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# Maximum likelihood identification in dynamic networks with rank-reduced noise

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## 1 Introduction

A typical assumption in multivariable and dynamic network system identification is that process noise is full rank. When a network becomes large-scale, e.g. when many sensors are placed physically close to each other, we have to consider the possibility that noise is rank-reduced<sup>2</sup>. Many of the typical steps in a prediction error identification framework can not immediately be taken in case noise is rank-reduced. Recently in a prediction error setting we have derived conditions under which consistent estimates of the network can be obtained in case noise is rank-reduced [1]. Our objective is to extend those results towards maximum likelihood estimates of the network dynamics, even when noise is of a reduced rank.

## 2 Approach

In order to estimate the network a prediction error approach is taken, leading to the multivariable prediction error  $\varepsilon(t, \theta)$  of dimension  $L$ , the number of measured node variables. The typical identification criterion is the weighted least squares (WLS)

$$\theta^* := \arg \min_{\theta} \bar{\mathbb{E}} \varepsilon^T(t, \theta) Q \varepsilon(t, \theta). \quad (1)$$

If  $Q$  is chosen as the identity matrix this criterion leads to consistent estimates under rather standard conditions, even when noise is rank-reduced [1]. A question that needs to be answered is whether this criterion leads to maximum likelihood estimates. Typically, taking  $Q$  as the inverse of the covariance matrix of the innovation process leads to a maximum likelihood estimate. When noise is rank-reduced the covariance matrix is singular, and therefore can not be inverted.

Due to the rank-reduced situation the covariance of the innovation can be written as  $B\Lambda B^T$ , with  $B$  an  $L \times p$  dimensional matrix and  $\Lambda$  the  $p \times p$  dimensional covariance matrix of  $e(t)$ . We use the left-inverse  $B^\dagger$  of  $B$  and

the nullspace of  $B$  spanned by the rows of matrix  $N$  to construct a generalized inverse of  $B\Lambda B^T$  as weighting matrix

$$Q_\lambda = (B^\dagger)^T \Lambda^{-1} B^\dagger + \lambda N^T N. \quad (2)$$

The  $B$  and  $N$  terms of the weight actually represent two orthogonal components of the innovation process.  $\lambda$  governs the balance between the two orthogonal components. When  $\lambda \rightarrow \infty$  the variance of the estimate reduces to some lower bound, possibly 0.

## 3 Maximum likelihood

When the innovation  $\epsilon$  is of rank  $p < L$  then it does not have a probability density function (pdf) in the usual sense. However a pdf for the innovation exists on a  $p$  dimensional subspace [2], which is then mapped to the  $L$  dimensional space by matrix  $B$ . This means that  $B^\dagger \epsilon(t) = e(t)$  does have a pdf, and moreover  $N\epsilon(t) = 0$  with probability 1 for all  $t$ . The distribution of  $\epsilon$  is specified by a pdf on the hyperplane  $N\epsilon(t) = 0$ .

When weight  $Q_\lambda$  is used in (1) with  $\lambda \rightarrow \infty$  then the second term of  $Q_\lambda$  ensures that the estimate is taken on the hyperplane  $N\epsilon(t) = 0$ . The first term of  $Q_\lambda$  maps the prediction error to dimension  $p$  via  $B^\dagger$ , and then applies the appropriately weight for estimation. When the appropriate  $B$ ,  $N$  and  $\Lambda$  are chosen then the estimate (1) with weight  $Q = Q_\lambda$  is a maximum likelihood estimate.

## References

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<sup>2</sup>A noise process  $v(t)$  of dimension  $L$  is called rank-reduced if it can be generated by filtering a white noise process  $e(t)$  of dimension  $p < L$ .