pressure loading with*
 182 *regulator N₂ supplied with through a cylinder reservoir, (d) magnified view of cross-section of a typical bulge*
 183 *sample with a thin film deposited in a silicon frame with an etched window in the center. The sample is*
 184 *fixed by gluing it onto the bulge tester sample holder.*

185
 186 The setup is placed under a commercially available optical profilometer, Sensofar Plμ 2300. The
 187 profilometer is used in confocal mode to obtain full-field topographical images. The highest
 188 magnification lens, with a magnification of 150x and Numerical Aperture (N.A.) of 0.95 is used to
 189 obtain high resolution images of the narrow ripples which are 20 – 30 μm wide. The resulting
 190 field of view is 84x63 μm² with in-plane spatial sampling (pixel size) of 0.11 μm, while the
 191 effective height resolution obtained is in the range of 25 nm.

192 Rectangular bulge test samples used for the proof of principle experiment were fabricated by
193 deposition of a (proprietary) multi-layered stack consisting of transition metals and oxides on a
194 monocrystalline $650\ \mu\text{m}$ thick silicon wafer. The freestanding window was created by wet etching
195 from the back side of the wafer with a $1 \times 5\ \text{mm}^2$ window. The sample has a total thickness of
196 $64\ \text{nm}$ and (volume averaged) Young's modulus (estimated by volume averaging using the rule
197 of mixtures) of $217\ \text{GPa}$ and Poisson's ratio of 0.35. In general, application of DIC requires a good
198 pattern providing sufficient image contrast. In case of digital height correlation this contrast is
199 achieved by local differences in height on the sample surface. Since the native surface roughness
200 of the samples is in the order of a few nanometers, below the resolution of the profilometer, a
201 'height' pattern is applied. For this purpose, $500\ \text{nm}$ mono-dispersed polystyrene microspheres
202 (by micromod®) were used. The microspheres were applied using the drop-casting method. The
203 particles are provided in a dense suspension and are further diluted (by a dilution factor of ~ 40)
204 in ethanol to achieve the required particle density on the sample surface (see Figure 4). A
205 relatively dense pattern is required to capture the expected inhomogeneous displacement fields.
206 Since the particles do not form a continuous layer, adhering to the sample surface (upon contact,
207 without the need for an adhesive) as single particles or homogeneously distributed aggregates
208 composed of few particles, their influence on the mechanics of the membrane is assumed to be
209 negligible. Moreover, during the pressure loading step, the particles adhere well to the sample
210 and no pattern change or degradation is observed, which is important for reliable application of
211 DIC.



212

213 *Figure 4: Images of ripples in a pressurized membrane with, optical image (left) and topographical image*
 214 *of an area imaged during the experiment, marked in the optical image by the red rectangle (right). The*
 215 *region of interest marked by the rectangle with black dashed lines is selected for DHC. The roughness*
 216 *pattern visible in the topographical image consists of clusters of 500 nm polystyrene microspheres, applied*
 217 *as a DHC pattern.*

218

219 **2.3 Numerical modelling**

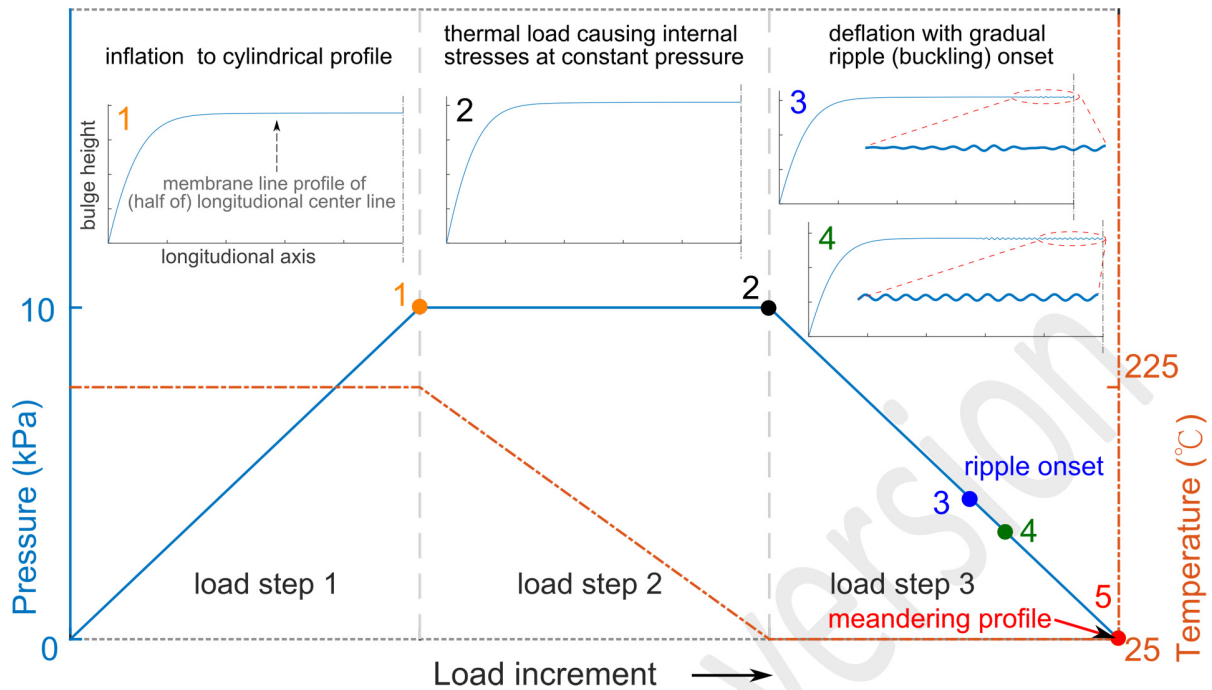
220 A non-linear FE model was set up to simulate the pressure loading of the test sample using the
 221 commercial FE program, MSC Marc/Mentat®. Since in the test specimens the meander profile
 222 starts transitioning to a ripple profile as soon as a slight pressure is applied [20] [see Figure 6], the
 223 analysis is focused on ripple and the cylindrical regime. As the substrate is many orders of
 224 magnitude stiffer than the film, it is modelled with rigid boundary conditions at the edges of the
 225 film. The rectangular membrane was meshed uniformly with quadrilateral 4-node thick-shell
 226 elements having three translational and three rotational degrees of freedom at each node. A

227 mesh convergence study was performed and it was found that the solution becomes mesh
228 independent at a size of 80×400 elements (used as the mesh size for the model).

229 Four batches of multi-layer test samples were available for testing, with varying material stack
230 thickness ratios and hence different overall membrane thickness and volume averaged Young's
231 modulus. The focus here is on developing a methodology for obtaining thickness-averaged
232 mechanical properties of the buckled membranes. This is also the case for conventional bulge test
233 which also provides thickness-averaged data only. Therefore, the Young's modulus and Poisson's
234 ratio in the FE model were set to 153.45 *GPa* and 0.351, respectively, based on a batch of
235 samples planned for fabrication but was never produced. Note, however, that the exact
236 properties used for the FE model are unimportant for the analysis and conclusion made and the
237 model serves as a general platform for virtual experiments. The dimensions of the membrane in
238 the FE model were taken as $1 \times 5 \text{ mm}^2$ in accordance with the size of the test samples, while the
239 membrane thickness was set to a value of 50 *nm* based on the median thickness batch.

240 To induce buckling in the membrane in-plane compressive load is required. The residual
241 compressive stress was simulated through a thermal loading step resulting in an in-plane stress
242 applied by the substrate (frame) on the freestanding membrane. This is caused by higher
243 contraction of the substrate w.r.t the membrane on cooling down from a high temperature. Since
244 the substrate was modelled by rigid membrane boundaries, the difference in coefficient of
245 thermal expansion (α) of the substrate and the membrane ($-2.6 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) was assigned to
246 the membrane, to simulate this effect.

247



248

249 *Figure 5: Three-step thermo-mechanical loading procedure for gradual onset of buckling, initially as a ripple*
 250 *profile, finally settling into a meandering profile at the last load increment. In load step 1, the pressure is*
 251 *increased while temperature is kept constant to result in an inflated cylindrical profile. Subsequently, in*
 252 *load step 2, the temperature is decreased while the pressure is kept constant, resulting in thermal stresses*
 253 *that (gradually) manifest themselves as increasing residual compressive stress while the pressure is*
 254 *reduced (as the temperature is kept constant) in load step 3, triggering the onset of ripples. Note that the*
 255 *top sub-figures represent membrane deflection for the corresponding load increment with a line profile of*
 256 *half of the longitudinal center line.*

257

258 In order to avoid typical numerical instabilities at the bifurcation point (i.e. suddenly going from
 259 a flat to meandering shape in a single increment), a method used to model strongly buckled
 260 square membranes from Ref. [23] was adopted here. This involves bypassing the bifurcation point
 261 by adopting the three-step loading procedure illustrated in Figure 5. First the membrane is bulged

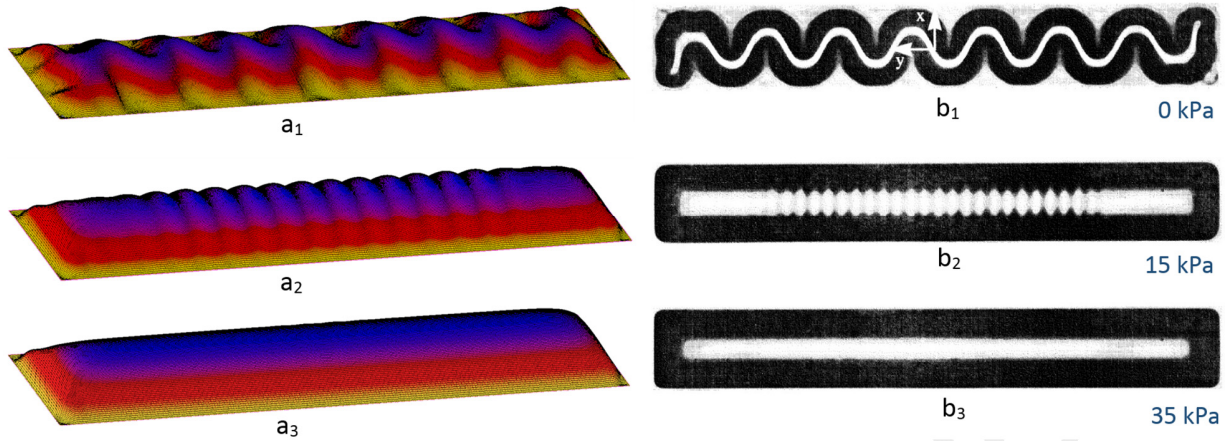
262 by application of a pressure load (Figure 6a₃). Then, compressive residual stresses are induced in
263 the bulged membrane by applying the thermal loading step. In the final step the pressures is
264 gradually decreased to let the membrane slowly settle down into a rippled profile (Figure 6a₂). If
265 the pressure is completely removed the membrane transitions from a rippled to a meandering
266 configuration (Figure 6a₁). Using this method, in addition to bypassing the bifurcation point, the
267 strong and sudden geometrical nonlinearities expected at the transition from planar to
268 meandering configuration are avoided by gradually settling from the cylindrical to the rippled
269 configuration.

270 **3. Numerical analysis**

271 **3.1 Simulation results**

272 Using the three step loading procedure, explained in the previous section, the numerical model
273 adequately captures the three different regimes seen in the experiments, i.e. the rippled,
274 meandering regime, and the cylindrical regime. Moreover, the evolution of the rippled regime,
275 with increasing pressure as well as the transitions between the different regimes seems are well
276 captured. Comparison with experimentally observed regimes shown in Figure 6 provides a
277 qualitative validation of the model.

278



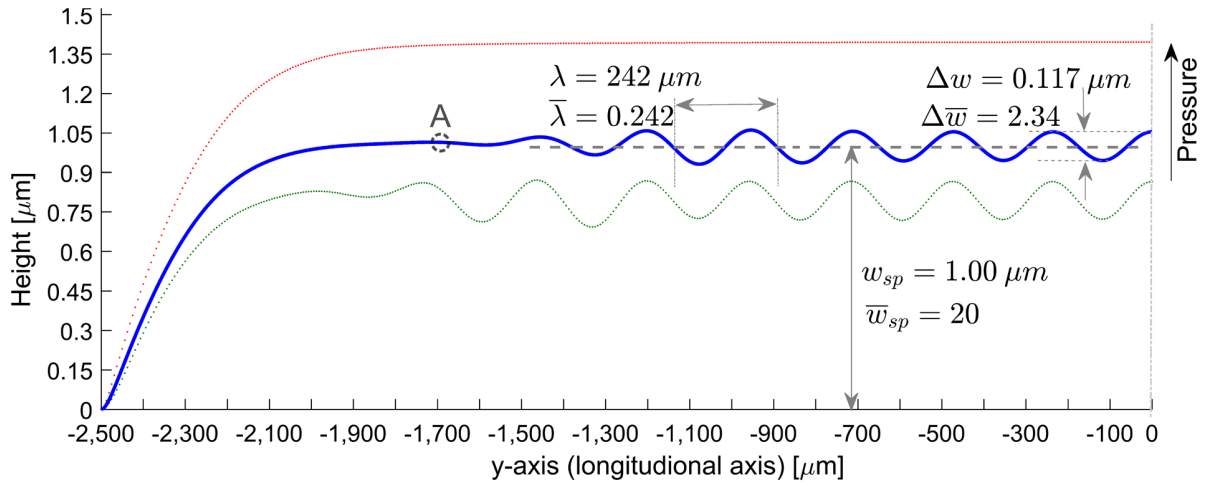
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280 *Figure 6: Comparison between numerically modelled (a) and experimentally observed [24] (b) buckling*
 281 *regimes: meandering regime (a_1, b_1), rippled regime (a_2, b_2) and cylindrical regime (a_3, b_3), with membrane*
 282 *size $1 \times 5 \text{ mm}^2$ (a) and $0.65 \times 4.5 \text{ mm}^2$ (b). Note that similar buckling (meandering and rippled) regimes as in*
 283 *Ref. [24] are observed in our samples (see Figure 4), however, since cylindrical regime is not reached, the*
 284 *images are not shown here.*

285

286 In order to also quantitatively validate the FE model, the Energy Minimization Method (EMM)
 287 based model reported by Kramer et al. ([24]) is exploited here. This model was developed to
 288 describe the rippled regime for a similar rectangular membrane with width a and thickness h , as
 289 discussed here. The model is able to predict the reduced ripple wavelength $\bar{\lambda} = \frac{\lambda}{a}$, the reduced
 290 peak-to-peak amplitude $\Delta\bar{w} = \frac{\Delta w}{h}$ and the reduced ripple free amplitude $\bar{w}_{ps} = \frac{w_{ps}}{h}$ (see Figure
 291 7), as a function of the reduced prestrain $\bar{\epsilon}_0 = \frac{\epsilon a^2}{h^2}$ and the reduced pressure $\bar{p} = \frac{p(1-\nu^2)a^4}{Eh^4}$.

292



293

294 *Figure 7: Centre line profile of (half of) the membrane along the longitudinal direction, extracted from FE*
 295 *simulation at increasing pressure values loads, with the middle profile (in blue) corresponding to $\bar{\epsilon}_0 = -500$*
 296 *and $\bar{p} = 7.48 \times 10^4$ displaying relevant parameters. The three profiles display the evolution of the buckling*
 297 *pattern with increasing pressure.*

298

299 For realistic values of the applied pressure and residual stress (due to thermal loading) the results
 300 for our test specimens lie significantly outside the boundaries of the plotted results presented in
 301 Ref. [25], due to their very small thickness. In order to validate the FE model, smaller temperature-
 302 load and pressure values are applied to enable a comparison with the EMM results. Based on the
 303 values of E , ν and the coefficient of thermal expansion (α), used in the FE model, a temperature
 304 difference of 0.347°C and a pressure load of 0.0818 Pa is calculated, which corresponds to a
 305 reduced strain, $\bar{\epsilon}_{0test}$ of -500 and a reduced pressure \bar{p}_{test} of 7.4811×10^4 , respectively, thus
 306 bringing the $\bar{\epsilon}_0$ and \bar{p} values within the bounds of the reported EMM results available (in Ref.
 307 [25]). A line profile along the longitudinal axis in the middle of the membrane from the FE
 308 simulations is displayed in Figure 7 with the calculated reduced parameters which are given in

309 Table 1, along with the corresponding parameters from the EMM results at the same $\bar{\epsilon}_0$ and \bar{p}
310 values.

311

312 *Table 1: Comparison of $\bar{\lambda}$, \bar{w}_{ps} and $\Delta\bar{w}$ predicted by EMM [25] and by the present FE analysis*

| | FE | EMM |
|-----------------|------------------|-------------------|
| $\bar{\lambda}$ | 0.242 ± 0.01 | 0.250 ± 0.005 |
| \bar{w}_{ps} | 20 ± 0.4 | 19.7 ± 0.2 |
| $\Delta\bar{w}$ | 2.3 ± 0.4 | 2.6 ± 0.1 |

313

314

315 As can be seen in Table 1, the FE and EMM results agree for all three parameters within readout
316 error from the FE and EMM results. However, the deviation for $\Delta\bar{w}$ is significant. On the one hand,
317 the higher variation for $\Delta\bar{w}$ can partially be attributed to a higher readout error from the EMM
318 plots, reflecting in the error bar in Table 1. On the other hand, the peak-to-peak amplitude varies
319 over the length of the line profile in the FE results (see Figure 7), thus an average value of a
320 relatively small magnitude is taken thereby, possibly contributing to the relatively high deviation.
321 Based on the adequate agreement for all three values, FE model is considered valid.

322 **3.2 Numerical analysis of a suitable regions of interest:**

323 As the model adequately captures the mechanics of the pressure loaded buckled membrane, it is
324 well suited to be used as a numerical framework to test the methodology developed here, i.e.

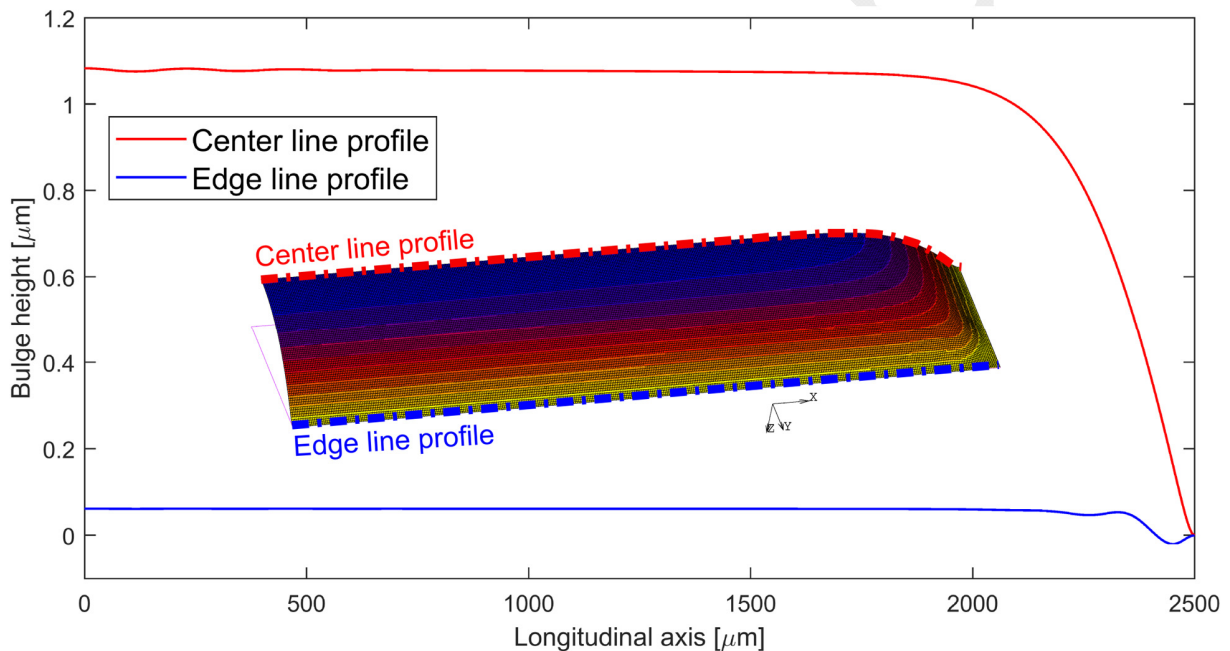
325 serving as a virtual experiment. Various ROIs are analyzed, where, with suitable modifications,
326 bulge test analysis can be applied. The criteria for choosing a suitable ROI are:

- 327 1. The presence of a simplified stress or strain state, e.g. plane-strain
- 328 2. Membrane stresses can be conveniently calculated using an analytical equation, e.g. Eq. (2)
- 329 3. Membrane strains (using DHC) can be accurately determined

330 While the cylindrical regime may seem promising from the analysis point of view as classical
331 simple plane strain bulge equations may be applicable there, most initially-buckled membranes
332 (of various types) fracture before reaching the cylindrical regime, while those membranes that
333 can sustain a high enough pressure typically go into plasticity before entering the cylindrical
334 regime. Alternatively, the meandering regimes is not interesting from a practical point of view, as
335 it is only accessible at minute pressures [20]. Since the rippled regime exists throughout almost
336 the whole pressure loading cycle, only the rippled regime is here considered for further analysis.
337 Therefore, the analysis must be performed in the ripple regime.

338 There are multiple reason for choosing the ROI along the longitudinal center line of the
339 membrane. First, it is observed in the FE simulations and the experiments that the magnitude of
340 the ripples close to the longitudinal edge is lower than that of the same ripples in the center of
341 member, see Figure 8. This is due to the boundary constraints provided by the edge (on the edge
342 ripples can, of course, not form). Second, considering the third criterion of accurate
343 determination of the membrane strains using DHC keeping the ROI always in the field of view is
344 important. Due to symmetric deformation, an ROI in the center of the membrane experiences the
345 least rigid body motion, while closer to the membrane edge the large out of plane rotations result

346 in large rigid body motion. Furthermore, these large out-of-plane rotations can affect the image
 347 residual, due to the change in effective viewing angle of the pattern, as discussed below. Finally,
 348 the boundary conditions at the edge are never as perfect as assumed in a Finite Elements
 349 simulation, especially when a dry etch is used to free the membrane, therefore, it is best to do
 350 the analysis far away from the edge, where these local boundary effects are negligible due to the
 351 well-known Saint-Venant's principle in solid mechanics. Therefore, the most suitable ROI is
 352 identified as an ROI along the longitudinal center line far away from the edges.



353
 354 *Figure 8: Comparison of simulated center line profile and edge line profile (calculated at the first free node*
 355 *from the edge), at the first FEM increment that shows ripples (i.e. point 3 in Figure 5).*

356
 357 It can be assumed that the membrane is in a state of plane stress with respect to the thickness
 358 direction, since the thickness of the membrane is very small relative to the other two dimensions

359 (with free contraction in the thickness direction). Indeed, the stress in the thickness direction due
360 to the applied pressure is negligible compared to the in-plane stress. Furthermore, isotropic
361 material behavior is assumed (which is common for thin films produced with thin film deposition
362 techniques). Therefore, the isotropic linear elastic plane stress equations (Eq. (16) and (17)) apply
363 for the present analysis.

$$\epsilon_{yy} = \frac{1}{E}\sigma_{yy} - \nu\frac{1}{E}\sigma_{xx}, \quad (16)$$

$$\epsilon_{xx} = -\nu\frac{1}{E}\sigma_{yy} + \frac{1}{E}\sigma_{xx}, \quad (17)$$

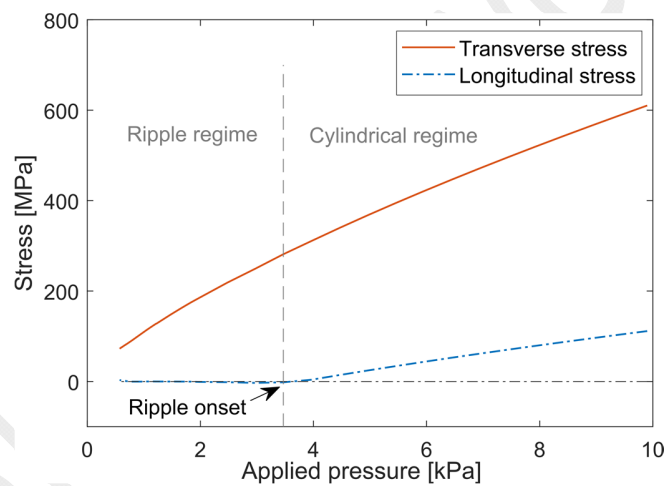
364

365 where ϵ_{yy} and ϵ_{xx} are the normal membrane strains in the longitudinal and transverse direction,
366 respectively, while, σ_{yy} and σ_{xx} are the membrane stresses in the longitudinal and transverse
367 direction respectively.

368 The FE analysis shows that in the cylindrical regime the magnitude of the transverse stress σ_{xx} is
369 much larger than the longitudinal stress σ_{yy} as expected for a rectangular geometry. In the last
370 loading step, as the pressure is being reduced, at a certain stage (labelled point 3 in Figure 5), the
371 longitudinal stress becomes compressive while the transverse stress is still tensile. As soon as the
372 stress state in the longitudinal direction becomes compressive, the membrane releases the
373 compressive stress by buckling, in the form of a rippled pattern. Given its small thickness and
374 consequently negligible bending stiffness, the membrane does not possess the ability to support
375 a compressive stress, which is therefore released in a buckling pattern. This phenomenon is
376 illustrated in Figure 9. After the emergence of the ripples, as the pressure is further decreased,

377 the longitudinal stress remains almost zero. The transverse stress however is still tensile and it
 378 keeps on decreasing with decreasing applied pressure. The small longitudinal stress varies over
 379 the width of a ripple due to the bending induced stress, and is therefore most compressive at the
 380 valley of the ripple while being least compressive at the peak of the ripple. At the crossover point
 381 in the middle of a ripple where the curvature is zero (marked with point C in Figure 11a) however,
 382 where bending effects do not contribute, the compressive stress is only due to the residual stress
 383 and ~ 300 times lower than σ_{xx} (at the ripple transition point), i.e. negligible.

384



385
 386 *Figure 9: Calculated transverse and longitudinal stress at the center in transverse direction and near the*
 387 *center in longitudinal direction, i.e. in the middle of a ripple where the curvature is zero, as a function of*
 388 *the applied pressure in the ripple and cylindrical regimes, revealing that the longitudinal stress stays close*
 389 *to zero in the rippled regime.*

390

391 Accordingly, this configuration is close to uniaxial tension, i.e. there is only a non-zero stress in
392 the transverse direction while the deformation in the longitudinal direction is governed by free
393 contraction. Therefore Eq. (16) and (17) reduce to:

$$\epsilon_{yy} = -\nu \frac{1}{E} \sigma_{xx}. \quad (18)$$

$$\epsilon_{xx} = \frac{1}{E} \sigma_{xx}. \quad (19)$$

394 Inserting Eq. (19) into Eq. (18):

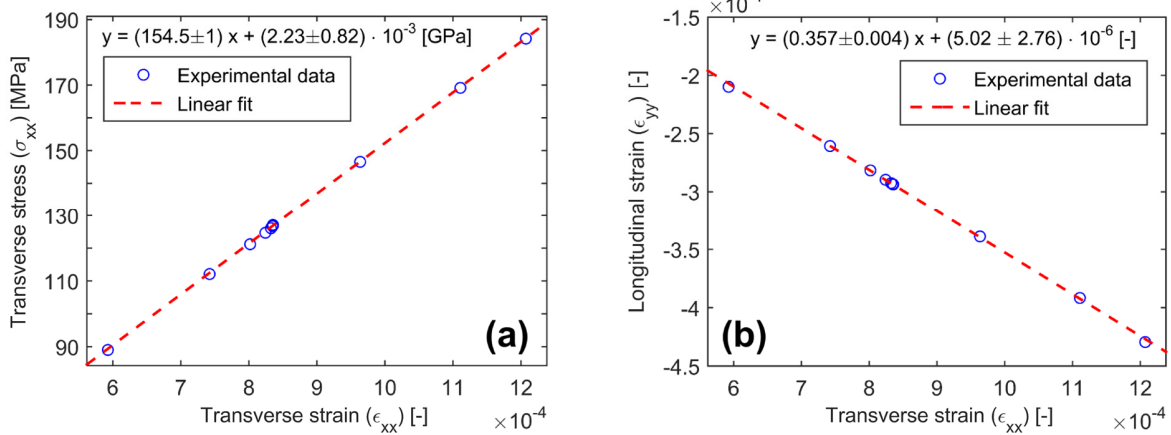
$$\epsilon_{yy} = -\nu \epsilon_{xx}. \quad (20)$$

395 Eq. (19) and (20) provide a direct (explicit) relationship between the relevant stress and strain
396 components, which is linear, similar to the conventional plane strain bulge test. Furthermore,
397 unlike the plane strain bulge test, where the Young's modulus and the Poisson's ratio are coupled
398 in the plane strain modulus ($\frac{E}{1-\nu^2}$), here E and ν can be obtained separately from a single
399 experiment.

400 At this point it is important to note that in the regions where the ripples have locally diminished
401 along the longitudinal direction, e.g. point A in Figure 7, neither plane strain, nor plane stress
402 condition holds with respect to the in-plane directions due to which Eq. (16) and (17) cannot be
403 reduced to a simpler form to yield E and ν , even in a coupled form. Therefore, the analysis is best
404 performed at the true center of the membrane (i.e. the middle in longitudinal direction), where
405 the ripples disappear for the highest pressure, making the above-mentioned analysis valid for the
406 largest pressure regime (another reason is that the rigid body motion is lowest at the true center
407 as discussed above).

408 The validity of the uniaxial tensile state (Eq. (19) and (20)) is assessed with a virtual experiment.
 409 The stress and strain state is extracted from a node at the cross-over (zero curvature) point
 410 (denoted as point C in Figure 11) in the ripple in the center of the membrane. The Young's
 411 modulus is extracted from the gradient of the $\sigma_{xx} - \epsilon_{xx}$ plot (see Figure 10a), whereas the
 412 Poisson's ratio is extracted from the $\epsilon_{yy} - \epsilon_{xx}$ plot (Figure 10b). The Young's modulus (with a
 413 value of 154.5 GPa) is extracted with an error of 0.68 %, while the resulting Poisson's ratio (with
 414 a value of 0.357) reveals an error of 1.7 % with respect to the respective reference (input) values
 415 in the FE model.

416



417

418 *Figure 10: Stress and strain values extracted from FE results in the rippled regime to obtain Young's*
 419 *modulus (E) and Poisson's ratio (ν): (a) Transverse stress (σ_{xx}) vs. transverse strain (ϵ_{xx}) with a linear fit*
 420 *to provide E, and (b) Longitudinal strain (ϵ_{yy}) vs. transverse strain (ϵ_{xx}) with a linear fit to provide ν.*

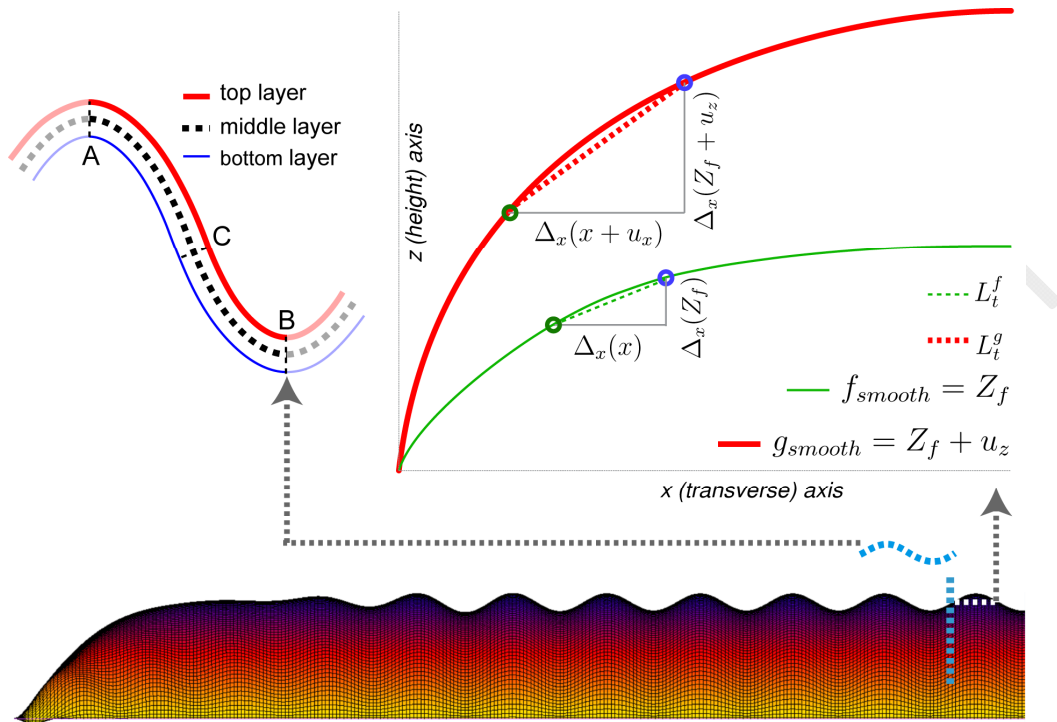
421

422 Note that Eq. (15) is still valid in the rippled regime, allowing to relate the local curvature and
423 applied pressure to the transverse stress at the crossover point. This is the case, since at the
424 crossover point the local curvature in the longitudinal direction is almost zero, whereas σ_{yy} is
425 negligible w.r.t σ_{xx} (as illustrated in Figure 9), thus the pressure applied to the face of the
426 membrane is entirely balanced by the transverse stress. The validity of Eq. (15) in the rippled
427 regime is numerically assessed by computing the transverse stress σ_{xx} from κ_{tt} (extracted from
428 the FE simulation) and the applied pressure P (known in the simulation). The resulting value was
429 plotted against the transverse strain ϵ_{xx} extracted from the FE simulation to determine the
430 Young's modulus, which has an error of 1.4 %.

431 Based on the relatively small errors in the extracted values, it is concluded that the proposed
432 method is promising. This analysis sets the basis of the characterization methodology. In a real
433 experiment, σ_{tt} can be determined using Eqs. (12) - (15) from the position field in the deformed
434 configuration. While ϵ_{yy} and ϵ_{xx} can be obtained from the 3D displacement fields using Eqs. (26)
435 - (29), as will be shown in the next section. Furthermore, for the membranes that have not
436 ruptured before reaching the cylindrical regime at higher pressure, this method can be used in
437 conjunction with the plane strain equation. In that case, the same ROI analyzed in the rippled
438 region can also be analyzed in the cylindrical plane strain state.

439 Practical application of DHC to the bulged buckled membranes requires highly accurate
440 determination of 3D displacement fields as well as the position field in the reference
441 configuration. This procedure is first tested in a virtual experiment. To this end, the displacement
442 fields are extracted from the FE simulation in a typical ROI, from peak to valley in the longitudinal
443 direction (marked in Figure 11a point A to B).

444



445

446 Figure 11: A schematic illustrating: top, middle and bottom layers of membrane long the longitudinal
 447 direction **(a)**; and L_t^g and L_t^f used for determining the transverse membrane strain (ϵ_{xx}) **(b)**; and
 448 longitudinal and transverse sections represented on FE simulation of rippled membrane **(c)**. $\Delta_x(\cdot)$ and $\Delta_y(\cdot)$
 449) denote operators that act on the specified field to produce the field's finite difference in, respectively, the
 450 x - and y - direction.

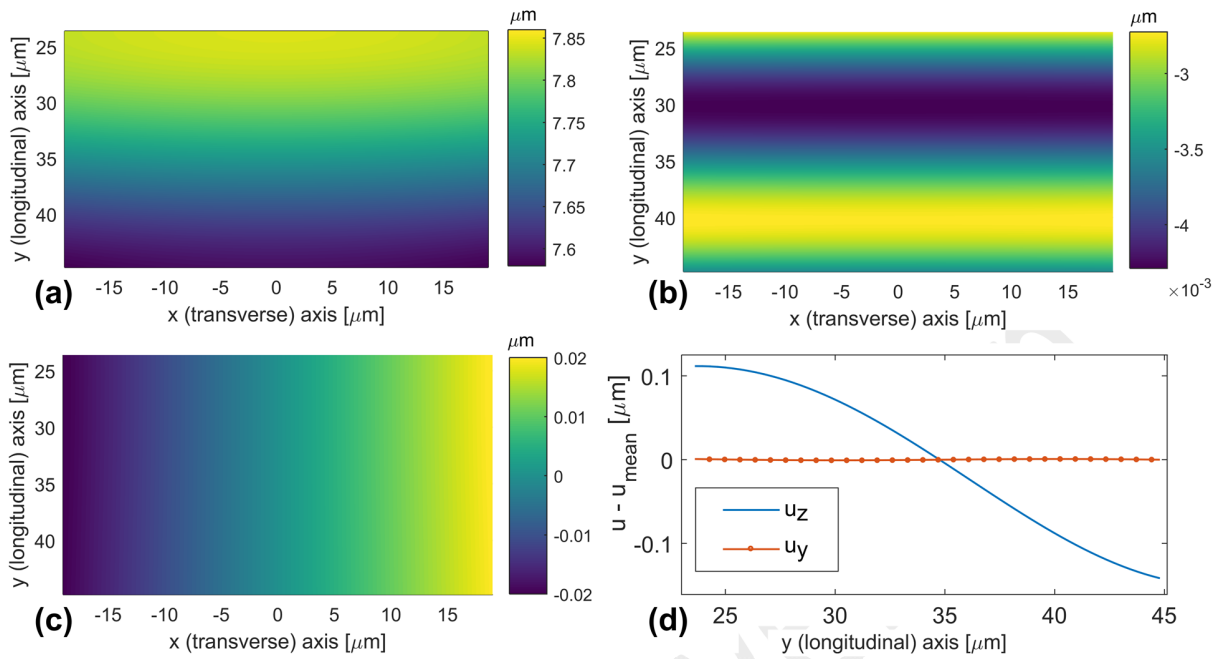
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452 These displacement fields and the resulting analysis correspond to the membrane mid-plane. In
 453 DHC however, only the membrane surface (top or bottom, see Figure 11a) can be analyzed and
 454 thus only displacements at the surface are determined. This causes a discrepancy of less than
 455 0.5 % between the transverse displacements extracted from the mid-plane and the outer surface

456 (top/bottom) plane in the FE simulation. However, for the longitudinal displacement, due to the
457 significant curvature in that direction, the displacement at the surface is found to be considerably
458 different with respect to the displacement at the mid-plane. Nevertheless, the average of the
459 longitudinal strain over a full and half ripple width (as considered here), on the top surface should
460 be the same as on the mid-plane, since the difference in the length of a line profile from the peak
461 to the valley of the ripple in the top surface, middle and bottom planes (Figure 11a) will be
462 negligible, especially for small strains. This has been validated by the FE simulation. Therefore,
463 the average longitudinal strain over half the ripple width is computed, and plotted against the
464 transverse strain in Figure 13a, resulting in a value of Poisson's ratio with an error of only 0.3 %.

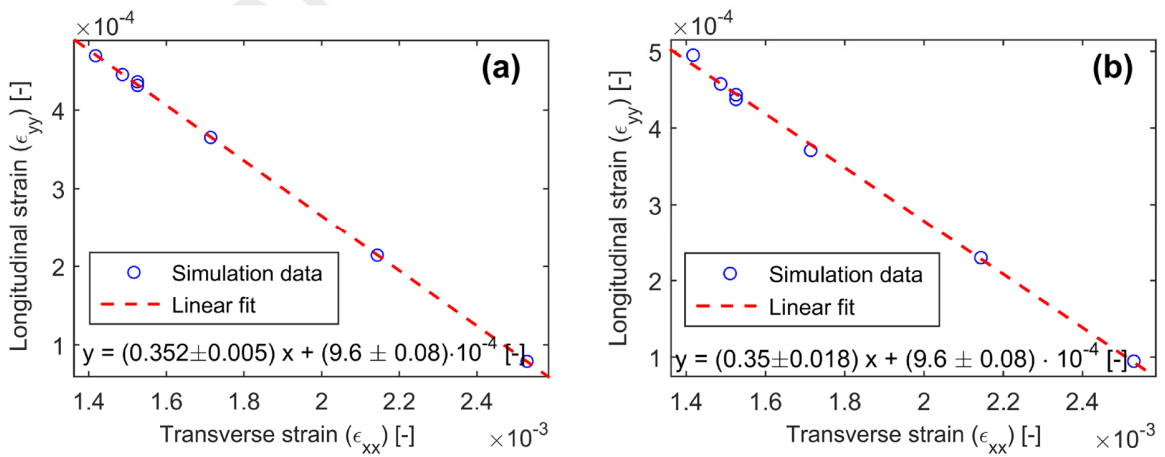
465 Another important observation can be made from Figure 12b, which shows that the longitudinal
466 displacement is in the order of a few nanometers. The best optical in-plane resolution is
467 approximately 250 nm. Even considering a subpixel resolution of ~ 0.01 pixels that can be
468 captured with DIC techniques, the longitudinal displacement cannot be captured accurately due
469 to the physical resolution limit. Furthermore it has been discussed in literature [26] that for large
470 out-of-plane rotations (high curvature changes), a systematic error in the displacement fields
471 might be introduced. To address this shortcoming, the longitudinal (in-plane) strain is disregarded
472 and only the rigid body motion in the longitudinal direction is used, together with the height
473 displacement field for determining the longitudinal membrane strain ϵ_{yy} , see Figure 13b. The
474 resulting Poisson's ratio still has an error limited to 0.3%. Therefore, only the rigid-body motion
475 will be included as the displacement description in DHC to capture the longitudinal displacement
476 field.

477



478
 479 *Figure 12: Displacement fields and line-profiles extracted from FE simulations from peak to valley along the*
 480 *longitudinal axis in the rippled regime: (a) out-of-plane displacement field (u_z), (b) longitudinal*
 481 *displacement field (u_y), (c) transverse displacement field (u_x), (d) mean-subtracted line-profiles of u_y and*
 482 *u_z at $y = 33 \mu\text{m}$.*

483



484

485 *Figure 13: Graphs of the longitudinal strain (ϵ_{yy}) as a function of the transverse strain (ϵ_{xx}) linear fits to*
486 *extract the Poisson's ratio, where ϵ_{yy} is averaged over a line-profile that ranges from ripple peak to valley,*
487 *for (a) ϵ_{yy} obtained from both \bar{u}_y and \bar{u}_z (from FE simulations) and (b) ϵ_{yy} obtained only from \bar{u}_z .*

488

489 **4. Proof of principle experiment**

490 Here, the results of a successful test serving as a proof of principle experiment to show the
491 feasibility of application of the method in a real experiment are shown. Pressure increments of
492 2.5 kPa were applied to the sample and kept constant while the topographical images were
493 acquired with the profilometer.

494 An ROI shown in Figure 4 from the peak to valley of a ripple is selected for DHC analysis. As
495 discussed in Ref. [8], a limited number of degrees of freedom (dofs) has to be used to capture the
496 deformation kinematics accurately. Too few dofs restrict the kinematics while too many can cause
497 the correlation to diverge while making it sensitive to image noise. Here, the expected order of
498 displacements is determined from the out-of-plane and transverse displacement fields extracted
499 from the FE simulations (Figure 12), defining the initial set of shape functions to be used for the
500 DHC analysis. More shape functions are subsequently added to optimize the correlation until the
501 residual fields cannot be further minimized. The longitudinal displacement only consists of a rigid-
502 body term.

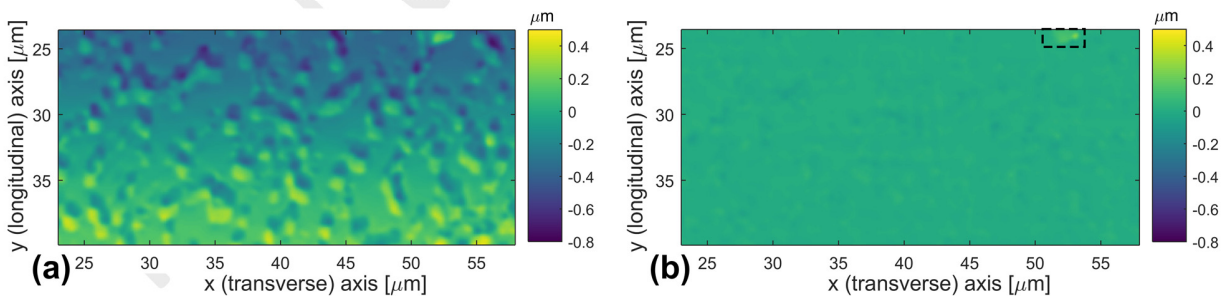
$$u_x = \lambda_1 x^0 y^0 + \lambda_2 x^0 y^1 + \lambda_3 x^1 y^0 + \lambda_4 x^1 y^1 \quad (21)$$

$$u_y = \lambda_5 x^0 y^0 \quad (22)$$

$$\begin{aligned} u_z = & \lambda_6 x^0 y^0 + \lambda_7 x^0 y^1 + \lambda_8 x^1 y^0 + \lambda_9 x^1 y^1 + \lambda_{10} x^2 y^0 + \lambda_{11} x^2 y^1 + \lambda_{12} x^2 y^2 \quad (23) \\ & + \lambda_{13} x^0 y^2 + \lambda_{14} x^1 y^2 + \lambda_{15} x^1 y^3 + \lambda_{16} x^0 y^3 + \lambda_{17} x^0 y^4 + \lambda_{18} x^0 y^5 \\ & + \lambda_{19} x^0 y^6 + \lambda_{20} x^0 y^7 + \lambda_{21} x^4 y^3 + \lambda_{22} x^1 y^4 + \lambda_{23} x^1 y^5 + \lambda_{24} x^1 y^6 \\ & + \lambda_{25} x^1 y^8 + \lambda_{26} x^3 y^8 + \lambda_{27} x^5 y^8 + \lambda_{28} x^6 y^8 + \lambda_{29} x^2 y^9 \end{aligned}$$

503 The number and order of shape functions needed to capture the out-of-plane displacement is
 504 significantly higher than those required for the in-plane (transverse) displacement field, see
 505 Eq.(21) and (23). This is expected since the predicted (from FE simulations) displacement field in
 506 the height direction is considerably more complex than the in-plane (transverse) displacement
 507 field, see Figure 12. This is not a problem since the optimization of the dofs related to out-of-
 508 plane shape functions is more robust than the in-plane shape functions, since the out-of-plane
 509 displacement field directly affects the image residual, whereas, the in-plane displacement field
 510 affects the image residual through the (noisy) image gradient (see Eq. (7)).

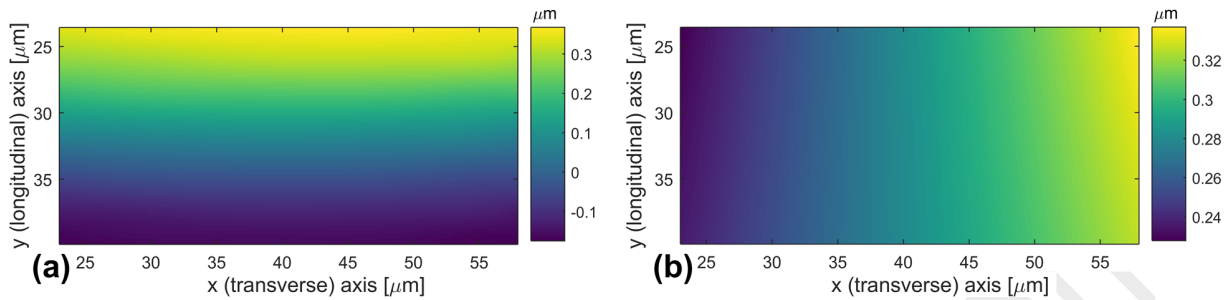
511



512

513 *Figure 14: Residual fields for selected region of interest: (a) before correlation, i.e. $f(\vec{x}) - g(\vec{x})$ and (b)*
 514 *after correlation, i.e. $f(\vec{x}) - g(\vec{x} + u_x(\vec{x}) + u_y(\vec{x})) + u_z(\vec{x})$. Note the region in the black rectangle in (b),*
 515 *which contains an irreproducible feature, was masked from the correlation.*

516



517

518 *Figure 15: Displacement fields for selected region of interest obtained by DHC: (a) height displacement field*
519 *(u_z) and (b) transverse displacement field (u_x).*

520

521 The quality of the correlation in DHC is typically assessed by analyzing the residual fields. Figure
522 14a, and Figure 14b show the residual before and after correlation, respectively. The signature of
523 the pattern is vaguely visible after convergence, due to the increasing rotation of the membrane
524 during bulging, resulting in an altered viewing angle. However, the residual field has a very low
525 RMS value indicating that the correlation was successful. Moreover, a good qualitative agreement
526 can be seen between the shape of the displacement fields extracted from the simulations (Figure
527 12) and the displacement fields obtained from DHC analysis (Figure 15), confirming that the
528 deformation kinematics has been adequately captured.

529 Subsequently, the required membrane strain ϵ_{xx} and ϵ_{yy} are calculated from the obtained 3D
530 displacement fields. This involves determination of strain with respect to a non-flat (rippled) initial
531 configuration (f). The stretch ratios and principal logarithmic strains resulting from the
532 deformation are given by:

$$\epsilon_{xx} = \ln(\lambda_t) \quad (24)$$

$$\epsilon_{yy} = \ln(\lambda_l) \quad (25)$$

533 where λ_t and λ_l are the stretch ratios ($\lambda = \frac{\text{current length}}{\text{original length}}$) in the transverse and longitudinal
 534 directions, respectively, and given by:

$$\lambda_t = \frac{L_t^g}{L_t^f} \quad (26)$$

$$\lambda_l = \frac{L_l^g}{L_l^f} \quad (27)$$

535 L_t^g and L_t^f are denoted in Figure 11b and defined as:

$$L_t^g = \sqrt{(\Delta_x(x + u_x))^2 + (\Delta_x(Z_f + u_z))^2} \quad (28)$$

$$L_t^f = \sqrt{(\Delta_x(x))^2 + (\Delta_x(Z_f))^2} \quad (29)$$

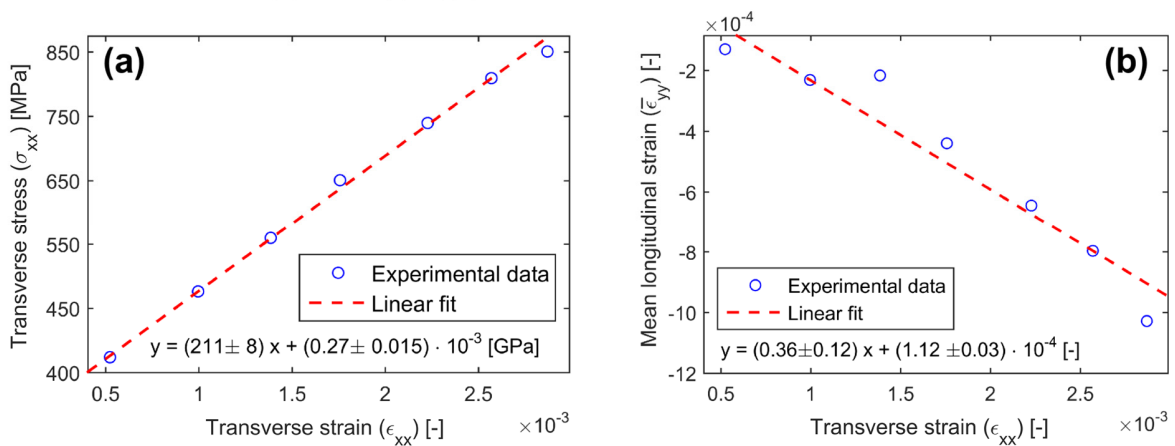
536 where Z_f is the height map of the reference image (i.e. $f(\vec{x})$), while $\Delta_x(\cdot)$ denotes an operator
 537 that acts on a field to produce the finite difference of the field in the x-direction. Since the
 538 reference surface is expected to be non-smooth due to the applied pattern and its intrinsic
 539 surface roughness, the height map and resulting displacement fields will be non-smooth. A
 540 surface polynomial fit of the reference image is used to smoothen this field. Similarly, for
 541 determining the surface normal and curvature values using Eq. (13), a smoothed position
 542 field ($Z_f + u_z$) is adopted instead of the deformed configuration (topographical image) $g(\vec{x})$.
 543 Likewise, L_l^g and L_l^f are given by:

$$L_l^g = \sqrt{(\Delta_y(y + u_y))^2 + (\Delta_y(Z_f + u_z))^2} \quad (30)$$

$$L_l^f = \sqrt{(\Delta_y(y))^2 + (\Delta_y(Z_f))^2} \quad (31)$$

544 where $\Delta_y(\cdot)$ is the operator that produces the finite difference of the specified field in the y -
 545 direction. Subsequently the transverse stress is determined using Eqs. (12) - (15). The transverse
 546 stress σ_{xx} is plotted again the transverse strain ϵ_{xx} in Figure 16a resulting in a Young's modulus
 547 of $211 \pm 8 \text{ GPa}$. In order to obtain the Poisson's ratio, the mean longitudinal strain $\bar{\epsilon}_{yy}$ (instead
 548 of ϵ_{ll} , as discussed in the previous section) is plotted against the transverse strain ϵ_{xx} resulting in
 549 a value of 0.36 ± 0.12 . These values are in adequate agreement with the predicted (volume
 550 averaged) Young's modulus and Poisson's ratio of respectively 217 GPa and 0.35 , respectively.
 551 Therefore, the proposed methodology can be applied to obtain the elastic properties from a
 552 buckled membrane.

553



554

555 *Figure 16: Stress and strains obtained from DHC results plotted to determine Young's modulus E and*
556 *Poisson's ratio ν : (a) transverse stress (σ_{xx}) vs. transverse strain (ϵ_{xx}) with a linear fit to obtain the Young's*
557 *modulus (E) and (b) mean longitudinal strain ($\bar{\epsilon}_{yy}$) vs. transverse strain (ϵ_{xx}) with a linear fit to obtain*
558 *the Poisson's ratio (ν).*

559

560 **5. Conclusions and Recommendations**

561 It is well known from literature that conventional bulge testing, which is frequently used to
562 characterize freestanding membranes, does not apply to the particular class of buckled
563 membranes. In this paper, the conventional bulge test methodology has been extended to
564 characterize the elastic properties of buckled membranes. This has been achieved by developing
565 a validated FE based numerical model, which captures the complex mechanics of the (pressure-
566 loaded) buckled membrane. A recently developed DHC approach has been employed to obtain
567 local, complex 3D displacement (strain) and curvature fields.

568 Interestingly, the virtual experiments revealed that, embedded in the seemingly complex ripple
569 configuration, a simplified state of uniaxial stress exists in the transverse direction. This implies
570 that the transverse stress can be directly related to the transverse strain, yielding Young's
571 modulus. Poisson's ratio can be extracted directly from the ratio of the longitudinal strain and the
572 transverse strain. This is a significant advantage over the conventional bulge test theory where
573 Young's modulus and Poisson's ratio are intrinsically measured in a coupled manner, either
574 through the plane strain modulus or the biaxial modulus. As a result, tests on two sample

575 geometries, typically rectangular and square shape, are needed to obtain Young's modulus and
576 Poisson's ratio independently.

577 The numerical model also showed that applying DHC to measure the displacements in the
578 longitudinal direction would require high-order shape functions and a displacement resolution
579 that cannot be achieved with an optical system. This problem was solved by exploiting the fact
580 that the contribution of the 'in-plane' longitudinal displacement to the longitudinal strain is
581 negligible, allowing to obtain an accurate value of Poisson's ratio by only considering the height
582 displacement to determine the longitudinal strain.

583 Proof of principle experiment clearly show that the method is applicable on real samples, even
584 with rather narrow ripples dimensions (20-30 μm) and only few data points, as the fragile samples
585 were very susceptible to deformation induced failure. Residual maps indicate proper convergence
586 and the shape of the displacement fields captured with DHC adequately match the predicted
587 displacement fields extracted from the FE simulation, confirming that the buckled membrane
588 kinematics are being properly captured. Moreover, the resulting values of Young's modulus and
589 Poisson's ratio are consistent with the expected values based on the material stack.

590 As both the stress and strains are measured under uniaxial tension, the method is not necessarily
591 confined to the elastic regime and should work as well in the plastic regime (yielding plasticity
592 parameters), if the specimens would plastically deform. Furthermore, by applying cyclic pressure
593 loading, fatigue testing, as suggested in Ref. [27] for plane strain loading, can be applied with
594 uniaxial tension. Using a feedback loop to maintain a constant membrane stress, as suggested in
595 Ref. [28], a creep test in uniaxial tension can be performed.

596

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603

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