

Interference in wireless networks : a game theory approach

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Interference in wireless networks A game theory approach

by

S. Vovchak, revised by T.G.J. Beelen



Centre for Analysis, Scientific computing and Applications Department of Mathematics and Computer Science Eindhoven University of Technology P.O. Box 513 5600 MB Eindhoven, The Netherlands ISSN: 0926-4507

Postmasters study program Mathematics for Industry

Interference in Wireless Networks A Game Theory Approach

Prepared by Svyatoslav Vovchak

Prepared for IMEC ¹⁾

22.08.2016

Revised by Theo Beelen, University of Eindhoven (TUE) ²⁾

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^{1), 2)} IMEC and TUE as members in the European CORTIF-project



Executive summary

In the current project, we address several aspects of wireless communication. Wireless communication is the type of communication in which information is transferred between two or more points without the use of a wire. It is present in everyday life in various applications such as mobile phone, personal digital assistants, garage door openers, wireless computer mice, keyboards and headsets, headphones, radio receivers, satellite television, and broadcast television.

Main advantages of wireless communication are flexibility and mobility, which allow, for example, to listen to the radio while traveling in a car. While wireless communication is convenient, its implementation creates many challenges. In this project, we study two challenges that arise in wireless communication – interference and power consumption. Interference is the phenomenon that a signal that is transmitted on one sender-receiver channel creates an undesired effect in another sender-receiver channel. A result of interference is low information throughput in the wireless network. Low power consumption is required to create small and light batteries that do not need frequent recharging for the portable wireless devices.

The goal of this project is to investigate if the design specification issues regarding the choice of optimal power, optimal speed, and optimal amount of information in a wireless network can be handled with the use of game theoretical techniques.

In this project we provide the theoretical evidence that game theory can be used to solve the design issues of the wireless network. Particularly, we consider the two scenarios of the wireless networks: non-cooperative and cooperative. We find the equilibriums of both networks for different locations of the (sender, receiver) pairs. We show that optimal power levels increases when the distances between pairs decrease and that optimal power level is lower in the cooperative case than in the non-cooperative case.

We recommend to use game theory approach to find optimal characteristics of the wireless networks. The model developed in this project should be test by empirical experiments. Further, this model can be used as a basis for the dynamic and incomplete information scenarios, and for the design question described in Appendix B.

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1.Management Introduction

1.1. Background of the project

The project was executed for the company IMEC that provides industry-relevant technology solutions in areas as wireless communication, body area networks, bio-electronic technology, image sensors, and smart energy grids. In the current project, we address several aspects of the area of wireless communication.

Wireless communication is the type of communication in which information is transferred between two or more points without the use of a wire. The information is transmitted using antennas and electromagnetic signals that are broadcasted through the air.

Wireless communication came at the turn of the 20th century with the invention of the radio and the radar. Nowadays, it is present in everyday life in various applications such as mobile phone, personal digital assistants, garage door openers, wireless computer mice, keyboards and headsets, headphones, radio receivers, satellite television, and broadcast television. From the moment we entered the era of Big Data, the increase in the use of wireless communication is gigantic. Many new technologies such as automated highways, personal health monitoring, smart homes, and smart cities are partly based on wireless communication.

Wireless communication is the alternative of wired communication. Wireless technologies supplement or replace wired technologies in many homes, businesses, and campuses. The advantages of wireless communication are

• *Mobility*. For example, it is possible to listen to the radio while traveling in a car and cables are not causing troubles while using the Internet with a laptop, because radio waves travel freely through the air.

• *Flexibility*. For example, there is the possibility to connect an additional device without installing an additional socket.

• *Lower cost*. With respect to installation and maintenance a wireless system is believed to be less expensive than a wired one.

While wireless communication is convenient, its implementation creates many challenges. The first challenge is how to allocate the full range of the radio spectrum, which is a scarce resource, to the many different applications and systems. The radio spectrum is divided into many frequency ranges and its division is well regulated. Frequencies of about 2.4 GHz are commonly used by applications as laptops, smartphones, tablets, wireless computer mice, and keyboards. Thus, this frequency range is very crowded and prone to interference.

The second challenge of wireless communication is the power consumption. The power refers to the energy per time unit of the electromagnetic signal that is used to transmit the information. For the portable wireless devices, the aim is to have small and light batteries that do not need frequent recharging. To achieve this aim, low power consumption is required.

The two mentioned challenges – interference and power consumption – constrain the design of the communication network with the objective to have high throughput and speed. We address these challenges in the current project as part of the CORTIF project (Coexistence Of Radio frequency Transmissions In the Future) in which IMEC is involved. CORTIF started in July 2014. The goal of this project is to enable concurrent use of the radio frequency (RF) spectrum by multiple, independent radio systems without harmful channel interference.

1.2. Problem description

A wireless network is a collection of senders, receivers, and channels such that wireless communication between senders and receivers is enabled. As an example of a wireless network, we consider an office of a fixed length and width. In the office, working places are installed. Each worker uses devices that apply wireless communication. A device that transmits information is called a sender and a device that receives information is called a receiver. We assume that each sender has one targeted receiver, so, they form a (sender, receiver) pair. The pairs in the network interfere with each other.

The sender's goal is to ensure that the send information is delivered correctly at the corresponding receiver with minimum power consumed. The probability that the information is delivered correctly depends on the interference level and the power level. The interference decreases this chance while the power level increases it. Each sender can control the power level that it is using. On one hand, the sender can increase its power level to increase the chance to send information correctly. On the other hand, by increasing its power level, the sender increases the interference with the other senders of the network. As a consequence, the other senders increase their power levels to increase their chance of sending information correctly. Thus, interference is increased implying that the chance of sending the overall information in the network correctly is decreased.

Additionally, a (sender, receiver) pair can choose the speed of sending the information and the amount of information to be sent. These two parameters influence the chance of sending information correctly.

In the network, (sender, receiver) pairs interact with each other. To choose the best design of the (sender, receiver) pair, that interaction has to be taken into account. Game theory looks at the interaction between participants in a particular model and predicts their optimal decisions. Thus, game theory is one of the tools that can be used to solve the design problems in the wireless network.

The goal of this project is to investigate if the design specification issues regarding the choice of optimal power, optimal speed, and optimal amount of information in a wireless network can be handled with the use of game theoretical techniques.

1.3. Strategy

To achieve the goal of the project, we adopt the following strategy. In the first step, we studied the literature on how game theory has been used to solve design issues in a wireless network. There are many different applications of game theory in this field depending on the scenario in the wireless network considered.

In the second step, we constructed a conceptual model. Here, we defined the concepts of the wireless network and the concepts of game theory.

In the third step, we constructed a mathematical model of the wireless network and showed how game theory can be used to optimize for power, speed, and amount of information.

In the fourth step, we analyzed our solution strategy and considered two wireless network scenarios: static non-cooperation and static cooperation. For these scenarios, we simulated the network behavior and presented results.

1.4. Results

We consider two scenarios in the wireless network. The first is a non-cooperative scenario. In this case, the pairs act independently from each other and make their choices simultaneously. We find that the optimal solution of this game depends on the location of the network users. Optimal power, optimal speed, and optimal amount of information increase when the distance between the network users decreases. The second is a cooperative scenario. In this case, the pairs maximize power efficiency together. Here, we find that the optimal solution depends on the location of the network users on the same way like in the non-cooperative scenario. However, in the cooperative scenario, optimal power is lower than in the non-cooperative.

1.5. Conclusions and recommendations

The model that we developed in this project is able to handle design issues of the wireless network. That is, we simulate the network behavior and receive results that are expected (for example, optimal power level is higher when the (sender, receiver) pairs are located closer). So, we conclude that game theory is an appropriate tool to solve interference problem in the wireless networks. Note that for each wireless network scenario we need to find an appropriate game that can be used to find optimal characteristics of the network.

Our main recommendation is to validate the model by doing empirical experiments. In current project, we provide only the theoretical evidence that the model works appropriately.

For the future research, additional scenarios should be considered such as dynamic scenario and incomplete information scenario.

2.Modeling

2.1. Literature review

In this section, we provide a literature review of game theory applications in wireless networks. Game theory has been used to tackle the challenges of the design of wireless networks since the early 1990s. The type of game that is used to design the network depends on the network configurations and purposes. There are several wireless network standards depending on the coverage area: personal area networks (PAN), local area networks (LAN), metropolitan area networks (MAN), and wide area networks (WAN)¹ (Han, Niyato, Saad, Basar, & Hjørungnes, 2012). In this project, we concentrate on the wireless PAN (WPAN) and wireless LAN (WLAN) types networks.

In Han et al (2012, Ch. 10), a wide range of game models is considered to analyze the performance of WLANs. The models are developed to address question related to power and rate control, access point selection, and service pricing. The authors consider the WiFi² standard that supports two major configurations: the distributed coordination function (DCF) and the point coordination function (PCF). In the first case, network nodes transmit independently checking the channel availability. That is, if the channel is busy, the node waits for a backoff period. If the channel is idle, it starts transmission. In the second case, nodes listen to the channel and wait for the permission to send signal from the central point.

In Han et al (2012, Ch. 10), performance of a network with a DCF configuration is discussed. In the DCF case, the user can adjust its backoff period to achieve a higher throughput. However, the throughput of the other users will become lower. The conflict is resolved by the use of static and dynamic game models, so that, an efficient and fair throughput for the users is ensured.

In Han et al (2012, Ch. 10), problems of the PCF configuration are discussed. In commercial WLAN access, a server maximizes its revenues by choosing a price charged to the user. The server does not know the users' preferences for service; therefore, an incomplete information game is used to find the optimum price. Another problem considered in Han et al (2012, Ch. 10)

¹ Personal Area Networks (PAN) are networks for interconnecting devices centered on an individual person's workspace; Local Area Networks (LAN) are used to interconnect computers within a limited area such as a residence, school, laboratory, or office building; Metropolitan Area Networks (MAN) are designed for a broader audience than the one for LAN, such as a large corporation or an entire city; Wide Area Networks (WAN) are used to transmit data over long distances and between different LANs and MANs.

² IEEE 802.11 standard. (for more information see IEEE Computer Society, "IEEE standard for local and metropolitan area networks," IEEE Standards Association, 2012)

is admission control. On one hand, the server decides whether a node can join the network. If there are too many nodes, then the network's performance degrades. On the other hand, the node decides whether to stay in the network or not. A non-cooperative game is proposed to resolve this problem.

In Bacci, Sanguinetti, and Luise (2015) and Bacci and Luise (2010), the power control problem is studied. The authors developed a model that can be applied to several network scenarios. For example, the model can be applied to a multicellular system, a cognitive radio system, and a device-to-device system. In their model, the user can choose transmission power to maximize its power efficiency that is defined as the ratio of the throughput and the power. The conflict of interests appears when the user increases its power and with that decreases the throughput of the other users. A non-cooperative static complete information game is developed to resolve this conflict so that all users find their optimum power levels.

Ginde, Neel, and Buehrer (2003) model interaction between network users similarly to Bacci and Luise (2010). The authors use a different throughput model than the one of Bacci and Luise (2010) and they allow for varying data rates and packet sizes. Thus, users have three controls: power level, data rate, and packet size. The authors use a non-cooperative static complete information game and find the equilibrium of the game. Since the equilibrium of the developed game is not unique, additional measures that accounts for tradeoff between throughput and power are used to select the appropriate equilibrium.

Meharouech, Elias, Paris, and Mehaoua (2015) study the interaction of a number of wireless body area networks (WBAN), which is known as a body-to-body networks (BBN). The wireless communication within the WBAN is based on the ZigBee³ technology while the communication between WBAN is based on WiFi technology. So, the authors use a game theoretic approach to analyze mutual and cross-technology interference problems. The interference problem arises when there is a limited number of transmitting channels available. They consider a two stage game relating to the BBN-stage and the WBAN-stage. For the WBAN-stage, a non-cooperative dynamic complete information game is used, where the sensors choose the ZigBee transmitting channel to minimize signal to interference ratio. For the BBN-stage, however, WBANs are separated into groups (sub-BBN) and each group chooses one WiFi transmitting channel to

³ see IEEE Computer Society, "IEEE standard for local and metropolitan area networks," IEEE Standards Association, 2012

minimize its signal to interference ratio under the assumptions of a non-cooperative dynamic complete information game.

In Zou, Liu, and Chen (2014), WBANs do not have the possibility to communicate with each other. Each WBAN has a star topology and consist of several end nodes and one coordinator. The coordinator gathers data from the end nodes by generating time slots within which each end node transmits its data. Thus, there is no interference within the WBANs instead there is inter-WBAN interference. The authors consider a non-cooperative static incomplete information game approach where WBANs maximize their throughput by choosing appropriate power levels. Zou et al (2014) prove the existence of a Nash equilibrium and provide conditions for its uniqueness.

We finish the literature review section with some observation regarding the commonalities of the wireless network studies. In every scenario considered above, a conflict of interests is present. That is, each user adversely affects the performance of the other users while trying to maximize its own performance. One of the widely used performance measures is information throughput. Throughput maximization in wireless networks has been addressed from different perspectives: resource allocation, scheduling, routing by using relay nodes, exploiting mobility of nodes, and channel characteristics. In the current project, we model a wireless network where users optimize their throughput by choosing a power level, data rate, and packet size.

2.2. Conceptual model

In this project, two different disciplines are connected: Wireless Network Theory and Game Theory. Thus, the related concepts are divided into two groups.

2.2.1. Concepts of Wireless Network Theory

Wireless communication is the type of communication in which information is transferred between two or more points without the use of a wire. That is, the information is transmitted by electromagnetic signals that are broadcasted through a physical environment like the atmosphere.

Sender is an electronic device with the ability to transmit information by propagating wireless signals. The sender consumes power while transmitting information. The sender is able to choose the level of power, by which the wireless signals are propagated.

Receiver is an electronic device that receives the information from the sender by capturing the wireless signals. The receiver converts the information carried by the send electromagnetic signals into a usable form.

Channel is used to convey a signal from one or several senders to one or several receivers. A channel has a certain capacity for transmitting information, often measured by its bandwidth in Hertz or its data rate in bits per second.

Interference is the phenomenon that a signal that is transmitted on one sender-receiver channel creates an undesired effect in another sender-receiver channel. So, interference is present because of the interactions between the signals from multiple senders.

Noise is any disturbance of data transmission and signal quality that is not due to interference.

Signal to interference plus noise ratio (SINR) is a measure of the quality of wireless connections. Due to the fact that the energy of a signal fades with distance and due to the presence of interference and noise, the information that the signal carries could not be transmitted successfully. It means that the signal cannot be converted into a usable form or that information is delivered with errors. Such situations occur rarely if the SINR values are high. It means that a high quality of wireless connections and high SINR values go together.

2.2.2. Concepts of Game Theory

Game theory creates a model of optimality taking into consideration the benefits, costs, and the interaction between participants.

Players are the participants in a game. They are the main actors in the problem, who have conflicting interests and affect each other's performance in the game.

Strategy is a set of actions available for each player. It determines the activities of each player during the game.

Utility measures the player's degree of satisfaction as a function of the combination of strategic choices of all players.

Rational players are players that maximize their utility. That is, a rational player is always capable of thinking through all possible outcomes and choosing the course of action that results in the best possible outcome.

Nash equilibrium is the outcome of a game such that no player has the incentive to deviate from its chosen strategy after considering the choices of the other players.

Pareto optimum is the situation that the resources are allocated such that it is impossible to make any player better off without making at least one player worse off.

Cooperative/non-cooperative game. Games can be classified in different levels. A game is called non-cooperative if its players act independently without being able to make a contract concerning each other's actual behavior. A game is called cooperative if agreements between players are possible.

Static/dynamic game. A game is called static if a single decision is made by each player, and each player has no knowledge of the decision made by the other players before making his/her own decision. That is, decisions are made simultaneously (or order is irrelevant). A game is dynamic if the game is repeated a number of times and its players know the outcome of the previous games before playing the next game.

Complete/incomplete information game. A game is called a complete information game if knowledge about players is available to all participants. Every player knows the payoffs and strategies available to the other players. In an incomplete information game, players may or may not have information about the other players, e.g., their "types," their strategies, or utilities.

2.3. Mathematical model

In this section, we develop a mathematical model to calculate network characteristics: throughput and SINR. We construct an objective function of senders such that there is the benefit of having high throughput and penalties for high power. Finally, we translate the network model into a game theoretic terminology.

2.3.1. Wireless network description

Assume there are n (sender, receiver) pairs. That is, there are n senders and n receivers. Each sender has one targeted receiver that forms a (sender, receiver) pair (see Figure 1).



Figure 1. The network architecture

The sender transmits information. The information is divided into packets. There are M bits per packet. When the sender transmits information, the signal that carries this information is distorted by the medium noise and by the signals of the other senders. The important characteristic of the sender is its Signal to Interference and Noise Ratio (SINR). Inspired by Bacci et al (2015), we model the SINR for sender *i* as follows:

$$\gamma_i(\boldsymbol{p}) = \frac{p_i/d_{ii}^{\alpha}}{N * N_c + \sum_{j \neq i} I_{\{d_{ij} < d\}} p_j/d_{ij}^{\alpha}}$$
(1)

in this formula,

 p_i is the power level of the i-th sender, measured in Watt, [W].

$$p = (p_1, \dots, p_n), p_{-i} = (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n).$$

N is the additive white Gaussian noise, measured in Watt [W].

r is the radius of the circle where the sender's signal is available (see Figure 2), measured in meter [m].

 d_{ij} is the relative (dimensionless) distance between receiver i and sender j (see Figure 2). d_{ij} is the relative distance because it is Euclidean distance divided by r (Ginde et al, 2003). We denote the collection of all distances by $D = [d_{ij}]_{i,j=1}^n$ and call this collection the distance matrix.

 $I_{\{d_{ij} < d\}}$ is equal to 1 if the condition $d_{ij} < d$ is satisfied and 0 otherwise. That is, we assume that sender j interferes receiver i if the distance between them is smaller than d * r. If the distance is greater than d * r then there is no interference from sender j (Ginde et al, 2003; Avin, Emek, Kantor, Lotker, Peleg, and Roditty, 2008).

 α is the path-loss exponent. That is, the strength of a signal is assumed to fade polynomially with the distance from the sender (Avin et al, 2008).

 N_c takes one of two values: 1 if the data rate equals to 1 Mbps (Megabits per second); 2 if the data rate equals 2 Mbps.



Figure 2. (Sender, receiver) pairs interactions

 ρ_i the power to noise ratio is denoted: $\rho_i = \frac{p_i}{N}$.

The model of the SINR can thus be presented as:

$$\gamma_i(\boldsymbol{\rho}) = \frac{\rho_i/d_{ii}^{\alpha}}{N_c + \sum_{j \neq i} I_{\{d_{ij} < d\}} \rho_j/d_{ij}^{\alpha}} \tag{1'}$$

Information can be delivered with mistakes because of the existence of noise and interference. It is true that the higher the SINR, the lower the error rate is. Thus, we can consider

set of SINRs of each (sender, receiver) pair as a model for their interactions. Each (sender, receiver) pair wants to have its SINR as high as possible. The pair can achieve that by increasing its power. By doing that, however, it decreases the SINR of the other pairs; therefore, the other pairs would increase their power levels, too.

Senders do not want to waste power. They want to be as efficient as possible in the transmission of information. Efficiency is measured in terms of bits per second sent correctly per power consumed (Bacci et al, 2015). It leads to the following model of efficiency:

$$u_i(\boldsymbol{p}, \boldsymbol{M}, \boldsymbol{R}) = \frac{T_i(\boldsymbol{p}, \boldsymbol{M}, \boldsymbol{R})}{p_i}, \quad \left[\frac{bit}{Joule}\right]$$
(2)

In this formula, $\mathbf{M} = (M_1, ..., M_n)$ is the vector containing the (sender, receiver) pair packet sizes, $\mathbf{R} = (R_1, ..., R_n)$ is the vector containing the (sender, receiver) pair data rates, and $T_i(\mathbf{p}, \mathbf{M}, \mathbf{R})$ is the throughput, i.e. the expected number of bits per second sent correctly. The throughput is modeled as follows:

$$T_i(\boldsymbol{p}, \boldsymbol{M}, \boldsymbol{R}) = I_i(M_i, R_i)F(\gamma_i(p)), \quad [bps]$$

where,

 $I_i(M_i, R_i)$ is the average data rate for the pair *i*, it is measured in bits per second [bps]. The relation between the average data rate I_i , the data rate R_i , and the packet size M_i is as follows:

$$I_i = \frac{M_i}{\frac{M_i}{R_i} + t_{interval}} = \frac{R_i}{1 + \frac{R_i}{M_i} t_{interval}}$$
(3)

where, $t_{interval}$ is the time needed to send two packets on the same data channel. $F(\gamma_i(\mathbf{p})) = (1 - G(\gamma_i(\mathbf{p})))^{M_i}$ is the probability that the packet is sent without an error. $G(\gamma_i(p))$ is the probability that one bit was sent incorrectly, so $1 - G(\gamma_i(\mathbf{p}))$ is the probability of succes for one bit. Since there are M_i bits in a packet, we have to take the success to the power M_i assuming the independence of the success events. In our model we use the following expression for the probability: $G(\gamma_i(\mathbf{p})) = e^{-\gamma_i(\mathbf{p})}$ as a function of the power.

In Table 1, we provide the values of the two data rates and the values of the three packet sizes that are used in our applications.

Table 1 Data rate and packet size values			
Data rate	Packet size		
1 Mhna	37 bytes		
1 Mbps	146 bytes		
2 wibps	255 bytes		

The throughput function takes one of six shapes depending on the combinations of the data rate and the packet size. In Figure 3, we show the shapes of the throughput function with respect to the dimensionless power ρ on a dB scale. We want to define the highest



Figure 3. Throughput as a function of power to noise ratio. The network consists of two (sender, receiver) pairs with $D = \begin{bmatrix} 0.1 & 2 \\ 2 & 0.1 \end{bmatrix}$, $p_{-1} = 5.94 \ db$, $\alpha = 1$.

throughput that can be achieved for each SINR value. If we look from left to right in Figure 3, we conclude that first, the 1^{st} graph has the highest SINR, second, the 2^{nd} has the highest SINR, third, the 3rd graph has the highest SINR, and fourth, the 6^{th} graph has the highest SINR. Therefore, we are interested in the points of intersection of the 1^{st} graph with the 2^{nd} graph, the 2^{nd} graph, and the 3^{rd} graph with the 6^{th} graph. The points of intersection depend on the distance matrix D.

We normalize the efficiency function in the same manner as in Bacci et al (2015). We divide the efficiency function $u_i(p_i)$ by the maximum average data rate I_{max} and multiply by the noise N. Thus, the normalized utility function is expressed as follows:



Similarly to the throughput function, the normalized efficiency function has the six shapes shown in Figure 4. We are interested in the same points of intersection as in the case of the throughput function. The points of intersection in Figure 4 are very much the same as the ones with the intersection in Figure 3.

The problem is to find power, data rate, and packet size such that the efficiency function is maximized.

2.3.2. Modeling a wireless network as a game

The situation where each (sender, receiver) pair wants to be as efficient as possible can be modeled as a game. So, a game theoretic approach is used to find the power level, the data rate and, the packet size that is used to maximize the efficiency.

In a game, three objects should be defined: (1) player, (2) strategy sets, and (3) utility function. In the approach towards wireless networks, we define these objects as follows:

Players are (sender receiver) pairs. There are *n* pairs and they are indexed by *i* ∈ {1, ..., n}.

- Strategy sets. The pair chooses optimal values for the following three variables : (1) power to noise ratio ρ_i ∈ [0, ρ_{i,max}], (2) packet size M_i ∈ {37 * 8, 146 * 8, 255 * 8} (measured in bits), and (3) data rate R_i ∈ {10⁶, 2 * 10⁶} (measured in bps).
- Utility functions are the normalized power efficiencies $\tilde{u}_i(\rho, M, R) =$

$$\frac{I_i(M_i,R_i)}{I_{max}} \frac{\left(1 - e^{-\gamma_i(\rho)}\right)^{M_i}}{\rho_i}$$

The next step is to find the equilibrium of the game. That is, we need to find the Nash equilibrium of the game:

Nash Equilibrium (NE) is the outcome of a game such that no player has the incentive to deviate from its chosen strategy after considering the choices of the opponents. More formally:

Power to noise ratio $\boldsymbol{\rho}^* = (\rho_1^*, \rho_2^*, \dots, \rho_n^*)$, packet size $\boldsymbol{M}^* = (M_1^*, \dots, M_n^*)$, and data rate $\boldsymbol{R}^* = (R_1^*, \dots, R_n^*)$ define the Nash equilibrium if

$$\tilde{u}_i(\boldsymbol{\rho}^*, \boldsymbol{M}^*, \boldsymbol{R}^*) \geq \tilde{u}_i(\rho_i \boldsymbol{\rho}_{-i}^*, M_i \boldsymbol{M}_{-i}^*, \boldsymbol{R}_i R_{-i}^*)$$

for all $p_i \in [0, p_{i,max}]$, $M_i \in \{37 * 8, 146 * 8, 255 * 8\}$, $R_i \in \{10^6, 2 * 10^6\}$ and i = 1, ..., n.

We make the following assumptions about the game:

- Players choose their strategy simultaneously
- Complete information is available
- Players are rational.

3.Analysis of the model

In this section, we consider two scenarios for a wireless network and develop the algorithms to find the Nash equilibrium (NE) for each scenario. The scenarios are as follows:

- Static non-cooperation. In this scenario, each (sender, receiver) pair maximizes its efficiency function individually. The distance between pairs is fixed and they make choice only once. So, the conditions of the network are always the same.
- Static cooperation. Cooperation is allowed. That is, the pairs maximize the joint utility function. As in the previous scenario, the conditions of the network are always the same.

3.1. Static non-cooperative game

In this section, in addition to the assumptions on the game, see Section 2.3.2, we make the following assumptions:

- The game is static. This means that each (sender, receiver) pair makes a single decision and has no knowledge of the decision made by the other players before making his/her own decision.
- The game is non-cooperative. This means that the (sender, receiver) pairs act independently without being able to make a contract concerning each other's behavior

Under these assumptions, we want to find the optimal power level, the optimal data rate, and the optimal packet size such that the efficiency function is maximized. This is equivalent to finding the NE of a static non-cooperative complete information game.

In order to find the NE, first, we need to find the optimum power level of each player:

$$\phi_i(\boldsymbol{\rho}_{-i}, \boldsymbol{M}, \boldsymbol{R}) = \arg \max_{\rho_i, M_i R_i} \frac{I_i(M_i, R_i) \left(1 - e^{-\gamma_i(\boldsymbol{\rho})}\right)^{M_i}}{I_{max}} \qquad (4)$$

Recall that, γ_i depends on ρ . This means that the solution ρ_i of the maximization problem depends on ρ_{-i} , M_{-i} , and R_{-i} . So, we have the following relations:

 $\rho_i = \phi_i(\rho_{-i}, M, R)$, for all i = 1, ..., n

This relation is called the *best response function of player i*. Second, we need to solve the system of equations that is formed by best response functions:

$$\begin{cases} \rho_1 = \phi_1(\boldsymbol{\rho}_{-1}, \boldsymbol{M}, \boldsymbol{R}) \\ \vdots \\ \rho_n = \phi_n(\boldsymbol{\rho}_{-n}, \boldsymbol{M}, \boldsymbol{R}) \end{cases}$$
(5)

The solution ρ^* , M^* , and R^* of this system of equations is the NE.

We do not know the analytical form of system (5); therefore, to find NE, we adopt the approach that consists of the following steps:

- 1) Choose initial power to noise ratios $\{\rho_1^0, ..., \rho_n^0\}$.
- Player 1 considers {ρ₂⁰, ..., ρ_n⁰} as given and solves his maximization problem (4) to obtain ρ₁', M₁', and R₁'. Player 2 considers {ρ₁', ρ₃⁰, ..., ρ_n⁰} as given and solves his maximization problem (4) to obtain ρ₂', M₂', and R₂'. Player 3 considers {ρ₁', ρ₂', ρ₄⁰ ..., ρ_n⁰} as given and solves his maximization problem (4), to obtain ρ₃', M₃', and R₃' and so on. In this way, we finally receive powers ρ', M' and R'.
- 3) Repeat 2) starting with power to noise ratios ρ' . As the result, we have ρ'' , M'' and R''.
- 4) Repeat 3) unless the stopping rule is satisfied:

$$\max|\boldsymbol{\rho}' - \boldsymbol{\rho}''| < \epsilon$$

where ϵ is a small number.

The NE of the static non-cooperative complete information game is ρ'', M'' and R''. That is, if senders use power to noise ratios ρ'' , packet sizes M'', and data rates R'' then a single sender cannot achieve higher efficiency by changing its power to noise ratio, packet size, or data rate.

3.2. Static cooperative game

In this section, we adopt the assumptions about the game made in Section 2.3.2 and the assumption that the game is static made in Section 3.1. However, we change the assumption of non-cooperation made in Section 3.1 into the assumption of cooperation. That is, we consider the static game where the cooperation is allowed.

We assume that n' (sender receiver) pairs are able to make a contract concerning each other's actual behavior ($n' \le n$). This means that the (sender receiver) pairs are optimizing their throughput efficiency together. That is, they have a joint utility function:

$$U(\boldsymbol{\rho}, \boldsymbol{M}, \boldsymbol{R}) = \sum_{i=1}^{n'} \omega_i u_i(\boldsymbol{\rho}, \boldsymbol{M}, \boldsymbol{R})$$
(6)

where, u_i the is individual utility given by formula (2) and ω_i is the weight that points to the importance of the (sender receiver) pair in the cooperation. Weights have to sum up to one: $\sum_{i=1}^{n'} \omega_i = 1, \omega_i > 0$. For example, unequal weights are used in the cognitive radio framework. This allows to distinguish between primary users and secondary users. To find optimal power, optimal data rate, and optimal packet size of the pairs in this framework, we use the following algorithm:

- 1) Choose initial power to noise ratios $\{\rho_1^0, ..., \rho_n^0\}$.
- 2) First *n'* players consider $\{\rho_{n'+1}^0, ..., \rho_n^0\}$ as given and solves the following maximization problem:

$$\max_{\{\rho_i, M_i, R_i\}_{i=1}^{n'}} U(\boldsymbol{\rho}, \boldsymbol{M}, \boldsymbol{R})$$
(7)

The solution of problem (7) is $\{\rho'_i, M'_i, R'_i\}_{i=1}^{n'}$ Player n' + 1 considers $\{\rho'_1, \rho'_3, \dots, \rho'_{n'}, \rho^0_{n'+2}, \dots, \rho^0_n\}$ as given and solves his maximization problem (4) to obtain $\rho'_{n'+1}, M'_{n'+1}$, and $R'_{n'+1}$ and so on. In this way, we finally receive powers ρ', M' and R'.

- 3) Repeat 2) starting with power to noise ratios ρ' . As the result, we have ρ'' , M'' and R''.
- 4) Repeat 3) unless the stopping rule is satisfied:

$$\max|\boldsymbol{\rho}' - \boldsymbol{\rho}''| < \epsilon$$

where ϵ is a small number.

The NE of the static cooperative complete information game is ρ'' , M'' and R''. That is, if senders use power to noise ratios ρ'' , packet sizes M'', and data rates R'' then a single sender cannot achieve higher efficiency by changing its power to noise ratio, packet size, or data rate.

4.Results

In this section, we illustrate how the algorithms that are described in section 3 work. For each scenario that is described in section 3, the parameters that are used in the calculations are given in Table 2. In Table 2, the distance matrix and the path-loss exponent take values from a continuous interval. We study how these parameters affect the NE of considered games.

Table 2 Parameters ⁴					
Notation	Value	Meaning			
Ν	{2,3,4,5}	number of (sender, receiver) pairs in the wireless network			
$[0, \rho_{max}]$	[0,20]	power to noise ratio takes value in a continuous interval (in dB)			
М	{37,146,255}	number of bits per packet (in bytes)			
R	{1,2}	data rate (in Mbps)			
$D = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$	$a \in [0.1, 1]$ $b \in (a \ 4]$	distance matrix (distance is relative, see section 2.3.1)			
α	$2 \le \alpha \le 4$	path-loss exponent ⁵			
ϵ	0.001	tolerance in the stopping criterion			

Using the values of the data rate and the packet size given in Table 2, we calculate the average data rate by the use of Formula (3). The average data rate takes the following values:

$$\begin{split} I_{max} &= 8.9868 * 10^5 \ bps \ , \ t_{interval} = 1.25 \ millisec \ (\text{using formula (3) and the data below}) \\ I_i(10^6 \ bps, 37 * 8 \ bits) &= 1.9146 * 10^5 \ bps \\ I_i \ (10^6 \ bps, 146 * 8 \ bits) = 4.8304 * 10^5 \ bps \\ I_i \ (10^6 \ bps, 255 * 8 \ bits) = 6.2006 * 10^5 \ bps \\ I_i(2 * 10^6 \ bps, 37 * 8 \ bits) = 2.1173 * 10^5 \ bps \\ I_i \ (2 * 10^6 \ bps, 146 * 8 \ bits) = 6.3686 * 10^5 \ bps \\ I_i(2 * 10^6 \ bps, 255 * 8 \ bits) = 6.3686 * 10^5 \ bps \\ I_i(2 * 10^6 \ bps, 255 * 8 \ bits) = 8.9868 * 10^5 \ bps \end{split}$$

⁴ We do not provide values of the parameter d in Table 2 because we consider mainly two players in our simulations and if the distance between players is greater than d then there is no interference, which is not an interesting case for this project.

⁵ See (Avin et al, 2008).

4.1. Static non-cooperative game

In this section, we study how the NE of a static non-cooperative complete information game that is described in Section 3.1 changes with the distance parameters a and b and the path-loss exponent α (see Table 2).

The effect of parameter b on NE.

We consider a wireless network with two (sender, receiver) pairs. We fix the parameters a and α to be 0.5 and 2, respectively. In Table 3, we provide distances between the senders and the receivers. In the upper left corner of Table 3, we have distance 0.5, meaning that the relative distance between the first sender and the first receiver is 0.5. In the first row, the second value 2.5 specifies the relative distance between the first sender the first sender and the second receiver. These distances are illustrated in Figure 5.

	Table 3 Distances					
	Sen	ders				
ivers	0.5	2.5				
Recei	2.5	0.5				



Figure 5 Distances between the second and the first (sender receiver) pairs

So, we have two players in a game that do not cooperate and simultaneously decide on their strategy. We find the NE of this game and provide the results in Table 4. As we see in Table 4, there are two NEs of the game for this parameter specification. In the first equilibrium point, both pairs use 1 Mbps and 255 bytes mode, their power to noise ratio is 6.14 dB and they achieve 0.56 Mbps throughput. In the second equilibrium point, both pairs use 2 Mbps and 255 bytes mode, their power to noise ratio is 9.15 dB and they achieve 0.81 Mbps throughput.

Table 4 The NE o	f the game with t	wo identical (send	ler, receiver) pairs	
Power to noise ratio [dB]	Data rate [Mbps]	Packet size [bytes]	SINR [dB]	Throughput [Mbps]

NE1	6.14	1	255	9.96	0.55993
NE2	9.15	2	255	9.96	0.81259

Next, we change the distance matrix that is given in Table 3. New distance matrices are given in the first column of Table 5. We fix the distance within the pairs and vary the distance between the pairs. In the first case, the pairs are put closer to each other as compared to the distance in Table 3 (see the second row of Table 5) and in the second case, they are put further from each other (see the third row in Table 5). We find NEs for each distance matrix and provide them in Table 5. As we see, in the case that the pairs are close to each other, it is optimal for them to use the higher data rate (2 Mbps > 1 Mbps), a higher power to noise ratio (11.15 dB > 5.34 dB), and they achieve a higher throughput (0.81 Mbps > 0.56 Mbps) than in the case when they are further away from each other. In the last row of Table 5, we see that the distance between the pairs is 1. In this case, the optimal power to noise ratio is 20 dB, the maximum allowed ratio. Additionally, we see that in each case presented in Table 5, there is one equilibrium.

	Tuble 5 THE of the game with two (sender, receiver) pairs						
Dist ma	ance trix	Power to noise ratio [dB]	Data rate [Mbps]	Packet size [bytes]	SINR [dB]	Throughput [Mbps]	Number of equilibrium points
$\big[{}^{0.5}_2$	2 0.5]	11.15	2	255	9.96	0.81266	1
$[0.5 \\ 3]$	$\binom{3}{0.5}$	5.34	1	255	9.96	0.56047	1
$\big[{}^{0.5}_{1}$	$\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$	20	1	37	5.93	0.000643	1

Table 5 NE of the game with two (sender, receiver) pairs

To sum up, there are four types of NE depending on the distance between the pairs: (1) one equilibrium with high data rate and high power to noise ratio, (2) one equilibrium with low data rate and low power to noise ratio, (3) two equilibrium points, and (4) maximum power to noise ratio is reached. These cases are illustrated in detail in Figure 6.

We consider the distance matrix $D = \begin{bmatrix} 0.5 & b \\ b & 0.5 \end{bmatrix}$, $b \in [0.6, 3]$. The fourth case occurs when the distance *b* is in the interval [0.6; 1.61). The interval [0.6; 3) consists of four subintervals. The first subinterval is [0.6, 1.16). For this interval the pairs use the maximum allowed power to noise ratio of 20 dB (see Figure 6.B) and work in the (1Mbps, 37bytes) mode (see Figure 6.A). In this case, if *b* is less than 0.9, then the throughput is less than 1 bps, and if *b* is greater than 0.9 then the throughput is increasing (see Figure 6.C). The situation in this subinterval is similar to the situation presented in the last row of Table 5. The other three subintervals are [1.16, 1.31), [1.31, 1.42), and [1.42, 1.61). For these intervals, maximum allowed power to noise ratio of 20 dB is used (see Figure 6.B), and the (2Mbps, 37bytes), (2 Mbps, 146bytes), and (2 Mbps, 255 bytes) modes are used, respectively (see Figure 6.A).

Another interval for *b* is [1.61; 2.13). It corresponds to the first type of NE. In this case, we have a unique equilibrium as represented by the first row of Table 5. In Figure 6, we see that the (2 Mbps, 255 bytes) mode is used (A), the power to noise ratio decreases from 20 dB to 10 dB as the distance between the pairs increases from 1.61 to 2.13 (B), the throughput is constant (C), and the normalized power efficiency is increasing (D). The next interval for *b* is [2.13; 2.55). It corresponds to the third type of NE. In this case, there are two equilibriums like in the case presented in Table 4. The last interval is $[2.55, d)^6$. It corresponds to the second type of NE. This case is similar to the case in the second row of Table 5.





⁶ Parameter d is introduced in formula (1).



Figure 6 The effect of distance between pairs on Nash Equilibrium. There are four cases: (1) one equilibrium with high data rate and high power to noise ratio, (2) one equilibrium with low data rate and low power to noise ratio, (3) two equilibrium points, and (4) maximum power to noise ratio is reached

The effect of parameter a on NE.

We fix the parameter $\alpha = 2$ and vary the parameters *b* and *a*. The parameter *b* takes only discrete values to account for the cases described above: {1.0, 2.0, 2.5, 3.0}; while the parameter *a* takes values in the continuous interval [0.1, 0.9]. That is, for each value of *b*, we run the program that finds the NE of the game for *a* running through the interval [0.1, 0.9]. The results are similar for each value of *b*; therefore, we show only the results when b = 3.0.

In this case, there are five types of NE (see Figure 7). The first four NE correspond to one of the types that we discussed above. The first – one equilibrium with high data rate and high power to noise ratio – happens when $a \in [0.58, 0.75)$; the second – one equilibrium with low data rate and low power to noise ratio – happens when $a \in [0.25, 0.49)$; the third – two equilibrium points – happens when $a \in [0.49, 0.58)$; and the fourth – maximum power to noise ratio is reached – happens when $a \in [0.75, 0.9)$



Figure 7 The effect of distance within a pair on Nash equilibrium. There are five cases: (1) one equilibrium with high data rate and high power to noise ratio, (2) one equilibrium with low data rate and low power to noise ratio, (3) two equilibrium points, and (4) maximum power to noise ratio is reached

A fifth case happens when $a \in [0.1, 0.25)$. This type of equilibrium is characterized by an optimal power of 0 dB (see Figure 7.B) and a throughput that reaches I_{max} (see Figure 7.C).

The parameters a and b have opposite effects on the equilibrium. When we increase the parameter b then the pairs become further from each other and the interference decreases. As a result, as b increases the optimal power decreases and throughput increases (see Figure 6). When we increases the parameter a then the sender becomes further from it's the targeted receiver. To deliver the information correctly it needs to increase power, so, the interference increases in this circumstances. Throughput decreases in this case.

The effect of the path-loss parameter α on NE.

We run the simulations of the wireless network with distance matrices that are given in Table 3 and Table 5. In the simulations, the path-loss parameter α takes values on the continuous interval [2, 4]. We observe five types of NE described above. In Fig. 8, we show the results for the case with distance matrix $D = \begin{bmatrix} 0.5 & 1.0 \\ 1.0 & 0.5 \end{bmatrix}$. In this case, only two types of NE are observed. The other types of NE are observed when we use different distance matrix. As we see in Figure 8, the effect of the path-loss parameter α is similar to the effect of distance parameter *b*.



Figure 8 The effect of the path-loss parameter on NE. There are five cases: (1) one equilibrium with high data rate and high power to noise ratio, (2) one equilibrium with low data rate and low power to noise ratio, (3) two equilibrium points, and (4) maximum power to noise ratio is reached

4.2. Static cooperative game

In this section, we study how the NE of a static cooperative complete information game that is described in Section 3.2 differs from the NE of a static non-cooperative complete information game that is described in Section 3.1. In this case, two (sender, receiver) pairs optimize a common utility function (6) with the weight vector $\omega = (0.5, 0.5)$.

In Table 6, we provide a result of the simulations of the cooperative scenario. In Table 6, we see that for the considered distance matrices, the senders work in (1 Mbps, 255 bytes) mode. In the cooperative scenario, the senders have lower power to noise ratio and lower throughput than in the non-cooperative scenario. However, in the cooperative scenario the senders are more efficient than in the non-cooperative. To see this, we need to do the following calculations:

Efficiency in cooperative scenario:

power to noise ratio = $10^{0.1*7.08} = 5.1$ efficiency = $\frac{0.4780}{5.1*8.9868*10^5} = 1.0429 * 10^{-7}$

Efficiency in non-cooperative scenario:

power to noise ratio $= 10^{0.1*11.15} = 13.03$

efficiency $=\frac{0.81266}{13.03 * 8.9868 * 10^5} = 6.9391 * 10^{-8}$

Table 6 Non-coo	perative game	e vs Coo	nerative game
	perative guin	2 13 000	perative guine

Distance matrix	nce matrix Network characteristics		Cooperative scenario
	Power to noise ratio [dB]	11.15	7.08
	Data rate [Mbps]	2	1
$\begin{bmatrix} 0.5 & 2\\ 2 & 0.5 \end{bmatrix}$	Packet size [bytes]	255	255
	SINR [dB]	9.96	9.52
	Throughput [Mbps]	0.81266	0.4780
	Power to noise ratio [dB]	5.34	5.19
	Data rate [Mbps]	1	1
$\begin{bmatrix} 0.5 & 3 \\ 3 & 0.5 \end{bmatrix}$	Packet size [bytes]	255	255
	SINR [dB]	9.96	9.65
	Throughput [Mbps]	0.56047	0.54430
	Power to noise ratio [dB]	(6.14, 9.15)*	5.8
	Data rate [Mbps]	(1, 2)	1
$\begin{bmatrix} 0.5 & 2.5 \\ 2.5 & 0.5 \end{bmatrix}$	Packet size [bytes]	(255, 255)	255
	SINR [dB]	(9.96, 9.96)	9.75
	Throughput [Mbps]	(0.55993, 0.81259)	0.52833

*In this case there are two NE as shown in Table 4

5.Conclusions

In this project, we investigate if the design issues of a wireless network can be handled by the use of game theory. To achieve this goal, we consider a general wireless network specification that can be considered as a multicellular network, a cognitive radio network, or a device-to-device network. (Sender receiver) pairs – the network users – maximize their power efficiency by choosing optimal power, optimal speed, and optimal amount of information. The choice of the (sender receiver) pairs depends on the choices of other pairs. Thus, game theoretic approach can be used in this case.

We consider two scenarios in the wireless network. The first is a non-cooperative scenario. In this case, the pairs act independently from each other and make their choices simultaneously. We find that the NE of this game depends on the location of the network users. Optimal power, optimal speed, and optimal amount of information increase when the distance between the network users decreases. The second is a cooperative scenario. In this case, the pairs maximize power efficiency together. Here, we find that NE depends on the location of the network users on the same way like in the non-cooperative scenario. However, in the cooperative scenario the optimal power is lower than in the non-cooperative.

Thus, we conclude that the design issues in a wireless networks can be handled by the use of game theory. There is no common game specification that can be applied to all wireless network cases; rather, for each wireless network specification we need to find the appropriate game.

6.Recommendations

The current project provide theoretical evidence that game theory can be used to solve the interference problem in a wireless networks. Thus, our main recommendation is to validate the developed model by doing the empirical experiments with wireless networks.

The model developed in this project can be used to solve interference problem in different scenarios. In this project, we consider only two scenarios: non-cooperative and cooperative. There are many more scenarios that can be considered. For example, the dynamic scenario ((sender, receiver) pairs chose their strategy every time period and the circumstances of the network change over time, the incomplete information scenario ((sender, receiver) pairs do not have the information about each other types and preferences). An additional design question is given in Appendix B.

References

- Zhu Han, Dusit Niyato, Walid Saad, Tamer Basar, Are Hjørungnes (2012). Game Theory in Wireless and Communication Networks: Theory, Models, and Applications. *New York: Cambridge University Press*
- Giacomo Bacci, Luca Sanguinetti, and Marco Luise (2015). Understanding Game Theory via Wireless Power Control. *Retrieved from http://arxiv.org/pdf/1504.06607.pdf*
- Giacomo Bacci and Marco Luise (2010). Game Theory in Wireless Communications with an Application to Signal Synchronization. Advances in electronics and telecommunications, 86-97
- Samir Ginde, James Neel, R. Michael Buehrer (2003). Game Theoretic Analysis of Joint Link Adaptation and Distributed Power Control in GPRS. ???,732-736
- Amira Meharouech, Jocelyne Elias, Stefano Paris, and Ahmed Mehaoua (2015). A Game Theoretical Approach for Interference Mitigation in Body-to-Body Networks. ???
- Lei Zou, Bin Liu, and Chang Chen (2014). Bayesian Game Based Power Control Scheme for Inter-WBAN Interference Mitigation. Globecom 2014 - Ad Hoc and Sensor Networking Symposium, 240-245
- Pierre Chevillat, Jens Jelitto, and Hong Linh Truong (2005). Dynamic Data Rate and Transmit Power Adjustment in IEEE 802.11 Wireless LANs. International Journal of Wireless Information Networks, 123-144
- R. Valli and P. Dananjayan (2010). A Non-cooperative Game Theoretical Approach for Power Control in Virtual MIMO Wireless Sensor Network. *International Journal Of UbiComp*, 44-55
- Rekha Menon, Allen B. MacKenzie, R. Michael Buehrer, Jeffrey H. Reed (2004). Game Theory and Interference Avoidance in Decentralized Networks. ???
- James O. Neel, Jeffrey H. Reed, Robert P. Gilles (2004). Game Model for Cognitive Radio Algorithm Analysis. ???
- Vivek Srivastava, James Neel, Allen B. MacKenzie, Rekha Menon, Luiz A. DaSilva, James E. Hicks, Jeffrey H. Reed, Robert P. Gilles (2005). Using Game Theory to Analyze Wireless Ad Hoc Networks. ???

Appendix A. Matlab code

In this Appendix, we provide the Matlab code that we used to calculate NE for each scenario.

Static non-cooperative game

```
classdef Transmitter %we define class Transmitter that has all
necessary properties
    properties
        p_opt %transmitter's optimal power to noise ratio
        p range %interval for power to noise ratio allowed
        dist %distance vector. Contains the distance from
transmitter to all receivers
        pComb %takes integer values from 1 to 6. Value 1 means
that (1Mbps, 37bytes) mode is used. Value 6 means that (2Mbps,
255bytes) is used
        u_max %optimal value of normalized utility
    end
    methods
        function [utility1, optPow, combParam, u_max1] =
OptimSol(obj, RestTr, indx, dataRate)
%this function calculate optimal response of the transmitter to
the behaviors of other transmitters
            alpha = 2;
            cnverRate = zeros(1, size(dataRate,1));
            for i = 1:length(cnverRate)
                cnverRate(i) =
dataRate(i,3)/(dataRate(i,3)/dataRate(i,2)+1.25/(10^3));
            end
            const = [1 1 1 2 2 2];
            througthput1 =
zeros(length(cnverRate),length(obj.p_range));
            utility1 =
zeros(length(cnverRate),length(obj.p_range));
            for i = 1:length(cnverRate)
                NandI = const(i);
                for j = 1:length(RestTr.tr)
                    NandI = NandI +
RestTr.tr{j}.p_opt/(RestTr.tr{j}.dist(indx)^alpha);
                end
                c1 = (1/(obj.dist(indx)^alpha)) / NandI;
                gamma1 = obj.p range * c1;
                througthput1(i,:) = cnverRate(i) * ((1 - exp(-
qamma1)).^dataRate(i,3));
                utility1(i,:) = througthput1(i,:) ./
(max(cnverRate)*obj.p_range);
```

```
end
opt_powl = zeros(1,size(utility1,1));
opt_ul = zeros(1,size(utility1,1));
for i = 1:length(cnverRate)
      [opt_ul(i), opt_powl(i)] = max(utility1(i,:));
end
[opt_ul1, opt_powl1] = max(opt_ul);
optPow = obj.p_range(opt_powl(opt_powl1));
combParam = opt_powl1;
u_max1 = opt_ul1;
end
```

end

end

Next, we create vector of cell type to store all transmitters in it. We add several functionalities to this vector like return vector of optimal power levels of all transmitters. So, we create new class:

```
classdef ArrayTrans
    properties
        tr %vector of transmitters
    end
    methods
        function ar = OptPow(obj)
            ar = zeros(1,length(obj.tr));
            for i = 1:length(obj.tr)
                ar(i) = obj.tr{i}.p_opt;
            end
        end
        function pC = OptParam(obj)
            pC = zeros(1,length(obj.tr));
            for i = 1:length(obj.tr)
                pC(i) = obj.tr{i}.pComb;
            end
        end
        function cl tr = initGuess(cellTR)
            cl_tr = cellTR;
            for i = 1:length(cl_tr.tr)
                al = rand;
                cl_tr.tr{i}.p_opt = al * cl_tr.tr{i}.p_min + (1)
- al) * cl_tr.tr{i}.p_max;
            end
```

```
end
function b = powChange(optVec,obj)
    b = abs(optVec(1) - obj.tr{1}.p_opt);
    for i = 2:length(obj.tr)
        if abs(optVec(i) - obj.tr{i}.p_opt) > b
            b = abs(optVec(i) - obj.tr{i}.p_opt);
        end
    end
end
function b = prmChange(optRate,obj)
    b = 0;
    for i = 1:length(obj.tr)
        if optRate(i) ~= obj.tr{i}.pComb
            b = 1;
            return;
        else
            b = 0;
        end
    end
end
function restTr = restTransm(obj, indx)
    restTr = ArrayTrans;
    restTr.tr = cell(1, length(obj.tr)-1);
    restTr.tr(1:indx-1) = obj.tr(1:indx-1);
    restTr.tr(indx:end) = obj.tr(indx+1:end);
end
```

end

These two classes are used to find equilibrium:

```
CellTransm.tr{i} = Transmitter;
    CellTransm.tr{i}.p_range = 1:0.1:100;
    CellTransm.tr{i}.ut =
zeros(size(DataRateParam,1),length(CellTransm.tr{i}.p_r
ange));
    CellTransm.tr{i}.p_opt = 2.0;%p_init(k);
    CellTransm.tr{i}.dist = distance1(i,:);
    CellTransm.tr{i}.pComb = 1;
    CellTransm.tr{i}.p_min =
min(CellTransm.tr{i}.p_range);
    CellTransm.tr{i}.p_max =
max(CellTransm.tr{i}.p_range);
end
OptPower = zeros(1,n);%OptPow(CellTransm);
OptRate = zeros(1,n);
%CellTransm = initGuess(CellTransm);
x = 0;
CellTransmCopy = CellTransm;
weight1 = [0.5 \ 0.5];
while (powChange(OptPower, CellTransm) >= 0.001) && (x
< 100)
    powChange(OptPower, CellTransm)
%
    %prmChange(OptRate,CellTransm)
    optPow1 = [];
    indx = 1;
    x = x + 1
    OptPower = OptPow(CellTransm);
    OptRate = OptParam(CellTransm);
    %step 1: choose optimal power
    for i = 1:length(CellTransm.tr)
        [CellTransmCopy.tr{i}.ut,
CellTransmCopy.tr{i}.p_opt, CellTransmCopy.tr{i}.pComb,
CellTransmCopy.tr{i}.u_max] =
OptimSol(CellTransm.tr{i}, restTransm(CellTransm, i),
i,DataRateParam);
    end
    CellTransm = CellTransmCopy;
end
```

Static cooperative game

In the cooperative case, class ArrayTrans has two additional functions:

```
function out = EfficiencyMatrix(obj, dataRate, dr1,
dr2, weight1)
            alpha = 2;
            cnverRate = zeros(1, size(dataRate,1));
            for i = 1:length(cnverRate)
                cnverRate(i) =
dataRate(i,3)/(dataRate(i,3)/dataRate(i,2)+1.25/(10^3))
;
            end
            const = [1 \ 1 \ 1 \ 2 \ 2 \ 2];
            out = zeros(length(obj.tr{1}.p_range),
length(obj.tr{2}.p_range));
            for i = 1:length(obj.tr{1}.p_range)
                obj.tr{1}.p_opt = obj.tr{1}.p_range(i);
                for j = 1:length(obj.tr{2}.p_range)
                    obj.tr{2}.p opt =
obj.tr{2}.p_range(j);
                    NandI1 = const(dr1) +
obj.tr{2}.p_opt/(obj.tr{2}.dist(1)^alpha);
                    gamma1 = (obj.tr{1}.p_opt /
(obj.tr{1}.dist(1)^alpha)) / NandI1;
                    througthput1 = cnverRate(dr1) * ((1)
- exp(-gamma1))^dataRate(dr1,3));
                    utility1 = througthput1 /
(max(cnverRate)*obj.tr{1}.p_opt);
                    NandI2 = const(dr2) +
obj.tr{1}.p_opt/(obj.tr{1}.dist(2)^alpha);
                    qamma2 = (obj.tr{2}.p opt /
(obj.tr{2}.dist(2)^alpha)) / NandI2;
                    througthput2 = cnverRate(dr2) * ((1)
- exp(-gamma2))^dataRate(dr2,3));
                    utility2 = througthput2 /
(max(cnverRate)*obj.tr{2}.p_opt);
```

```
out(i,j) = utility1*weight1(1) +
utility2*weight1(2);
                end
            end
        end
        function out = OptInCoop(obj, weight1,
dataRate)
            %UTILIITY = cell(size(dataRat,1),
size(dataRat,1));
            maxU = 0; maxU1 = 0;
            maxI = 0; maxI1 = 0;
            maxJ = 0; maxJ1 = 0;
            maxP1 = 0; maxP11 = 0;
            maxP2 = 0; maxP21 = 0;
            maxData1 = 0; maxData11 = 0;
            maxData2 = 0; maxData21 = 0;
            for i = 1:size(dataRate,1)
                for j = 1:size(dataRate,1)
                     a = EfficiencyMatrix(obj,
dataRate, i, j, weight1);
                      [U, indx] = max(a);
                     [U1, indx1] = max(U);
                     maxU1 = U1(1);
                     maxI1 = indx(indx1(1));
                     maxJ1 = indx1(1);
                     maxP11 = obj.tr{1}.p_range(maxI1);
                     maxP21 = obj.tr{2}.p range(maxJ1);
                     maxData11 = i;
                     maxData21 = ii
                      if maxU < maxU1
                         maxU = maxU1
                         maxI = maxI1;
                          maxJ = maxJ1;
                          maxP1 = maxP11;
                          maxP2 = maxP21;
                          maxData1 = maxData11;
                          maxData2 = maxData21;
                      end
                end
```

```
end
out = [maxP1 maxP2 maxData1 maxData2];
end
```

This two functions are used to find equilibrium in the wireless network:

```
clear
clc
distance1 = importfile1('distance2.txt', 1, 2);
n = size(distance1,1);
DataRateParam = [1 \ 10^{6} \ 37^{8}]
                 2 10^6 146*8
                 3 10^6 255*8
                 4 2*10^6 37*8
                 5 2*10^6 146*8
                 6 2*10^6 255*8];
CellTransm = ArrayTrans;
CellTransm.tr = cell(1,n);
for i = 1:n
    CellTransm.tr{i} = Transmitter;
    CellTransm.tr{i}.p_range = 1:0.1:100;
    CellTransm.tr{i}.ut =
zeros(size(DataRateParam,1),length(CellTransm.tr{i}.p_r
ange));
    CellTransm.tr{i}.p_opt = 2.0;
    CellTransm.tr{i}.dist = distance1(i,:);
    CellTransm.tr{i}.pComb = 1;
    CellTransm.tr{i}.p_min =
min(CellTransm.tr{i}.p_range);
    CellTransm.tr{i}.p_max =
max(CellTransm.tr{i}.p_range);
end
OptPower = zeros(1,n);
OptRate = zeros(1,n);
x = 0;
CellTransmCopy = CellTransm;
weight1 = [0.5 \ 0.5];
```

```
while (powChange(OptPower, CellTransm) >= 0.001) && (x
< 100)
    optPow1 = [];
    indx = 1;
    x = x + 1
    OptPower = OptPow(CellTransm);
    OptRate = OptParam(CellTransm);
    optPowPar = OptInCoop(CellTransm, weight1,
DataRateParam);
    CellTransmCopy.tr{1}.p_opt = optPowPar(1);
    CellTransmCopy.tr{2}.p_opt = optPowPar(2);
    CellTransmCopy.tr{1}.pComb = optPowPar(3);
    CellTransmCopy.tr{2}.pComb = optPowPar(4);
    CellTransm = CellTransmCopy;
end</pre>
```

Appendix B. Additional Design Question

In this Appendix, we formulate an additional design question of wireless communications that can be solved by the use of game theory.

Find the best locations of players in the room

For a given room size and a given number of (sender-receiver) pairs, we need to find the location of the pairs so that all pairs have maximum utility. For this task, two scenarios could be considered:

- There exists a coalition between some players and we find the best location for all players in the room.
- The dynamic case for the on-off random pattern is considered and we calculate the best location based on average utility from simulation. That is, for each location we simulate the on-off pattern and calculate the average utility from simulation. We choose the location with the highest average utility.

Modeling the first scenario: coalition

By N the number of (sender-receiver) pairs is denoted and by L and W we denote the length and the width of the room respectively. N, L, and W are fixed numbers in the model.

Each (sender-receiver) pair has a location that is defined by the two pairs of numbers (l_i^s, w_i^s) and (l_i^r, w_i^r) that are room coordinates of a sender and a receiver with $l_i^s, l_i^r \in [0, L]$ and $w_i^s, w_i^r \in [0, W], i = 1 \dots N$ (see Figure 1). We define *system location* as a collection of the locations of all pairs. The set of all system locations is denoted by L^{sr} . We show the example of the element loc^{sr} of L^{sr} set in Figure 1. So, we construct the following set:

$$L^{sr} = \{\{(l_i^s, w_i^s), (l_i^r, w_i^r)\}_{i=1}^{N} : \ l_i^s, l_i^r \in [0, L], w_i^s, w_i^r \in [0, W]\}$$

Next, we construct a function defined on the set L^{sr} . We denote this function $LEF^{c}(loc^{sr})$ and call it the *Location Efficiency Function (LEF) for the coalition case:*

$$LEF^c: L^{sr} \to \mathbb{R}.$$

For each system location loc^{sr} , $LEF^{c}(loc^{sr})$ is calculated as follows.

Step 1: we use the Euclidean distance to calculate the distance matrix for the given system location loc^{sr} :

$$D = \left[d_{ij}\right]_{i,j=1}^{N},$$

where $d_{ij} = \sqrt{(l_i^r - l_j^s)^2 + (w_i^r - w_j^s)^2}$ is the distance between the receiver *i* and the sender *j*.



Figure 9 Positions of sender receiver pairs

We assume that the distance d_{ii} between the sender *i* and its targeted receiver is fixed. Step 2: we define a game and find the Nash Equilibrium (NE) of the game. In the game, players are (sender-receiver) pairs, the strategy set of each (sender-receiver) pair is the continuous interval $[0, p_{i,max}]$ of power levels, and the utility function of each pair is its power efficiency. NE of the game is the collection of the power levels of all pairs

$$(p_1^*, p_2^*, \dots, p_N^*).$$

Step 3: we calculate the efficiencies of (sender-receiver) pairs at NE and define $LEF^{c}(loc^{sr})$ to be the minimum value of these efficiencies:

$$LEF^{c}(loc^{sr}) = \min\{u_{1}(p_{1}^{*}), u_{2}(p_{2}^{*}), \dots, u_{N}(p_{N}^{*})\}.$$

The solution of the problem for the first scenario is the solution of the following optimization problem:

$$loc_{opt}^{sr} = \arg\max_{loc^{sr}} \{LEF^{c}(loc^{sr})\}$$

Note: There are many solutions of the maximization problem. For example, if N=2 then there are two solutions. The first solution is the location in the opposite corners of the

room and the second solution is the location in the other opposite corners. There are multiple solutions of the optimization problem because in the game, which is considered here, the same distance corresponds to different locations.

Modeling the second scenario: on-off random pattern

We consider the same set of system locations:

$$L^{sr} = \{\{(l_i^s, w_i^s), (l_i^r, w_i^r)\}_{i=1}^{\mathbb{N}} : \ l_i^s, l_i^r \in [0, L], w_i^s, w_i^r \in [0, W]\}$$

We have the same goal as in the first scenario to construct the *Location Efficiency Function for the on-off random pattern case*:

$$LEF^{on}: L^{sr} \to \mathbb{R}$$

For each system location loc^{sr} , $LEF^{on}(loc^{sr})$ is calculated as follows.

Step 1 is the same as in the previous case, we use the Euclidean distance to calculate the distance matrix for the given system location loc^{sr} :

$$D = \left[d_{ij}\right]_{i,j=1}^{N}$$

Step 2: we define a dynamic game with on-off random pattern for the given system location and for the fixed number of time periods T. Each (sender-receiver) pair can take one of two states *on* (sending signal) or *off* (not sending signal, using zero power level). Therefore, we consider the (sender-receiver) pair's behavior as the two states Markov chain. Each time period, (sender-receiver) pairs switch between states randomly according to transition probability matrix. We run Monte Carlo simulation for T periods and calculate average efficiencies and its standard deviation of each (sender-receiver) pair:

$$(u_1^a, \dots, u_N^a) = \left(\frac{1}{T} \sum_{t=1}^T u_1(p_{t|t-1}, t), \dots, \frac{1}{T} \sum_{i=1}^T u_{Ni}(p_{t|t-1}, t)\right)$$

where, $p_{t|t-1}$ is the collection of the power levels that is chosen at time *t* depending on the collection of the power levels used at time *t*-1. That is, at time *t*, each (sender-receiver) pair that is in state *on* finds power that is the best respond to the situation at time *t*-1.

Step 3: we define $LEF^{on}(loc^{sr})$ to be the minimum value of average efficiencies:

$$LEF^{on}(loc^{sr}) = \min\{u_1^a, u_2^a, \dots, u_N^a\}.$$

The solution of the problem for the second scenario is the solution of the following optimization problem:

$$loc_{opt}^{sr} = \arg\max_{lsr} \{LEF^{on}(l^{sr})\}$$

Note: the optimization problem has multiple solutions because of the same reason as in the first scenario and also because of an additional reason. $LEF^{on}(loc^{sr})$ is the estimation of the expected value of random variable. So, we need to check if the estimations are statistically different from each other for different system locations.

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