Analysis of three conductor coaxial systems : computer-aided determination of the frequency characteristics and the impulse and step response of a two-port consisting of a system of three coaxial conductors terminating in lumped impedances

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by

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Contents

1. Summary

A theoretical analysis is presented of a system consisting of three coaxial conductors of a certain length terminating in concentrated elements. This is a configuration which is applied in coaxial shunts.
Starting with Maxwell's equations, expressions are derived, describing the field intensities in the conductors and in the intermediate dielectrics.
The four quasi TEM propagation modes, inherent in three parallel conductors, are derived from the characteristic determinant of the system and are used to obtain general expressions for the currents and the voltages in the system.
After the introduction of the terminal conditions in conformity with the use of the system as a coaxial shunt, the eigenvalue $H(\omega)$ of the two-port concerned is determined. The inverse Fourier transform then leads to the impulse response of the two-port from which the step response is derived by integration.
Finally, computational results are given of the transfer impedance and the responses in the time domain.

2. Introduction

2.1 Object of the analysis

The object of the investigations described in this paper is to derive the responses in the frequency domain as well as in the time domain of a structure consisting of two coaxial pairs coupled to each other by a common cylindrical conductor.

The direct motive for these investigation was the possibility of analysing the behaviour of the coaxial shunt making use of the experience gained in the author's research group in treating coaxial structures.

A coaxial shunt is a device with the structure mentioned and is used for measuring high short-circuit currents [3].

Because a reliable interpretation of a measured transient response of a coaxial shunt requires a precise knowledge of the behaviour of the system, a general fundamental analysis of the structure seemed justifiable.

Moreover, the same basic configuration is used to measure the transfer impedance of coaxial cables, an important quantity related to crosstalk in coaxial cable systems [4].

The treatment is distinguished from others [3],[6],[7] in several ways :

a. It takes as a starting point the very coaxial structure and not a substitute with concentrated elements.
b. There is no restriction as regards the thickness of the common conductor.
c. There are no restrictions as to frequency intervals.
d. Complete transient responses are calculated with discrete Fourier transforms [5].

2.2 Description of the system

The investigated system consists of three parallel conductors of the same length, at both ends arbitrarily terminating in concentrated elements. The system is excited by one or more voltage and/or current sources at one or both ends of the system.

Fig. 1 shows a possible situation with a single voltage source.

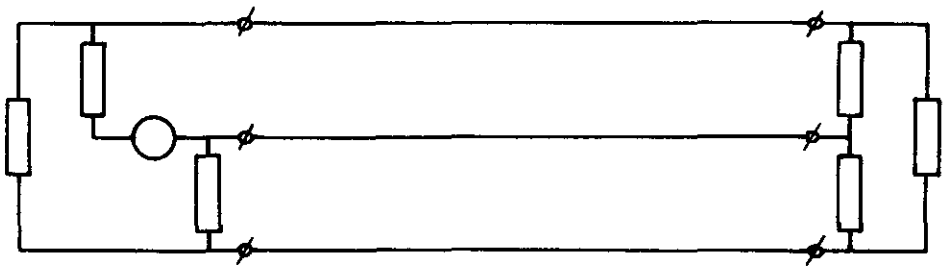

Fig. 1. Principle of the system, consisting of three parallel conductors, terminating in concentrated elements and driven by one or more voltage and/or current sources

With regard to the conductors, it is assumed that there are three mutually isolated, coaxial, cylindrical conductors, one massive one surrounded by two hollow conductors, as shown in fig. 2.

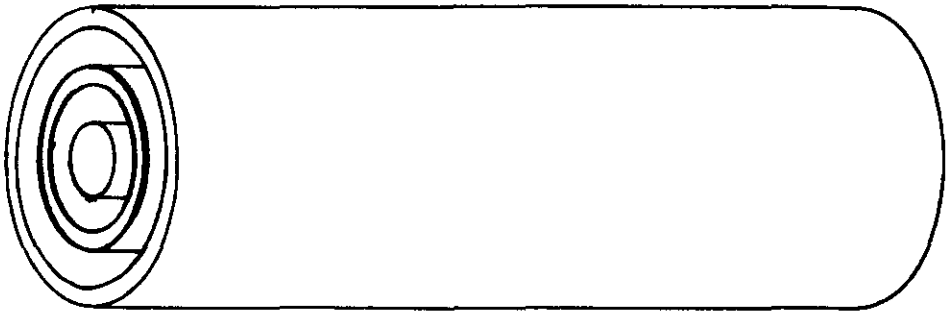

Fig. 2. The physical construction of the three parallel conductors

With regard to the terminations and the kind of excitation, we will restrict ourselves to the situation as sketched in fig. 3.

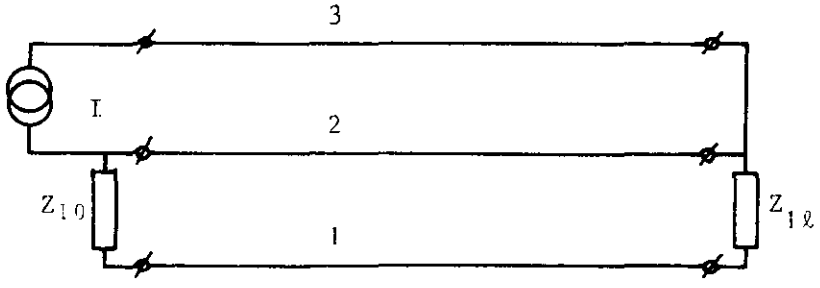

Fig. 3. The assumed terminal conditions

The outer coaxial pair, consisting of the conductors 2 and 3, is short-circuited at one end and excited by a current source at the other.
The main part of the injected current I takes its way through the common conductor 2 and the rest takes its way through the inner conductor 1; the total current I flows back through the outer conductor 3.
The inner coaxial pair (conductors 1 and 2) in general terminates in impedances Z_{10} and $Z_{1\ell}$. In practice, Z_{10} will be zero (a short circuited end) or will be taken equal to the high frequency characteristic impedance of the inner coaxial pair.
$Z_{1\ell}$, the impedance across which the output voltage is measured if the system is used as a coaxial shunt, can be an open end or, for instance, a resistor, equalling the high-frequency characteristic impedance of the inner pair.
Other kinds of terminations than the ones treated do not give rise to new aspects and can be solved in the same way.

2.3 Nomenclature

The most elegant way to describe the rotational symmetrical system concerned is by means of cylindrical polar co-ordinates (ρ, ϕ, z).
Let the z-axis coincide with the axis of the system. Distances to the axis are then represented by ρ. As a result of the rotational symmetry all the physical magnitudes are independent of ϕ.
From the axis to the outside of the system the conductors will be indicated by 1, 2 and 3 and the dielectrics by a and b (fig. 4).

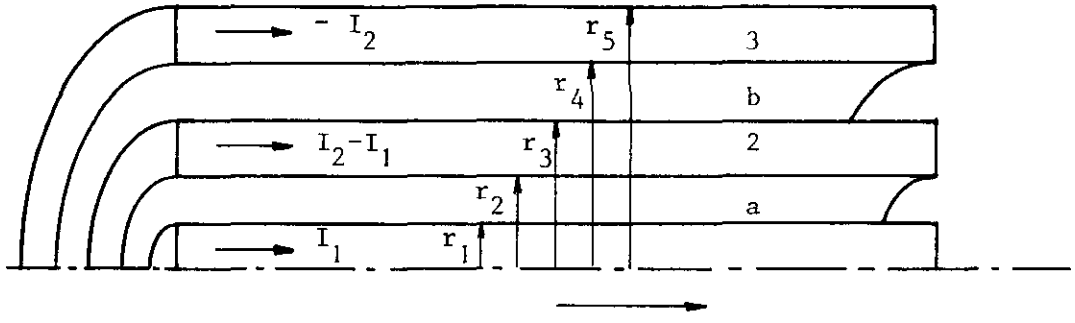

Fig. 4. The nomenclature of the system of three coaxial conductors

The respective radii will be denoted by r_1 to r_5.
The currents in the positive z-direction are denoted by I_1 in conductor 1, I_2-I_1 in conductor 2, and $-I_2$ in conductor 3 respectivily.
The sum of these currents being zero, there will be no resulting field outside the system.

3. Derivation of the field intensities from Maxwell's equations

3.1 General formulae describing the system.

The starting point is found in Maxwell's equations :

$$\nabla \times H = gE + \frac{\delta D}{\delta t} \tag{1}$$

$$\nabla \times E = - \frac{\delta B}{\delta t} \tag{2}$$

$$B = \mu H \tag{3}$$

$$D = \varepsilon E \tag{4}$$

If all the time-dependent magnitudes are supposed to be sinusoidal functions of time and independent of ϕ, a change to cylindrical co-ordinates leads to :

$$\frac{\delta H_\phi}{\delta z} = -(g+j\omega\varepsilon)E_\rho \tag{5}$$

$$\frac{\delta(\rho H_\phi)}{\delta \rho} = (g+j\omega\varepsilon)\rho E_z \tag{6}$$

$$\frac{\delta E_z}{\delta \rho} - \frac{\delta E_\rho}{\delta z} = j\omega\mu H_\phi \tag{7}$$

The formulae (5) (6) and (7) are valid in the conductors as well as in the dielectrics.

Substitution of (5) and (6) in (7) results in :

$$\frac{\delta}{\delta \rho}\left[\frac{1}{\rho}\frac{\delta(\rho H_\phi)}{\delta \rho}\right] + \frac{\delta^2 H_\phi}{\delta z^2} = (j\omega\mu g - \omega^2\varepsilon\mu)H_\phi \tag{8}$$

Using the normal method of searching for particular solutions, H_ϕ is written as a product of two functions, the first a function of z alone and the other a function of ρ alone. The dependence on z is then found to be given by :

$$H_\phi(\rho,z) = H_\phi(\rho).\varepsilon^{-\gamma z} \tag{9}$$

supposing (for the sake of simplicity) only for the time being a wave in the positive z-direction.

3.2 Field intensities in the conductors

As the displacement current in metals is very small compared with the conduction current, it is allowed to put $\varepsilon=0$; this is certainly allowed for all the frequencies that play a role in the calculations to be made in this paper.

Substituting $\varepsilon=0$ and the time-dependence given by (9) in (8) gives :

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d(\rho H_\phi)}{d\rho}\right] = (\sigma^2-\gamma^2)H_\phi \tag{10}$$

with $\sigma^2=j\omega\mu g$ (11)

In all practical situations, if the conductors are made of metals (for instance copper with $g=5{,}8.10^7\ \Omega^{-1}m^{-1}$)

γ^2 can be ignored compared with σ^2, so (10) can be written as :

$$\frac{d^2H_\phi}{d\rho^2} + \frac{1}{\rho}\frac{dH_\phi}{d\rho} - \frac{1}{\rho^2}H_\phi = \sigma^2 H_\phi \tag{12}$$

The solution of (12) yields

$$H_\phi = AI_1(\sigma\rho)+BK_1(\sigma\rho) \tag{13}$$

which is a summation of two Bessel functions.

From (6) and (5) respectivily follows :

$$E_z = \frac{\sigma}{g}\left[AI_o(\sigma\rho)-BK_o(\sigma\rho)\right] \tag{14}$$

$$E_\rho = \frac{\gamma}{g}H_\phi \simeq 0$$

3.3 Field intensities in the dielectrics

If the isolating media between the conductors are of such a quality that the conductivity can be ignored (for instance air, polyethylene), the equations (5), (6) and (7), after introducing the z-dependence given by (9), become

$$E_\rho = \frac{\gamma}{j\omega\varepsilon}H_\phi \tag{16}$$

$$\frac{d(\rho H_\phi)}{d\rho} = j\omega\varepsilon\rho E_z \tag{17}$$

$$\frac{dE_z}{d\rho} + \gamma E_\rho = j\omega\mu H_\phi \tag{18}$$

The conduction current resulting from E_z being very small compared with the current in the conductors, it can be put that the magnetomotive intensity H_ϕ in the dielectrics is only determined by the currents in the conductors, leading to :

$$H_\phi = \frac{I_t}{2\pi\rho} \tag{19}$$

with I_t the total enclosed current in the conductors by the field line concerned. It is thus supposed that :

$$\frac{d(\rho H_\phi)}{d\rho} = 0 \tag{20}$$

an approximation of (17).

Substitution of (16) in (18) and solution of the resulting differential equation gives :

$$E_z = \frac{1}{2\tau}\left(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}\right) I_t \ln\frac{\rho}{r_n} + E_z(r_n) \tag{21}$$

$$\text{for } r_n \leqslant \rho \leqslant r_{n+1}$$

with r_n the radius of the nearest enclosed metal-insulator interface.

The expression derived can now be used in the particular case of the three mutually isolated coaxial conductors, as described below.

4. The field intensities in the three conductors and the two dielectrics expressed in terms of the currents I_1 and I_2.

The general expressions derived in part 3 can now be used to find specific solutions to the particular case of the coaxial conductors. This means that the constants in the formulae have to be expressed in terms of the currents I_1 and I_2. It is assumed that the conductors 1, 2 and 3 have the constants σ_1, σ_2 and σ_3 because it is not necessary for the conductivity of the conductors to be equal.

Used as a coaxial shunt, for instance, it is imaginable that the common conductor 2 is made of a material with a high specific resistance and the conductors 1 and 3 of a material with a low specific resistance.

As in the general case the constants in formulae (13) and (14) were denoted by A and B, in the specific cases they will be represented by A_1 and B_1 in conductor 1, A_2 and B_2 in conductor 2, etc.

4.1 Field intensities in conductor 1

With the restriction that the formulae hold only for $0 \leq \rho \leq r_1$, (13) becomes

$$H_\phi = A_1 I_1(\sigma_1 \rho) + B_1 K_1(\sigma_1 \rho) \tag{22}$$

$$\sigma_1^2 = j\omega\mu g_1 \tag{23}$$

Because $K_1(\sigma_1 \rho)$ is infinite for $\rho=0$, it follows that $B_1=0$. With $\rho=r_1$ in (19) it follows from (22) that :

$$A_1 = \frac{I_1}{2\pi r_1} \cdot \frac{1}{I_1(\sigma_1 r_1)} \tag{24}$$

With (24) and $B_1=0$ substituted in (22) and (14), the latter read

$$H_\phi = I_1 \frac{I_1(\sigma_1 \rho)}{2\pi r_1 I_1(\sigma_1 r_1)} \tag{25}$$

$$E_z = \eta_1 I_1 \frac{I_0(\sigma_1 \rho)}{2\pi r_1 I_1(\sigma_1 r_1)} \tag{26}$$

$$\text{with} \quad \eta_1 = \frac{\sigma_1}{g_1} \tag{27}$$

On the interface between 1 and a we have :

$$E_z(r_1) = \eta_1 I_1 \frac{I_0(\sigma_1 r_1)}{2\pi r_1 I_1(\sigma_1 r_1)} = W I_1 \tag{28}$$

with $$W = \frac{I_0(\sigma_1 r_1)}{2\pi r_1 I_1(\sigma_1 r_1)} \cdot \eta_1 \tag{29}$$

4.2 Field intensities in dielectric a

In the interval $r_1 \leq \rho \leq r_2$ (19) becomes :

$$H_\phi = \frac{I_1}{2\pi\rho} \tag{30}$$

From (16) and (30) it follows that :

$$E_\rho = \frac{\gamma I_1}{2\pi j\omega\varepsilon\rho} \tag{31}$$

And from (21)

$$E_z = \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}) I_1 \ln\frac{\rho}{r_1} + E_z(r_1) \tag{32}$$

With (28) substituted in (32) :

$$E_z = \{\frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}) \ln\frac{\rho}{r_1} + W\} I_1 \tag{33}$$

4.3 Field intensities in conductor 2

The magnetic field intensity in conductor 2 follows from (13) by substituting A_2, B_2 and σ_2 for A, B and σ:

$$H_\phi = A_2 I_1(\sigma_2\rho) + B_2 K_1(\sigma_2\rho) \tag{34}$$

$$\text{for } r_2 \leq \rho \leq r_3$$

In the special cases in which $\rho = r_2$ and $\rho = r_3$ (34) becomes with (19)

$$A_2 I_1(\sigma_2 r_2) + B_2 K_1(\sigma_2 r_2) = \frac{I_1}{2\pi r_2} \qquad \text{and} \tag{35}$$

$$A_2 I_1(\sigma_2 r_3) + B_2 K_1(\sigma_2 r_3) = \frac{I_2}{2\pi r_3} \tag{36}$$

respectivily.

Solving A_2 and B_2 from (35) and (36) we obtain :

$$A_2 = \frac{I_1}{2\pi r_2} \cdot \frac{K_1(\sigma_2 r_3)}{D_2} - \frac{I_2}{2\pi r_3} \cdot \frac{K_1(\sigma_2 r_2)}{D_2} \tag{37}$$

$$B_2 = -\frac{I_1}{2\pi r_2} \cdot \frac{I_1(\sigma_2 r_3)}{D_2} + \frac{I_2}{2\pi r_3} \cdot \frac{I_1(\sigma_2 r_2)}{D_2} \tag{38}$$

With $D_2 = I_1(\sigma_2 r_2)K_1(\sigma_2 r_3) - I_1(\sigma_2 r_3)K_1(\sigma_2 r_2)$ (39)

From (14) it follows that :

$$E_z = \eta_2\{A_2 I_0(\sigma_2\rho) - B_2 K_0(\sigma_2\rho)\} \tag{40}$$

On the interface between a and 2 :

$$E_z(r_2) = FI_1 - GI_2 \tag{41}$$

With :

$$F = \frac{I_0(\sigma_2 r_2)K_1(\sigma_2 r_3) + I_1(\sigma_2 r_3)K_0(\sigma_2 r_2)}{2\pi r_2 D_2} \cdot \eta_2 \tag{42}$$

$$G = \frac{I_0(\sigma_2 r_2)K_1(\sigma_2 r_2) + I_1(\sigma_2 r_2)K_0(\sigma_2 r_2)}{2\pi r_3 D_2} \cdot \eta_2 \tag{43}$$

The axial component of the electric field intensity on the interface between a and 2 can also be found from (33) :

$$E_z(r_2) = \{\frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})\ln\frac{r_2}{r_1} + W\}I_1 \tag{44}$$

Elimination of $E_z(r_2)$ from (41) and (44) gives an equation in terms of I_1 and I_2 with parameter γ:

$$\{\frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})\ln\frac{r_2}{r_1} - F + W\}I_1 + GI_2 = 0 \tag{45}$$

4.4 Field intensities in the dielectric b

For the dielectric b, if $r_3 \leq \rho \leq r_4$, it follows from (19) that :

$$H_\phi = \frac{I_2}{2\pi\rho} \tag{46}$$

And from (16) together with (46) :

$$E_\rho = \frac{\gamma I_2}{2\rho j\omega\varepsilon\rho} \tag{47}$$

From (21) it follows that :

$$E_z = \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}) I_2 \ln \frac{\rho}{r_3} + E_z(r_3) \tag{48}$$

The component of the electric field-intensity $E_z(r_3)$ follows from (40) :

$$E_z(r_3) = \eta_2\{A_2 I_0(\sigma_2 r_3) - B_2 K_0(\sigma_2 r_3)\} \tag{49}$$

(49) in (48) gives :

$$E_z = \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}) I_2 \ln \frac{\rho}{r_3} + \eta_2\{A_2 I_0(\sigma_2 r_3) - B_2 K_0(\sigma_2 r_3)\} \tag{50}$$

4.5 Field intensities in conductor 3

The magnetic field intensity in conductor 3 follows from (13) by substituting A_3, B_3 and σ_3 for A, B and σ:

$$H_\phi = A_3 I_1(\sigma_3\rho) + B_3 K_1(\sigma_3\rho) \tag{51}$$

$$\text{for } r_4 \leq \rho \leq r_5$$

In the special cases in which $\rho = r_4$ and $\rho = r_5$ (51) becomes with (19) :

$$A_3 I_1(\sigma_3 r_4) + B_3 K_1(\sigma_3 r_4) = \frac{I_2}{2\pi r_4} \qquad \text{and} \tag{52}$$

$$A_3 I_1(\sigma_3 r_5) + B_3 K_1(\sigma_3 r_5) = 0 \tag{53}$$

respectivily.

Solving A_3 and B_3 from (52) and (53) we obtain :

$$A_3 = \frac{I_2}{2\pi r_4} \cdot \frac{K_1(\sigma_3 r_5)}{D_3} \tag{54}$$

$$B_3 = -\frac{I_2}{2\pi r_4} \cdot \frac{I_1(\sigma_3 r_5)}{D_3} \tag{55}$$

With :

$$D_3 = I_1(\sigma_3 r_4) K_1(\sigma_3 r_5) - I_1(\sigma_3 r_5) K_1(\sigma_3 r_4) \tag{56}$$

From (14) it follows that :

$$E_z = \eta_3\{A_3 I_0(\sigma_3\rho) - B_3 K_0(\sigma_3\rho)\} \tag{57}$$

On the interface between b and 3 :

$$E_z(r_4) = LI_2 \tag{58}$$

With :

$$L = \frac{I_0(\sigma_3 r_4)K_1(\sigma_3 r_5)+I_1(\sigma_3 r_5)K_0(\sigma_3 r_4)}{2\pi r_4 D_3} \cdot \eta_3 \tag{59}$$

$E_z(r_4)$ can also be found from (50) :

$$E_z(r_4) = \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})I_2 \ln\frac{r_4}{r_3} + MI_1 - NI_2 \tag{60}$$

with

$$M = \frac{I_0(\sigma_2 r_3)K_1(\sigma_2 r_3)+I_1(\sigma_2 r_3)K_0(\sigma_2 r_3)}{2\pi r_2 D_2} \cdot \eta_2 \tag{61}$$

$$N = \frac{I_0(\sigma_2 r_3)K_1(\sigma_2 r_2)+I_1(\sigma_2 r_2)K_0(\sigma_2 r_3)}{2\pi r_3 D_2} \cdot \eta_2 \tag{62}$$

Elimination of $E_z(r_4)$ from (59) and (60) gives a second equation in terms of I_1 and I_2 with γ as parameter.

$$MI_1 + \{\frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})\ln\frac{r_4}{r_3} - L - N\}I_2 = 0 \tag{63}$$

5. The propagation modes

5.1 Derivation of the propagation constants belonging to the different possible propagation modes

The equations (45) and (63) form a set of two homogeneous linear equations in the unknowns I_1 and I_2. A necessary and sufficient condition for this set of equations to have a solution other than the trivial one $I_1=I_2=0$, is that the characteristic determinant of the coefficients must vanish, that is :

$$\begin{vmatrix} \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})\ln\frac{r_2}{r_1} - F + W & G \\ M & \frac{1}{2\pi}(j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon})\ln\frac{r_4}{r_3} - L - N \end{vmatrix} = 0 \qquad (64)$$

With $$Q = \frac{j\pi}{\omega\mu \ln\frac{r_2}{r_1} \ln\frac{r_4}{r_3}} \qquad (65)$$

$$O_1 = (L+N)\ln\frac{r_2}{r_1} \qquad (66)$$

$$O_2 = (F-W)\ln\frac{r_4}{r_3} \qquad (67)$$

$$O = (O_1-O_2)^2+4GM \ln\frac{r_2}{r_1} \ln\frac{r_4}{r_3} \qquad (68)$$

$$P = O_1+O_2 \qquad (69)$$

$$B_i = \omega\sqrt{\mu\varepsilon} \qquad (70)$$

the solutions of (64) become :

$$\gamma_1 = \pm jB_i\{1+Q(P-O^{\frac{1}{2}})\}^{\frac{1}{2}} \qquad (71)$$

$$\gamma_2 = \pm jB_i\{1+Q(P+O^{\frac{1}{2}})\}^{\frac{1}{2}} \qquad (72)$$

Each solution of γ corresponds to a propagation mode [1]. The currents and the voltages in the system are linear superpositions of the component waves of all the four modes of propagation, two of which propagate in the +z direction and two in the -z direction.

5.2 Current ratios and voltage ratios corresponding to the four propagation modes

To each mode corresponds a certain ratio between the currents I_1 and I_2. Substitution of γ_1^2 in (45) or in (63) gives us an expression for the corresponding current ratio k_1 :

$$k_1 = \frac{I_2}{I_1} = \frac{jf\mu\{Q(P-O^{\frac{1}{2}})\}\ln\frac{r_2}{r_1} - F + W}{-G} \tag{76}$$

$$k_1 = \frac{I_2}{I_1} = \frac{-M}{jf\mu\{Q(P-O^{\frac{1}{2}})\}\ln\frac{r_4}{r_3} - L - N} \tag{77}$$

Substitution of γ_2^2 in (45) or in (63) gives us an expression for the corresponding current ratio k_2 :

$$k_2 = \frac{I_1}{I_2} = \frac{-G}{jf\mu\{Q(P+O^{\frac{1}{2}})\}\ln\frac{r_2}{r_1} - F + W} \tag{78}$$

$$k_2 = \frac{I_1}{I_2} = \frac{jf\mu\{Q(P+O^{\frac{1}{2}})\}\ln\frac{r_4}{r_3} - L - N}{-M} \tag{79}$$

(The current ratios have been so chosen that $|k_1| < 1$ and $|k_2| < 1$)

The voltage V_a between conductor 1 and conductor 2 follows from an integration of E_ρ to ρ :

$$V_a = \int_{r_1}^{r_2} E_\rho \, d\rho \tag{80}$$

With (31) this becomes :

$$V_a = \int_{r_1}^{r_2} \frac{\gamma I_1}{2\pi j\omega\varepsilon_\rho} \, d\rho = \frac{\gamma I_1}{2\pi j\omega\varepsilon} \ln\frac{r_2}{r_1} \tag{81}$$

The voltage V_b between conductor 2 and conductor 3 follows with (47) :

$$V_b = \frac{\gamma I_2}{2\pi j\omega\varepsilon} \ln\frac{r_4}{r_3} \tag{82}$$

The voltage ratio corresponding with γ_1 :

$$\frac{V_b}{V_a} = \frac{I_2}{I_1} \cdot \frac{\ln \frac{r_4}{r_3}}{\ln \frac{r_2}{r_1}} = k_1 \frac{\ln \frac{r_4}{r_3}}{\ln \frac{r_2}{r_1}} \tag{83}$$

The voltage ratio corresponding with γ_2 :

$$\frac{V_a}{V_b} = \frac{I_1}{I_2} \cdot \frac{\ln \frac{r_2}{r_1}}{\ln \frac{r_4}{r_3}} = k_2 \frac{\ln \frac{r_2}{r_1}}{\ln \frac{r_4}{r_3}} \tag{84}$$

6. The currents and voltages in the system

6.1 General equations of currents and voltages

In general,there will be a linear combination of the different voltages, respectivily currents,corresponding to the four propagation modes.
This gives us the following general expressions for the voltages and the currents :

$$V_a = Z_{a_1} A\varepsilon^{-\gamma_1 z} + BZ_{a_1}\varepsilon^{\gamma_1 z} + k_2 Z_{a_2} C\varepsilon^{-\gamma_2 z} + k_2 Z_{a_2} D\varepsilon^{\gamma_2 z} \tag{80}$$

$$V_b = k_1 Z_{b_1} A\varepsilon^{-\gamma_1 z} + k_1 Z_{b_1} B\varepsilon^{\gamma_1 z} + Z_{b_2} C\varepsilon^{-\gamma_2 z} + Z_{b_2} D\varepsilon^{\gamma_2 z} \tag{81}$$

$$I_1 = A\varepsilon^{-\gamma_1 z} - B\varepsilon^{\gamma_1 z} + k_2 C\varepsilon^{-\gamma_2 z} - k_2 D\varepsilon^{\gamma_2 z} \tag{82}$$

$$I_2 = k_1 A\varepsilon^{-\gamma_1 z} - k_1 B\varepsilon^{\gamma_1 z} + C\varepsilon^{-\gamma_2 z} - D\varepsilon^{\gamma_2 z} \tag{83}$$

where

$$Z_{a_1} = \frac{\gamma_1}{2\pi j\omega\varepsilon} \ln \frac{r_2}{r_1} \tag{84}$$

$$Z_{b_1} = \frac{\gamma_1}{2\pi j\omega\varepsilon} \ln \frac{r_4}{r_3} \tag{85}$$

$$Z_{a_2} = \frac{\gamma_2}{2\pi j\omega\varepsilon} \ln \frac{r_2}{r_1} \tag{86}$$

$$Z_{b_2} = \frac{\gamma_2}{2\pi j\omega\varepsilon} \ln \frac{r_4}{r_3} \tag{87}$$

Z_{a_1} and Z_{b_1} are the characteristic impedances (V_a/I_1 and V_b/I_2) of the inner and outer coaxial pairs, respectivily if the propagation mode with propagation constant γ_1 is the only mode present in the system.
Z_{a_2} and Z_{b_2} are these characteristic impedances with γ_2 as propagation constant.

6.2 Introduction of a certain set of terminal conditions

A solution of equations (80) to (83) to find the voltages V_a and V_b and of the currents I_1 and I_2, is only possible if the constants A, B, C and D are known.

Because the values of these constants follow from the terminal conditions, we now choose the concrete situation that is shown in fig. 5. These conditions correspond with the use of the system as a coaxial shunt for current measuring purposes and were also the starting point for the calculations that led to the results described in section 8 of this paper.

From fig. 5 we see that the terminal conditions read :

For z=0 $\quad I_2 = I$ (88)

$$V_a = 0 \tag{89}$$

For z=ℓ $\quad V_b = 0$ (90)

$$I_1 R = V_a \tag{91}$$

(88) substituted in (83) gives us :

$$k_1 A - k_1 B + C - D = I \tag{92}$$

(89) substituted in (80) :

$$Z_{a_1} A + Z_{a_1} B + k_2 Z_{a_2} C + k_2 Z_{a_2} D = 0 \tag{93}$$

(90) substituted in (81)

$$k_1 Z_{b_1} A\varepsilon^{-\gamma_1 \ell} + k_1 Z_{b_1} B\varepsilon^{\gamma_1 \ell} + Z_{b_2} C\varepsilon^{-\gamma_2 \ell} + Z_{b_2} D\varepsilon^{\gamma_2 \ell} = 0 \tag{94}$$

(91) in (80) :

$$R(A\varepsilon^{-\gamma_1 \ell} - B\varepsilon^{\gamma_1 \ell} + k_2 C\varepsilon^{-\gamma_2 \ell}) = Z_{a_1} A\varepsilon^{-\gamma_1 \ell} + BZ_{a_1}\varepsilon^{\gamma_1 \ell} + k_2 Z_{a_2} C\varepsilon^{-\gamma_2 \ell} + k_2 Z_{a_2} D\varepsilon^{\gamma_2 \ell} \tag{95}$$

The constants A, B, C and D can now be determined from (92) to (95) and substituted in the equations (80) to (83),giving explicit expressions for the voltages and currents as functions of z.

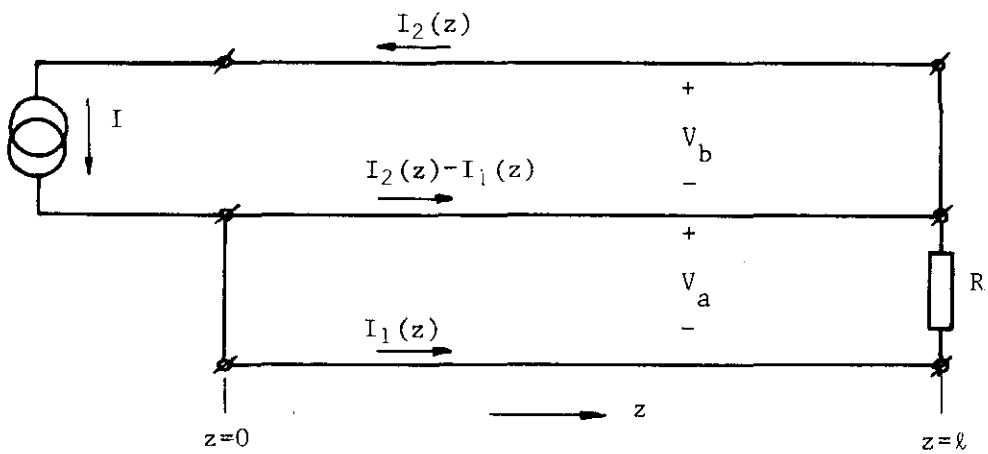

Fig. 5. The terminal conditions chosen

7. Description of the overall behaviour of the system in the frequency and in the time domains

7.1 Eigenvalue of the system, the transfer impedance H(ω)

Referring to fig.5, an impressed current I at z=0 in the outer coaxial pair causes a voltage $V_a(\ell)$ at $z=\ell$ across the resistor R that terminates the inner coaxial pair. The system can thus be seen as a two-port (fig. 6) that can be described in the frequency domain by an eigenvalue, the transfer impedance H(ω) defined as :

$$H(\omega) = \frac{V_a(\ell)}{I} \tag{96}$$

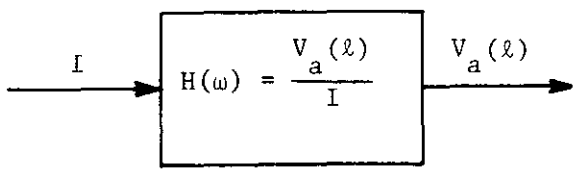

Fig. 6. The coaxial shunt as a two-port with driving current I and resulting voltage $V_a(\ell)$. The transfer impedance is defined as the quotient $V_a(\ell)$ over I.

7.2 Impulse response and step response of the system

The impulse response h(t) of the system follows from the inverse Fourier transform of the transfer impedance :

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega)\varepsilon^{j\omega t} d\omega \tag{97}$$

It is the response of the system to the generalised function δ(t), the Dirac impulse.

The step response of the system follows by integrating the impulse response :

$$a(t) = \int_{0}^{\infty} h(t)dt \tag{98}$$

Because it is impossible to write $H(\omega)$ in a closed mathematical form, the values of the transfer impedance are calculated for the discrete frequencies : $r.\Delta f$, where $r = 0, 1, 2, \ldots\ldots N-1$, and Δf the spacing between sample points in the frequency domain.

$f_g = (N-1)\Delta f$ is the highest frequency that is taken into account.

This is schematically illustrated in fig. 7.

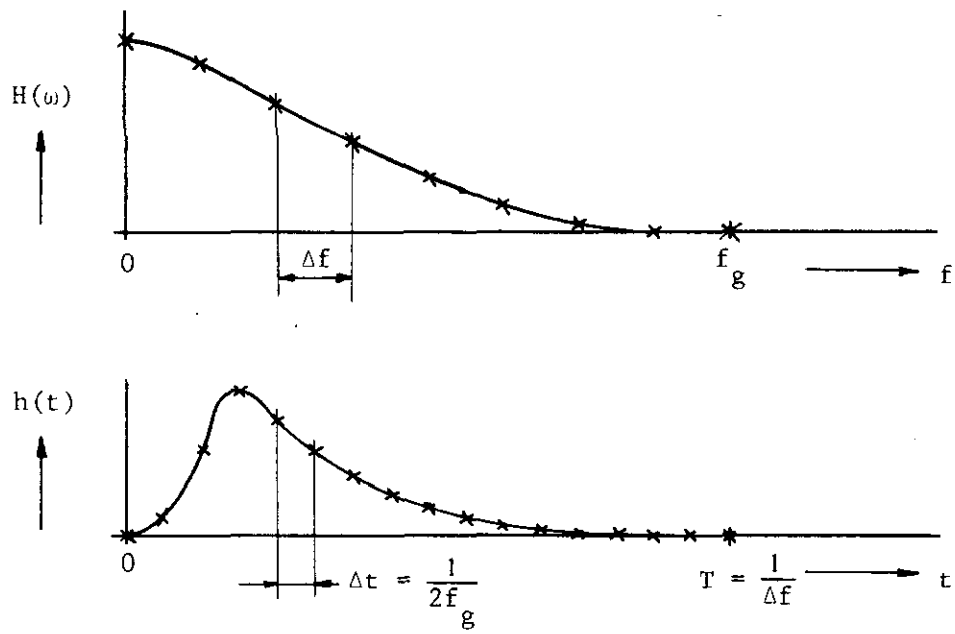

Fig. 7. The relations between Δt and f_g and between T and Δf

The inverse discrete Fourier transform results in sample values of the impulse response, with a distance between the samples of

$$\Delta t = \frac{1}{2f_g}$$

The desired fine-structure of the impulse response can be achieved by taking the value of f_g sufficiently high.

Because the transform results in a periodic time-function, with a period

$$T = \frac{1}{\Delta f}$$

it is necessary to take care that the neighbouring pulses do not interfere. To avoid this "aliasing distortion" it is necessary to take Δf sufficiently low.

The Fourier transform mentioned is executed as a "Fast Fourier Transform", an efficient algorism of the discrete Fourier transform [5].

8. Computational results

8.1 A coaxial shunt 1 metre in length

In this section graphical results are given of computations on a coaxial shunt with the following dimensions and constants :

$\ell = 1$ m

$g = 1{,}16.10^{6}$ S/m

$R = 50\ \Omega$

$r_1 = 4.13\ 10^{-3}$ m

$r_2 = 9.5\ 10^{-3}$ m

$r_3 = 10.0\ 10^{-3}$ m

$r_4 = 23.0\ 10^{-3}$ m

$r_5 = 23.5\ 10^{-3}$ m

The ratios r_2/r_1 and r_4/r_3 are so chosen that the high frequency characteristic impedances of the inner and outer pairs equal 50 Ω, with the idea that, at least for the higher frequencies, the system can be matched to a 50 Ω extension cable.

Fig. 8a gives an overall impression of the calculated amplitude characteristic that is a plot of $\left|\frac{V_a(\ell)}{I}\right|$ as a function of frequency.

Fig. 8b gives the low frequency part of the amplitude characteristic.

An interesting part of this characteristic is found around the frequency of 75 MHz, corresponding with the coaxial shunt as a quarter-wavelength line.

Fig.9a and fig.9b show the real and imaginary parts of the transfer impedance respectivily, and fig. 10 is the polar plot of the transfer impedance in the complex plane.

Fig. 11a gives an overall impression of the derived impulse response of the two-port, and fig. 11b the first part of this response.

The plot of fig. 11a makes clear that the sample density in the frequency domain is high enough to avoid aliasing distortion of the impulse response.

To avoid unnecessary extra labour in the computation of the responses in the time domain, it is assumed that the transfer impedance equals zero for frequencies higher than 100 MHz, an assumption which is not entirely justifiable in the neighbourhood of frequencies corresponding with an odd number of quarter wavelengths.

The high frequency components which are thus ignored would slightly modify the appearance of the ripple on the impulse response.
Finally, fig. 12 shows the step response derived by integrating the impulse response.

8.2 A coaxial shunt 3 metres in length

To illustrate the influence of the length of the shunt on the responses, results are given of computations on a coaxial shunt with a length of 3 m. All the other dimensions and constants are kept the same as in the example of section 8.1.
Figures 13 to 17 give a clear insight into the behaviour of the coaxial shunt.

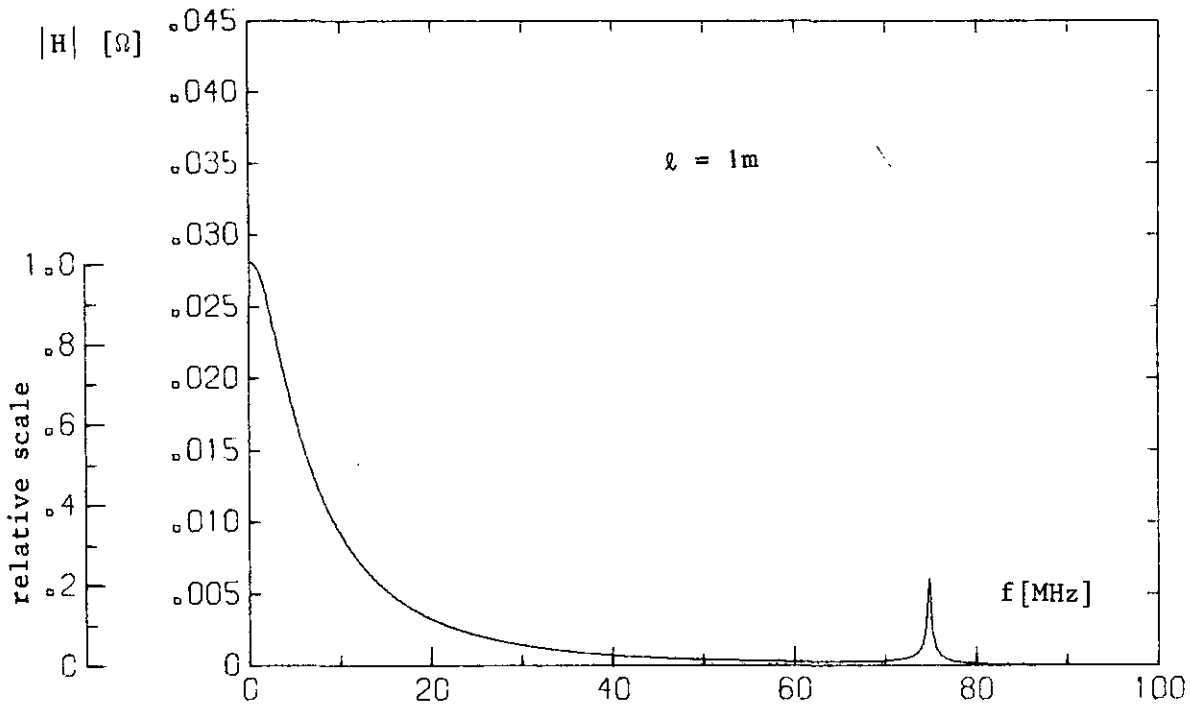

Fig.8a. The amplitude characteristic of the transfer impedance, that is $|H|=|V_a(\ell)/I|$ as a function of frequency in 1000 steps of 10^5 Hz. Note the peak at 75 MHz, corresponding with the shunt as a quarter wavelength line.

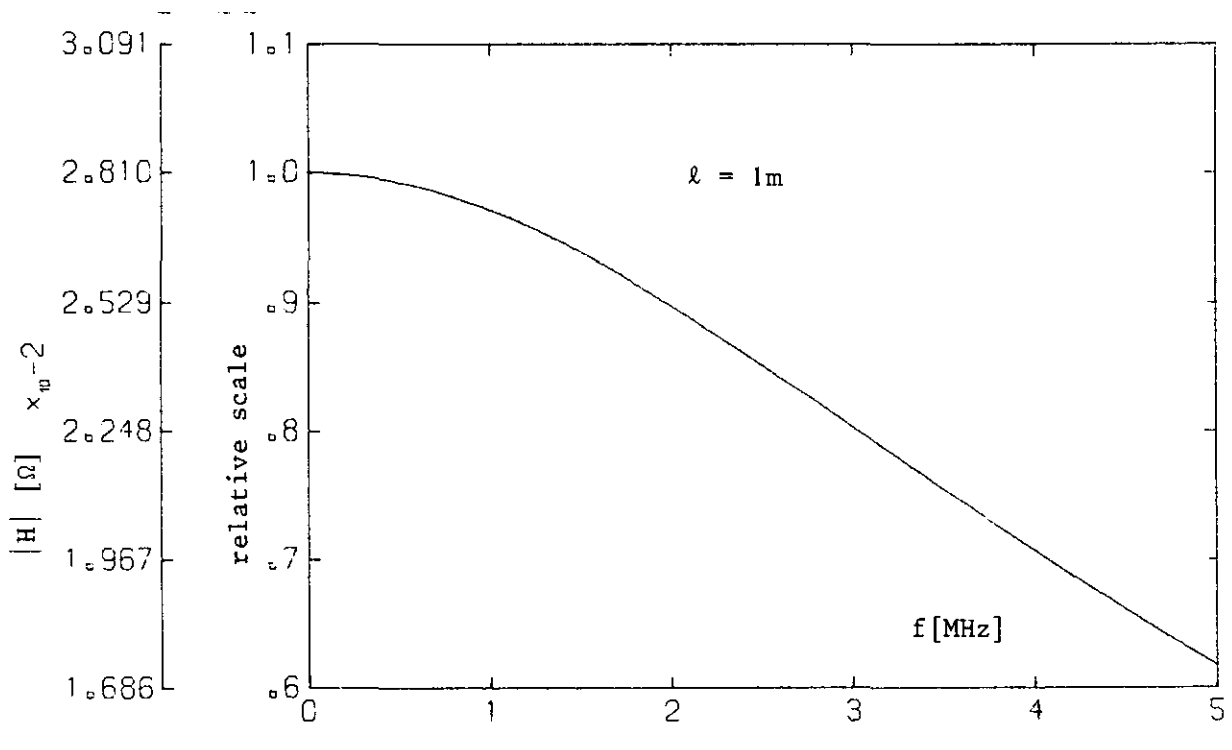

Fig.8b. The low frequency part of the amplitude characteristic calculated in 100 steps of 5.10^4 Hz

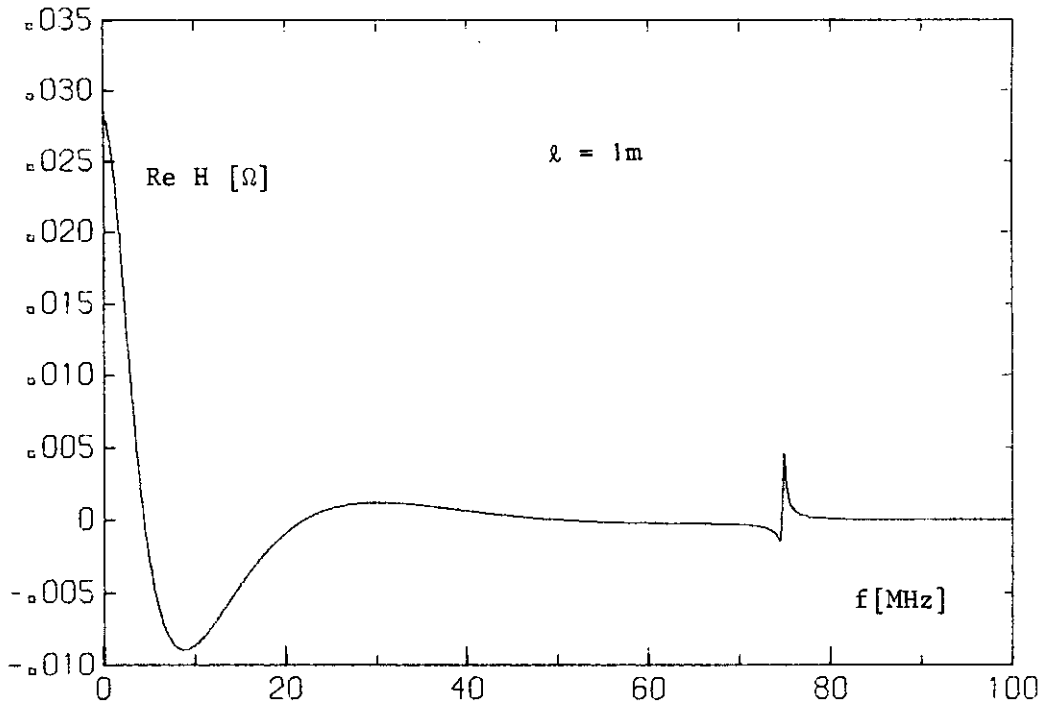

Fig.9a. The real part of the transfer impedance as a function of frequency in 1000 steps of 10^5 Hz

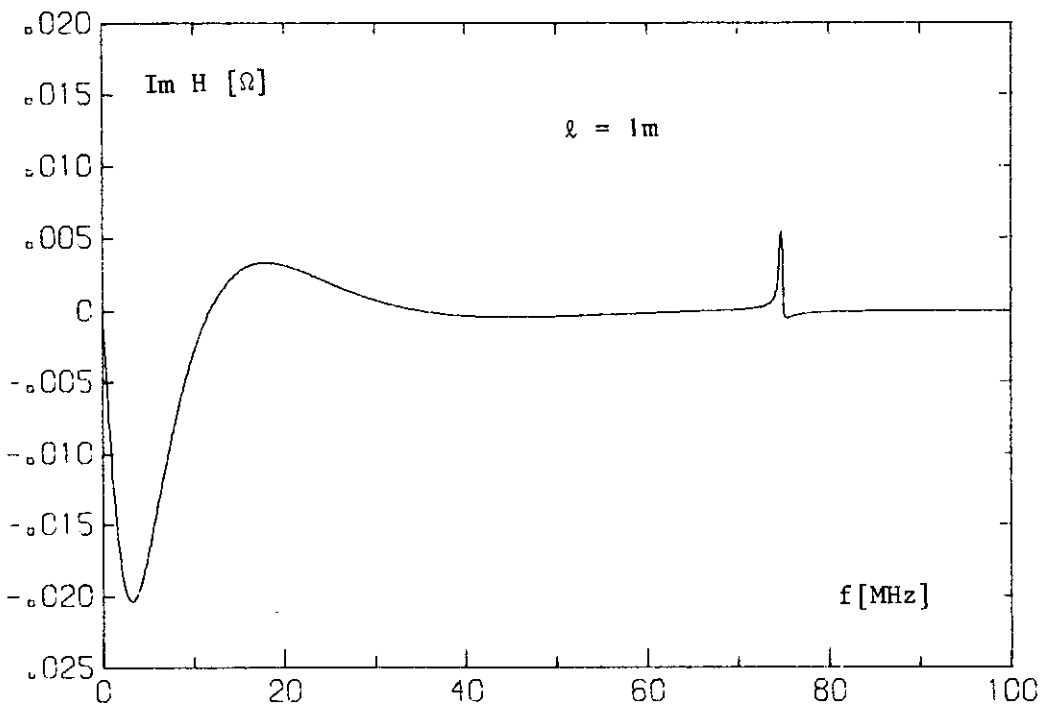

Fig.9b. The imaginary part of the transfer impedance as a function of frequency in 1000 steps of 10^5 Hz

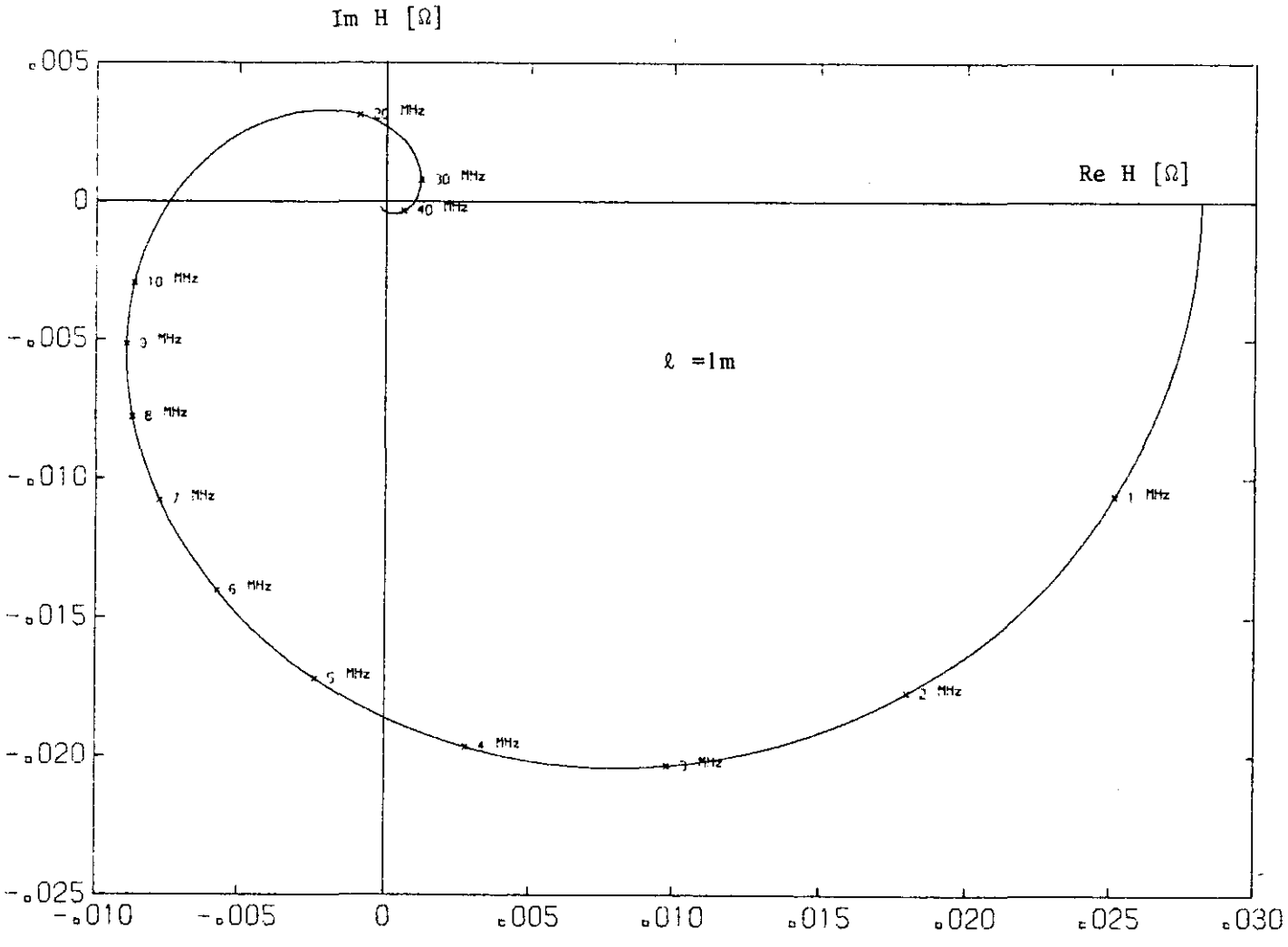

Fig.10. The polar plot of the transfer impedance in the complex plane with the frequency as a parameter

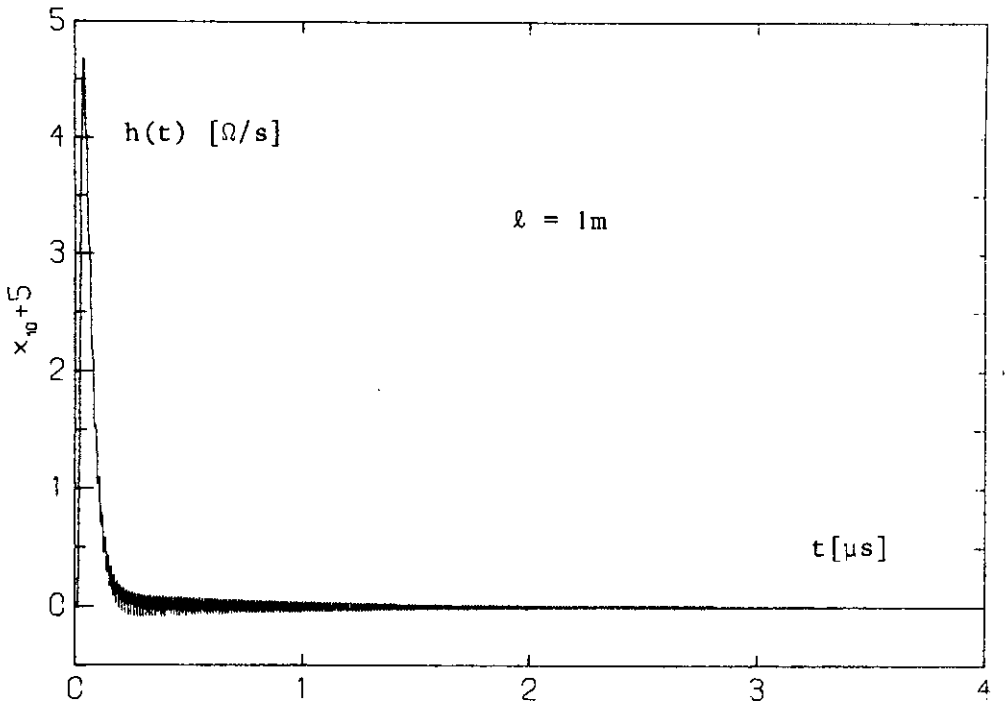

Fig.11a. The impulse response of the coaxial shunt; the result of an FFT of $H(\omega)$. The transform was performed with f_g = 500 MHz and N=2048. Note that Δf is chosen small enough to ensure a negligible aliasing distortion.

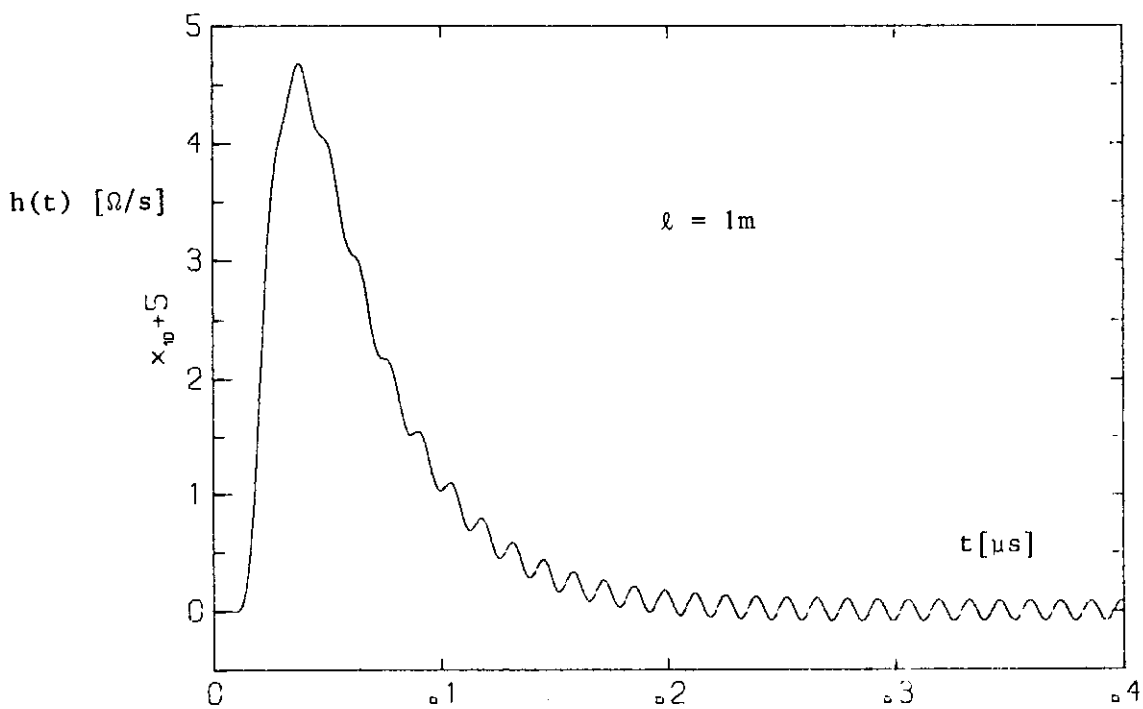

Fig.11b. The first part of the impulse response. Note the ripple of 75 MHz corresponding with the peak in the amplitude characteristic (fig.8a)

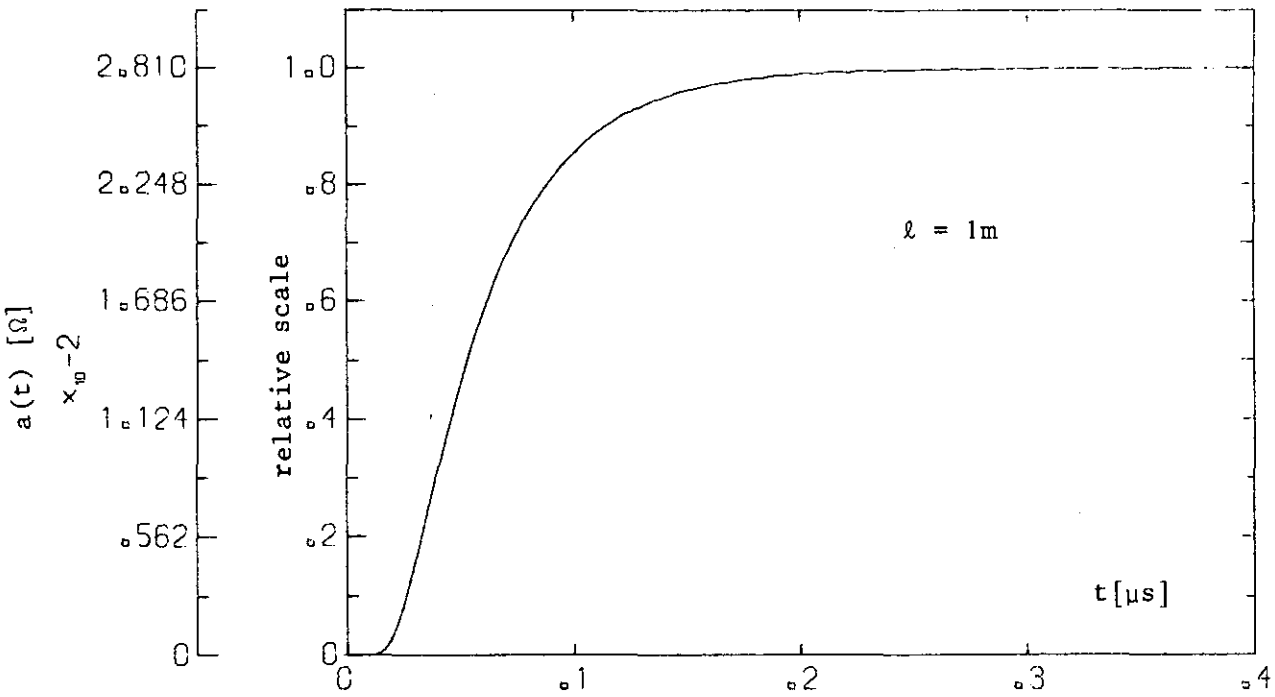

Fig.12. The step response of the coaxial shunt; the result of the integration of h(t)

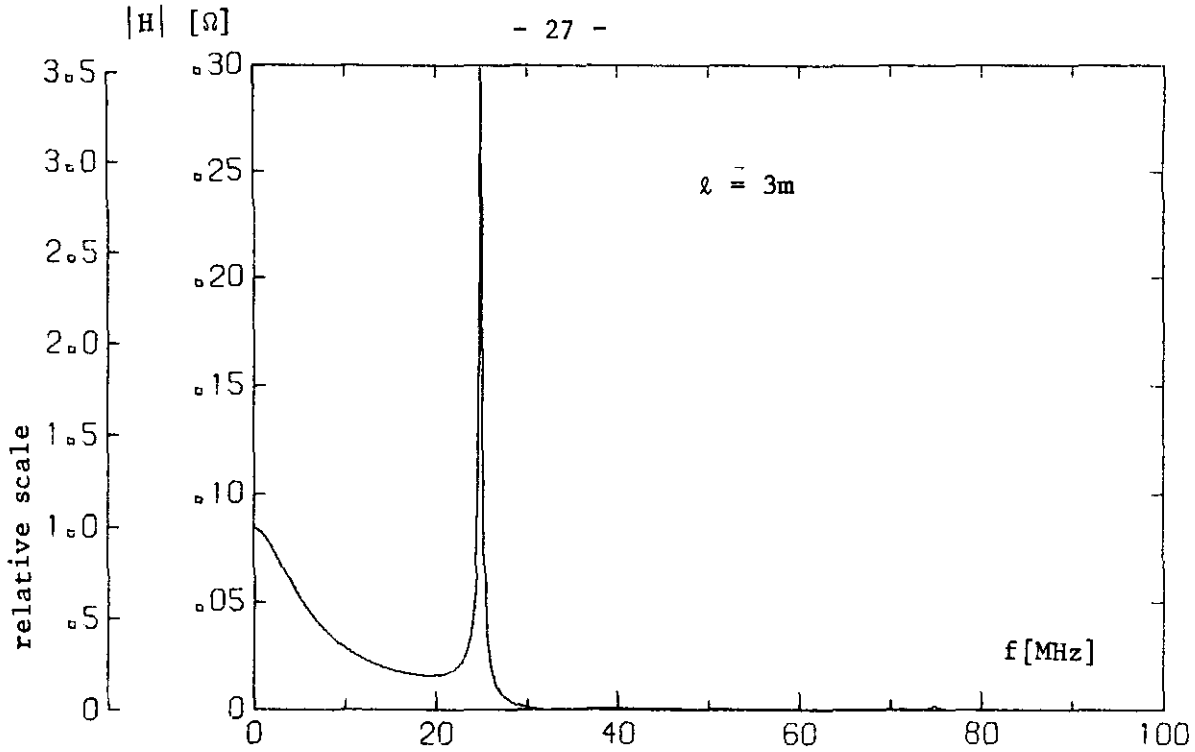

Fig. 13a. The amplitude characteristic of the transfer impedance, that is $|H|=|V_a(\ell)/I|$ as a function of frequency in 1000 steps of 10^5 Hz. Note the peak at 25 MHz, corresponding with the shunt as a quarter wavelength line

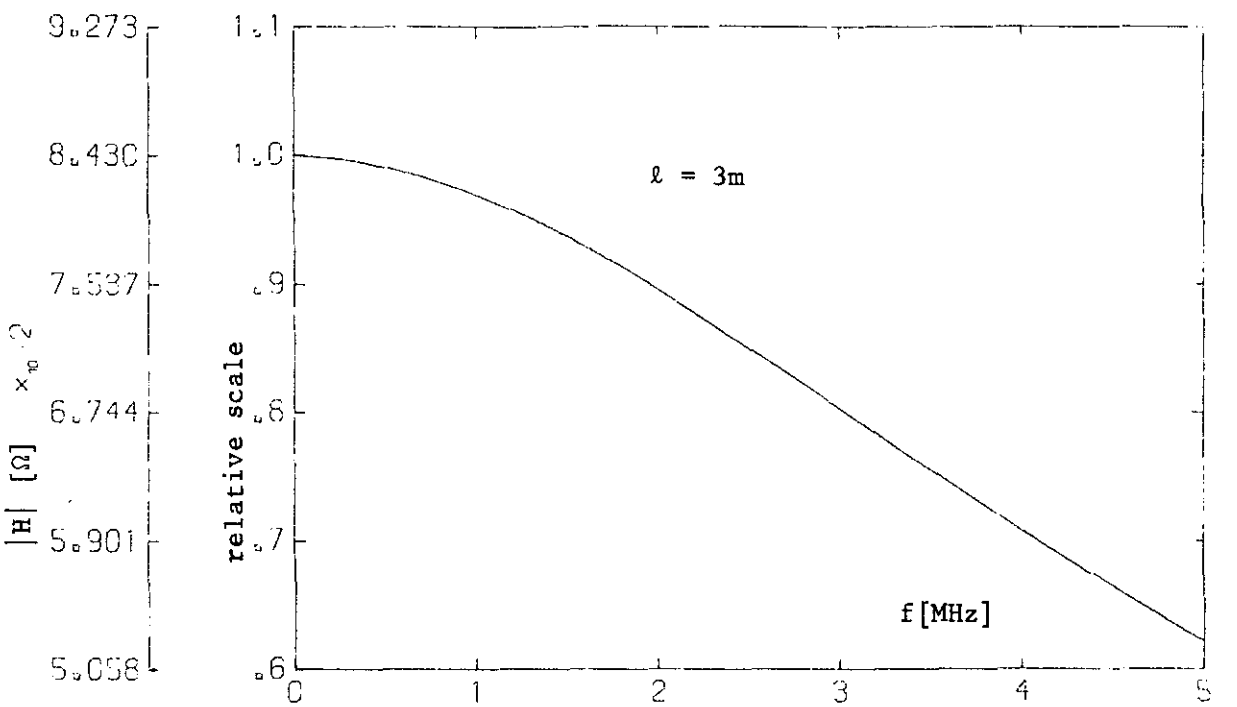

Fig. 13b. The low frequency part of the amplitude characteristic calculated in 100 steps of 5.10^4 Hz

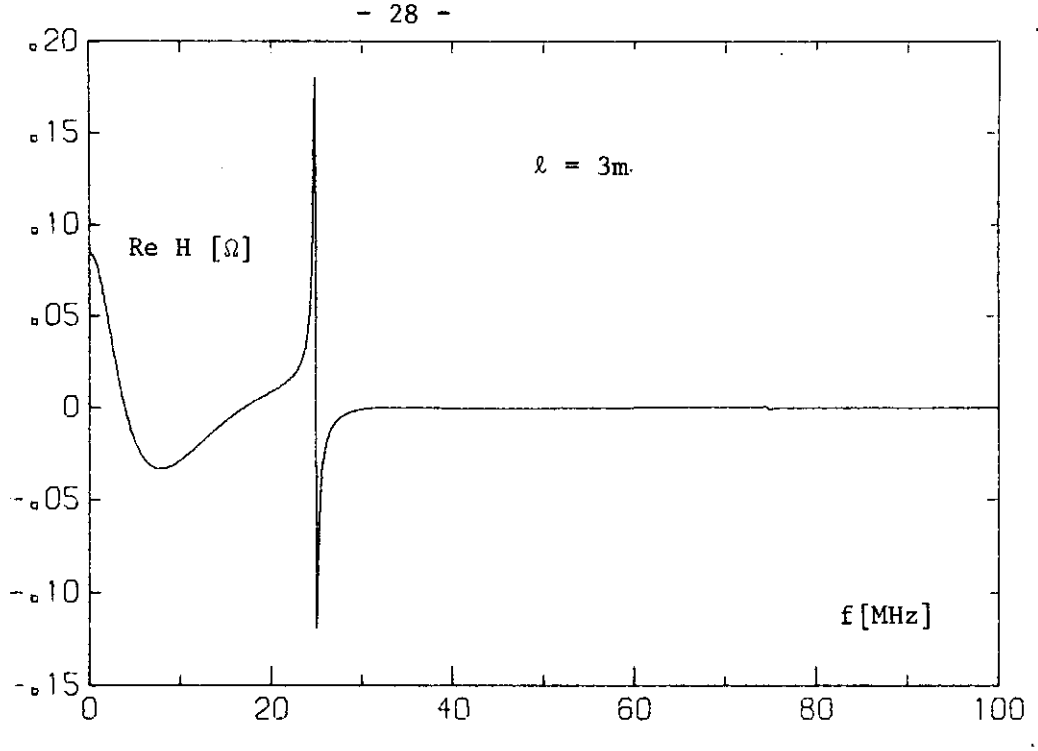

Fig. 14a. The real part of the transfer impedance as a function of frequency in 1000 steps of 10^5 Hz

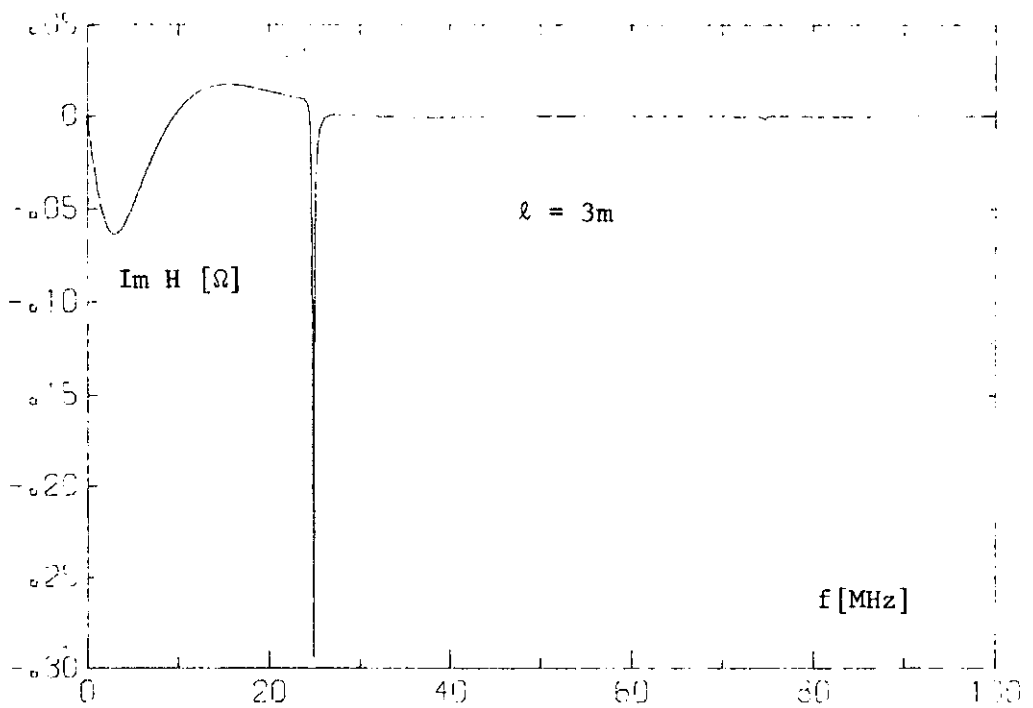

Fig. 14b. The imaginary part of the transfer impedance as a function of frequency in 1000 steps of 10^5 Hz

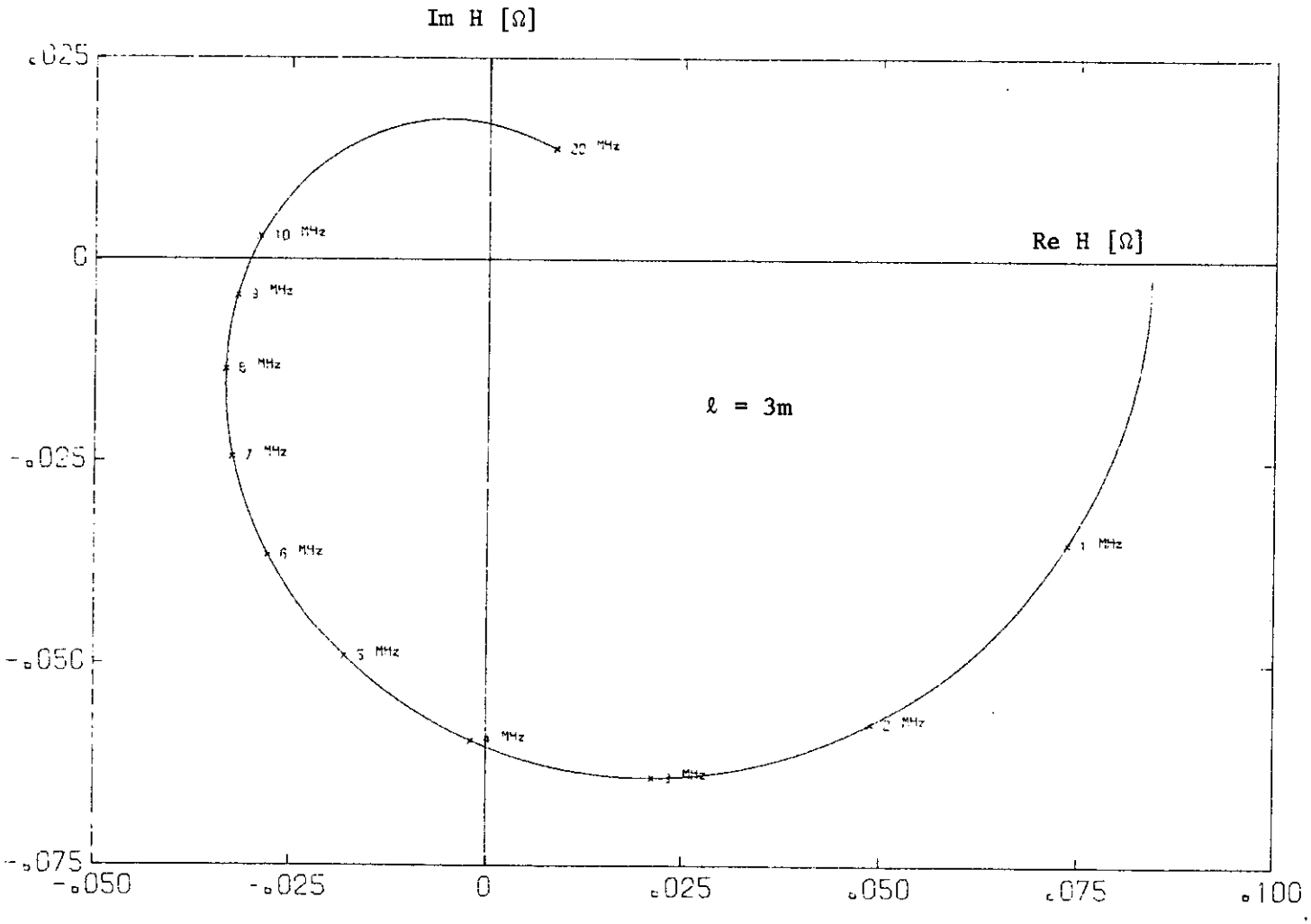

Fig. 15. The polar plot of the transfer impedance in the complex plane with the frequency as a parameter

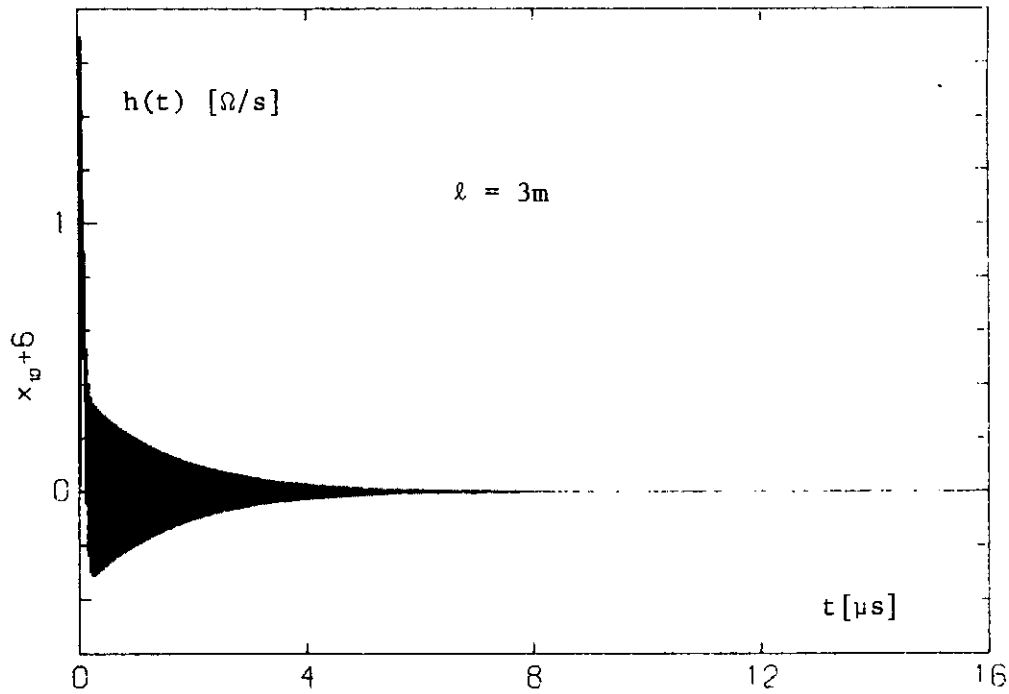

Fig. 16a. The impulse response of the coaxial shunt; the result of an FFT of $H(\omega)$. The transform was performed with f_g = 500 MHz and N=8192. Note that Δf is chosen small enough to ensure a negligible aliasing distortion.

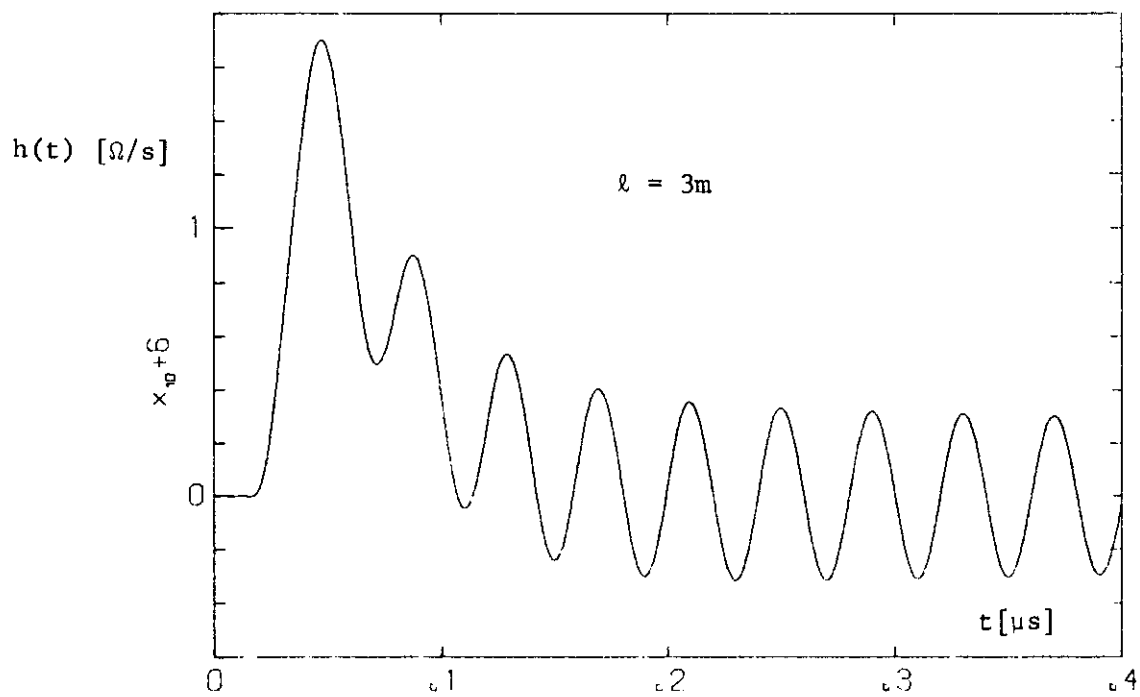

Fig. 16b. The first part of the impulse response. Note the ripple of 25 MHz corresponding with the peak in the amplitude characteristic (fig.13a)

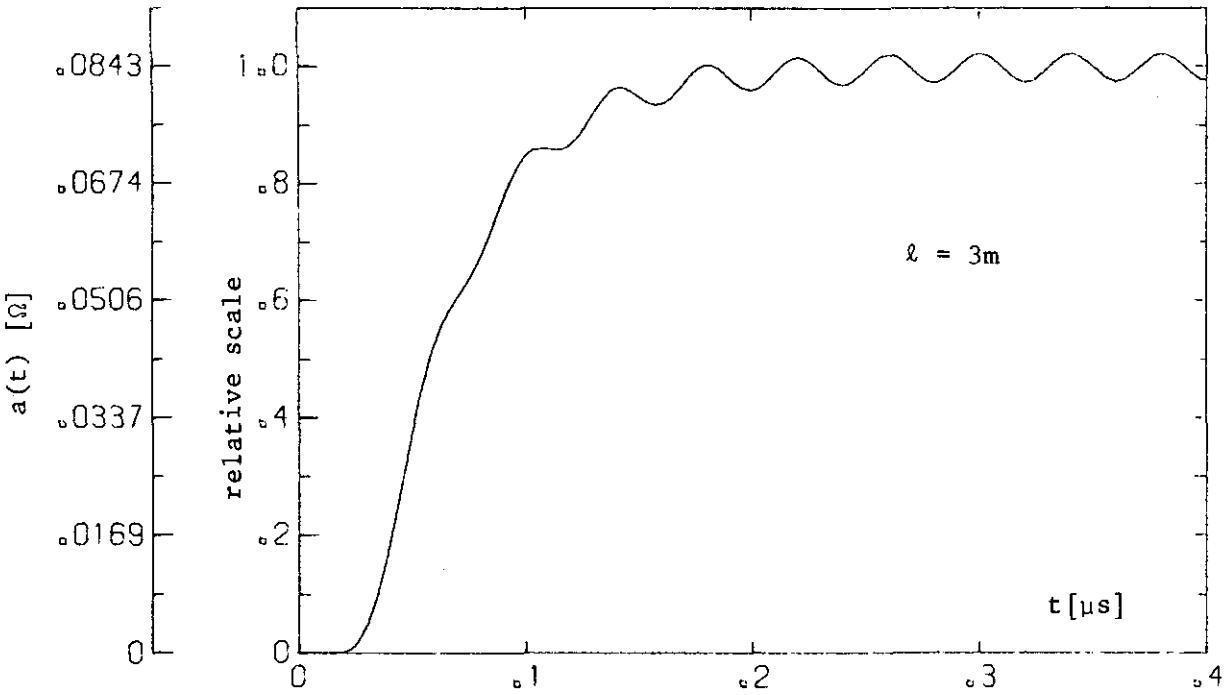

Fig. 17. The step response of the coaxial shunt; the result of the integration of h(t)

9. Conclusions

1. The paper shows the possibility of a rigourous treatment of the coaxial shunt with the aid of the computer, without losing the possibility of physical interpretation.

2. The results of section 8 show clearly that the behaviour of the coaxial shunt in the high-frequency part of the frequency domain and related to this the first part of the impulse and step responses is influenced by the length of the coaxial shunt and is not exclusively a function of the wall thickness and the resistivity of the common conductor.

3. If the length of a coaxial shunt is of the order of the wavelength of the frequency concerned, it is not allowed to derive the transfer impedance of the two-port by simply multiplying the transfer impedance per unit length by the length of the shunt [3].
   (For the definition of the transfer impedance per unit length as a field intensity E divided by a current I, see Schelkunoff [2]).

10. Aknowledgement

The author appreciates the continued help and the interest of his colleague W.C. van Etten and is indebted to L. van der Waals whose competent and devoted efforts made it possible to conclude this work successfully.

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