Simplifying the uncertainty of linear models in terms of h2 performance using LMIs

Simplifying the uncertainty of linear models in terms of $\mathscr{H}_2$ performance using LMIs

Gustavo S. Mazzoccante and Ricardo C. L. F. Oliveira
School of Electrical and Computer Engineering – University of Campinas – UNICAMP
13083-852, Campinas, SP – Brazil – Email: {gsmazzo, ricfow}@dt.fee.unicamp.br

1 Introduction

For at least four decades the model order reduction (MOR) problem has received great attention. This fact is not surprising since, in general, a simplified model is required to design controllers for systems with many states, such as power systems. Basically, the problem consists of determining a reduced-order model that appropriately approximates the original system. Compared to a high-order model, the benefit of a low-order model is that it is easier to analyze, simulate and implement. Moreover, there is a continuous demand from industry to improve performance while reducing costs. To achieve this, various applications require precise mathematical models, that consider, for instance, parametric uncertainties and structural constraints.

2 Problem formulation

Consider the uncertain robustly stable discrete-time linear system of order n described by

$$\begin{aligned} x(k+1) &= \big(A(\alpha)+\Delta A(\alpha)\big)x(k)+\big(B(\alpha)+\Delta B(\alpha)\big)u(k) \\ y(k) &= C(\alpha)x(k)+D(\alpha)u(k) \end{aligned} \tag{1}$$

where $x(k)\in\mathbb{R}^n$, $u(k)\in\mathbb{R}^m$ and $y(k)\in\mathbb{R}^p$ respectively denote the state, input and output vectors. The matrices $A(\alpha)$, $B(\alpha)$, $C(\alpha)$ and $D(\alpha)$ have a polynomial dependency of fixed degree on the bounded time-invariant parameter $\alpha\in\mathbb{R}^N$. The terms $\Delta A(\alpha)$ and $\Delta B(\alpha)$ represent unstructured uncertainties that are limited as $\| \Delta A(\alpha) \|_2\leq \sigma_A$ and $\| \Delta B(\alpha) \|_2\leq \sigma_B$, where σ_A and σ_B are given values. This particular structure of the matrices of the system is obtained, for instance, when discretizing an uncertain polytopic continuous-time system using a Taylor series expansion [2].

The classical MOR problem is to design a reduced model for system (1), of order $r\leq n$, with state-space representation given by

$$\begin{aligned} x_r(k+1) &= A_r(\alpha)x_r(k)+B_r(\alpha)u(k) \\ y_r(k) &= C_r(\alpha)x_r(k)+D_r(\alpha)u(k) \end{aligned} \tag{2}$$

where $x_r(k)\in\mathbb{R}^r$, $u(k)\in\mathbb{R}^m$ and $y_r(k)\in\mathbb{R}^p$ are the state, input and output vectors, respectively, and $A_r(\alpha)$, $B_r(\alpha)$, $C_r(\alpha)$ and $D_r(\alpha)$ are the uncertain matrices of the reduced system to be determined. Although the classical MOR problem only considers a reduction of the number of states, this work provides the following generalizations: • the norm-bounded terms are removed by incorporating their effect in the parametric uncertainty α; • the degree of the reduced system matrices can be chosen arbitrarily (e.g., constant, affine or polynomial); • each parameter α_i can be independently removed, resulting in a reduced system depending on less parameters. All these capabilities are particularly useful when dealing with robust control of discretized uncertain systems. Namely, the resulting model can be synthesized according to the constraints or limitations imposed by the existing robust control design conditions.

3 $\mathscr{H}_2$ Model order reduction approach

Using the $\mathscr{H}_2$ norm as performance criterion, the MOR problem naturally cast in terms of parameter-dependent bilinear matrix inequalities, which are hard to solve numerically. To relieve the latter issue, sufficient parameter-dependent LMIs are derived by, amongst others, application of the projection lemma to introduce additional variables, and by imposing structure on these variables. Subsequently, a numerically tractable (i.e., finite) set of LMIs that guarantees feasibility of the parameter-dependent LMIs is derived by imposing a polynomial parameterization and employing positive basis functions. Such manipulations are obtained with the aid of a robust LMI parser [1]. Numerical examples illustrate the viability of the proposed approach when obtaining a discretized model from an uncertain continuous-time model. Additionally, in the particular case of affine parameter-dependent uncertain discrete-time systems free of norm-bounded uncertainty, where other conditions from the literature can be used, it is shown that the proposed conditions can be more effective.

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