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Padé Approximation of Delays in CACC-Controlled String-Stable Platoons

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ABSTRACT: Cooperative Adaptive Cruise Control (CACC) improves road throughput by employing inter-vehicle wireless communications. The inherent communication time delay significantly limits the minimum inter-vehicle distance in view of string stability. Applying a Padé approximation leads to a model with a rational transfer function, which allows many control methods to compensate for this delay to a certain extent. Our objective is to find the lowest possible order of Padé approximations, which is sufficiently accurate in view of string stability. To this end, the minimum string-stable time gaps are chosen as the main criterion against which the quality of the Padé approximations is measured. The results indicate that it is feasible to apply a 2$^{\text{nd}}$-order Padé approximation in the given CACC system.

1 INTRODUCTION

Advanced Driver Assistance systems have significantly developed in the last decades. Adaptive Cruise Control (ACC) systems equipped with radar or camera, which relieve the drivers' task by automatically keeping a desired inter-vehicle distance (Marsden, McDonald, & Brackstone 2001), are penetrating into the market. To realize a shorter inter-vehicle distance and, consequently, to improve highway capacity, Cooperative ACC (CACC) systems have been developed, which use wireless inter-vehicle communications (Naus et al. 2010, Shladover et al. 2012). String stability, which refers to the attenuation of the effects of disturbances in upstream direction of the string of vehicles, is a primary requirement for a vehicle platoon. CACC systems can significantly improve string stability, that prevent traffic jams while increasing highway throughput (Ploeg et al. 2011).

However, string stability can be affected by wireless communication impairments, such as time delays, sampling intervals, packet loss and communication constraints (Heemels & van de Wouw 2010). Thus, it is critical to take communication delays into account to design and analyze a CACC string (Liu et al. 2001, Naus et al. 2010, Ploeg et al. 2011, among others).

However, considering time delays leads to a non-rational transfer function, which poses a problem for many controller design methods. Consequently, many studies have paid attention to rational approximations of time delays. Padé approximation is widely used, since it often gives a better approximation of the delay system than other series of the same order (Golub & Van Loan 1989). However, the order of the Padé approximation, used in CACC systems (Kianfar et al. 2012, Öncü et al. 2014, Ploeg et al. 2014), was not explicitly motivated. Although a higher order Padé approximation leads to a more accurate representation, the resulting system is more complex. In addition, a too high-order Padé approximation may cause numerical problems.

This paper focusses on finding a suitable (low) order for the Padé approximation for the communication delay in view of string stability. To this end, the minimum string-stable time gap is chosen as the string stability property, in the sense that the minimum time gap for the approximated CACC system

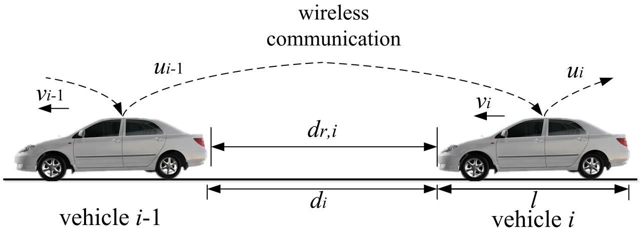

Figure 1. CACC-equipped string of vehicles.

with selected order should be sufficiently close to the one of the original system. The selected order Padé approximation of a PD controller is quite possibly suitable to other linear controllers approximately realizing the same bandwidth as the PD controller, which suffices CACC (string) stability.

The outline of this paper is as follows. The next section introduces a model of a CACC-controlled vehicle string with wireless communication delay. Section 3 presents a frequency-domain method to pursue the appropriate order of the Padé approximation in view of string stability, followed by time-domain analysis in Section 4. The last section summarizes the conclusions.

2 CACC-CONTROLLED PLATOON

2.1 *Platoon with communication delay*

A homogeneous CACC string is considered in this paper, as shown in Fig. 1, where l_i, v_i and u_i are the length, the velocity, and the desired acceleration of vehicle i, respectively; $d_{r,i}$ and d_i represent the desired distance and the actual distance between vehicle i and its preceding vehicle $i-1$, respectively.

In a CACC system, the wireless inter-vehicle link is employed for feedforward purposes. In our case the feedforward input is the desired acceleration u_{i-1} of the preceding vehicle. Although the wireless communication time delay has decreased largely with the recent developments of both software and hardware (Ploeg et al. 2014), it still has a significant effect on string stability of CACC. Thus, considering the wireless communication θ_c, the true feedforward input of vehicle i is

$$u_{i-1,c}(t) = u_{i-1}(t-\theta_c), \tag{1}$$

where θ_c consists of the total time delay due to queueing, contention, transmission, and propagation. Note that other network effects in wireless communication, such as packet loss and sampling effects, are not taken into account in this paper. The string is assumed to be homogeneous, which is why the communication delay θ_c is independent of the vehicle index i.

Considering non-linear dynamics of engine and drive train, and aerodynamic drag and rolling resistance, feedback linearization is often adopted to arrive at a suitable model for CACC design (Hedrick, Tomizuka, & Varaiya 1994). As a result, the acceleration a_i of vehicle i follows

$$\tau \dot{a}_i(t) + a_i = u_i(t), \tag{2}$$

where the vehicle actuator time constant τ represents the longitudinal vehicle response, which is set identical for both acceleration and brake situations, for the sake of simplicity. In addition, the transfer function $G(s)$ from the desired acceleration to the position reads,

$$G(s) = \frac{1}{s^2(\tau s + 1)}, \tag{3}$$

where $s \in \mathbb{C}$ is the Laplace variable.

2.2 *CACC Controller*

In a CACC string, each vehicle (except the lead vehicle) aims to keep a desired distance $d_{r,i}$ to its preceding vehicle. The constant time gap policy is utilized, which is the most common one to improve string stability, see Naus et al. (2010) and the references contained therein. Thus, the desired inter-vehicle distance involves a standstill distance and a velocity-dependent part:

$$d_{r,i}(t) = r + hv_i(t) \qquad i > 1, \tag{4}$$

where r represents the standstill distance, and h is a constant time gap. We pursue a small value of h to increase traffic throughput. However, inherent time delays in the wireless communication yield a lower bound for the time gap from a string-stability perspective (Naus et al. 2010, Ploeg et al. 2011).

To realize the vehicle-following objective, the inter-vehicle distance error e_i, defined as

$$e_i(t) = d_i(t) - d_{r,i}(t), \tag{5}$$

should asymptotically converge to zero. To this end, a variety of controllers have been proposed (Hedrick, Mcmahnon, & Swaroop 1993, Huang & Ren 1997, Stanger & del Re 2013, among others). However, a linear PD controller is most widely adopted, especially in experimental applications (Ploeg et al. 2011, Milanés et al. 2014). Adopting the controller structure in the work of Ploeg et al. (2011), a pre-compensator is used, introducing a new input ξ_i is as,

$$\xi_i(t) := h\dot{u}_i(t) + u_i(t). \tag{6}$$

Together with applying a standard PD controller, given by

$$K(s) = \omega_p + \omega_d s, \tag{7}$$

where ω_p and ω_d represent the proportional and derivative parameters, respectively, it leads to the controller of vehicle $i > 1$ to be,

$$\xi_i(t) = u_{i-1}(t-\theta_c) + \omega_p e_i(t) + \omega_d \dot{e}_i(t). \tag{8}$$

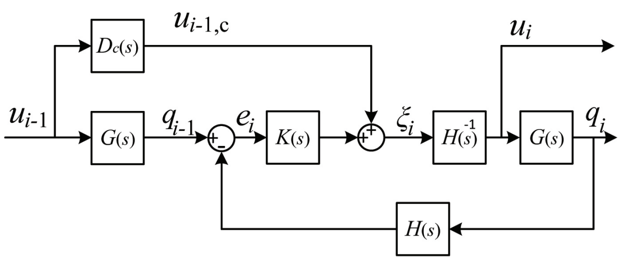

Figure 2. Block scheme of the CACC system.

For analytic convenience,

$$\omega_p = \omega_d^2 \tag{9}$$

is chosen, which reduces an overflow of controller parameters in the sequel (Naus et al. 2010, Öncü et al. 2014). Consequently, the control structure with wireless communication delay θ_c can be depicted as in Fig. 2, where $H(s)$ and $D_c(s)$ are given by

$$H(s) = hs + 1 \qquad D_c(s) = e^{-\theta_c s}. \tag{10}$$

Thus, the transfer function $C(s)$ from the external input u_{i-1} to the inter-vehicle distance error e_i reads,

$$C(s) = \frac{G(s)(1 - D_c(s))}{1 + G(s)K(s)}. \tag{11}$$

Individual vehicle stability requires that all the roots of the denominator of (11) should have negative real parts, where the Routh-Hurwitz stability criterion can be applied, resulting in that the CACC string can be stabilized for $0 < \omega_d < \frac{1}{\tau}$. The PD controller (7) can provide the freedom to choose the bandwidth of (11) as the frequency at which $|G(j\omega)K(j\omega)|$ crosses 0 dB in the downwards sense.

Note that the communication delay θ_c occurring in the feedforward loop does not influence individual vehicle stability in the presented CACC structure.

3 STRING STABILITY BASED ON PADÉ APPROXIMATIONS

3.1 *Padé approximations of delays*

Padé approximation of a time delay yields a finite-dimensional state-space model with a rational transfer function:

$$e^{-\theta_c s} \cong \frac{\sum_{k=0}^{p} \beta_k (-\theta_c s)^k}{\sum_{k=0}^{p} \beta_k (\theta_c s)^k}, \tag{12}$$

where p represents the approximation order, and the coefficients β_k are given by

$$\beta_k = \frac{(2p-k)!p!}{(2p)!k!(p-k)!}. \tag{13}$$

Increasing the order can lead to a more accurate approximation, while the resulting model is more complex. The 1st-, 2nd-, and 3rd-order Padé approximations of a delay are represented as

$$P_1(s) = \frac{1 - \frac{1}{2}\theta_c s}{1 + \frac{1}{2}\theta_c s} \tag{14a}$$

$$P_2(s) = \frac{1 - \frac{1}{2}\theta_c s + \frac{1}{12}(\theta_c s)^2}{1 + \frac{1}{2}\theta_c s + \frac{1}{12}(\theta_c s)^2} \tag{14b}$$

$$P_3(s) = \frac{1 - \frac{1}{2}\theta_c s + \frac{1}{10}(\theta_c s)^2 - \frac{1}{120}(\theta_c s)^3}{1 + \frac{1}{2}\theta_c s + \frac{1}{10}(\theta_c s)^2 + \frac{1}{120}(\theta_c s)^3} \tag{14c}$$

To arrive at a suitable order of the Padé approximation, we apply a large range of system parameters, to find an order leading to similar string stability property as the aforementioned CACC system with pure delays.

3.2 *String stability*

Since the constant time gap policy (4) is introduced to achieve string stability, we choose the minimum string-stable time gap as the string stability property to arrive at the lowest possible Padé approximation order. In other words, the minimum string-stable time gaps for the CACC system with approximated delays and with pure delays should be sufficiently close. To this end, a range of values for the parameters of the vehicle actuator lag τ, the controller parameter ω_d, and the communication delay θ_c are considered.

In the frequently applied performance-oriented approach, string stability is characterized by the amplification in upstream direction of the signal of interest (Naus et al. 2010, Ploeg et al. 2011). Denote the string stability transfer function as $S(s)$, which describes the relation between a relevant (scalar) signal of vehicle i and the corresponding signal of its preceding vehicle $i-1$. In CACC systems, the signals of interest generally involve the inter-vehicle distance error, the acceleration, the velocity, and the distance. Then the system of interconnected vehicles is string stable if and only if

$$\sup_{\omega} |S(j\omega)| \leq 1, \tag{15}$$

with the frequency $\omega \in \mathbb{R}^+$. Note that $S(s)$ is independent of the vehicle index i due to the homogeneity assumption.

Let $E_i(s)$, $U_i(s)$, $V_i(s)$, and $Q_i(s)$ represent the Laplace transforms of the inter-vehicle distance error, the vehicle input (desired acceleration), the velocity, and the position of vehicle i, respectively. Under the assumption of homogeneous traffic, the string stability transfer function $S(s)$ does not depend on the choice of the signal, yielding,

$$\begin{aligned} S(s) &= \frac{E_i(s)}{E_{i-1}(s)} = \frac{U_i(s)}{U_{i-1}(s)} = \frac{V_i(s)}{V_{i-1}(s)} = \frac{Q_i(s)}{Q_{i-1}(s)} \\ &= \frac{1}{H(s)} \frac{D_c(s) + G(s)K(s)}{1 + G(s)K(s)} \end{aligned} \tag{16}$$

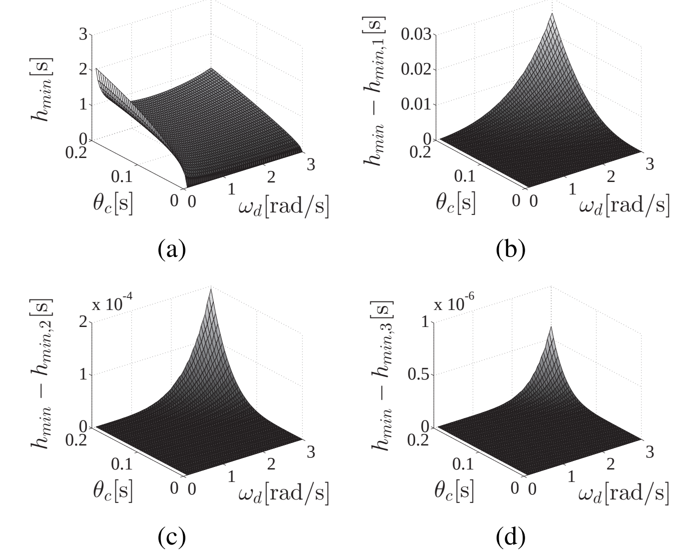

Figure 3. (a) The minimum string-stable time gap h_{min} for CACC with pure delays; differences $h_{min} - h_{min,p}$ of (b) the 1st-order, (c) the 2nd-order, and (d) the 3rd-order Padé approximations, with $\tau = 0.2$ s, $\theta_c \in [0, 0.2]$ s, and $\omega_d \in [0.1, 3]$ rad/s.

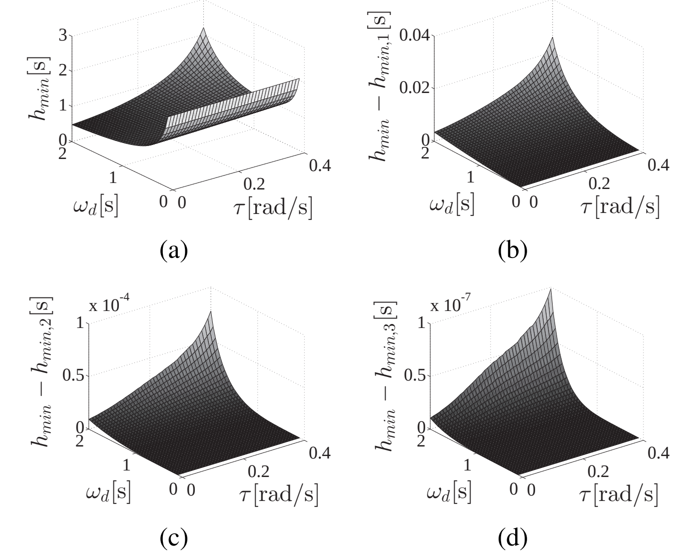

Figure 4. (a) The minimum string-stable time gap h_{min} for CACC with pure delays; differences $h_{min} - h_{min,p}$ of (b) the 1st-order, (c) the 2nd-order, and (d) the 3rd-order Padé approximations, with $\theta_c = 0.2$ s, $\tau \in (0, 0.4]$ s, and $\omega_d \in [0.1, 2]$ rad/s.

Substituting (3), (7), (10) and (16) in (15) yields

$$\sup_{\omega}(\left|\frac{1}{hj\omega+1}\right| \cdot \left|\frac{e^{-\theta_c j\omega} + \frac{\omega_p + \omega_d j\omega}{(j\omega)^2(\tau j\omega+1)}}{1 + \frac{\omega_p+\omega_d j\omega}{(j\omega)^2(\tau j\omega+1)}}\right|) \leq 1 \tag{17}$$

to realize string stability.

In the case without communication delay, (17) reduces to

$$\sup_{\omega}\left|\frac{1}{hj\omega+1}\right| \leq 1, \tag{18}$$

which is fulfilled for any non-negative time gap, i.e., $h \geq 0$ s. However, as previously stated, in reality a communication delay exists, which plays a significant role in designing the time gap for string stability.

When $\theta_c > 0$ s, the magnitude of the string stability transfer function can be expressed as

$$|S(j\omega)| = \frac{1}{\sqrt{(h\omega)^2+1}}\left|\frac{M(j\omega)}{N(j\omega)}\right| \tag{19}$$

where

$$M(j\omega) = e^{-\theta_c j\omega} + \frac{\omega_p + \omega_d j\omega}{(j\omega)^2(\tau j\omega+1)} \tag{20a}$$

$$N(j\omega) = 1 + \frac{\omega_p + \omega_d j\omega}{(j\omega)^2(\tau j\omega+1)} \tag{20b}$$

from which it follows that string stability can be guaranteed for $h \geq h_{min}$, where

$$h_{min} = \sup_{\omega}(\frac{\sqrt{|\frac{M(j\omega)}{N(j\omega)}|^2 - 1}}{\omega}). \tag{21}$$

Substituting (14) and (20) in (21), results in an approximated minimum time gap $h_{min,p}$, where p represents the order of Padé approximation.

Thus, our objective is to find the lowest possible order p such that $h_{min,p}$ is sufficiently close to h_{min}. In practice, the time gap h is changed with increments of 0.1 s. Hence, a controller designed on the basis of Padé approximations should be significantly more accurate regarding string-stable time gap, which is why we assume a maximum error of 0.001 s, i.e.,

$$|h_{min} - h_{min,p}| < 0.001. \tag{22}$$

Note that $h_{min,p}$ is not guaranteed to be larger than h_{min}, i.e., string stability of the approximated system does not imply string stability of the system with pure delay.

Firstly, choose vehicle actuator lag $\tau = 0.2$ s, and controller parameter range $\omega_d \in [0.1, 3]$ rad/s, meeting the individual vehicle stability requirement $0 < \omega_d < \frac{1}{\tau}$. The time delay range $\theta_c \in [0, 0.2]$ s is considered, which covers possible communication delay of CACC systems, even when mild packet loss occurs. Fig. 3 shows the minimum time gap h_{min} and the differences $h_{min} - h_{min,p}$, with $p = 1, 2, 3$, as a function of θ_c and ω_d. Time gaps above the surface in Fig. 3(a) will guarantee string stability of the CACC string. Fig. 3(b) indicates that the differences $h_{min} - h_{min,1}$ are nearly 0.03 s for the 1st-order Padé approximation. In Fig. 3(c) and (d), for 2nd- and 3rd-order Padé approximations, $h_{min} - h_{min,p}$ are less than 2×10^{-4} s and 1.0×10^{-6} s, respectively, which meet the accuracy requirement (22). Hence, there is no necessity to consider a higher order Padé approximation.

Then, Fig. 4 shows the numerical results of the minimum time gap as a function of ω_d and τ, with a quite large value of $\theta_c = 0.2$ s, since the approximation of a large time delay yielding a sufficiently similar time gap h_{min}, is very likely to also suffice for smaller time delays. The value of the vehicle actuator lag is extended to a large range for passenger vehicles as $\tau \in (0, 0.4]$ s. A controller parameter in

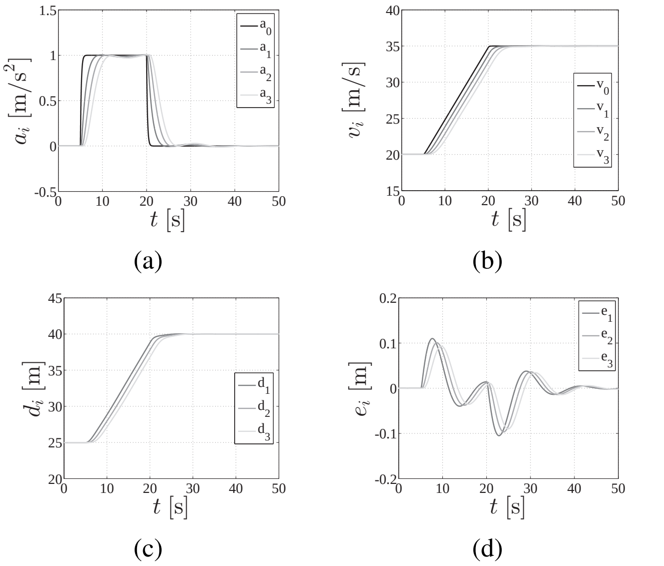

Figure 5. Time responses of (a) the acceleration a_i, (b) the velocity v_i, (c) the inter-vehicle distance d_i, (d) the distance error e_i of CACC with pure delays.

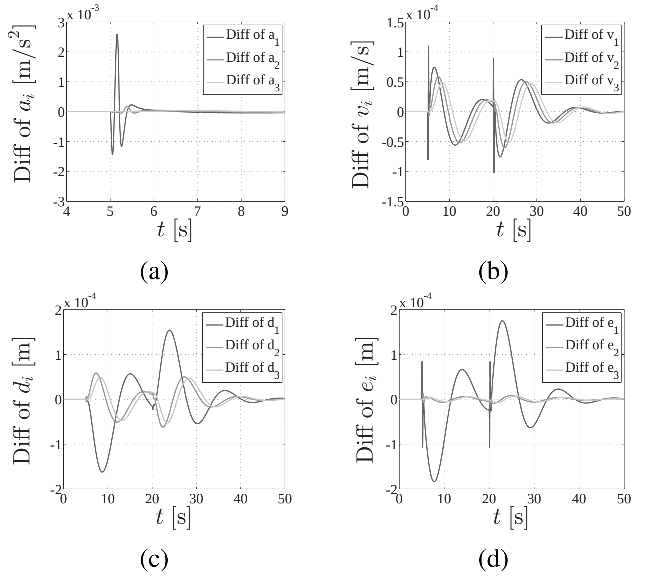

Figure 6. Differences of time responses of (a) the acceleration $a_i - a_{i,2}$, (b) the velocity $v_i - v_{i,2}$, (c) the inter-vehicle distance $d_i - d_{i,2}$, and (d) the distance error $e_i - e_{i,2}$, between the CACC models with pure delays and with 2$^{\text{nd}}$-order Padé approximated delays.

the range $\omega_d \in [0.1, 2]$ rad/s is adopted, where individual vehicle stability is guaranteed. Fig. 4(a) shows how the minimum time gap h_{min} depends on τ and ω_d with pure delays. In Fig. 4(b), (c) and (d), the differences of the minimum time gap between the model with pure delays and 1$^{\text{st}}$-, 2$^{\text{nd}}$- and 3$^{\text{rd}}$-order Padé approximations are less than 3.0×10^{-2} s, 1.0×10^{-4} s and 1.0×10^{-7} s, respectively. Therefore, adopting 2$^{\text{nd}}$- or 3$^{\text{rd}}$-Padé approximations can obtain a sufficiently small value of the difference $h_{min} - h_{min,p}$. Note that the same process can be applied with independent proportional and derivative controller parameters, whereas the resulting differences $h_{min} - h_{min,p}$ have the close values for the corresponding orders of Padé approximations to these as shown in Fig.3 and Fig. 4.

In summary, taking a 2$^{\text{nd}}$-order Padé approximation for communication delays leads to similar minimum string-stable time gap for the original system with exact delays and for the system with the approximated delays.

4 SIMULATION IN TIME-DOMAIN

To validate the accuracy of the 2$^{\text{nd}}$-order Padé approximation of the communication delays, time-domain simulations have been carried out for a CACC string with four vehicles with the standstill distance $r = 5$ m, the length of vehicle as $l = 3$ m, the vehicle actuator lag $\tau = 0.2$ s, and communication delay $\theta_c = 0.2$ s. According to Fig. 3(a), the controller gain is chosen as $\omega_d = 0.8$ rad/s and the time gap as $h = 1$ s to guarantee both string stability and individual vehicle stability.

The external input to the string is desired acceleration u_0 of the leading vehicle, and approximations of delays are most sensitive to a step response. Thus, the desired acceleration for the leading vehicle is set as follows,

$$u_0 = \begin{cases} 1 & 5\,\text{s} \leqslant t \leqslant 20\,\text{s} \\ 0 & \text{other} \end{cases}, \tag{23}$$

where the acceleration interval is chosen such that all follower vehicles can reach the desired acceleration. The initial velocity of all vehicles in this CACC string is $v_{initial} = 20$ m/s. All vehicles start with the desired distance 25 m.

Fig. 5 shows the time responses of the CACC string with pure delays. There is a slight overshoot in the acceleration responses in Fig. 5(a), fulfilling the frequency-domain string stability criterion (15) does not guarantee the absence of overshoot in time-domain (Ploeg et al. 2011, Ploeg et al. 2014). The velocity, inter-vehicle distance, and the distance error responses clearly show that this system is string stable from in Fig. 5(b), (c), and (d).

Applying the 2$^{\text{nd}}$-order Padé approximation for communication delays, results in the approximated responses $a_{i,2}$, $v_{i,2}$, $d_{i,2}$ with the same input (22). The differences between the CACC string responses with pure delays and the ones with approximated delays are shown in Fig. 6, for vehicle 1, 2, and 3, which are CACC controlled. In Fig. 6, all differences of vehicle signals decrease with vehicle number i in the CACC string, i.e., the largest difference occurs in the responses of the first follower vehicle 1. We only zoom in around $t = 5$s in Fig. 6(a), when the acceleration changes, to show the differences clearly. Similar results around $t = 20$s are omitted here. The acceleration differences reach 3.0×10^{-3} m/s^2, which is much smaller than the step input of 1 m/s^2. Fig. 6(b) shows that the velocity differences due to the 2$^{\text{nd}}$-order Padé approximation of delays are less than 1.5×10^{-4} m/s. The differences of inter-vehicle distances and the distance errors are less than 2.0×10^{-4} m as shown in

Fig. 6(c) and (d), which are even less than the possible measurement noise of the inter-vehicle distance. Consequently, it is suitable to select a 2nd-order Padé approximation for communication delays in the string of CACC-equipped vehicles.

5 CONCLUSIONS

In CACC-controlled vehicle platoons, communication time delay inherently exists. Adopting a (low order) Padé approximation for the communication delay leads to a rational transfer function representation of the CACC string, which allows many controller design methods. To arrive at a suitable Padé approximation order, the string stability properties of the system with pure delay and with approximated delay are compared. In particular, the minimum string-stable time gaps are selected as string stability property. With a PD controller, we obtain a satisfying behaviour in terms of bandwidth, as also shown in practice (Naus et al. 2010, Öncü et al. 2014). Hence, for linear controllers approximately realizing the same bandwidth, the results of the selection of Padé approximation orders are probable to hold as well. In this paper, the minimum string-stable time gap for the system with pure delay and for the system with approximated delay of various orders have been determined as a function of controller parameter ω_d, communication delay θ_c and vehicle actuator lag τ. With the frequency-domain analysis, a 2nd-order approximation appears to give satisfying results, providing a good balance between the (low) order and the accuracy in terms of minimum string-stable time gap. Time-domain simulations have been conducted to validate the results.

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