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*DOI:*
10.1103/PhysRevLett.120.100401

*Document status and date:*
Published: 09/03/2018

*Document Version:*
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

# Dynamics of Three-Body Correlations in Quenched Unitary Bose Gases

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(Received 29 October 2017; published 9 March 2018)

We investigate dynamical three-body correlations in the Bose gas during the earliest stages of evolution after a quench to the unitary regime. The development of few-body correlations is theoretically observed by determining the two- and three-body contacts. We find that the growth of three-body correlations is gradual compared to two-body correlations. The three-body contact oscillates coherently, and we identify this as a signature of Efimov trimers. We show that the growth of three-body correlations depends nontrivially on parameters derived from both the density and Efimov physics. These results demonstrate the violation of scaling invariance of unitary bosonic systems via the appearance of log-periodic modulation of three-body correlations.

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*Introduction.*—In the ultracold regime of bosonic gases, where the interaction is well characterized by the $s$-wave scattering length, $a$, macroscopic theories of matter can be formulated from microscopic Hamiltonians. These theories relate few-atom physics to their manifestations in macroscopic observables. At the heart of this link is a set of universal relations attributed to Tan [1–3]. The Tan relations provide an alternative path to calculate thermodynamical properties of an ultracold quantum gas by studying analytic solutions of the two-body problem and extracting the extensive two-body contact density $\mathcal{C}_2$, which characterizes two-body correlations at short distances within the system. These relations are well understood for two-component Fermi gases even in the unitary regime $n|a|^3 \gg 1$, where $n$ is the atomic density, and they have been verified experimentally [4,5]. For strongly interacting Bose gases, there is an additional complication due to the existence of the three-body Efimov effect. At unitarity ($a \to \infty$) an infinity of three-body trimers emerges, which strongly alters the scattering observables at ultracold energies [6–9]. Here, universal relations between few-body physics and macroscopic observables also involve the three-body contact density $\mathcal{C}_3$ [10,11], central to the many-body theory, whose properties are not yet theoretically known for the degenerate Bose gas in the unitary regime.

Unlike two-component unitary Fermi gases, strongly interacting Bose-condensed gases are plagued by an enhanced three-body loss rate growing as $n^3a^4$, limiting the development of correlations. By quenching the interactions from weak to unitarity, Makotyn *et al.* [12] observed saturation of the single-particle momentum distribution—an observable sensitive to few-body correlations—of the quenched unitary degenerate Bose gas on a time scale shorter than the observed atom loss rate. It has been suggested that the observed tail of the saturated momentum distribution oscillates log periodically, the signature of Efimov physics, and therefore is a measurement of a nonzero $\mathcal{C}_3$ [13,14]. For the thermal unitary Bose gas, $\mathcal{C}_3$ has been measured interferometrically in Fletcher *et al.* [15] and approaches the theoretical saturation value from Ref. [14].

Introducing additional length scales due to Efimov physics can break the continuous scale invariance of system properties with the interparticle spacing $n^{-1/3}$. Within the universality hypothesis, all properties of unitary quantum gases depend solely on the density [16]. For bosons or fermions, the only relevant scales in the universal theory are set by the momentum $\hbar k_n = \hbar(6\pi^2 n)^{1/3}$, the energy $E_n = \hbar^2 k_n^2/2m$, and the time $t_n = \hbar/E_n$, where $m$ is the atomic mass. Although $\mathcal{C}_2$ within the nonequilibrium regime is well studied [17–19], predicting the time evolution, scaling properties, and saturation value of $\mathcal{C}_3$ remains an open problem, limiting our full understanding of the role of Efimov physics in quenched unitary Bose gases.

In this Letter, we theoretically observe the growth of the *dynamical* three-body contact density $\mathcal{C}_3$ immediately following the quench to unitarity. We have developed a simple model that describes the early correlation dynamics of the quenched unitary Bose gas using analytic solutions of the three-body problem [20]. At the earliest stages of evolution, we find that the three-body contact grows slowly compared to the two-body contact and exhibits coherent oscillations at the frequency of Efimov trimers. Our results demonstrate that the violation of the continuous scale invariance of $\mathcal{C}_3$ at early times is maximized whenever the size of an Efimov trimer is comparable to the interparticle spacing.

*Relations at short distances.*—We begin by establishing short distance connections between two- and three-body correlations of a Bose gas, two- and three-body contacts, and solutions of the few-body problem. These connections are made at distances larger than the van der Waals length, $r_{\rm vdW}$, but smaller than other length scales of the problem ($a$, $n^{-1/3}$, etc.) This is done for a uniform gas of $N$ particles in volume $V$ with the density $n = N/V$, which can be generalized to trapped gases using the local-density approximation, where $n$ is the average density $\langle n \rangle$.

Within the zero-range model for the interatomic interactions, the short distance behavior of the two- and three-body correlation functions is determined exclusively by the two- and three-body contacts (see Ref. [11])

$$g^{(2)}(\mathbf{r}, \mathbf{t}) \underset{|\mathbf{r}|\to 0}{=} \frac{1}{16\pi^2 n^2 r^2} \mathcal{C}_2, \tag{1}$$

$$g^{(3)}(R, \mathbf{\Omega}, t) \underset{R\to 0}{=} |\Psi_{\rm sc}(R, \mathbf{\Omega})|^2 \frac{8}{n^3 s_0^2 \sqrt{3}} \mathcal{C}_3, \tag{2}$$

where $s_0 \approx 1.006\,24$ is Efimov's universal constant for three identical bosons [21]. Center of mass dependence in the equations above has been suppressed due to translational invariance. The relative atomic configuration is parametrized by Jacobi vectors $\mathbf{r} \equiv \mathbf{r}_2 - \mathbf{r}_1$ and $\boldsymbol{\rho} \equiv (2\mathbf{r_3} - \mathbf{r_1} - \mathbf{r_2})/\sqrt{3}$. Alternatively, it can be parametrized by the hyper-radius $R^2 \equiv (r^2 + \rho^2)/2$ and the set of hyperangles $\mathbf{\Omega} = \{\alpha, \hat{\mathbf{r}}, \hat{\boldsymbol{\rho}}\}$, containing the hyperangle $\alpha = \arctan(r/\rho)$ and spherical angles for each Jacobi vector. The limit notation in Eqs. (1) and (2) indicates $|\mathbf{r}| \to 0$ for a fixed $\hat{\mathbf{r}}$, and $R \to 0$ for a fixed $\mathbf{\Omega}$, respectively. $\Psi_{\rm sc}(R, \mathbf{\Omega})$ is the zero-energy three-body scattering wave function

$$\Psi_{\rm sc}(R, \mathbf{\Omega}) = \frac{1}{R^2} \sin\left(s_0 \ln \frac{R}{R_t}\right) \frac{\phi_{s_0}(\mathbf{\Omega})}{\sqrt{\langle \phi_{s_0} | \phi_{s_0} \rangle}}, \tag{3}$$

where $R_t/r_{\rm vdW} \in [1, e^{\pi/s_0}]$ is the three-body parameter, setting the phase of log-periodic oscillations, and $\phi_{s_0}(\mathbf{\Omega})$ is the hyperangular wave function for three identical bosons in the state of lowest total angular momentum. [For analytic expressions of $\phi_{s_0}(\mathbf{\Omega})$ and $\langle \phi_{s_0} | \phi_{s_0} \rangle$, we refer the reader to Refs. [22,30,31].]

After the interaction quench—amounting within our model to a quench of the Bethe-Peierls contact boundary condition at $r = 0$—the contact dynamics occur exclusively at short distances. Therefore, the short-time short-range behavior of few-body wave functions can yield quantitatively correct predictions for the contact dynamics of a quenched many-body system [18,32]. Generally, if a particle is measured at a location defining the origin of a coordinate system, then $ng^{(2)}(\mathbf{r}, t)$ is the probability density for measuring another particle at $\mathbf{r}$ [33]. In a three-body model, that probability density is given in terms of the three-body wave function $\Psi(\mathbf{r}, \boldsymbol{\rho}, t)$. We are interested in this probability density at short distances where $\mathcal{C}_2$ is defined, suggesting the relation

$$ng^{(2)}(\mathbf{r}, t) \underset{|\mathbf{r}|\to 0}{=} 2 \int d^3 r_{3,12} |\Psi(\mathbf{r}, \boldsymbol{\rho}, t)|^2, \tag{4}$$

where $\mathbf{r}_{3,12} = \boldsymbol{\rho}\sqrt{3}/2$. Additionally, the quantity $n^2 g^{(3)}(R, \mathbf{\Omega}, t)$ is the probability density of finding two other particles at locations defined by the three-body configuration $(R, \mathbf{\Omega})$. The analogous relation between the three-body correlation function and the three-body wave function is

$$n^2 g^{(3)}(R, \mathbf{\Omega}, t) \underset{R\to 0}{=} 2 |\Psi(R, \mathbf{\Omega}, t)|^2. \tag{5}$$

The factor of 2 in Eqs. (4) and (5) is due to the indistinguishability of particles not fixed at the origin.

*Initial conditions.*—To make the links in Eqs. (4) and (5) quantitatively correct, we employ an unambiguous calibration scheme. The three-body model yields correct short-time predictions of the contacts if and only if the initial wave function satisfies Eqs. (4) and (5) at $t = 0$. To model the quench, we start from the noninteracting limit where $g^{(2)}(\mathbf{r}, 0) = g^{(3)}(R, \mathbf{\Omega}, 0) = 1$. There is freedom of choice for the initial three-body wave function satisfying these initial conditions. Here, we choose

$$\Psi_0(R, \mathbf{\Omega}) = A e^{-R^2/2B_1^2} \left[1 - \left(\frac{R}{B_2}\right)^2\right], \tag{6}$$

where the analytic expression for the normalization constant $A$ is given in Ref. [22]. Setting $B_1 \approx 0.6009 n^{-1/3}$ and $B_2 \approx 1.1278 n^{-1/3}$ satisfies both initial conditions. With this calibration scheme, predictions for short-time short-distance correlation phenomenon for quenched many-body systems should not depend on the long-range part of the few-body wave function. This was demonstrated for two-body correlations in one and three dimensions in Refs. [18,32].

*Three-body model at unitarity.*—After the quench to unitarity, the initial wave function [Eq. (6)] is projected onto eigenstates at unitarity, for which we utilize solutions for three harmonically confined bosons given in Ref. [20]. These eigenstates serve only as a convenient basis on which to expand the problem. At unitarity within the zero-range model, the relative three-body eigenstates can be factorized as $\Psi_{s,j}(R, \mathbf{\Omega}) = \mathcal{N} F_j^{(s)}(R) \phi_s(\mathbf{\Omega}) / R^2 \sin 2\alpha$, where $\mathcal{N}$ is a normalization factor, and $s$ is a solution of a transcendental equation resulting from the Bethe-Peierls contact condition taken at unitarity (see Ref. [22]). The hyper-radial wave functions $F_j^{(s)}(R)$ obey [20]

$$\left[-\frac{\hbar^2}{2m}\left(\frac{d^2}{dR^2} + \frac{1}{R}\frac{d}{dR}\right) + U_s(R)\right] F_j^{(s)}(R) = E F_j^{(s)}(R), \tag{7}$$

where $U_s(R) = \hbar^2 s^2/(2mR^2) + m\omega_0^2 R^2/2$ is a sum of the effective three-body potential in channel $s$ and of the local harmonic trap with frequency $\omega_0$ and trap length $a_{\rm ho} = \sqrt{\hbar/m\omega_0}$. The index $j$ labels eigenstates within a channel, and $E$ is the three-body relative energy.

To connect with the short-distance behavior of three-body correlations, we consider the $R \to 0$ behavior of the hyper-radial eigenstates. For $s > 0$, the limiting behavior is $F_j^{(s)}(R) \propto O(R^s)$, which does not contribute to the short-range three-body correlations. The only channel contributing to three-body correlations at short distances is associated with the lone imaginary solution of the transcendental equation denoted $s = is_0$, giving rise to the attractive $1/R^2$ three-body potential that produces the Efimov effect. The limiting behavior of the hyper-radial eigenstates in the Efimov channel is $F_j^{(s_0)} \propto \sin[s_0 \ln(R/R_t)]$, and the eigenenergies $E_{3b}^{(j)}$ are obtained from solving

$$\arg\Gamma\left[\frac{1+is_0-E_{3b}^{(j)}/\hbar\omega_0}{2}\right]+s_0\ln\frac{R_t}{a_{\rm ho}}=\arg\Gamma[1+is_0], \qquad (8)$$

which is evaluated mod $\pi$. In the free-space limit ($\omega_0 \to 0$), there exist an infinite number of bound Efimov trimers whose energies and sizes are characterized by the log-periodic geometric scaling [6–9,34]:

$$E_{3b}^{(j)} = \frac{E_{3b}^{(0)}}{(e^{\pi/s_0})^{2j}} \quad \text{and} \quad R_{3b}^{(j)} = \sqrt{\frac{2(1+s_0^2)}{3}}\frac{(e^{\pi/s_0})^j}{\kappa_*}, \qquad (9)$$

where $j = 0, 1, \ldots, \infty$ [35]. We choose $R_t$ such that there is a trimer with the energy $E_{3b}^{(0)} = \hbar^2\kappa_*^2/m \approx 0.051\hbar^2/mr_{\rm vdW}^2$ in the free-space limit of Eq. (8). $\kappa_*$ is the universal three-body parameter found in Ref. [36].

*Postquench dynamics of* $\mathcal{C}_3$.—Given the initial condition in Eq. (6), the solution after quenching is $\Psi(R,\boldsymbol{\Omega},t) = \sum_{s,j} c_{s,j}\Psi_{s,j}(R,\boldsymbol{\Omega})e^{-iE_{3b}^{(j)}t/\hbar}$, with overlaps $c_{s,j} = \langle\Psi_{s,j}|\Psi_0\rangle$ (see Ref. [22].) This sum runs over all channels; however, the Efimov channel makes the sole contribution to the short-range behavior of three-body correlations at unitarity. Dominant contributions come from only a few trimers ($E_{3b} \leq 0$) and trapped states ($E_{3b} > 0$), with eigenenergies comparable in magnitude to $E_n$ [17]. At short range, the relevant behavior of each eigenstate in the Efimov channel is captured by the extensive three-body contact $C_3^{(j)}$, which we have calculated analytically (see Ref. [22]). Intuitively, the dynamical three-body contact density can be written as a superposition of $C_3^{(j)}$ by combining Eqs. (2) and (5) and integrating over the hyperangles

$$\mathcal{C}_3 = \frac{n}{3}\left|\sum\nolimits_j c_{s_0,j} \times e^{i\phi_j}\sqrt{|C_3^{(j)}|}e^{-iE_{3b}^{(j)}t/\hbar}e^{-\Gamma_j t/2\hbar}\right|^2, \qquad (10)$$

where $\phi_j \underset{R\to 0}{=} \arg[\Psi_{s_0,j}/\Psi_{\rm sc}]$. Here, we account for three-body losses by utilizing a relation from Refs. [11,14] to estimate finite widths $\Gamma_j = C_3^{(j)}4\hbar\eta/ms_0$ valid in the limit where the inelasticity parameter satisfies $\eta \ll 1$. We assume that this relation is satisfied in the remainder of this Letter. As a result of the finite width, the time evolution of three-body eigenstates at unitarity is updated to $E_{3b}^{(j)} \to E_{3b}^{(j)} - i\Gamma_j/2$, which leads to a decay of the norm and the form of Eq. (10).

Hidden in Eq. (10) is a dependence of $\mathcal{C}_3$ on long-range details of the three-body model. However, the postquench three-body contact dynamics should depend only on the behavior of the three-body wave function at short distances. We therefore require our results to be robust to variations of both the trapping parameters [see Eq. (7)] and the arbitrary functional form of $\Psi_0$ [Eq. (6)] provided that the initial boundary conditions are satisfied. By investigating the sensitivity of our results to these variations (see Ref. [22]), we find that these criteria are satisfied at the earliest stages of evolution even for loose traps supporting more than a few trimers. Beyond $t/t_n \lesssim 0.5$ our results develop dependence on the long-range details of the model, and we truncate the analysis. Additionally, at later times our model loses physical significance as we expect genuine many-body effects to play a role in the correlation dynamics. These constraints echo the findings of Refs. [17,18].

Early-time evolution of the two- and three-body contacts in the unitary regime is shown in Fig. 1 over a range of densities. Our three-body contact results are specific to $^{85}$Rb, depending on $r_{\rm vdW}$, $m$, and $\eta$. We take $\eta = 0.06$ from the experimental measurements in Ref. [37]. Qualitatively, the contact dynamics agree with the experimental observation in Ref. [15] that the three-body contact develops gradually compared to the two-body contact. Interpreting $\mathcal{C}_2$ as the number of pairs per (volume)$^{4/3}$ and $\mathcal{C}_3$ as the number of triples per (volume)$^{5/3}$ [1–3,14], we find support

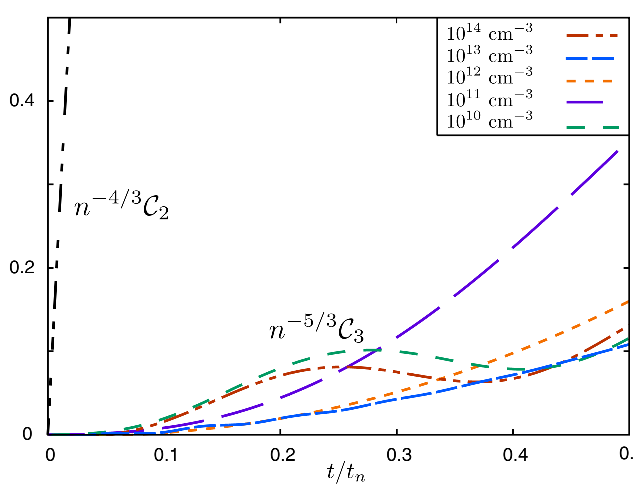


FIG. 1. Postquench dynamics of dimensionless, scaled two- and three-body contacts over a range of densities. Evolution of the two-body contact is given by the universal growth rate $n^{-4/3}\mathcal{C}_2 = 128\pi/(6\pi^2)^{2/3}t/t_n$ from Ref. [18], which quickly increases beyond the plotted range. This behavior is known and is therefore not shown.

for the sequential buildup of clusters [38,39]. Unlike the early-time behavior of $n^{-4/3}\mathcal{C}_2$ obtained in Refs. [17,18], the behavior of $n^{-5/3}\mathcal{C}_3$ in Fig. 1 varies for different densities. This is a strong indication of scaling violations in the dynamics of three-body correlations at short distances discussed below.

Curiously, for densities $n = 10^{10}$ and $10^{14}$ cm$^{-3}$, the corresponding $n^{-5/3}\mathcal{C}_3$ curves in Fig. 1 exhibit a visible oscillation on a time scale shorter than $t_n$. By eliminating contributions of specific eigenstates to Eq. (10), their origin can be isolated to the Efimov trimer with binding energy nearest $E_n$ satisfying $|E_{3b}^{(j)}| \gg E_n$. Specifically, the oscillation is due to coherences between this trimer and states with energy comparable to $E_n$, resulting in a *beating* phenomenon [40]. As the energy of this trimer approaches $E_n$ for an increasing density, the frequency of the visible oscillations, as well as their amplitude, increases as shown in Fig. 2. Empirically, we observe that the frequency and damping rate of the oscillations correspond roughly to the frequency $\omega_{3b}^{(j)} = E_{3b}^{(j)}/\hbar$ and the width $\Gamma_j$ of this trimer, respectively. The trimer oscillations are therefore underdamped and are theoretically observable provided $|E_{3b}^{(j)}| > \Gamma_j$, obtained whenever $\eta < s_0/4$ (see Ref. [22]). Oscillation maxima occur at fixed values of the phase $|E_{3b}^{(j)} t|/\hbar = 1.33(11)\pi \bmod 2\pi$. For the highest and lowest densities in Fig. 1, oscillation is due to the $j = 0$ and $j = 1$ Efimov trimers, respectively. Populations of the $j = 1$ trimer in the unitary Bose gas were recently observed through a double exponential decay of the molecular gas in Ref. [41]. Here, we find additional theoretical evidence for three-body bound-state signatures as coherent beats in the early-time correlation dynamics.

*Scaling violations.*—How does Efimov physics alter the density dependence of the early-time evolution of the three-body contact? The dynamical surface in Fig. 2 displays a "rippling" effect due to the density independence of the trimer oscillation phase discussed previously. A pair of pronounced "peaks" in Fig. 2 are due to the variation of the trimer oscillation amplitude with density. We find identical results for $n^{-5/3}\mathcal{C}_3$ for densities rescaled by powers of $(e^{\pi/s_0})^3$ when plotted as a function of $t/t_n$. Therefore, the surface in Fig. 2 represents only a single log period, demonstrating that $n^{-5/3}\mathcal{C}_3$ has a *discrete* scale invariance as a consequence of Efimov physics.

With this in mind, we study the envelope of the log-periodic modulation of $n^{-5/3}\mathcal{C}_3$ shown in Fig. 3. To characterize the correlation trends, we propose a functional form for the growth of three-body correlations which is quadratic in time to leading order

$$n^{-5/3}\mathcal{C}_3 = A[1 + B \times H(n, \kappa_*, t)](t/t_n)^2, \tag{11}$$

where $H(n, \kappa_*, t) = H(ne^{3j\pi/s_0}, \kappa_*, t) \in [0, 1]$ is an unknown log-periodic function reflecting the influence of Efimov physics. The first term above, proportional to $A$, captures the continuous scale invariant part of the three-body contact, corresponding to the floor $[\min(n^{-5/3}\mathcal{C}_3)]$ of the curves in Fig. 3. In the second term above, the quantity $B$ is the fractional amplitude of the log-periodic modulation $[\max(n^{-5/3}\mathcal{C}_3) - \min(n^{-5/3}\mathcal{C}_3)]/\min(n^{-5/3}\mathcal{C}_3)$ at a fixed $t/t_n$, quantifying the violation of the continuous scale invariance. From fitting our data at early times $t/t_n \ll 1$, we find $A \approx 0.55$ and $B \approx 3.09$. Therefore the early-time evolution of three-body correlations is, *in general*, poorly captured by fitting to a universal function with continuous scaling invariance. In Ref. [14] the restrictive assumption was made that the saturated value of $\mathcal{C}_3$ scales continuously

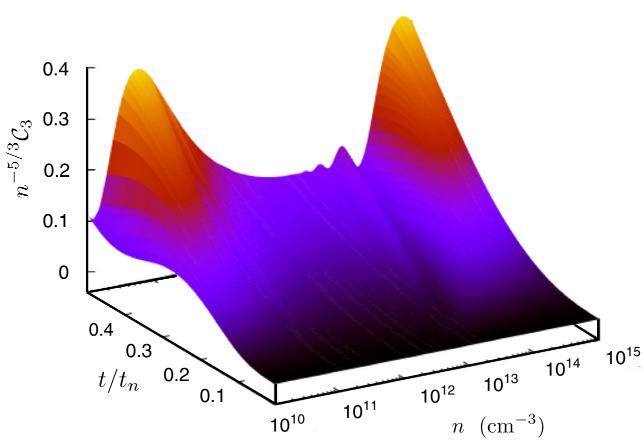


FIG. 2. Dynamical surface of $n^{-5/3}\mathcal{C}_3$ over a range of densities. A rippling effect due to the coherent trimer oscillations occurs as the peaks are approached from lower densities. This behavior is repeated for densities rescaled by powers of $e^{3\pi/s_0}$.

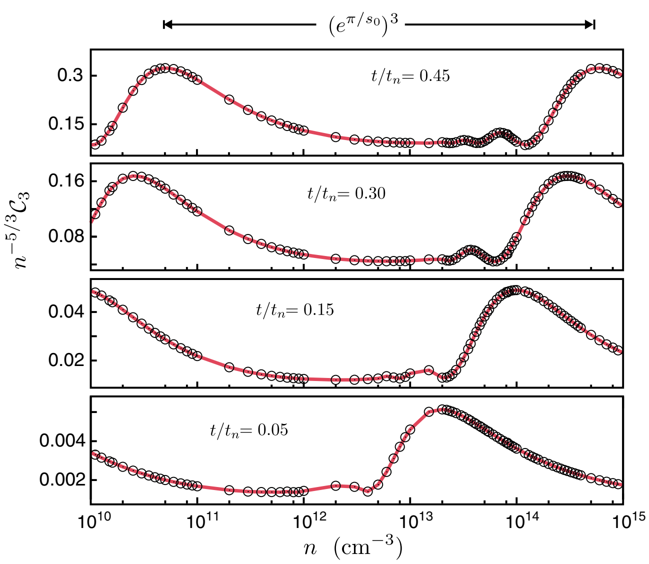


FIG. 3. Profiles of $n^{-5/3}\mathcal{C}_3$ at a fixed $t/t_n$. The solid lines are guides for the eye connecting the data from Fig. 2.

as $n^{5/3}$ with numerically suppressed log-periodic effects. Over the density range $(1.6–5.5) \times 10^{12}$ cm$^{-3}$ fit in Ref. [14], we find that $H(n, \kappa_*, t)$ is slowly varying with the density, providing only a minor correction to the continuous scaling law. Hence, fitting to a universal scaling law is sufficient over this limited density range, failing over a broader range as $H$ becomes significant.

Comparing Eq. (11) to our data, we infer the behavior of $H(n, \kappa_*, t)$, quantifying the violation of scale invariance at particular densities and times. Within our model, the maximum of this unknown function occurs for densities satisfying

$$R_{3b}^{(j)} \times k_n = 0.74(5), \tag{12}$$

where $j$ is any trimer index, as in Eq. (9). When Eq. (12) is satisfied, the size of the trimer responsible for the coherent oscillation is comparable to $k_n^{-1}$, and therefore the interparticle spacing. This results in a correlation enhancement. Similarly, recent results in Refs. [42,43] for the Bose polaron problem suggest that when the size of Efimov trimers becomes comparable to the interparticle scaling, signatures of Efimov physics become visible in the polaron spectrum.

*Conclusion.*—We have studied the early-time dynamics of the three-body contact density for the quenched unitary Bose gas. The relative growth of the two- and three-body contacts indicates that triples are generated slower than pairs of atoms immediately after the quench. Efimov physics arises through coherent oscillations of the three-body contact, a bound-state signature of trimers, and through the violation of continuous scale invariance. Our methodology can be extended to analyze three-body contact dynamics for quench scenarios away from unitarity within the zero-range model, which is beyond the scope of this Letter (see Ref. [22]). It is of interest to extend this analysis to later times beyond the range of our model and to observables depending functionally on the three-body contact. These investigations may suggest regimes of interest for experiments, which have covered, to date [12,15,41,44], a fraction of the log period studied in this Letter. Preliminary observations of the decay rate over a wider range of densities display oscillations [45]. With an increase of signal to noise of the measurements in Ref. [12], it may be possible to observe the scale violations and coherent trimer oscillations predicted in this Letter or through time-resolved rf spectroscopy (see Ref. [46]).

The authors thank John Bohn and Servaas Kokkelmans for the critical feedback. J. P. D. acknowledges support from National Science Foundation (NSF) Grant No. PHY-1607204 and from the National Aeronautics and Space Administration (NASA). V. E. C. and J. P. C. acknowledge support from the NSF under Grant No. PHY-1734006. V. E. C. is supported also by the Netherlands Organisation for Scientific Research (NWO) under Grant No. 680-47-623.

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