Electrical and Magnetic Model Coupling of Permanent Magnet Machines Based on the Harmonic Analysis

M. Merdzan, S. Jumayev, A. Borisavljevic, K. O. Boynov, J. J. H. Paulides, and E. A. Lomonova

Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven 5612 AZ, The Netherlands

A widely used method for the magnetic field calculation in permanent magnet (PM) machines is the harmonic modeling (HM) method. Despite its many advantages, the application of this method is limited to machines in which armature currents, as a source of the magnetic field, are known. Since most of PM machines are supplied with three phase voltages, any variation of parameters in the armature electric circuit could lead to the uncertainty in knowing the armature currents and therefore limit use of the harmonic modeling method. In this paper an approach for overcoming this limitation is presented which enables the calculation of the magnetic field in PM machines without a priori knowledge of the armature currents.

Index Terms—**Harmonic modeling, permanent magnet machines, high-speed machines, rotor eddy currents.**

I. Introduction

Harmonic modeling (HM) technique is a fast and precise tool for the magnetic field analysis in electrical machines [1]. Different types and configurations of machines can be analyzed using this approach, among them rotating permanent magnet (PM) machines [2]-[3].

Harmonic analysis is based on Fourier representation of sources of the magnetic field which have to be expressed in terms of magnetization and current density [1]. In PM machines magnetization source terms correspond to permanent magnets, while current density source terms correspond to armature currents. However, PM machines are usually supplied by voltage sources and the armature currents are not known in advance. Therefore, they have to be extracted from the applied voltage and parameters of the machine electric circuit, such as resistance and inductance.

The resistance and inductance of the armature circuit cannot be considered constant. Because of the skin and proximity effects occurring in armature conductors [4], the resistance increases with frequency. The armature inductance, on other hand, decreases with frequency [5]. This is caused, especially in high-speed PM machines, by eddy currents flowing in conducting parts of the machine which influence (mostly) the magnetizing inductance. In the work presented here, the change of the armature winding resistance is not analyzed and it is assumed to be known in advance.

To overcome problems related to uncertainty in knowing the inductance, the magnetic field and armature currents in voltage-fed PM machines could be obtained in several steps [6]. The inductance can be calculated separately for a given frequency and then used to calculate the armature currents from the supply voltages. Using the calculated currents the armature magnetic field can be solved. This method has several steps and is not suitable for use in a design procedure where fast tools are necessary.

In this paper a method for simultaneous solving of the magnetic field and the armature currents in voltage-fed PM machines is presented. The method is based on a coupling of the harmonic model solution with the equation describing the armature electric circuit.

Corresponding author: M. Merdzan (e-mail: m.merdzan@tue.nl).

II. Harmonic modeling of permanent magnet machines

Harmonic modeling approach is based on a direct solution of Maxwell's equations. By introducing magnetic vector potential, the field behavior is described by a single second order partial differential equation which is solved by the method of separation of variables [7]. The domain in which magnetic field needs to be solved is divided into regions with different electromagnetic properties [1] in which the governing equation takes different forms (Laplace, Poisson, or Helmholtz equation). Field sources are expressed as infinite series of harmonics and magnetic vector potential solution is assumed to be in the same form.

Equation (1) is the governing equation in its most general form. Magnetic vector potential is indicated with $\vec{A}$ [Wb/m], the current density with $\vec{J}$ [A/mm^2] and remanent flux density with $\vec{B}_{rem}$ [T], while μ [H/m] and σ [S/m] represent permeability and conductivity in considered regions. The magnetic field in PM machines has two independent sources: magnets and armature currents. Field components originating from each source are solved separately and the total field is obtained by the summation. From the point of view of the permanent magnet field the machine can be divided in two types of regions: with magnetization (permanent magnet) and without magnetization (all others). From the armature field point of view, the machine can be divided into a current carrying region (armature winding), a non-conducting region (air gap) and conducting regions (magnet and retaining sleeve if present). In Table I the form of the governing equation in every region type is shown. In 2D analysis which is considered in this paper, the remanence can have a radial and azimuthal component, while the current density and magnetic vector potential have only a component in the axial direction.

$$-\nabla^2\vec{A}+\mu\sigma\frac{\partial\vec{A}}{\partial t}=\mu\vec{J}+\nabla\times\vec{B}_{rem} \quad (1)$$

Using Fourier decomposition, the spatial distribution of conductors in one phase of the armature winding can be represented as:

TABLE I
GOVERNING EQUATION IN DIFFERENT MACHINE REGIONS

Region	Governing equation	Region	Governing equation
PERMANENT MAGNET FIELD		ARMATURE FIELD	
Magnet	$-\nabla^2\vec{A}=\nabla\times\vec{B}_{rem}$	Conducting	$\nabla^2\vec{A}=\mu\sigma\frac{\partial\vec{A}}{\partial t}$
Others	$\nabla^2\vec{A}=0$	Air gap	$\nabla^2\vec{A}=0$
		Winding	$-\nabla^2\vec{A}=\mu\vec{J}$

$$n(\varphi_s)=\sum_{k_a=1}^{\infty}\hat{n}\cos\left(k_a(p\varphi_s+\alpha)\right) \quad (2)$$

where $\hat{n}$ is the peak of the winding distribution which depends on the number of turns and harmonic winding factor, k_a is the harmonic order, p is the number of pole pairs, φ_s [rad] the angular coordinate in the stator reference frame, while α takes three different values $(0,-2\pi/3,2\pi/3)$ in three stator phases. If balanced 3-phase currents of angular frequency ω_a [rad/s] flow through the armature windings, the equivalent source of the armature field (composed of infinite number of rotating waves) can be expressed in complex harmonic form as:

$$J=\sum_{k_a=1}^{\infty}\hat{J}e^{j(\omega_a t\pm pk_a\varphi_s)} \quad (3)$$

In (3) $+$ indicates waves rotating opposite of the rotor direction, while $-$ stands for waves rotating in the rotor direction. Term $\hat{J}$ depends on the amplitude of the armature currents, the peak of the winding distribution and the winding geometry. In slotless PM machines J represents the current density [A/mm^2] imposed in the winding region, while in slotted machines J is often approximated by a current sheet [A/mm] distributed over the stator inner surface and it is taken into account through boundary conditions [2].

The angular coordinate in the rotor reference frame (φ_r) [rad] can be expressed using φ_s as:

$$\varphi_s=\varphi_r+\omega_m t+\theta_0 \quad (4)$$

where ω_m [rad/s] is the angular velocity of the rotor and θ_0 [rad] is the initial rotor position. Using Table I and (4), the general solution for the magnetic vector potential of the armature field can be written for conducting, air gap and winding regions, respectively, as:

$$A_a(r,\varphi_r,t)=\sum_{k_a=1}^{\infty}(cI_{pk_a}(\tau r)+dK_{pk_a}(\tau r))e^{j\beta_r} \quad (5)$$

$$A_a(r,\varphi_r,t)=\sum_{k_a=1}^{\infty}(cr^{pk_a}+dr^{-pk_a})e^{j\beta_r} \quad (6)$$

$$A_a(r,\varphi_r,t)=\sum_{k_a=1}^{\infty}(cr^{pk_a}+dr^{-pk_a}+A_{pa}(r))e^{j\beta_r} \quad (7)$$

where I_{pk_a} and K_{pk_a} are the modified Bessel functions of first and second kind of order pk_a, respectively. Terms τ and β_r are given by (8), ω_{ar} [rad/s] is the angular frequency in rotating regions, given by (9), while A_{pa} is the particular part of the solution in the winding region. Sign $\pm$, originating in (3) indicates that frequency in the rotor increases for the spatial harmonics rotating in the direction opposite of the rotor. Terms indicated with c and d are unknown constants obtained from the boundary conditions. If any of the regions contains a point $r=0$, the corresponding constant d equals to zero. If term ω_{ar} has value zero (the field component rotates in the synchronism with the rotor), the solution (5) takes form of (6).

$$\tau=j^{\frac{3}{2}}\sqrt{\omega_{ar}\mu\sigma}\quad ;\quad \beta_r=\omega_{ar}t\pm pk_a(\varphi_r+\theta_0) \quad (8)$$

$$\omega_{ar}=\omega_a\pm pk_a\omega_m \quad (9)$$

The solution for the magnetic vector potential of the permanent magnet field can be written in the following way (for the magnet and all other regions, respectively):

$$A_{pm}(r,\varphi_r)=\sum_{k_{pm}=1}^{\infty}(A_{hpm}(r)+A_{ppm}(r))\cos(pk_{pm}\varphi_r) \quad (10)$$

$$A_{pm}(r,\varphi_r)=\sum_{k_{pm}=1}^{\infty}A_{hpm}(r)\cos(pk_{pm}\varphi_r) \quad (11)$$

where k_{pm} is the order of the spatial harmonics in the magnet magnetization and A_{ppm} is the particular solution in the magnet region. The term A_{apm} is the homogeneous part of the solution given by:

$$A_{hpm}(r)=ar^{pk_{pm}}+br^{-pk_{pm}} \quad (12)$$

Terms a and b are unknown constants determined from the boundary conditions. In the region which contains the point $r=0$ corresponding constant b equals to zero.

For a domain having n regions including the point $r=0$ there is in total $2n-1$ unknown constants and boundary conditions. All equations from the boundary conditions are combined into a matrix equation for the magnet field and, if the armature currents are known, for the armature field. The unknown constants are contained in a vector column $\mathbf{X}$ and are calculated by solving:

$$\mathbf{E}_{(2n-1)\times(2n-1)}\cdot\mathbf{X}_{(2n-1)\times 1}=\mathbf{Y}_{(2n-1)\times 1} \quad (13)$$

From the solution of the vector potential the flux linkage can be calculated as [5]:

$$\psi=\oint_{coil}\vec{A}\cdot d\vec{l}=pl_s\int_{-\pi/p}^{\pi/p}n(\varphi_s)A(r=r_c,\varphi_s)d\varphi_s \quad (14)$$

where l_s is the axial length and r_c is the radius at which the integration is performed, usually in the middle of the winding region for slotless machines, or stator inner radius for slotted machines. After applying (14) flux linkages of the armature field, ψ_a [Vs], and the magnet, ψ_{pm} [Vs], can be written as:

$$\psi_a = \sum_{k_a=1}^{\infty} \hat{\psi}_a e^{j(\omega_a t \mp k_a \alpha)} \tag{15}$$

$$\psi_{pm} = \sum_{k_{pm}=k_a=1}^{\infty} \hat{\psi}_{pm} e^{j(k_{pm}(p(\omega_m t+\theta_0)+\alpha))} \tag{16}$$

Flux linkages amplitudes $\hat{\psi}_a$ and $\hat{\psi}_{pm}$ contain the unknown constants from the vector potential solution. The lower summation limit in (16) indicates that out of all harmonics in the permanent magnet field only those of the same order as armature harmonics can be linked by the armature winding.

III. Coupling of the harmonic model with the armature electric circuit

The proposed method is based on the analysis of the armature circuit voltage equation, where the leakage inductance L_σ [H] is assumed to be independent on frequency and known. The voltage equilibrium equation of one phase of the armature winding of any PM machine can be written as:

$$v(t) = Ri(t) + L_m \frac{di(t)}{dt} + L_\sigma \frac{di(t)}{dt} + \frac{d\psi_{pm}(t)}{dt} \tag{17}$$

where $v(t)$ [V] is the supply voltage, $i(t)$ [A] the armature current of the angular frequency ω_a, R [Ω] the armature winding resistance and L_m [H] the magnetizing inductance. By using (15) and (16), equation (17) can be rewritten in the complex notation as:

$$\underline{U} = \underline{Z}\ \underline{I} + j\omega_a \sum_{k_a=1}^{\infty} \hat{\psi}_a e^{\mp j k_a \alpha} \tag{18}$$

The impedance and the resultant voltage acting on the armature winding are given by (19) and (20).

$$\underline{Z} = R + j\omega_a L_\sigma \tag{19}$$

$$\underline{U} = \underline{V} - jp\omega_m \sum_{k_{pm}=k_a=1}^{\infty} k_{pm} \hat{\psi}_{pm} e^{j(k_{pm}(p\theta_0+\alpha))} \tag{20}$$

where $\underline{V}$ and $\underline{I}$ in other two phases are shifted for α. In (18) the magnetizing inductance is included in the flux linkage of the armature field, and eddy currents are accounted for through the field solution in conducting regions. Permanent magnet flux linkage does not depend on other terms in (18) and can be obtained by solving the unknown constants using (13).

To solve the unknown constants in the armature field solution (5) - (7), the armature currents, as a source of the armature magnetic field, need to be known. At the same time, to obtain the armature currents from (18) constants from the field solution (contained in the flux linkage amplitude) need to be known. These opposing requirements impose simultaneous solving of both the armature field and the armature currents.

The solving procedure is the extension of previously presented approach for solving the boundary conditions in the harmonic model. A similar analysis for induction machines was presented in [8].

If the total number of harmonics included in (18) is k_{tot}, it is possible to define k_{tot} vector rows:

$$\mathbf{F}^{k_a}_{1\times(2n-1)} = j\omega_a K[0\ 0\ \ldots r_c^{pk_a}\ r_c^{-pk_a}] \tag{21}$$

where K is a constant dependent on the machine geometry which multiplies terms $cr_c^{pk_a}$ and $dr_c^{-pk_a}$ in the armature flux linkage amplitude. Furthermore, it is possible to define k_{tot} square matrixes $\mathbf{E}^{k_a}$ corresponding to $\mathbf{E}_{(2n-1)\times(2n-1)}$ from (13) for every considered harmonic of the armature field and in similar way k_{tot} vector columns $\mathbf{X}^{k_a}$ corresponding to $\mathbf{X}_{(2n-1)\times 1}$ from (13). Additionally, by dividing $\mathbf{Y}_{(2n-1)\times 1}$ from (13) by unknown armature current $\underline{I}$, it is possible to define k_{tot} vector columns $\mathbf{Y}^{k_a}$. Moreover $\mathbf{N}$ is defined as the zero square matrix with dimensions $2n-1$ and $\mathbf{M}$ is the zero vector column of the length $2n-1$.

Using introduced matrixes and vectors it is possible to write the following matrix equation:

$$\begin{bmatrix} \underline{Z} & \mathbf{F}^1 & \mathbf{F}^2 & \ldots & \mathbf{F}^{k_{tot}} \\ -\mathbf{Y}^1 & \mathbf{E}^1 & \mathbf{N} & \ldots & \mathbf{N} \\ -\mathbf{Y}^2 & \mathbf{N} & \mathbf{E}^2 & \ldots & \mathbf{N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\mathbf{Y}^{k_{tot}} & \mathbf{N} & \mathbf{N} & \mathbf{N} & \mathbf{E}^{k_{tot}} \end{bmatrix} \cdot \begin{bmatrix} \underline{I} \\ \mathbf{X}^1 \\ \mathbf{X}^2 \\ \vdots \\ \mathbf{X}^{k_{tot}} \end{bmatrix} = \begin{bmatrix} \underline{U} \\ \mathbf{M} \\ \mathbf{M} \\ \vdots \\ \mathbf{M} \end{bmatrix} \tag{22}$$

Multiplication of the first row in (22) with the vector containing the unknown current and vectors $\mathbf{X}^{k_a}$ gives equation (18). Only two constants from every $\mathbf{X}^{k_a}$ vector are present in the armature flux linkage expression. Their position in these vectors determines in which positions the non-zero terms appear in (21). Two required constants for every harmonic have to be solved together with other $2n-3$ unknown constants, which requires extension of the matrix to the form shown in (22).

After solving (22) both the armature currents and the unknown constants are calculated simultaneously. By inserting obtained constants in (5) - (7) the magnetic vector potential of the armature field can be calculated and the field in every region can be obtained using:

$$\vec{B} = \nabla \times \vec{A} \tag{23}$$

IV. Results

The method is applied to a high-speed slotless PM machine with cross section shown in Fig. 1 and parameters shown in Table II. The considered machine has a cylindrical magnet and a retaining sleeve made of stainless steel [3]. The results are compared to those from a 2D transient finite element model (FEM). In Fig. 2 the armature current of one phase obtained by two methods is shown for a same phase voltage. It can be seen that a good agreement is achieved in both amplitude and phase which confirms correctness of the proposed method.

Additionally, the radial component of the flux density in

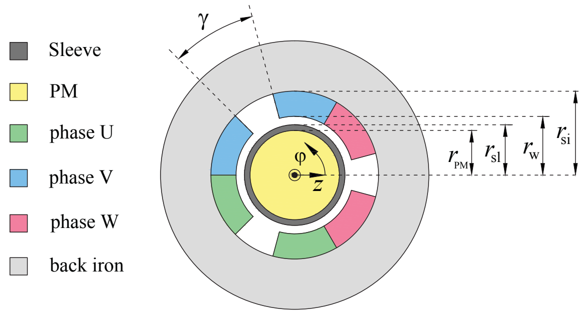

Fig. 1. The slotless high-speed PM machine used for verification of the model

TABLE II
PARAMETERS OF THE USED SLOTLESS HIGH-SPEED PM MACHINE

PARAMETER	SYMBOL	VALUE
Stator inner radius	r_{si}	4.25 mm
Winding inner radius	r_{w}	3.25 mm
Sleeve outer radius	r_{sl}	2.75 mm
Magnet radius	r_{m}	2.25 mm
Stack length	l_{s}	20 mm
Shift between coils of same phase	γ	30°
Magnet remanence	B_{rem}	1.3 T
Magnet recoil permeability	μ_{rec}	1.05
Nominal rotational speed	n	80000 rpm

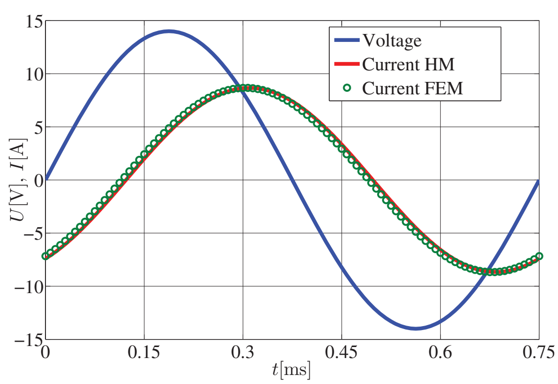

Fig. 2. Waveforms of the voltage and the armature current obtained using the harmonic model (HM) and 2D FEM

the middle of the winding region is shown in Fig. 3. In this machine type, the flux density component produced by the permanent magnet is dominant. To clearly see the armature field obtained by two models the magnet remanence is set to zero. A good agreement is achieved. The shape of the flux density waveform is a consequence of second harmonic in the armature winding spatial distribution which is significant.

V. CONCLUSION

In this paper, a method for solving the magnetic field in voltage-supplied permanent magnet machines has been proposed. The presented approach allows use of the harmonic modeling technique in any PM machine regardless of the supply type and frequency dependence of the inductance. By

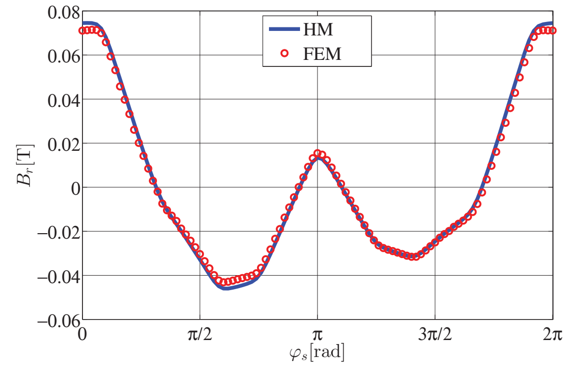

Fig. 3. The spatial distribution of the radial flux density in the middle of the winding region for the considered PM machine

including the magnetic field solution in the voltage equation of the armature winding both the armature currents and the magnetic field are solved simultaneously. Therefore, the magnetic field is obtained directly from applied voltages, without explicit inductance calculation. This makes the method very useful as an optimization tool for use in PM machines design.

The method is applicable to PM machines with arbitrary windings and magnets topology. It can be used for machines with multiple rotor magnets and account for field solution in separate armature slots. Due to increased number of considered regions, the model complexity would increase, which, however, would not limit the suitability of the presented method.

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