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Mixed-integer linear program for an optimal hybrid energy network topology

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Abstract—Existing networks do not have the quantitative and qualitative capacity to facilitate the transition towards distributed renewable energy sources. Irregular production of energy over time at different locations will alter the current patterns of energy flow, necessitating the implementation of short- and long-term changes in the energy distribution network. To determine the optimal topology of a future multi-carrier hybrid energy distribution network an optimization model is currently under development.

Keywords—multi-carrier energy networks; network topology optimization; energy conversion; linear programming

I. INTRODUCTION

The world is changing, and with it the energy markets are changing. Fossil fuel reserves are gradually depleting and therefore the extraction of oil, coal and natural gas will become more complicated, more expensive and more unreliable. Furthermore the consumption of fossil fuels has a major impact on our environment and climate [1]. The transition towards renewable energy technologies reduces the need for fossil fuels and therefore also the impact on our environment and climate at the cost of fluctuations in energy production, which are inextricably linked to renewable sources. The current energy distribution system, which was designed to distribute energy produced by few producers among many consumers, is not able to cope with many production units, is not able to cope with units simultaneously or alternately both producing and consuming energy and is also not able to cope with fluctuations in energy availability and energy prices [2]. The development of a network that can cope with these problems and provides the flexibility to switch from carrier anywhere in the network is of great importance to secure the future supply of energy.

Hybrid energy networks provide such flexibility [3]. Unlike a conventional energy distribution network, i.e. an electric power network, a natural gas network or a district heating network, which operates independently from others, a hybrid energy network consists of multiple energy carriers between which conversion is possible [4]. When one of the energy carriers suddenly becomes unavailable, hybrid energy networks introduce the ability to adapt by local or central conversion of an available form of energy into the required

form of energy. When local conversion is possible, hybrid energy networks also introduce the ability to select a specific form of energy for transportation independent from the form of energy produced or the form of energy required.

This paper presents the ongoing PhD research project in which an approach is being developed to determine the optimal layout of a hybrid energy distribution network. This approach determines the location of energy distribution lines, conversion and storage units, given the location of energy producers and consumers in order to find the optimal balance between capital, operational and maintenance costs on the one hand and revenue on the other hand.

This paper is divided into two parts. The first part describes the topology optimization of single-carrier energy distribution networks. Both algorithms found in literature and algorithms developed within the PhD project are covered. The second part describes the topology optimization of multi-carrier networks. Algorithms described in the first part are extended in order to support more than one carrier and in order to support conversion between different carriers.

II. SINGLE-CARRIER ENERGY DISTRIBUTION NETWORK TOPOLOGY OPTIMIZATION

Although the goal of this PhD project is to optimize the topology of multi-carrier energy networks, first the topology optimization of single-carrier networks was addressed. Two different optimization methods were applied, the cross-entropy method, which is a heuristic optimization method and the linear programming method, which requires all equations to be linear. The optimization algorithm under development is versatile and therefore the linear program for the optimization of multi-carrier networks in section 3 is described in more detail than the linear program for single-carrier networks in this section.

A. Cross-entropy method

The cross-entropy method [5] relies on the consecutive generation of collections of random data samples or in this case, collections of random district heating networks. The quality of each randomly generated network is evaluated and the parameters of the random mechanism are updated based on

the outcome of that evaluation. Therefore the quality of the last collection is most likely higher than the quality of any of the preceding collections.

The cross-entropy method allows for separation between the optimization algorithm itself and the algorithms that determine the quality of a solution generated by this heuristic method. This separation provides the opportunity to apply more detailed, non-linear physical and economical equations to determine the quality of a randomly generated network. These are equations to determine (1) investment costs, (2) operational costs, (3) thermal losses, (4) heat generation costs and (5) heat revenue.

For each randomly generated network investment costs, operational costs, costs related to thermal losses, heat generation costs and heat revenue are added to determine the quality of the network in order to generate a higher quality collection in the next iteration. Here quality is related to costs, however quality can also be related to any other objective, e.g. environmental impact or comfort. The cross-entropy method is stopped when the quality of the generated networks no longer increases.

B. Linear programming method

The linear programming method relies on the optimization of linear objective function, subject to linear equality and linear inequality constraints, which means that the non-linear equations described in the cross-entropy section need to be linearized. Compared to the cross-entropy method in which a clear separation between the optimization algorithm and the model is observed, which provides flexibility during development of the model, this separation is less clear in the case of linear programming.

[6] describes how the linear programming method is applied to find the optimal layout of a district heating network. This publication however does not explain how to linearize the equations to determine investment costs, operational costs and thermal losses. In order to determine the optimal layout of a district heating network under different conditions a straightforward method was developed to linearize these non-linear equations.

In [6] investment costs, operational costs and thermal losses vary when the thermal capacity of a distribution line varies. For each specific scenario (1) supply temperature, (2) return temperature, (3) maximum pressure drop (4) thermal properties of the distribution pipe and (5) relation between the diameter and the costs of the distribution pipe are determined in advance.

C. Conclusions

Both the cross-entropy method as the linear programming method return plausible results, however validation is required to confirm this and also to determine the error as a result of the linear approximation. Although the cross-entropy method allows for the optimization of a more detailed model, process time increases exponentially with the size of problem. Also, a heuristic optimization approach, in contrast to the linear

programming method, is not guaranteed to find the optimal solution. For these reasons the linear optimization approach will be applied to find the optimal topology of multi-carrier hybrid energy distribution networks, although some small experiments have been conducted in which the cross-entropy method was applied.

III. MULTI-CARRIER ENERGY DISTRIBUTION NETWORK TOPOLOGY OPTIMIZATION

The algorithms applied to find the optimal topology of single-carrier networks can be applied to find the optimal topology of multi-carrier networks. In order to do so, these algorithms are expanded to cope with multiple energy carriers and conversion of energy, which results in an increased solution space.

The solution space of a single-carrier network consists of a single set of edges (fig. 1, left). Each edge represents a possible location for an energy distribution line. The solution space of a multi-carrier network, with in this example two carriers, consists of three sets of edges (fig. 1, right). Each red edge represents a possible location for an energy line, which distributes the first energy carrier. Each blue edge represents a possible location for an energy line, which distributes the second energy carrier. Each green line represents a possible location for an energy converter, which transfers energy between carriers.

In fig. 1 each edge represents two opposite directed edges. A directed edge allows energy to flow in only one direction. Accordingly each green edge in fig. 1 represents two potential converters. The first converter transfers energy from the first carrier to the second carrier. The second converter transfers energy from the second carrier to the first carrier.

In reality the red and blue edges share the same location, however, for clarity, they are drawn separately. Also, in contrast to the lengths of the red and the blue edges, the lengths of the green edges, representing the converters, have no meaning.

A. Cross-entropy method

First the cross-entropy method is applied to find the optimal topology of a multi-carrier energy system. Although process time increases exponentially with the size of the problem, this method is applicable to smaller problems.

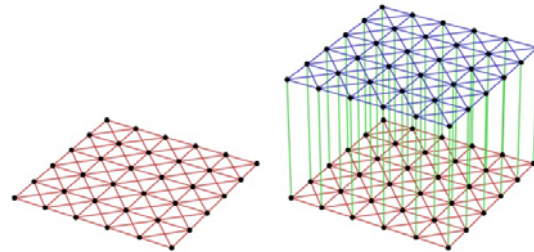


Figure 1 Solution space of a single-carrier (left) and a multi-carrier network (right)

In section 2.1 equations relating to (1) investment costs, (2) operational costs, (3) thermal losses, (4) heat generation costs and (5) heat revenue were introduced. These equations apply to district heating networks. Similar equations are required for the second network, in his case the electric power network. These equations relate to (6) investment costs, (7) operational costs, (8) energy loss to resistance, (9) electric power generation costs and (10) electricity power revenue. Finally, similar equations are required for the conversion units. These equations relate to (11) investment costs, (12) operational costs and (13) energy conversion losses.

Fig. 6 represents a problem that can be solved by applying the cross entropy method. The edges part of the upper half represent possible locations for heat distribution lines. The edges part of the lower half represent possible location for electric power lines. The vertical lines represent possible locations for conversion units.

The magenta colored node (+) part of the upper mesh represents an electric energy production unit. The cyan color nodes (x) part of the upper mesh represent electric energy customers. The magenta colored node (+) part of the lower mesh represent a heat production unit. The cyan colored nodes (x) part of the lower mesh represent heat customers.

Four hypothetical scenarios were examined by the cross-entropy method. In scenario (a) conversion units are relatively expensive and energy revenues are high. In scenario (b) conversion units are relatively inexpensive and energy revenues are high. In scenario (c) energy revenues are low. In scenario (d) conversion units are reasonably priced compared to energy revenues.

Fig. 7, 8, 9 and 10 shows the multi-carrier networks generated by the cross-entropy method for the four scenarios. In scenario (a) expensive energy units and high energy revenues result in two separate, fully connected networks (fig. 7). In scenario (b) inexpensive energy units and high revenues

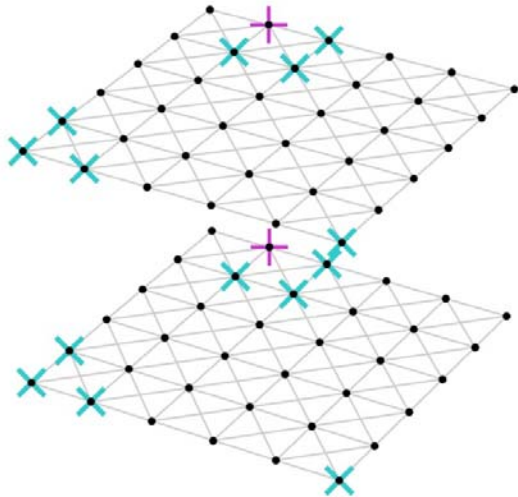


Figure 2 Multi-carrier network problem. Magenta colored nodes (+) represent production units. Cyan colored nodes (x) represent customers

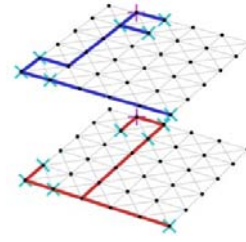


Figure 3 Optimal network in scenario (a)

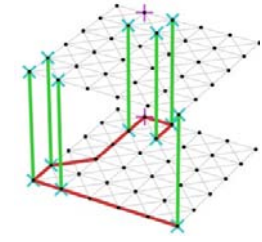


Figure 4 Optimal network in scenario (b)

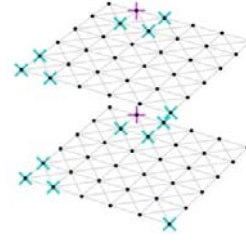


Figure 5 Optimal network in scenario (c)

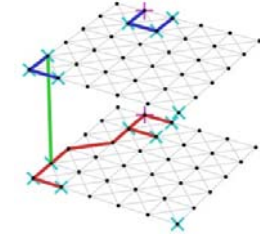


Figure 6 Optimal network in scenario (d)

result in one fully connected network. Instead of constructing a second network, energy conversion units provide the second form of energy (fig. 8). In scenario (c) energy revenues are too low to make an energy distribution network profitable (fig. 9). In scenario (d) customers close to the production unit are directly connected to both production units. A distant group of consumers is directly connected to one production unit. The other form of energy is obtained through conversion. The single, distant consumer is not connected, because that connection would not be profitable (fig. 10).

B. Linear programming

A district heating network with a high-temperature, high-pressure primary network and a low-temperature, low-pressure secondary network can be regarded as a multi-carrier network. Area substations transfer energy between the primary and the secondary network. The multi-carrier topology optimization algorithms described in this paper can be applied to any combination of carriers, including a multistage district heating network.

1) Sets

Let $G(V,E)$ be the graph that represents the solution space. Let V be the set of vertices v_i , representing starting points, end points and crossings of possible future energy distribution lines and energy converters. Let E be the set of directed edges e_{ij} representing possible future energy distribution lines and energy converters. Every possible future energy distribution line and energy converter is represented by two direct edges, e_{ij} and e_{ji} (fig. 5, right).

2) Parameters

Table 1 summarizes all model parameters. Different types of parameters can be distinguished, e.g. economical (c , r), technical (θ) and parameters that describe the existing situation (l , ϵ).

Table 1 Model parameters

Name	Description
$c_{so^c}^{fix}$	Source, fixed investment costs
$c_{so^c}^{var}$	Source, variable investment costs
$c_{so^c}^{om}$	Source, operation and maintenance costs
c_d^{fix}	Drain, fixed investment costs
c_d^{var}	Drain, variable investment costs
c_d^{om}	Drain, operation and maintenance costs
c_c^{fix}	Converter, fixed investment costs
c_c^{var}	Converter, variable investment costs
c_c^{om}	Converter, operation and maintenance costs
c_e^{fix}	Edge, fixed investment costs
c_e^{var}	Edge, variable investment costs
c_e^{om}	Edge, operation and maintenance costs
$c_{so_i^c}^{en}(t_n)$	Energy generation costs
$r_{d_i}^{en}(t_n)$	Energy revenue
k	Derived cost parameter
u	Derived revenue parameter
$\theta_{c_i}^{fix}$	Converter, fixed energy losses
$\theta_{c_i}^{var}$	Converter, variable energy losses
$\theta_{e_{ij}}^{fix}$	Edge, fixed energy losses
$\theta_{e_{ij}}^{var}$	Edge, variable energy losses
α	Annuity factor for investment costs
τ	Time factor
$l_{e_{ij}}$	Edge length
$\epsilon_{so_i^c}$	Source is already connected
ϵ_{d_i}	Drain is already connected
ϵ_{c_i}	Converter already exists
$\epsilon_{e_{ij}}$	Edge already exists
$p_{so_i^c}^{out,lim}$	Source, available capacity
$p_{c_i}^{lim}$	Converter, available capacity
$p_{e_{ij}}^{lim}$	Edge, available capacity
$p_{d_i}^{demand}(t_n)$	Drain, demand

3) Variables

Table 2 summarizes all model variables. Different types of variables can be distinguished, e.g. variables that represents the decision to connect a source or drain and the decision to install a converter or distribution line (x), the power in and out of system components as a function of time ($p(t_n)$) and the peak powers (p^{max}).

Table 2 Model variables

Name	R	Description
z	\mathbb{R}	System costs
$x_{so_i^c}$	$\{0,1\}$	Source, decision variable
$x_{d_i^{nc}}$	$\{0,1\}$	Drain, decision variable
x_{c_i}	$\{0,1\}$	Converter, decision variable
$x_{e_{ij}}$	$\{0,1\}$	Edge, decision variable
$p_{so_i^c}^{out}(t_n)$	\mathbb{R}_0^+	Source, power out
$p_{d_i^{nc}}^{in}(t_n)$	\mathbb{R}_0^+	Drain, power in
$p_{c_i}^{in}(t_n)$	\mathbb{R}_0^+	Converter, power in
$p_{c_i}^{out}(t_n)$	\mathbb{R}_0^+	Converter, power out
$p_{e_{ij}}^{in}(t_n)$	\mathbb{R}_0^+	Edge, power in
$p_{e_{ij}}^{out}(t_n)$	\mathbb{R}_0^+	Edge, power out
$p_{so_i^c}^{max}$	\mathbb{R}_0^+	Source, maximum power
$p_{d_i^{nc}}^{max}$	\mathbb{R}_0^+	Drain, maximum power
$p_{c_i}^{max}$	\mathbb{R}_0^+	Converter, maximum power
$p_{e_{ij}}^{max}$	\mathbb{R}_0^+	Edge, maximum power

4) System costs function

The system costs function (6) sums the costs of the system components, the energy production costs and the energy revenues.

$$z = k_{so^c} + k_{D^{nc}} + k_C + k_E + k_{so^c}^{en} - u_{D^{nc}}^{en} \quad (6)$$

5) Component costs functions

Function (7) returns the sum of the fixed and variable investment costs of the sources. The fixed and variable investment costs depend on the existence and capacity of the sources. Function (8) and (9) return the annualized fixed investment costs and the annualized variable investment costs of the sources.

$$k_{so^c} = \sum_{so_i^c \in SO^c} [k_{so_i^c}^{fix} * x_{so_i^c} + k_{so_i^c}^{var} * p_{so_i^c}^{max}] \quad (7)$$

$$\forall so_i^c \in SO^c: k_{so_i^c}^{fix} = c_{so_i^c}^{fix} * \alpha * (1 - \epsilon_{so_i^c}) + c_{so_i^c}^{om} \quad (8)$$

$$\forall so_i^c \in SO^c: k_{so_i^c}^{var} = c_{so_i^c}^{var} * \alpha * (1 - \epsilon_{so_i^c}) \quad (9)$$

Function (10) returns the sum of the fixed and variable investment costs of the drain. The fixed and variable investment costs depend on the existence and capacity of the drains. Function (11) and (12) return the annualized fixed investment costs and the annualized variable investment costs of the drains.

$$k_{D^{nc}} = \sum_{d_i^{nc} \in D^{nc}} [k_{d_i^{nc}}^{fix} * x_{d_i^{nc}} + k_{d_i^{nc}}^{var} * p_{d_i^{nc}}^{max}] \quad (10)$$

$$\forall d_i^{nc} \in D^{nc}: k_{d_i^{nc}}^{fix} = c_{d_i^{nc}}^{fix} * \alpha * (1 - \epsilon_{d_i^{nc}}) + c_{d_i^{nc}}^{om} \quad (11)$$

$$\forall d_i^{nc} \in D^{nc}: k_{d_i^{nc}}^{var} = c_{d_i^{nc}}^{var} * \alpha * (1 - \epsilon_{d_i^{nc}}) \quad (12)$$

Function (13) returns the sum of the fixed and variable investment costs of the converters. The fixed and variable

investment costs depend on the existence and capacity of the converters. Function (14) and (15) return the annualized fixed investment costs and the annualized variable investment costs of the converters.

$$k_C = \sum_{c_{ij} \in C} [k_{c_{ij}}^{fix} * x_{c_{ij}} + k_{c_{ij}}^{var} * p_{c_{ij}}^{max}] \quad (13)$$

$$\forall c_{ij} \in C: k_{c_{ij}}^{fix} = c_c^{fix} * \alpha * (1 - \epsilon_{c_{ij}}) + c_c^{om} \quad (14)$$

$$\forall c_{ij} \in C: k_{c_{ij}}^{var} = c_c^{var} * \alpha * (1 - \epsilon_{c_{ij}}) \quad (15)$$

Function (16) returns the sum of the fixed and variable investment costs of the edges. The fixed and variable investment costs depend on the existence and capacity of the edges. Function (17) and (18) return the annualized fixed investment costs and the annualized variable investment costs of the edges.

$$k_E = \sum_{e_{ij} \in E} [k_{e_{ij}}^{fix} * x_{e_{ij}} + k_{e_{ij}}^{var} * p_{e_{ij}}^{max}] \quad (16)$$

$$\forall e_{ij} \in E: k_{e_{ij}}^{fix} = c_e^{fix} * l_{e_{ij}} * \alpha * (1 - \epsilon_{e_{ij}}) + c_e^{om} * l_{ij} \quad (17)$$

$$\forall e_{ij} \in E: k_{e_{ij}}^{var} = c_e^{var} * l_{e_{ij}} * \alpha * (1 - \epsilon_{e_{ij}}) \quad (18)$$

6) Energy costs function

For each source function (19) returns annualized energy production costs.

$$\forall so_i^c \in SO^c: \forall t_n \in T: p_{so_i^c}^{en}(t_n) = c_{so_i^c}^{en}(t_n) * \tau * \Delta t \quad (19)$$

7) Energy revenues function

For each drain function (20) returns the annualized energy revenues.

$$\forall d_i \in D: \forall t_n \in T: u_{d_i}^{en}(t_n) = r_{d_i}^{en}(t_n) * \tau * \Delta t \quad (20)$$

8) Vertex constraints

Constraint (21) ensures that for every vertex or energy hub [7] and for every moment in time the amount of power that leaves that vertex at a certain moment in time is at least as large as the amount of power that enters that vertex at the same moment in time. N_i is the set of vertices adjacent to vertex i .

$$\begin{aligned} \forall v_i \in V: \forall t_n \in T: \\ \sum_{m \in N_i^e} [p_{e_{im}}^{in}(t_n)] + \sum_{m \in N_i^c} [p_{c_{im}}^{in}(t_n)] + p_{d_i}^{in}(t_n) \leq \\ \sum_{m \in N_i^e} [p_{e_{mi}}^{out}(t_n)] + \sum_{m \in N_i^c} [p_{c_{mi}}^{out}(t_n)] + p_{so_i^c}^{out}(t_n) \end{aligned} \quad (21)$$

9) Edge constraints

Constraint (22) represents the difference in power into and out of an energy distribution line. Function (23) returns a derived parameter which relates to the variable energy losses and function (24) returns a derived parameter which relates to the fixed energy losses in the energy distribution line.

$$\forall e_{ij} \in E: \forall t_n \in T: \eta_{e_{ij}} * P_{e_{ij}}^{in}(t_n) - P_{e_{ij}}^{out}(t_n) = \delta_{e_{ij}} * x_{e_{ij}} \quad (22)$$

$$\forall e_{ij} \in E: \eta_{e_{ij}} = 1 - l_{e_{ij}} * \theta_{e_{ij}}^{var} \quad (23)$$

$$\forall e_{ij} \in E: \delta_{e_{ij}} = l_{e_{ij}} * \theta_{e_{ij}}^{fix} \quad (24)$$

10) Power constraints

Constraint (25) registers the peak power of every source. The peak power of a source has great influence on the variable investment costs. Constraint (26) ensures a source's peak power never exceeds its predefined limit.

$$\forall so_i^c \in SO^c: \forall t_n \in T: p_{so_i^c}^{out}(t_n) \leq p_{so_i^c}^{out,max} \quad (25)$$

$$\forall so_i^c \in SO^c: p_{so_i^c}^{out,max} \leq p_{so_i^c}^{out,lim} * x_{so_i^c} \quad (26)$$

Constraint (27) registers the peak power of every converter. The peak power of a converter has great influence on the variable investment costs. Constraint (28) ensures a converter's peak power never exceeds its predefined limit.

$$\forall c_{ij} \in C: \forall t_n \in T: p_{c_{ij}}(t_n) \leq p_{c_{ij}}^{max} \quad (27)$$

$$\forall c_{ij} \in C: p_{c_{ij}}^{max} \leq p_{c_{ij}}^{lim} * x_{c_{ij}} \quad (28)$$

Constraint (29) registers the peak power of every energy distribution line. Constraint (30) ensures a line's peak power never exceeds its predefined limit.

$$\forall e_{ij} \in E: \forall t_n \in T: p_{e_{ij}}^{in}(t_n) \leq p_{e_{ij}}^{max} \quad (29)$$

$$\forall e_{ij} \in E: p_{e_{ij}}^{max} \leq p_{e_{ij}}^{lim} * x_{e_{ij}} \quad (30)$$

Constraint (31) ensures that when a consumers is connected demand will always be met.

$$\forall d_i^{nc} \in D^{nc}: \forall t_n \in T: p_{d_i^{nc}}^{in}(t_n) = p_{d_i^{nc}}^{demand}(t_n) * x_{d_i} \quad (31)$$

Equations and constraints (6) up to (31) define a mixed integer linear program which was implemented in MathWorks MATLAB and solved by IBM ILOG CPLEX Optimization Studio.

In fig. 11, which shows the result of the optimization process, analogous to previous examples, the magenta node (+) represents the heat production units, the cyan nodes (x) represent the heat customers, the red lines represent the primary network, the blue lines represent the secondary network and the green lines represent the area substations.

IV. CONCLUSIONS

Both the cross-entropy method as the linear programming method return plausible results, however validation is required to confirm this and also to determine the error as a result of the linear approximation. The solution space of a multi-carrier problem increases with the number of carriers and is therefore at least twice as large as the solutions space of a single carrier problem. Accordingly, as a result of better performance, the

linear programming method is more applicable than the cross-entropy method when dealing with multi-carrier problems.

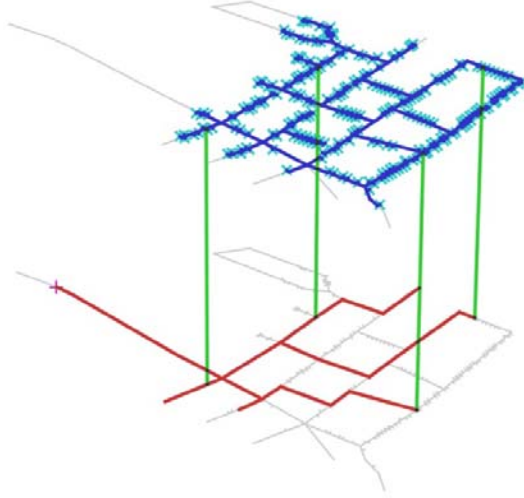


Figure 7 Optimal multi-stage district heating network. The red graph represents the primary network, the blue graph represents the secondary network. The green graph represents the area substations.

V. OUTLOOK

The optimization methods described in this paper return plausible results. Introduction of other carriers, e.g. natural or hydrogen gas, requires collecting data on those carriers and related conversion techniques. Introduction of many distributed renewable energy production units, e.g. rooftop pv-

panels, requires collecting data on the location and power of those techniques.

Implementation of renewable energy techniques is inseparable from energy storage. Dynamic modelling and optimization over time is required to determine the location, capacity and power of potential storage units. To make this possible, first, the existing optimization method needs to be extended in order to remember a storage unit's state. Second, charge and discharge profiles need to be added to the optimization model.

These extensions provide the possibility to determine the optimal topology of a future multi-carrier hybrid energy network, taking into account distributed generation with the possibility to decide between local and central storage.

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