Design of Da Vinci's bridge in ice

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ABSTRACT: The starting point for the design of the bridge in ice is based on a design made by Leonardo Da Vinci. The original design from the 16th century was designed to span the Golden Horn in Istanbul. This design would have had a span of 240 metres and a width of 24 metres. The bridge would have had the largest span at that time, if the sultan of Turkey had approved the design. This design was the inspiration for this project, a bridge consisting of ice with the largest span until now. Arno Pronk and two master students (Roel Koekkoek and Thijs van de Nieuwenhof) started this project in continuation of the previous ice-building projects (Sagrada Familia in Ice 2015 and the Pykrete dome 2014). Accompanied by a group of master students from Eindhoven University of Technology, the project team worked on this project in collaboration with the structural ice association, the municipality of Juuka and more than 15 international universities and local volunteers in Finland.

1 INTRODUCTION

In the previous years the Pykrete Dome and the 'Sagrada Familia in ice' were realised, with respectively the largest ice dome and the highest tower in ice. In the winter of 2015/2016 the design focused on reaching the largest span in ice ever made.
The structures in previous years were constructed with pykrete, ice reinforced with sawdust or wood pulp. The ice bridge was constructed with cellulose as fiber material to strengthen the ice. By doing so the material should be 3 times stronger and 20 times more ductile than ordinary ice. This paper shows the design and the material properties, the structural decisions and calculations that were made. The main goal was to build a bridge of ice by spraying layers of pykrete on an inflatable. To design the optimal shape we looked at the internal stresses and the deformation; the bridge was designed to have the optimal force distribution and a minimum of deformations. This paper shows several models with different dimensions that were modelled to analyze the possible results and give a proper conclusion for the final design of the bridge.

2 ASSUMPTIONS

2.1 *Soil properties*

The soil under the bridge consists of stone blocks with a diameter from 200-1000 mm, where the major part of the stones have a dimension between 200-500 mm. There might be some cavities between the rocks. The top layer of 200 mm consists of gravel with a maximum diameter of 32 mm. As there the soil on the building site has not been fully investigated, some assumptions need to be made about the properties. The forces that occur at the base of the bridge are mainly caused by the own load of the bridge, these forces need to be distributed at the foundation. To guarantee the reliability of the bridge, either in its final stage as during construction, the soil stiffness is assumed to have a value between 2.5 – 20 MN/m^3. The soil stiffness of a standard gravel/sand mixture lays between 50 – 100 MN/m^3. Because of the uncertainty we have chosen to reduce this stiffness in the calculations.

Since there has been little research, there are more unknown factors, such as the behaviour between the ice and the ground. The friction between the ice and the surface is unclear because of many factors like the temperature and the roughness. To make a conservative decision we have chosen to calculate with a low value for the stiffness. When the soil stiffness increases, it is expected that this has a positive result on the models.

2.2 *Material properties and safety*

The material used for the bridge is 'Pykrete' – ice reinforced with 2% paper fibres (cellulose). To obtain this material, water is mixed with the cellulose and then sprayed onto the bridge surface in thin layers, which can freeze subsequently. Different tests were done under semi-controlled circumstances, from where the following material properties were obtained:

Table 1. Material properties

Material properties Pykrete, 2% cellulose		
Compression strength	Average	
	Characteristic	5.90 N/mm^2
Bending strength	Average	
	Characteristic	0.91 N/mm^2
Young's Modulus		500 M/mm^2

Table 2. Some other material properties of the Pykrete:

Material properties Pykrete	
Density	980 kg/m^3

The material tests were done under circumstances that were not very representative. Therefore, the results for the strength tests are not assumed to be very reliable. To make sure that there is little chance to have a lower material strength than according to the calculations, the following strength properties are used:

Table 3. Design values

Material properties Pykrete, 2% cellulose, design values	
Compression strength	1.0 N/mm^2
Tension strength	0.9 N/mm^2
Young's Modulus	500 N/mm^2

2.3 *Given geometry balloon*

To construct the bridge, a large inflatable will be used as a mould to spray the ice on. Therefore, the ice will adapt the geometry of the balloon. The only way to influence the geometry of the bridge is to control the thickness of the ice.
The balloon is pulled into shape by two large, steel cables. These cables form two arcs, which have a distance from each other of 15 metres at the base and 3 metres at the top. The balloon

surface between these cables form the underside of the bridge. The main part of the balloon will be covered by a rope net, this is done to guide the balloon in its shape. In the first phases of the building period this balloon will have a support function, while the bridge needs to reach a certain thickness to bear its own loads. The balloon surface outside of the cables has no influence on the shape, only on the loading capacity of the balloon itself.

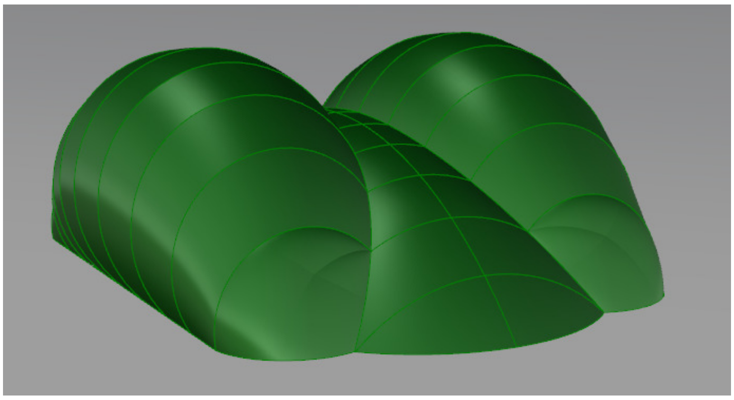

Figure 1: Balloon

2.4 *Balloon pressure*

The balloon will have a support function in the first phases of the building period. In this period the upward pressure of the balloon will be assumed 0.5 kN/m^2. Air pumps will be used to keep the balloon inflated and the balloon will constantly be kept on overpressure to ensure its structural function. The air pressure in the balloon will be constant over the entire surface of the balloon. However, not all air pressure will assist in supporting the bridge. In Figure the top view and side view are shown. Red marks which part will be taken into account to support the bridge. The occurring vertical pressure will be applied in the calculations that will be made. This pressure will be present until the end of the construction.

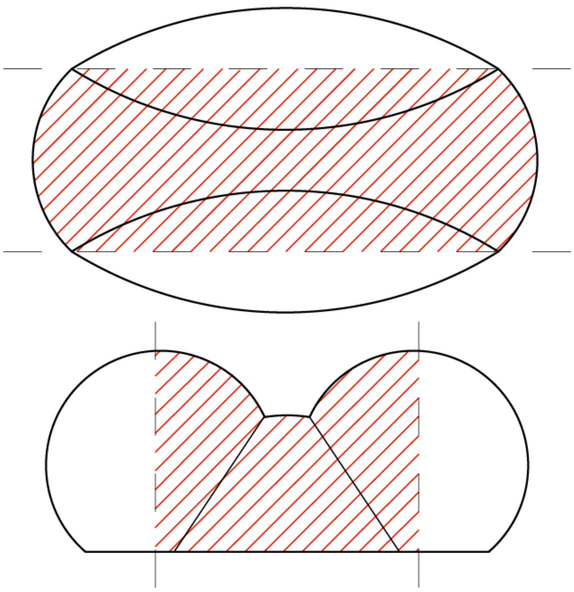

Figure 2: Schematization balloon

2.5 *Balloon*

When the bridge is under construction the balloon will support and stabilize the loads on the inflatable. This is of special importance when the shell of the bridge is still thin. The air pressure in the balloon of 0.5 kN/m^2 will be implemented in the model.

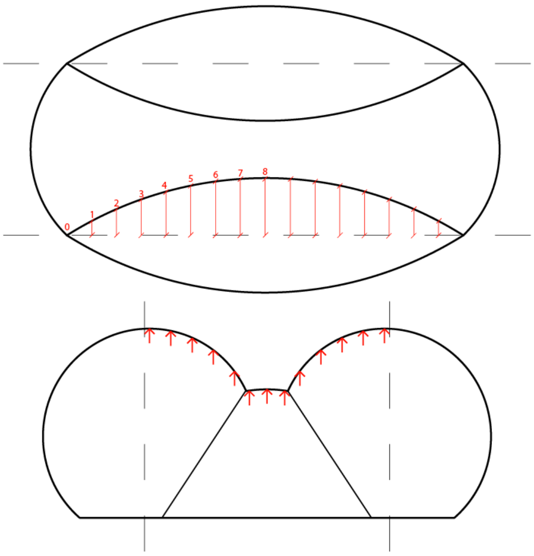

Figure 3: Schematization air pressure

The forces occurring in the model consist of two components: (1) the air pressure directly under the deck of the bridge and (2) the component that will arise from the pre-stress in the steel cables, which results in an upward force along the sides of the bridge. These force components are modelled in the program Abaqus to see the effect of the balloon on the bridge. The air pressure that is located under the bridge deck can be easily modelled by applying a pressure of 0.5 kN/m^2 perpendicular on the bottom surface of the bridge. It is more difficult to determine the value of the pressure that acts on the sides of the bridge due to the steel cables. As mentioned in the assumptions of the balloon pressure only a part of the balloon will be taken into account to contribute to the supporting function of the balloon.

First, the sides of the bridge are fictitiously divided into sixteen equal points in order to make a proper estimation of the force distribution. Only the vertical component of the air pressure contributes in supporting the bridge. A schematization of the air pressure is given in Figure 3. The value of the air pressure differs over the length of the bridge; the pressure will be higher near the centre of the bridge. To determine the pressure that needs to be applied, the length of the balloon from where the pressure exerts needs to be defined. When the top view is examined red lines are drawn which represent the length of the balloon over which the air pressure is taken into account. The air pressure of 0.5 kN/m^2 is multiplied by the length over which the vertical component works. These different pressures can be found below. X represents the different points in which the balloon is divided, starting with X = 0 at the bottom of the bridge (Figure).

Table 4 air pressure

X	0	1	2	3	4	5	6	7	8
Air pressure (kN/m)	0,00	0,65	1,23	1,75	2,19	2,54	2,79	2,95	3,00

As soon as the air pressure on the side of the bridge is known, the next step can be taken on how to model the line load under the bridge. There are two different options observed: (1) where the line load is applied as a pressure under the bridge (see Figure) and (2) to define the line load as several point loads on the bridge (see Figure). In the first option a line load is created that imitates the reality. The downside of this option is the manner in which the force is applied.

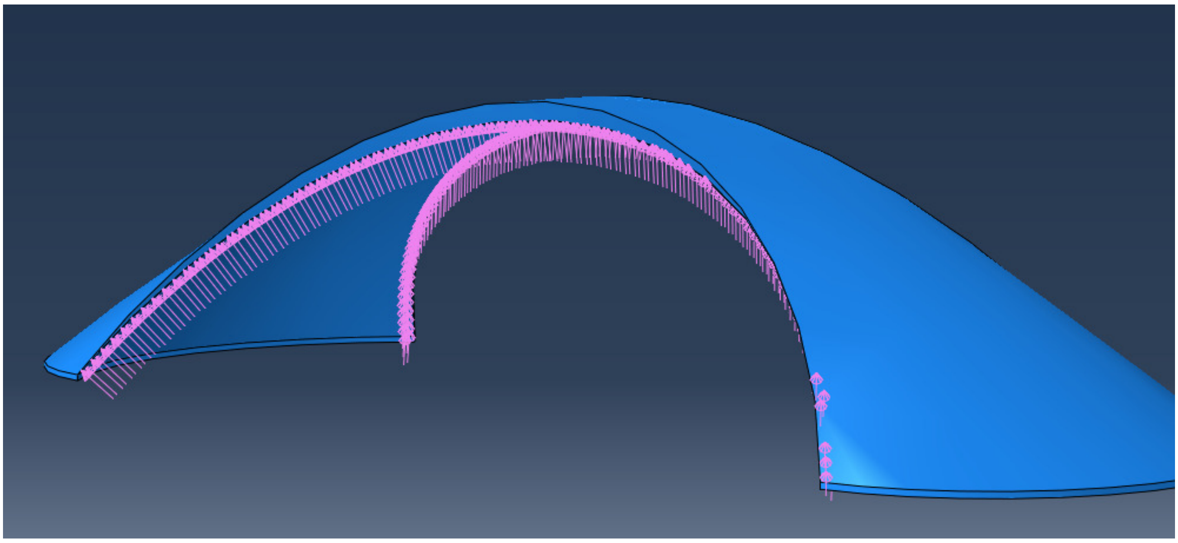

Figure 4: Air pressure, Line load

The load needs to be applied on the mesh of the bridge. In this case the size of the mesh plays a role of great importance in the value of the pressure. The width of the mesh determines the final value of the applied pressure. However, the width of the mesh cannot be determined precisely. Due to this uncertain width factor a proper conclusion cannot be made about the different models, because the mesh size may vary between these various models.

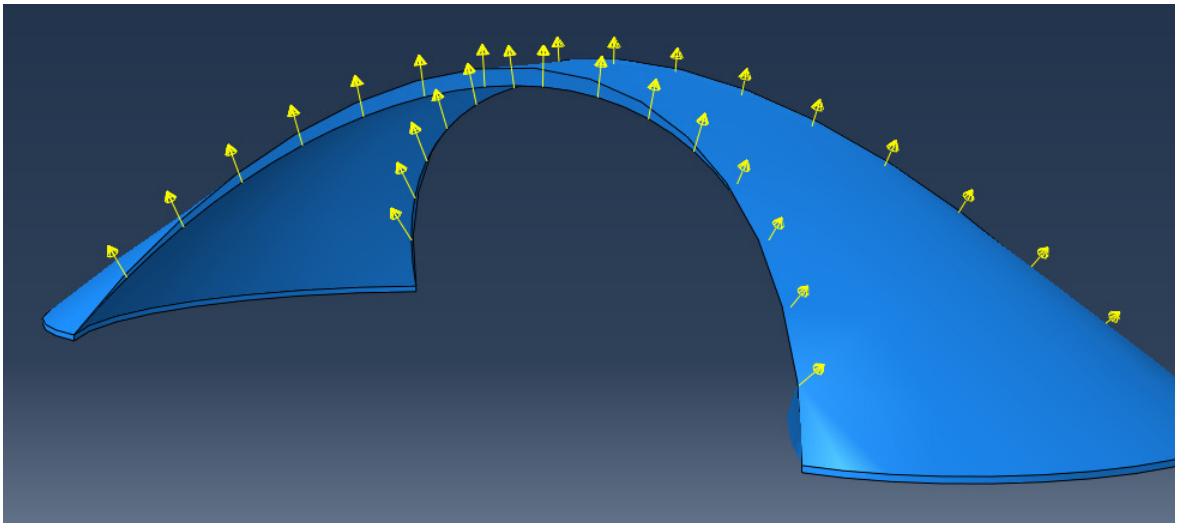

Figure 5: Air pressure, Point load

In the second option the line loads as earlier determined, can be multiplied by the distance between the several points. In this case the exact value of the air pressure can be calculated. The big difference between this method and the previous is the certainty of the occurring pressure, for a concentrated force can be introduced exactly. A disadvantage of this method may be the formation of extreme tension on the position where the point load arises. In reality the force of

the balloon will be exerted as a line load, so these extreme tension will not occur while building. The possibility of an appearance of the extreme tensions is tested in several models. This resulted in the conclusion that these extreme tensions will only occur in bridges with a small thickness with approximately a maximum thickness of 200 millimetre. This means that in the major part of the models this method will be a realistic way to apply the air pressure on the balloon.
Because of the uncertainty of the first method there is chosen to apply the air pressure as several point loads, while investigating different models this methods seemed to be a realistic way to apply the pressure. To get a uniform manner in testing different models there is chosen to create a general model of the pressure that is exerted by the balloon.
To translate the line load correctly to several point loads the distance between these points needs to be determined, this distance has to be multiplied by the average line load calculated earlier which will results in point loads as defined in the details below. The X represents again the different points on the balloon which can be found in Figure .

Table 5. Air pressure

X	0	1	2	3	4	5	6	7	8
Air pressure (kN)	0,00	0,99	2,25	3,64	5,03	6,27	7,25	7,88	8,10

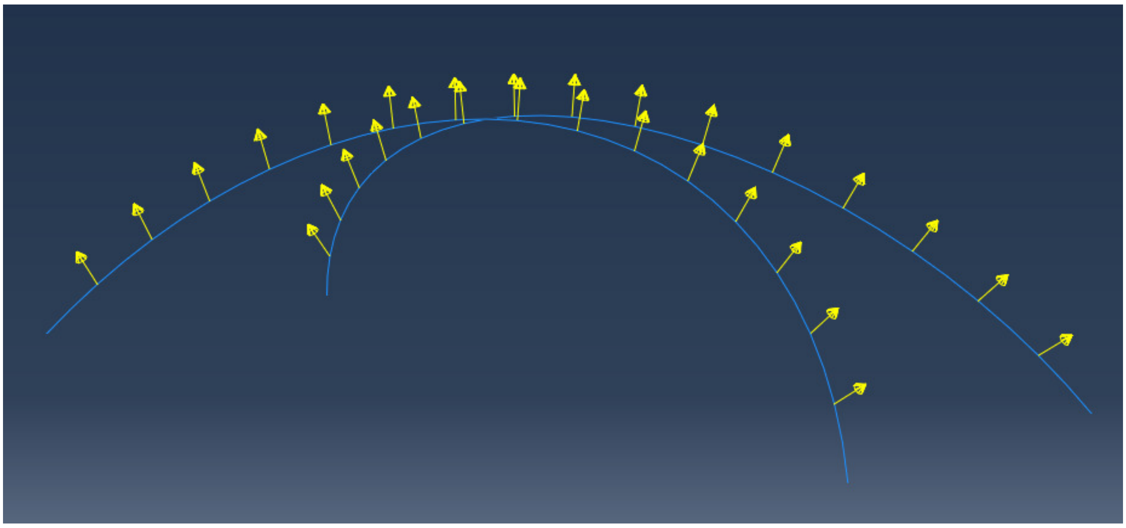

Figure 6. Air pressure, Wire model

With these point loads a standard model can be made that can be used to test different shapes of the bridge, a standard model will ensure that the application of the loads will be similar in all models. The air pressure under the bridge can easily be modelled, therefore there is chosen to apply this force in each model separately. The defined point loads however take some more effort to model correctly. The steel cables will be modelled as wires in Abaqus, the thickness of the wires will correspond with the real thickness of 21 millimetres. As well as the bridge was divided in 16 equal parts, so will the wires be divided in 16 equal parts. The air pressure that is determined for each node will be applied perpendicular on the bridge, like is shown in Figure. This model can be used to test several models of the bridge while the shape of the balloon is already determined in this phase of the design. To make sure the wires correctly transfer their loads to the bridge, the wires will be tied to the sides of the bridge, in this manner the two components will fully cooperate and work as one single element. This will also be the case in reality while the cables will be pressed against the bridge by the balloon. So by using these applications of the air pressure exerted by the balloon it is possible to give conclusion based on what will happen when the bridge is build.

3 APPROACH

3.1 *Use of Grasshopper*

In order to generate different models for calculating different sizes of the ice layer, a Grasshopper model was made. Grasshopper is a plugin for Rhinoceros, in which a script can be made for generating a shape in Rhinoceros. In this script, some parameters were kept variable, so the shape of the 'bridge' can easily be adjusted. The basic approach in this script is:

- Importing the balloon surface;
- Generating the shape of the top surface with variable heights;
- Extruding both the top surface and the balloon surface;
- Apply a 'solid difference' (Boolean) operation on these two extrusions, which should leave only the ice layer;
- The solid ice layer is 'baked' into Rhino, from where the geometry can be further processed for calculation.

Note: because of the complex geometry of the 'top surface', the Boolean operation does not always give a solution. Because of this problem, it was not possible to generate a bridge shape with all parameter combinations. However, there were enough different shapes generated to calculate the effect of the thickness on the internal stresses.
Note: an easier script was used to model the bridge in final situation. This model does give the possibility to generate all possible geometries, but has less parameters and has a flat top surface.

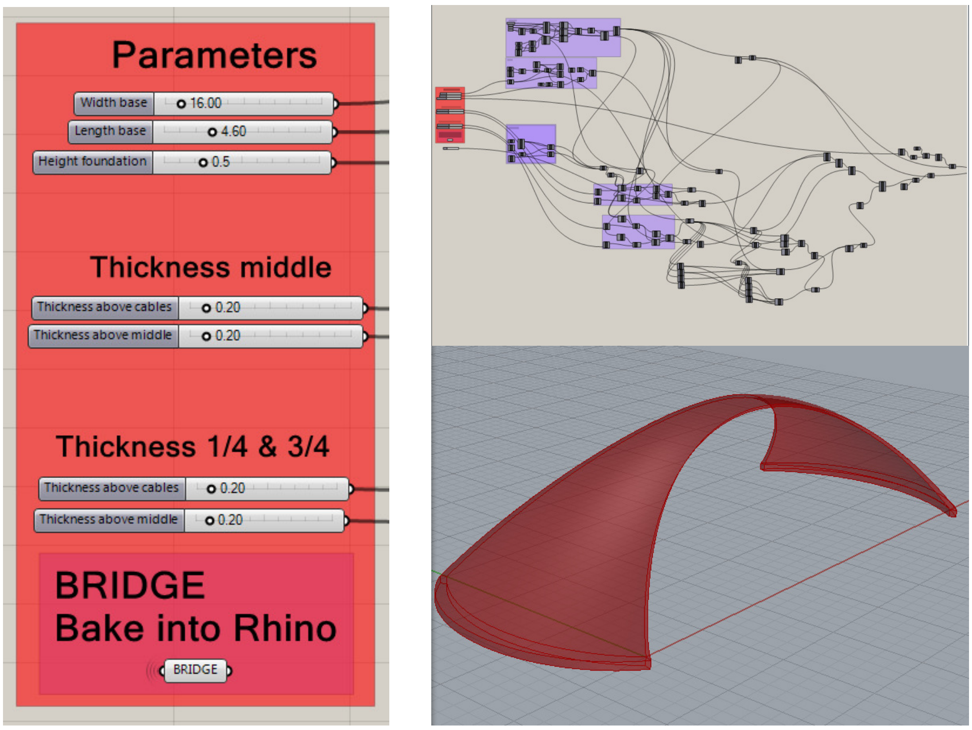

Figure 7: Input rhino

3.2 *Calculations using Abaqus*

Calculations on the bridge are made in Abaqus, a Finite Element Modelling program. In this section, there will be explained how the models are made and how the calculations are done.

3.3 *Input*

For the geometry, a Rhino model is made (using the Grasshopper model). This Rhino model should be a 'solid' shape, which is called a 'closed polysurface' in Rhino. If this is the case, the geometry can be saved as filetype '.iges', which can be imported in Abaqus.

3.4 *Units and values*

Abaqus does not use standard units, but the units depend on the model and on the values that are used. Therefore, awareness of the input values is important and the user should use consistent units. The imported geometry was in millimetres, so the used values and units are:

Table 6 used values

	value	**unit**
Soil stiffness	1,00E-02	*N/mm^3*
Density	9,80E-10	*$Tonne/mm^3$*
Young's modulus	500	*N/mm^2*
Gravity	9810	*mm/s^2*
Balloon pressure	5,00E-04	*N/mm^2*
Poisson ratio	0,15	-
Force	-	*N*
Pressure	-	*N/mm^2*
Stress	-	*N/mm^2*
Displacements	-	*mm*

3.5 *Mesh*

The way the mesh is generated has a large influence on the calculations. In general, when the element size is smaller, the results are more precise. However, using a finer mesh also causes large calculations, which takes much time and needs much processor capacity. The mesh size that was used was based on the smallest thickness of the bridge. The mesh is more precise when there are at least two elements above each other, so the aim was to use element sizes that equals half of the smallest thickness. Because time and processor capacity are limited, the minimum mesh size used was 200 mm.
There are different basic shapes for the elements. Because of the complex geometry of the bridge and the round surfaces, the chosen element shape is a tetrahedron.

4 FINAL BRIDGE

4.1 *Optimal shape*

With the grasshopper model as described, a starting position was made to do a research for the optimal form to be build. The starting position consisted of an arch which has a thickness at the top of 1 meter. From there on the length of base was varied to determine the different forces and tensions in the different designs. The following designs were tested:

Length of base:
5 m, 6 m, 7 m, 8 m, 9 m, 10 m, 11 m, 12 m

These dimensions (length of base/thickness at the top) refer to the following locations of the arch:

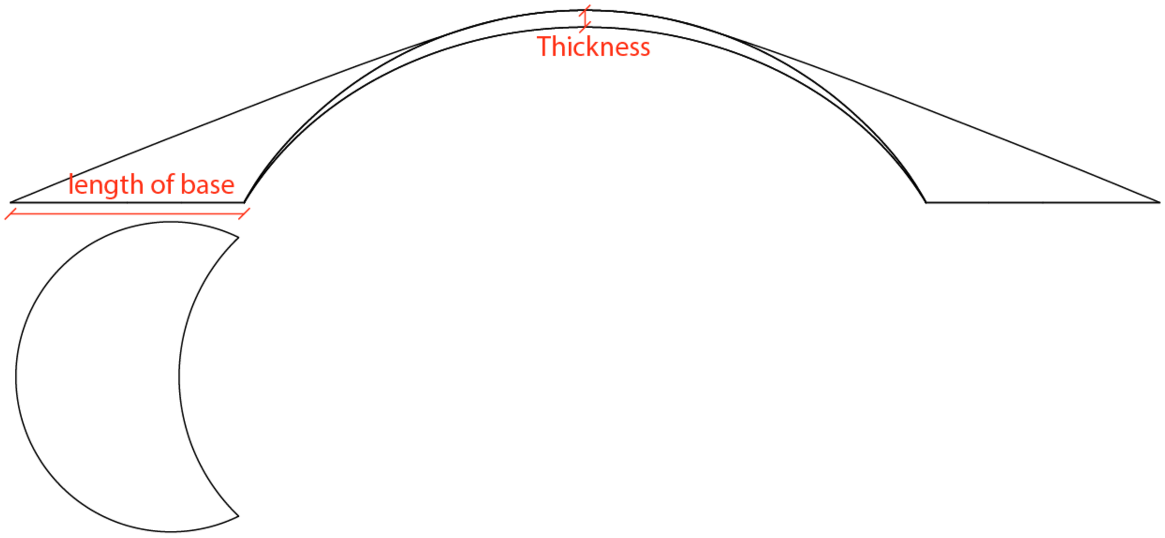

Figure 1: Dimensions of bridge

Importing the grasshopper-model, from Rhino into Abaqus, results in the following tensions for the 5m model:

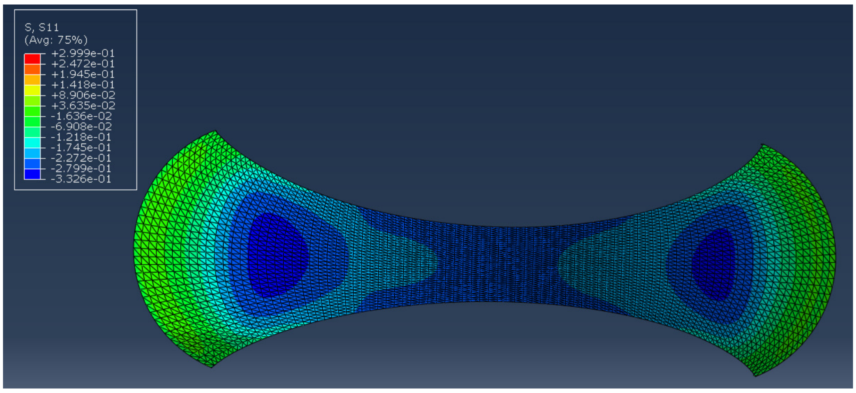

Figure 2: Compressive/tensile stresses, thickness of top 1000mm

And the following deformations:

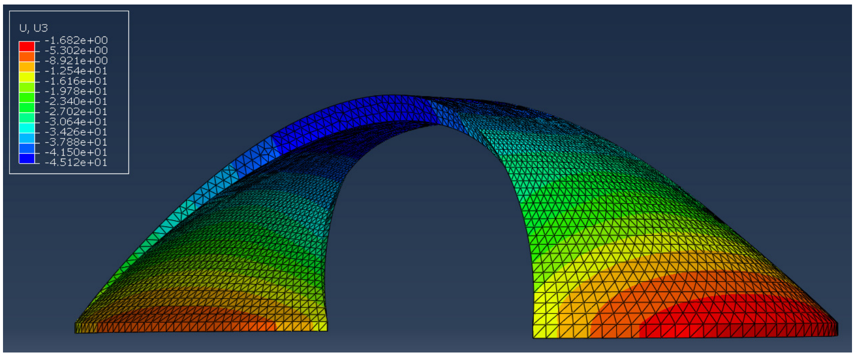

Figure 3: Vertical deformations, Thickness of top 1000mm

To be able to compare the different variations, we put the found maximum tension and deformations in one table, to be able to find the optimal result.

Table 7 output several options

	Length of base [m]							
	5		**6**		**7**		**8**	
Compressive stress	-0,33	N/mm^2	-0,37	N/mm^2	-0,41	N/mm^2	-0,42	N/mm^2
Tensile stress	0,30	N/mm^2	0,23	N/mm^2	0,15	N/mm^2	0,12	N/mm^2
Vertical deformation -	-45,12	mm	-45,16	mm	-49,28	mm	-50,69	mm
Vertical deformation +	1,68	mm	1,18	mm	2,23	mm	2,22	mm
Horizontal deformation at base	14,51	mm	17,85	mm	19,83	mm	20,26	mm
	9		**10**		**11**		**12**	
Compressive stress	-0,43	N/mm^2	-0,43	N/mm^2	-0,37	N/mm^2	-0,43	N/mm^2
Tensile stress	0,14	N/mm^2	0,15	N/mm^2	0,23	N/mm^2	0,16	N/mm^2
Vertical deformation -	-51,13	mm	-51,15	mm	-45,16	mm	-50,54	mm
Vertical deformation +	1,77	mm	1,36	mm	1,18	mm	0,77	mm
Horizontal deformation at base	20,5	mm	20,07	mm	17,86	mm	18,49	mm

These results show that the tensions and deformations of the arch, stay within the given limits. Next step, is to check thinner versions of the same arch. Once again, using Abaqus to determine the tension, we get the next results:

Table 8 output several options

Thickness			Length of base [m] 8		10		12	
	650 mm	**Compressive stress**	-0,47	N/mm^2	-0,48	N/mm^2	-0,49	N/mm^2
		Tensile stress	0,19	N/mm^2	0,18	N/mm^2	0,17	N/mm^2
		Vertical deformation -	-50,99	mm	-49,56	mm	-48,19	mm
		Vertical deformation +	2,59	mm	1,51	mm	0,84	mm
		Horizontal deformation at base	17,26	mm	13,78	mm	12,63	mm
	750 mm	**Compressive stress**	-0,46	N/mm^2	-0,47	N/mm^2	-0,47	N/mm^2
		Tensile stress	0,18	N/mm^2	0,17	N/mm^2	0,15	N/mm^2
		Vertical deformation -	-50,90	mm	-49,84	mm	-48,56	mm
		Vertical deformation +	2,52	mm	1,48	mm	0,83	mm
		Horizontal deformation at base	15,97	mm	15,47	mm	14,26	mm
	850 mm	**Compressive stress**	-0,45	N/mm^2	-0,46	N/mm^2	-0,46	N/mm^2
		Tensile stress	0,16	N/mm^2	0,15	N/mm^2	0,16	N/mm^2
		Vertical deformation -	-51,06	mm	-50,40	mm	-49,30	mm
		Vertical deformation +	2,44	mm	1,45	mm	0,81	mm
		Horizontal deformation at base	17,78	mm	17,26	mm	15,91	mm
	1000 mm	**Compressive stress**	-0,42	N/mm^2	-0,43	N/mm^2	-0,43	N/mm^2
		Tensile stress	0,12	N/mm^2	0,15	N/mm^2	0,16	N/mm^2
		Vertical deformation -	-50,69	mm	-51,15	mm	-50,54	mm
		Vertical deformation +	2,22	mm	1,36	mm	0,77	mm
		Horizontal deformation at base	20,26	mm	20,07	mm	18,49	mm

These values show a number of things. For example, the compressive stress seems to become less once more mass at the top is added. This is probably the result of dividing a relatively same force over a greater surface. The differences in maximum compressive stresses within the same thickness, but with different base lengths, are of such a small amount that these differences are neglected.

Besides that it seems to show that the deformations become less, when the length of the base is increased. An explanation for this can be found in the increase of the moment of inertia, which causes the deformations to decline.

4.2 *Construction*

Starting with the construction of the base, prior to the pouring of arch of the ice bridge will have an very positive effect on the deformations during construction. Especially on deformations as a result of asymmetric loading. This effect is the largest at a shell thickness of 100 mm. Therefore the first layers of ice should be poured with extreme caution and height indicators are definitely advised. When a thickness of 200 mm is reached, the effects of this asymmetrical loading will become far less. A most ideal sequence of pouring the ice is shown in Figure11: Sequence of construction.

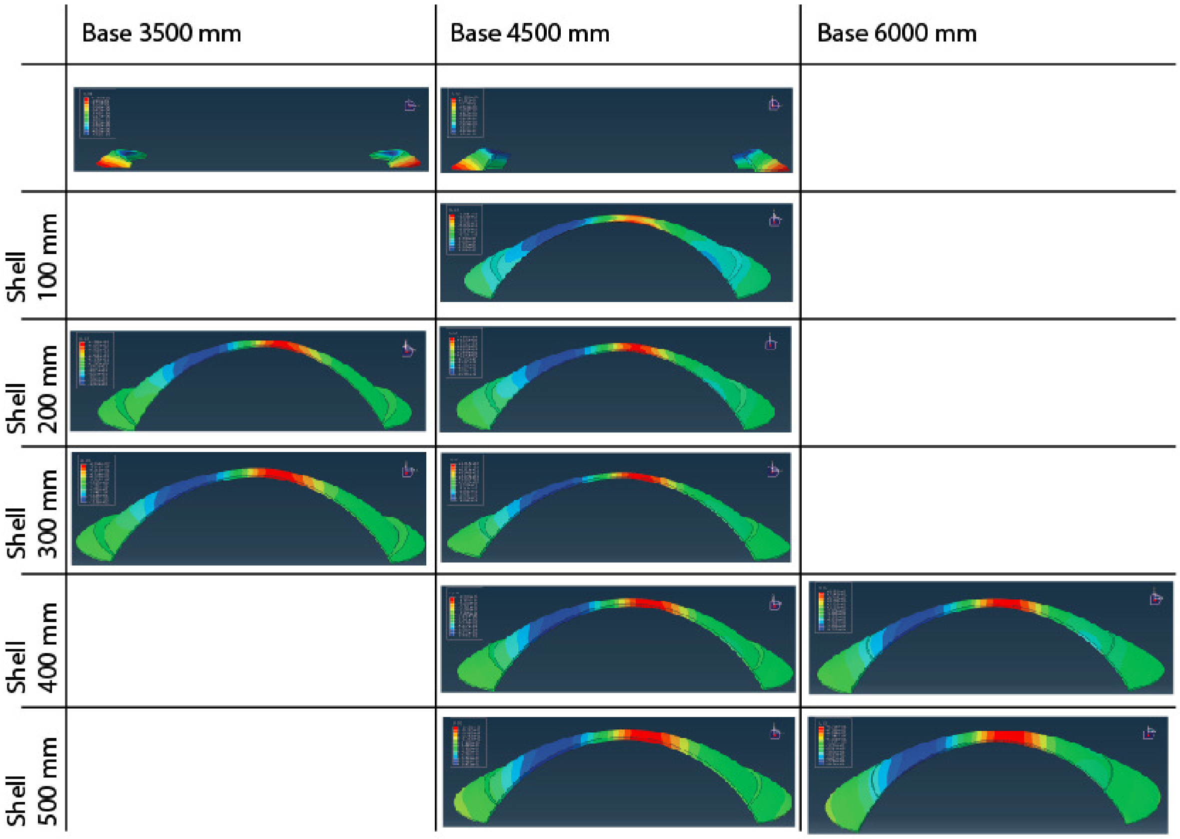

Figure11: Sequence of construction

Because of the uncertainties of creep, the overhang of base in the starting phase should be limited. When the base starts moving downwards, the middle of the bridge will start moving upwards. When this effect becomes too large, this will result in large deformations of the inflatable and therefore in an asymmetric shape of the downside of the bridge. To avoid this problem several struts were used to support the overhang of the base. The volume of one base is approximately 70 m³. What means at least 140 m³ ice should be poured before the construction of the arch of the bridge can begin.

5 CONCLUSION

The optimal shape of the bridge depends on several factors such as the maximum stress and maximum deformation. However, practical parameters also have an influence, for example the accessibility of the bridge. The goal of the project is that people are able to walk over the bridge. Therefore the slope of the bridge cannot be too large. Due to the building method of spraying layers of Pykrete the construction phase should also be taken into account; not all shapes can be built. The balloon sets the boundary conditions for the bridge. With the chosen building method it is also necessary to check the model for eccentric load cases.. It is almost impossible to prevent an asymmetric shape. It is thus necessary to select a range for which the model will still have enough load-bearing capacity. Because of the large amount of unknown properties several safety factors are applied. Also for the final shape there will be a large range for which the model needs to be sufficient.
When the conclusion is solely based upon a perfect, symmetric load a shell thickness of 650 millimetre should easily be able to bear the load. However, as mentioned, it will be impossible

to reach this shape as it is a theoretical shape. Therefore there are several loading combinations tested with different material properties to get an overview of the effects of each parameter.
A maximum compressive stress of 1.0 N/mm^2 will be tolerated; the maximum tensile stress may be 0.9 N/mm^2. To determine the maximum deformation a rule of thumb will be used, the maximum allowed deformation will be 0.004 times the length. This results in a maximum deflection of 140 millimetre. For almost all parameters a conservative value is chosen, for example the Young's Modulus or the soil stiffness. There is, however, an unknown parameter that plays an important role: the creep of pykrete. There is still some research necessary on this topic to properly form a conclusion. Until there are results of a creep test, the design of the model will have a wide range in which it will fulfil for the abovementioned maximum values. Creep depends on several factors: material properties, time exposure, temperature changes and the structural load. The first three parameters have values that cannot be changed or controlled easily. Therefore the only possibility is to reduce the force that occurs on the bridge. The bridge will constantly endure stresses causing the construction to deform slowly. By reducing the internal stresses the creep can be diminished.

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