A constitutive equation of Al-Li powder for a simulation program of dynamic compaction

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- PREFACE.

When it was time to do my first traineeship, I contacted my professor, Prof. Ir. J.A.G. Kals. He told me that it was possible to go to Japan to do my traineeship. This was around June 1992. August the 31st I already flew to Japan. In Japan I was picked up by K. Kato who at the time was a master course student and became my co-researcher. I stayed at the Mechanical Engineering Laboratory (M.E.L.) in Tsukuba Science City. M.E.L. belongs to the Agency of Industrial Science and Technology (A.I.S.T.) a part of the Ministry of International Trade and Industry (M.I.T.I.). At that time was the plasticity and forming division of Dr. Sano, my coach, working on several projects involving AlLi-powder. It became my task to find a constitutive equation of this powder during static compaction. This equation should be used in a simulation program for dynamic compaction. In front of you lays the report of this research.
My total stay in Japan lasted for three months till the end of November. I hereby want to say thanks to my professor, Prof. Ir. Kals, Dr. Matsuno and Dr. Sano for giving me the opportunity to do my traineeship at M.E.L. in Tsukuba, Japan and the effort they made to make my stay as pleasant as possible. I also want to thank K. Kato for his effort in the research and for introducing me to the Japanese culture of everyday. At last but not least I want to thank the rest of the Plasticity and Forming Division of M.E.L. for their daily support.

J.H.M.G. (John) Schroen.

- INTRODUCTION.

In order to get a simulation program for the dynamic compaction of Aluminium-Lithium powder, it's necessary to know more about the behaviour of this kind of metal powder. Therefore we started with trying to get a static constitutive equation of this material. Wé did not only lubricated compaction, but also dry compaction. This constitutive equation then would be put into the simulation program that is based on the method of characteristics to describe the propagation of elastic and (visco)plastic waves through the material. The program also needs Young's modulus to simulate the unloading part of the waves. There will be instantaneous as well as non-instantaneous response and therefore we adjusted the program concept to this. By trying to get the constitutive equation we learned more about the problems of compacting the powder.

- Brief introduction to the method of characteristics.

This program works with a one dimensional wave propagation and by that you can represent the waves in a x-t-plane. A characteristic line is a curve that represents the wave propagation by pointing out the points were the velocity (v) and the strain (ε) are continuous but their derivatives are discontinuous.

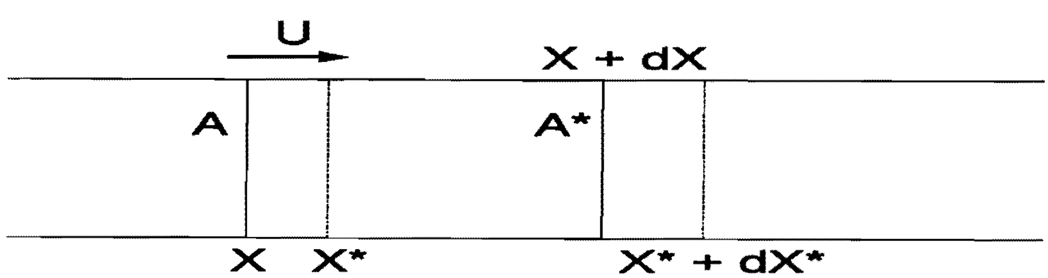

Figure 1

Suppose we have a small part of the material with a cross sectional area A, density ρ, and a length dX (see figure 1). After a certain time the left end of this part has undergone a displacement U.
By that $X^* = X + U$. The right end, $X + dX$ with area A^*, has a displacement of $U + dU$ to $X^* + dX^*$. Therefore $dX^* = dX + dU = (1 + \partial U / \partial X) * dX$.
From the law of conservation of mass it follows that:

$$\rho A = \rho^* A^* (1+\varepsilon) \tag{1}$$

where $\varepsilon = \partial U / \partial X$ represents the strain.

If F is the force that acts on the cross section X at time t, we get the following equation of motion for the element of length dX:

$$\rho A \frac{\partial^2 U}{\partial t^2} = \frac{\partial F}{\partial X} = \frac{\partial (A\sigma)}{\partial X} \tag{2}$$

At the compaction of the powder we assume a constant cross section area and thereby equation (2) becomes:

$$\rho \frac{\partial V}{\partial t} = \frac{\partial \sigma}{\partial X} \tag{3}$$

where $V = \partial U / \partial t$ is the particle velocity.

This is the equation of motion we used in the simulation program.

The second equation we need is of course a constitutive equation. This usually looks as follows:

$$\sigma = f(\varepsilon) \tag{4}$$

With equation (4), equation (3) looks :

$$\frac{\partial^2 U}{\partial t^2} = \frac{1}{\rho} \frac{d\sigma}{d\varepsilon} \frac{\partial^2 U}{\partial X^2} \tag{5}$$

This second order quasi-linear equation can also be written as a first order system.

$$\frac{\partial V}{\partial t} = c^2(\varepsilon)\frac{\partial \varepsilon}{\partial X}$$

$$\frac{\partial V}{\partial X} = \frac{\partial \varepsilon}{\partial t} \tag{6}$$

$$c^2(\varepsilon) = \frac{1}{\rho}\frac{d\sigma}{d\varepsilon} \tag{7}$$

is the velocity of propagation of the wave.

The system can be written in matrix notation too:

$$\frac{\partial}{\partial t}\mathbf{B} + \mathbf{A}\frac{\partial}{\partial X}\mathbf{B} = 0$$

$$\mathbf{B} = \begin{bmatrix} \varepsilon \\ V \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} 0 & -1 \\ -c^2 & 0 \end{bmatrix} \tag{8}$$

Further we need the directional derivatives in the characteristic directions (e.g. s). We will write dV instead of $\partial V / \partial s$.

$$dV = \frac{\partial V}{\partial X} dX + \frac{\partial V}{\partial t} dt$$

$$d\varepsilon = \frac{\partial \varepsilon}{\partial X} dX + \frac{\partial \varepsilon}{\partial t} dt \tag{9}$$

By combining equations (6) and (9) we get:

$$\frac{\partial V}{\partial t} = \frac{c^2(dV\,dt - d\varepsilon\,dX)}{-dX^2 + c^2\,dt^2}$$

$$\frac{\partial V}{\partial X} = -\frac{c^2\,d\varepsilon\,dt - dV\,dX}{-dX^2 + c^2\,dt^2} = \frac{\partial \varepsilon}{\partial t} \qquad (10)$$

$$\frac{\partial \varepsilon}{\partial X} = \frac{dV\,dt - d\varepsilon\,dX}{-dX^2 + c^2\,dt^2}$$

These now are the derivatives of V and ε, which have to be discontinuous for pointing out the characteristic curves. To obtain this the denominator has to be zero. Therefore is

$$\frac{dX}{dt} = \pm c(\varepsilon) \qquad (11)$$

the definition of the characteristic curves. But not only the denominator, but also the numerator has to be zero. By that we get the consistency conditions:

$$dV = \pm c(\varepsilon)d\varepsilon \qquad (12)$$

- **Interpretation of the characteristic curves.**

If the constitutive equation is $\sigma = E\varepsilon$, with E Young's modulus, then the slope of the characteristic curves, $\pm c(\varepsilon)$, will be a constant, namely $\sqrt{\frac{E}{\rho}}$. The characteristics become straight lines (see figure 2).

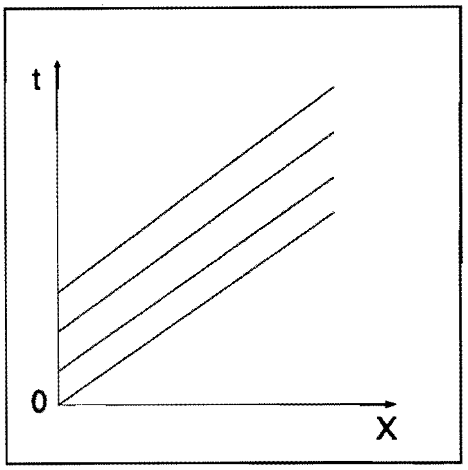

Figure 2

The same will occur when the material is linear workhardening. The stress-strain relationship then is $\sigma = \sigma_y + E_l(\varepsilon - \varepsilon_y)$ where E_l is the constant work-hardening modulus. For a perfectly plastic material, there will be only the same elastic waves going through the material and as soon as the stress becomes at the yield point, wave propagation stops.

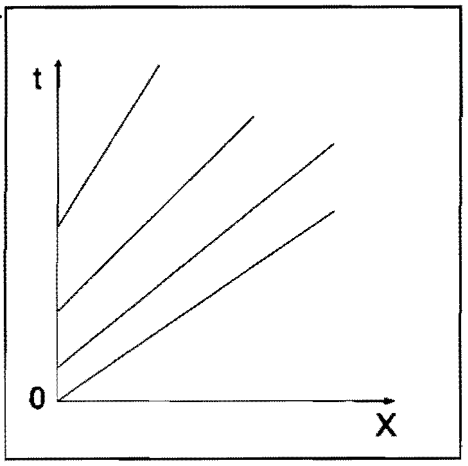

Figure 3

Although for most metals it holds that the stress-strain curve is convex towards the stress axis. The slope of the characteristics will then decrease as the stress grows, because $d\sigma / d\varepsilon$ decreases. The curves will be divergent (see figure 3).
In the case of powder compaction the constitutive equation is concave towards the stress axis, as we will see at our constitutive equation too. Now the derivative of σ to ε will increase and the curves will be convergent (see figure 4). This implies that there can occur shock waves, because waves which start later can catch up with earlier waves and interfere with them. When shock waves occur, not only the derivatives of ε and V will be discontinuous, but also ε and V themselves. In the section: more about the constitutive equation, we will discuss the rate of increase of the slope of the characteristics.

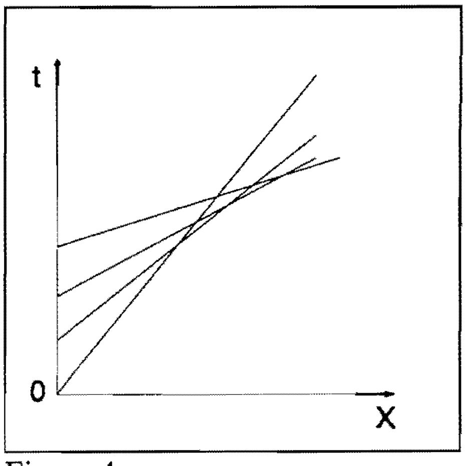

Figure 4

- **The unloading parts of the constitutive equation.**

The unloading part of the stress-strain curve differs form the loading part in that way that it in most cases can be represented by a straight line. It starts at the stress where the loading stopped. For most the metals the slope can be presented by Young's modulus. By that the constitutive equation will look as follows:

$$\sigma = \sigma_m + E(\varepsilon - \varepsilon_m) \tag{13}$$

where σ_m and ε_m are the stress and the strain at the starting point, respectively. As we will show later in this paper, depends Young's modulus of powder compactions on the apparent density. Therefore will each part of the specimen unload in a different way. This is the reason that the process of powder compaction only can be described by using a simulation program that makes a grid of the specimen and calculates the various variables on the grid points.

- **Solution method for the method of characteristics.**

The differential equations of the method of characteristics can be solved in a numerical way using a computer. We used the Hartree-method because this is the most convergent method for the iteration calculations that have to be done.

From the method of characteristics we derived three equations:

1) The definition of the characteristic curves: $dX / dt = \pm c(\varepsilon)$ ...(1')

2) The consistency conditions: $dV = \pm c(\varepsilon) d\varepsilon$ ...(2')

3) The constitutive equation: $\sigma = f(\varepsilon)$ ...(3')

We now put a grid on the X-t-plane and start to calculate on the impact surface, because there we know σ, V, ε and c. The calculation will be similar for each grid point, except the points at the left and right end.

At time t1 we know σ, V, ε and c (see figure 5). Now we are going to calculate these variables at point Q. The left slope we get from:

$$cl = \{ c(ix) + c(ix-1) \} / 2$$

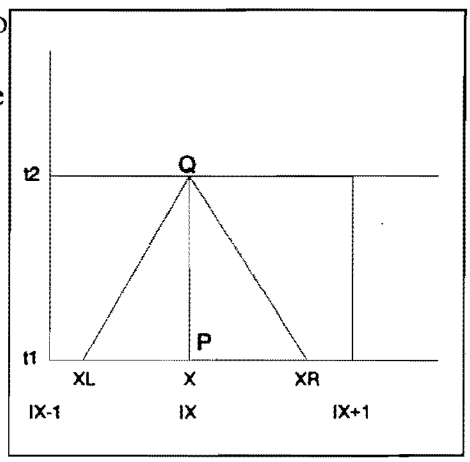

Figure 5

The right one from:

cr = { c(ix) + c(ix+1) } / 2

We are now able to calculate the intersection points with t1 (xl, xr). By interpolation we get the values of σ, V, ε and c at xl and xr.

With the use of the equations

$$V2 - V1 = cl\ (\varepsilon 2 - \varepsilon 1) \tag{14}$$

$$V2 - V1 = -cr\ (\varepsilon 2 - \varepsilon 1) \tag{15}$$

we can calculate V2 and $\varepsilon 2$. Then with the use of equation (3'), $\sigma 2$ and with

$c = \sqrt{\frac{1}{\rho}\frac{\partial \sigma}{\partial \varepsilon}}$, c2. Now we start a iteration process. We determine cl and cr again with the following equations:

cr = (cr + c2) / 2

and

cl = (cl + c2) / 2

and calculate $\sigma 2$, V2 and $\varepsilon 2$. While doing this we check also if the iteration is convergent. This is done by the equations:

$$\frac{\varepsilon_N - \varepsilon_{N-1}}{\varepsilon_N} < d\varepsilon \ \wedge \ \frac{V_N - V_{N-1}}{V_N} < dV \tag{16}$$

N is the iteration step.

If the righthand line hits the first wave front, the fastest elastic wave, than the values on the intersection point (xr) will equal the values of this wave front (see figure 6). These values are $V = 0$, $\varepsilon = 0$, $\sigma = 0$ and $c = c_e = \sqrt{\frac{E}{\rho}}$.

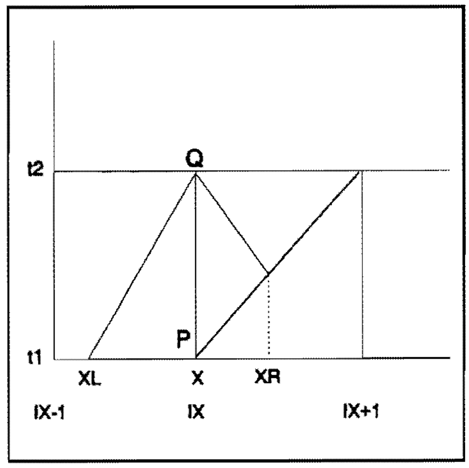

Figure 6

At the left end, the side of the punch, are the boundary conditions the stress $\sigma 2$ or velocity V2 (see figure 7). From these it's possible to derive the other values:

$$\sigma 2 \rightarrow \varepsilon 2 \rightarrow V2 \rightarrow c2$$

or

$$V2 \rightarrow \varepsilon 2 \rightarrow c2 \rightarrow \sigma 2$$

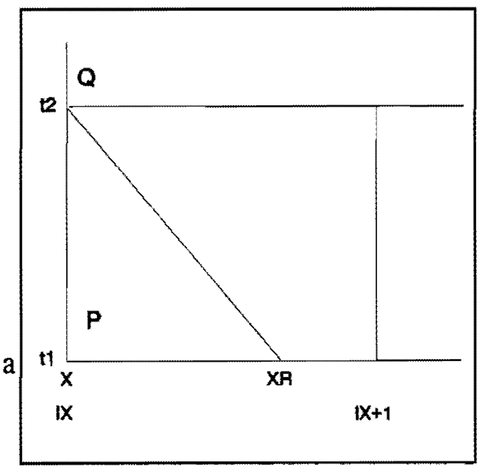

Figure 7

At the right side, the bottom of the container, are the boundary conditions V2 (= 0) or σ (see figure 8). The powder can not pass on a positive stress, only when it already has a higher density. Therefore we will assume that a stress above zero can not occur. The values at the righthand side are calculated in the same way as at the lefthand side.

If the slope is to small so that it doesn't hit the t1-line before ix-1, then we will use an interpolation between ix-n and ix-(n-1) to calculate the values of xl.

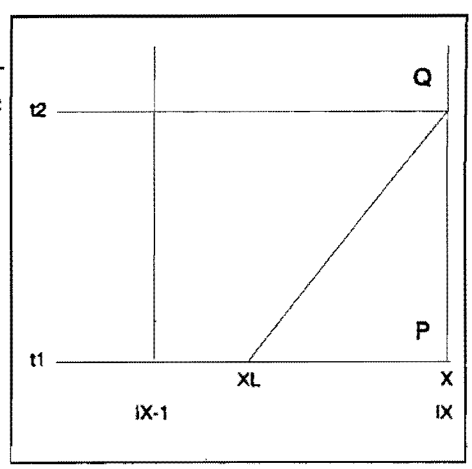

Figure 8

If the line hits the punch surface then we use a interpolation of the t-axis.

The size of the next time-step will be decided by the Courant number. This number decides what size the next time-step has to be, if the minimum slope has to cross the next t-line in a certain place. For example if the minimum slope has to cross the next t-line in the middle between xl and x, then the Courant number will be 0.5.

$$\Delta t = (dX / c_{max}) \times \text{Courant number} \qquad (17)$$

- **<u>The equations of the method of characteristics for friction at the wall of the container.</u>**

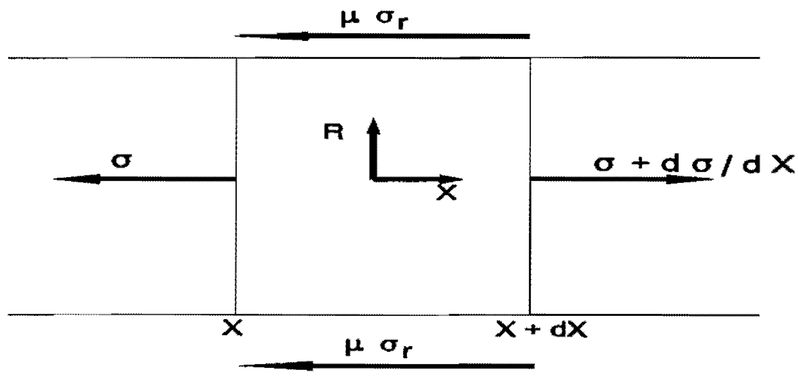

Figure 9

The balance of the piece of material is:

$$\pi r^2 dX\{-\sigma + (\sigma + \frac{\partial\sigma}{\partial X})\} - 2\pi r dX \mu \sigma_r = \rho \pi r^2 dX \frac{\partial V}{\partial t} \qquad (18)$$

By putting $\dfrac{\partial\sigma}{\partial X} = \dfrac{\partial\sigma}{\partial\varepsilon} \times \dfrac{\partial^2 U}{\partial X^2}$ and $\sigma_r = \alpha\sigma$, we get the equation of motion:

$$\frac{\partial V}{\partial t} = \frac{1}{\rho}\frac{\partial\sigma}{\partial\varepsilon}\frac{\partial^2 U}{\partial X^2} - \frac{2\mu\alpha\sigma}{\rho r} \qquad (19)$$

The constitutive equation stays the same, only it has to be a equation for compaction with friction.
We can write this as a first order system.

$$\frac{\partial V}{\partial t} = c^2(\varepsilon)\frac{\partial \varepsilon}{\partial X} - \frac{2\mu\alpha\sigma}{\rho r}$$

$$\frac{\partial V}{\partial X} = \frac{\partial \varepsilon}{\partial t} \tag{20}$$

The definition of the characteristic curves is the same as in the previous section:

$$\frac{dX}{dt} = \pm c(\varepsilon) \tag{21}$$

However the consistency conditions change. They get an extension.

$$dV = \pm c(\varepsilon)d\varepsilon + \frac{2\mu\alpha\sigma}{\rho r}dt \tag{22}$$

- **The method of characteristics for instantaneous and non-instantaneous response.**

Until now we only discussed the method of characteristics for instantaneous response. But with dynamic compaction there is often also non-instantaneous response, visco-plastic material behaviour. Therefore we also have to take a time dependency of σ into our calculations. From this moment on we will write $\varepsilon = \varepsilon_e + \varepsilon_p$, where ε_p is the strain that remains after spring back and ε_e the spring back strain or elastic strain.
We now have eight unknowns, the derivatives of σ, ε, ε_p and V to t and X. So we also need eight equations. The first one is the equation of motion, which still remains the same:

$$\rho\frac{\partial V}{\partial t} = \frac{\partial \sigma}{\partial X} \tag{23}$$

Then we have the constitutive equation:

$$\frac{\partial \sigma}{\partial t} - E \frac{\partial \varepsilon_e}{\partial t} = 0 \tag{24}$$

$$\frac{\partial \sigma}{\partial t} - \Phi(\sigma,\varepsilon)\frac{\partial \varepsilon_p}{\partial t} = \Psi(\sigma,\varepsilon) \tag{25}$$

$E = \partial\sigma / \partial\varepsilon = f(\sigma_m)$ is Young's modulus, that depends on the maximum stress so far.

Φ is a function for the instantaneous plastic response.
Ψ is a function for the non-instantaneous plastic response.
The second function of the first order system in the method of characteristics is also called the equation of compatibility, because it connects the particle velocity with the strain. This is the fourth equation:

$$\frac{\partial V}{\partial X} - \frac{\partial \varepsilon_e}{\partial t} - \frac{\partial \varepsilon_p}{\partial t} = 0 \tag{26}$$

The last four equations are the directional derivatives in a characteristic direction:

$$d\beta = \frac{\partial \beta}{\partial X} dX + \frac{\partial \beta}{\partial t} dt \tag{27}$$

where β stands for σ, ε_p, ε_e or V.

If we calculate the definition of the characteristic curves again, we obtain:

$$\frac{dX}{dt} = \pm\sqrt{\frac{E\ \Phi(\sigma,\varepsilon)}{\rho(E + \Phi(\sigma,\varepsilon))}} = \pm c\,(\sigma,\varepsilon) \qquad \text{(A),(B)}$$

$$dX = 0 \qquad \text{(twice)} \qquad \text{(C),(D)}$$

The consistency conditions look as follows:

Along with (A), (B):

$$d\sigma = \pm \rho c \left\{ \pm \frac{\Psi}{\Phi} c\,dt + dV \right\} \tag{28}$$

Along with (C):

$$d\sigma = E\,d\varepsilon_e \tag{29}$$

Along with (D):

$$d\sigma = \Phi(\sigma,\varepsilon)\,d\varepsilon_p + \Psi(\sigma,\varepsilon)\,dt \tag{30}$$

In all these equations is σ the real dynamic stress. The static stress at a certain strain will be written from now on as $f(\varepsilon)$.
The instantaneous and the non-instantaneous functions still have to be completed. Of the non-instantaneous we only know that it depends on the over-stress, defined by $(\sigma - f(\varepsilon))$ [5]. Until the moment of writing this report we were not able to find a proper function.
We have chosen the Prandtle-Reuss' equation for the instantaneous function.

$$\Phi = \frac{\delta\sigma}{\delta\varepsilon} \tag{31}$$

By this the characteristic curves (A) and (B) become:

$$\frac{dX}{dt} = \pm \sqrt{\frac{E\frac{\delta\sigma}{\delta\varepsilon}}{\rho(E + \frac{\delta\sigma}{\delta\varepsilon})}} = \pm\, c(\sigma,\varepsilon) \tag{32}$$

The consistency conditions along (A) and (B) become:

$$d\sigma = \pm\, \rho c\, \{\frac{\delta\varepsilon}{\delta\sigma}\, g(\sigma - f(\varepsilon))\, c\, dt + dV\} \tag{33}$$

The consistency along (C) stays the same, but that along (D) changes into:

$$d\sigma = \frac{\delta\sigma}{\delta\varepsilon}\, d\varepsilon_p + g(\sigma - f(\varepsilon))\, dt \tag{34}$$

- **Approximation of the grid-point values.**

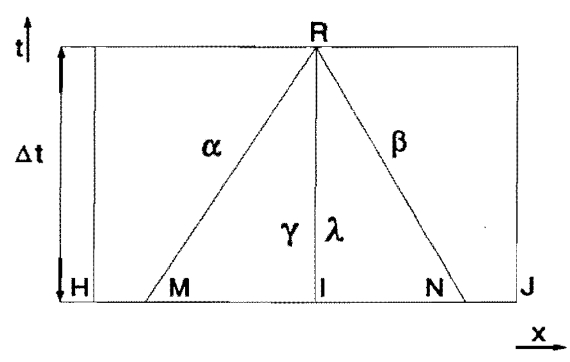

Figure 10

The slope of the characteristic curves will be determined by:

$$c_\alpha = \frac{1}{2}(c_I + c_H) \tag{35}$$

and

$$c_\beta = \frac{1}{2}(c_I + c_J) \tag{36}$$

The other values can be calculated as follows:

(1) Unloading (elastic) $(\sigma \leq \sigma_{max})$

$$(\alpha) \qquad \sigma_R - \sigma_M = \rho c (V_R - V_M)$$

$$(\beta) \qquad \sigma_R - \sigma_N = -\rho c (V_R - V_N) \tag{37}$$

$$(\gamma) \qquad \sigma_R - \sigma_I = E (\varepsilon_{e,R} - \varepsilon_{e,I})$$

where $E = \dfrac{\delta\sigma}{\delta\varepsilon} = f(\sigma_{max})$, $\varepsilon = \varepsilon_e$ and $\rho = \rho_0\, e^{-\varepsilon}$. ρ_0 is the initial density.

(2) Loading (elastic + plastic) $(\sigma > \sigma_{max})$

$$(\alpha) \qquad \sigma_R - \sigma_M = \rho c \{(V_R - V_M) + \frac{\delta\varepsilon}{\delta\sigma} \cdot \frac{g_R + g_M}{2} \cdot c\Delta t\}$$

$$(\beta) \qquad \sigma_R - \sigma_N = -\rho c \{(V_R - V_N) + \frac{\delta\varepsilon}{\delta\sigma} \cdot \frac{g_R + g_N}{2} \cdot c\Delta t\}$$

$$(\gamma) \qquad \sigma_R - \sigma_I = E (\varepsilon_{e,R} - \varepsilon_{e,I}) \tag{38}$$

$$(\lambda) \qquad \sigma_R - \sigma_I = \frac{\delta\sigma}{\delta\varepsilon} (\varepsilon_{p,R} - \varepsilon_{p,I}) + \frac{g_R + g_I}{2} \cdot \Delta t$$

where $g_R = g(\sigma_R - f(\varepsilon_R))$.

The initial conditions for the calculations for $t = 0, 0 \leq x \leq l$ will be:

$V(x,0)$, $\varepsilon(x,0)$ and $\sigma(x,0)$.

The boundary condition: $x = 0, t > 0 \Rightarrow \sigma = \sigma(0,t)$.

For further information about the method of characteristics and dynamic compaction we refer to [1], [2] and [3].

- **The experiments to get the constitutive equation.**

We started with measuring the elasticity of the punches. This included also the support plates. So now we knew how many micrometers we had to add to the measured height of the powder column at each load.
Then we filled the container to a certain height with the lower punch in it. After putting the upper punch in the container we measured the height of the powder column with a vernier callipers. This was a first indication of the starting height. If it was more than 1 mm besides the desired height we adjusted the amount of powder. The machine was turned on slowly. Immediately when the load started to rise the micrometer was set zero. This was about 25 kg. We then wrote down the displacement at every hundred kg until 500 kg and thereafter at every five hundred kg. After the desired load was reached we started with slowly unloading. We still measured the displacement of the upper punch at the same loads only we skipped 400 and 200 kg. Then we pulled the punches out and pushed out the specimen with a ejection punch. The specimens were given a certain code and were weighed. The height was measured with the same vernier callipers.
If a lubricant was used we sprayed the punches and the inside of the container with this lubricant. Then they were dried with a dryer.
After each test the container and the punches were cleaned with acetone and polishing paper.
This test complies with the standard test method for compressibility of metal powders in uniaxial compaction according to ASTM [7] [8].
For further details see the experimental details section and figure 11.

- **Experimental details.**

- Experimental devices.

We used a Shimadzu universal testing machine, type REH-50. This is a hydraulic machine with a moveable table. The capacity is 50 ton. The range can be set on 50, 25, 10, 5, 2.5 and 1 ton.

For measuring the displacement we used a digital micrometer of Mitutoyo, type ID-110. The range is 10 mm and the resolution 0.001 mm.

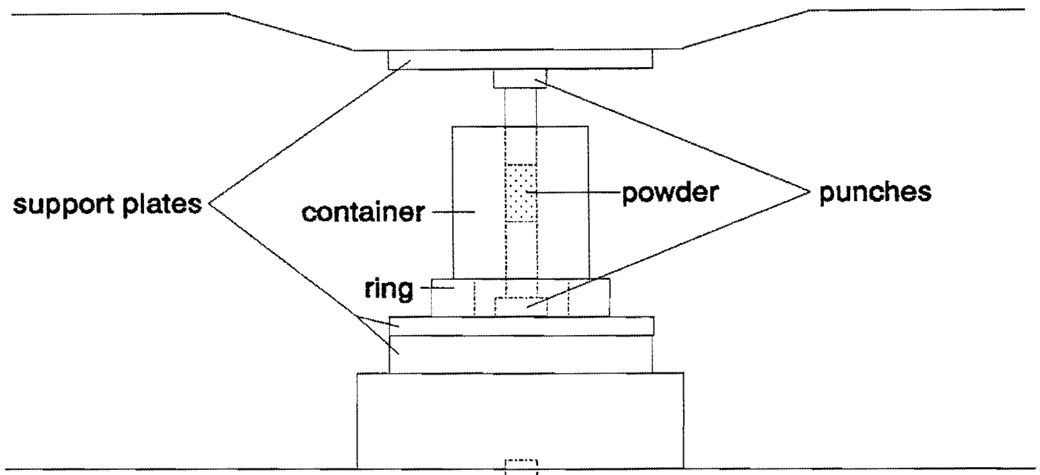

Figure 11

The weight of the specimens was decided by a Shimadzu AEL 200 balance. Resolution 0.0001 g.

The tools we used were a container of an outer diameter of 50.0 mm, an inner diameter of 10.0 mm and a height of 50.0 mm. Also two punches of a diameter of 9.95 mm and a length of 40.0 and 40.2 mm. The punches stood on three square support plates with a length of 80.5 mm and a total height of 41.5 mm. Between the container and the support plates we placed a ring of an outer diameter of 76.6 mm, an inner diameter of 38.9 mm and a height of 25.0 mm. All these tools were made of SKD-11 steel, except the ring that was made of iron. For the ejection we used a punch of a diameter of 9.80 mm.

We used two kinds of lubricants. DM-100 of NipponMolybdeen, a dry lubricant on base of MoS_2 and Emralon 327 of Acheson, a dry lubricant on base of PTFE. After a few tests we decided to use only the last one for further tests.

- Test powder.

For this research we used a Aluminium-Lithium powder from the AA 8090 range [6]. The producer gave this powder the name: L1. This powder is an atomized powder and thereby a rapid solidified powder. The grain size is small ($< 100 \mu m$, with a median of $22 \mu m$) and that improves the material properties. The weight percentages of this powder are: AL 94.93%, Li 2.54%, Cu 1.49%, Mg 0.91%, Zr 0.13%. From this it follows that the theoretical density is 2.734 * 10^3 kg/m^3.

- **Processing of the experimental results.**

After deciding the mass and the volume of the specimen we calculated the relative density by dividing the measured density by the theoretical density. The starting density varies from test to test. So we decided to calculate the starting height of the specimen at 54% of the theoretical density after measuring the mass. By setting this value as zero strain we were able to compare all the different tests. We choose 54% because this was the average relative starting density of the first test serie.

Then the strain has been calculated as follows:

$$\varepsilon = \ln\frac{h}{h_0} \tag{39}$$

h = specimen height.
h_0 = starting height of the powder (54%).

The reference of the specimen height was the end height (measured) plus the registered displacement at the end. This was the real starting height. Then the other heights were calculated by distracting the displacement from this reference height. Now we have the heights which not yet are corrected for the shortening of the punches. So we have to add this shortening at each load to the height to get the real height.

The shortening of the punches looks as in figure 12. There is a clear hysteresis in it. So in the loading parts we add the squares to the heights and in the unloading parts the crosses.

****
Intermezzo:

The shortening of the punches only can be calculated theoretically by

$$\varepsilon = \ln\frac{l}{l_0} = \frac{\sigma}{E} = \frac{500}{210000}$$

$$l_0 = 80.2 \text{ mm}$$

$$\text{shortening} = l - l_0 = 0.19 \text{ mm}$$

This applies for a load of 4000 kg.

This shows that also the frame has a displacement. This is included in our measurements of the shortenings of the punches.

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Elasticity of punches and support plates.

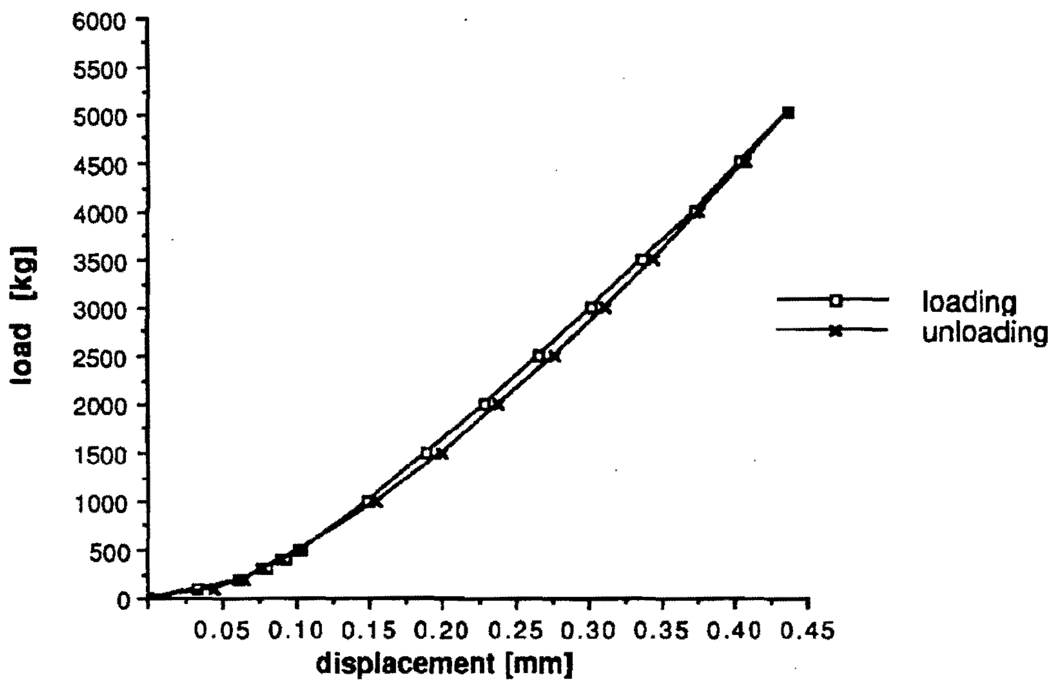


Figure 12

The stress is calculated by dividing the force by the specimen surface.

Now we have a stress - strain relation for different starting heights and with or without the use of a lubricant.

- **The constitutive equations.**

We started to do experiments without a lubricant and for three desired starting heights, namely 1, 2 and 4 cm. These experiments are done because a lubricant may prevent a very high realtive density. As figure 13 shows is there a significant influence of the friction on the stress - strain relation. These curves can be described by the best fit of the 2 cm curve and a height influence factor. The constitutive equation then looks as follows:

$$\sigma = (11 + 707\varepsilon - 2084\varepsilon^2 + 10032\varepsilon^3) * (0.5961 + 9.2208 * 10^{-3} H_{cs} + 6.2922 * 10^{-4} H_{cs}^{2}) \qquad (40)$$

H_{cs} = corrected starting height [mm].

Loading lines for different starting heights without the use of a lubricant.

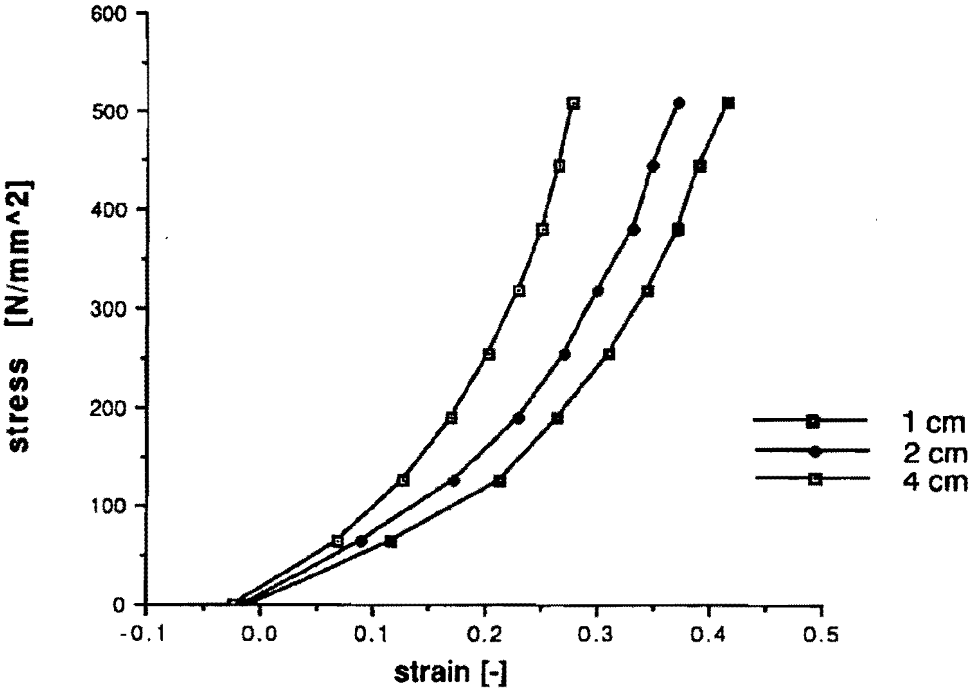


Figure 13

While doing the tests without the use of a lubricant, we noticed a couple of problems. First of all became the ejection force for a starting height of 4 cm a lot higher than the load force (once even 110 kN). The second problem was that it became likely that the container could be damaged severely, due to the sticking of particles in the inside of the container. Removing these particles was a time-consuming task, which wasn't without the risk of making grooves in the container.
These problems were the reason that we went on doing the experiments with the use of a lubricant and only for 1 and 2 cm of starting height.
The use of a lubricant has, as you can expect, an advantageous influence on the compaction (See figure 14 and 15).

Loading lines with and without lubrication.

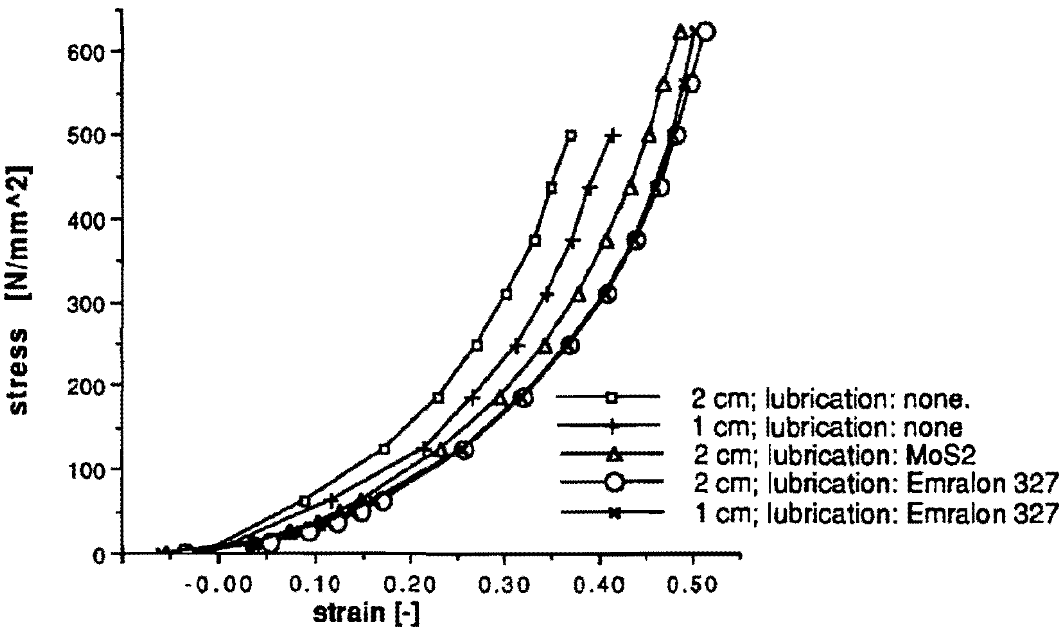


Figure 14

Constitutive equations for 2 cm of powder.

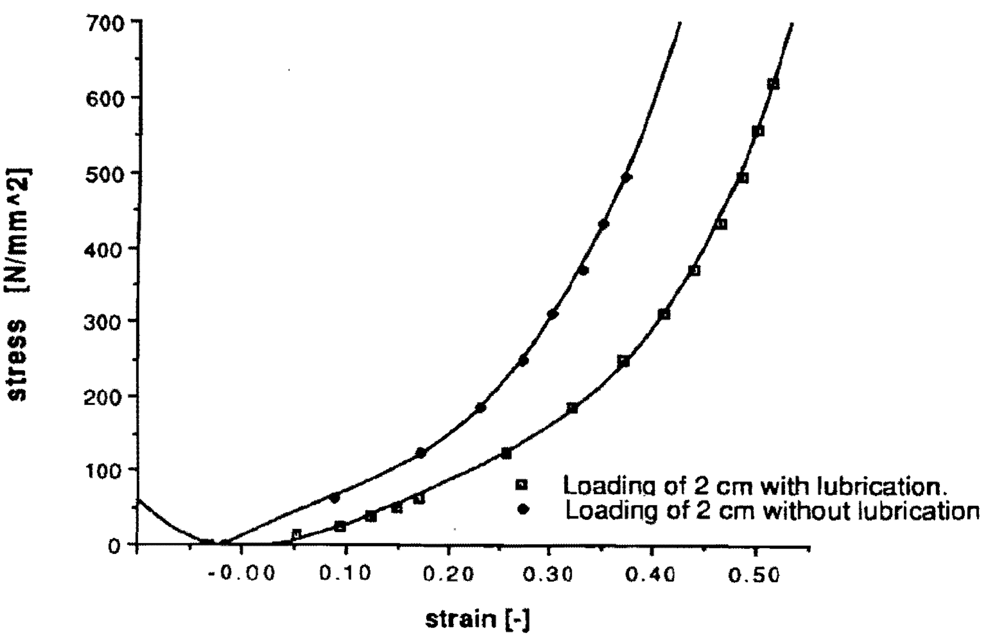


Figure 15

Figure 14 shows also that the lubricant Emralon 327 the better lubricant is and that the friction, due to the lubricant, was decreased to almost zero. Because when we doubled the possible friction surface (starting height of 1 and 2 cm), the stress-strain relations still coincide with each other.
The little difference at the top could be explained by the non-uniaxial character of the compaction. Namely if the height of the specimen becomes smaller than the diameter, then the process will deflect from being uniaxial. This and the reason that the height is no longer of influence, made us decide to go on with specimens of only 2 cm of starting height.

The constitutive equation with the use of a lubricant looks as follows:

$$\sigma = -4 - 18\varepsilon + 4540\varepsilon^2 - 15303\varepsilon^3 + 21694\varepsilon^4 \qquad (41)$$

Figure 15 shows us how well the equations coincide with the measurements (squares).
If you know the end density and end height, it's easy to calculate the density during the test.

Because: $$\varepsilon = \ln\frac{l}{l_0} = \ln\frac{\rho}{\rho_0} \Rightarrow \rho = \rho_0\, e^{\varepsilon} = 0.54\, e^{\varepsilon} \qquad (42)$$

If we do this we get the stress-density relation. This looks as follows:

$$\sigma = 13639 - 82066\rho + 183140\rho^2 - 180750\rho^3 + 67460\rho^4 \qquad (43)$$

$$0.54 < \rho < 1$$

If we put this in a graph we get figure 16.

Stress - density relation for 2 cm of powder.

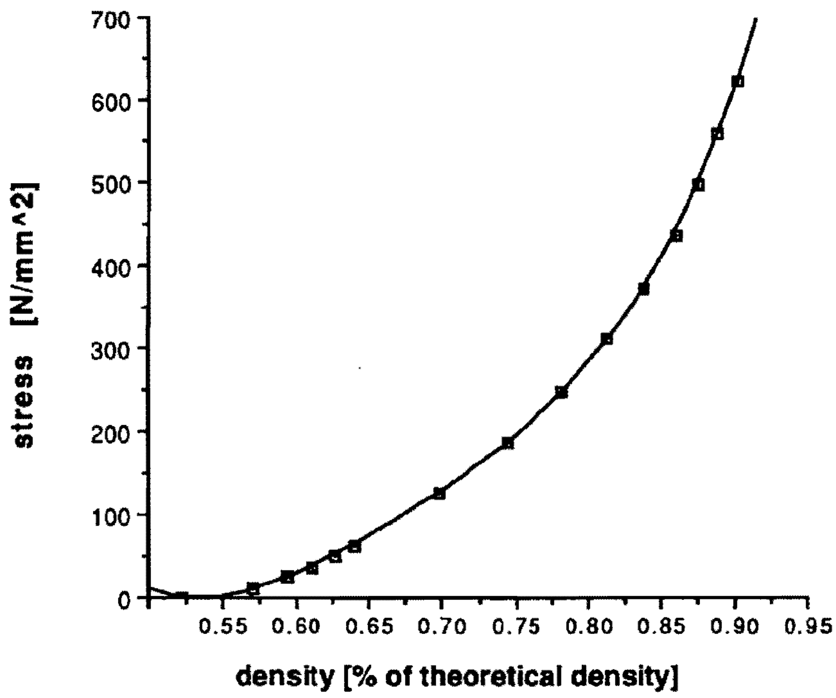
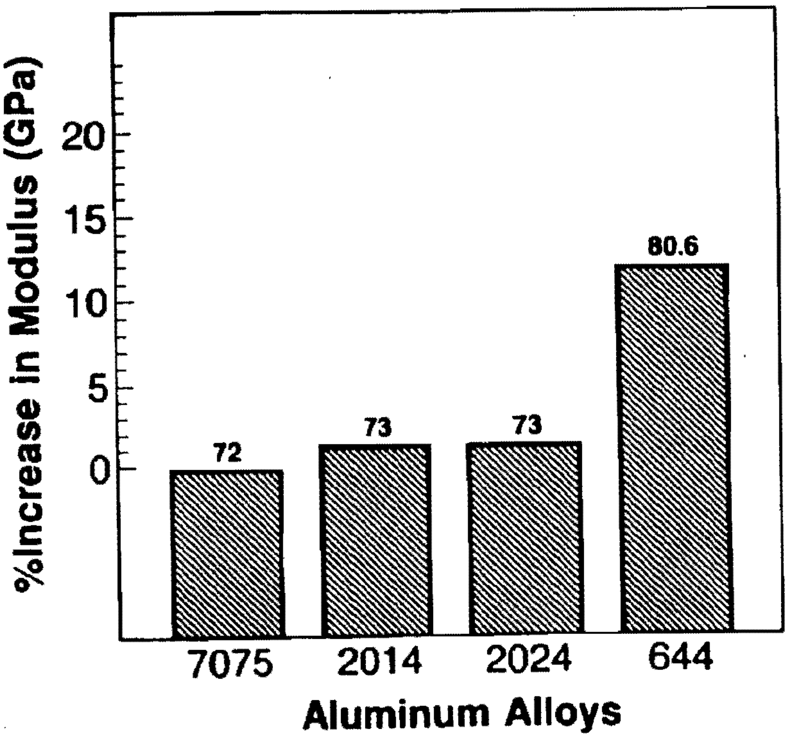


Figure 16

- **The unloading equation.**

In dynamic compaction there will be stress waves going through the material. Therefore we need also the unloading relation of the material for the simulation program. The unloading can be described with a linear equation like $\sigma = E\varepsilon$, where E is Young's modulus. For a solid material is Young's modulus a constant. However in case of a porous material, depends E on the density. So we need a relation between Young's modulus (E) and the relative density (ρ). Young's modulus will always be lower than Young's modulus for solid AlLi. This gives us a top asymptote of circa 83 GPa. The high E is one of the advantages of AlLi alloys (see figure 17).[4]



The increase in tensile modulus of RS Al-Li alloys in comparison with conventional alloys. For RS Al-Li alloys the minimum value is typically 80.6 GPa (11.7 Msi)

To get the Young's modulus - density relationship we searched first for the Young's modulus - stress relationship. Therefore we compacted specimens to five different loads. 1000, 2000, 3000, 4000 and 5000 kg. After reaching the desired load we decreased the load slowly. While doing this we wrote down the displacement at the same loads as for compaction, only we skipped 400 and 200 kg. After correcting we got the figures 18 to 22. The tangent of the line in the figures is Young's modulus.

Unloading line of 1000 kg (124.283 MPa).

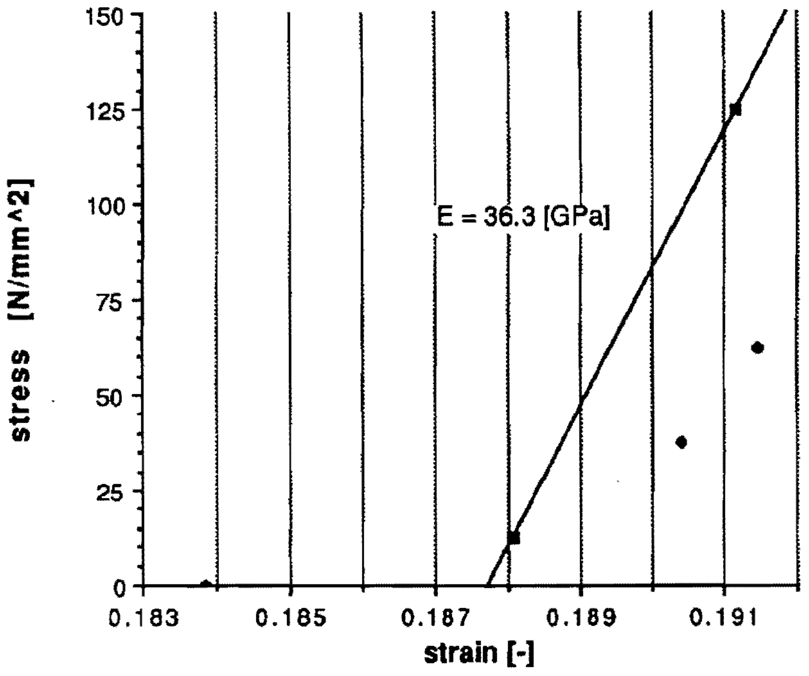


Figure 18

Unloading line of 2000 kg (248.565 MPa).

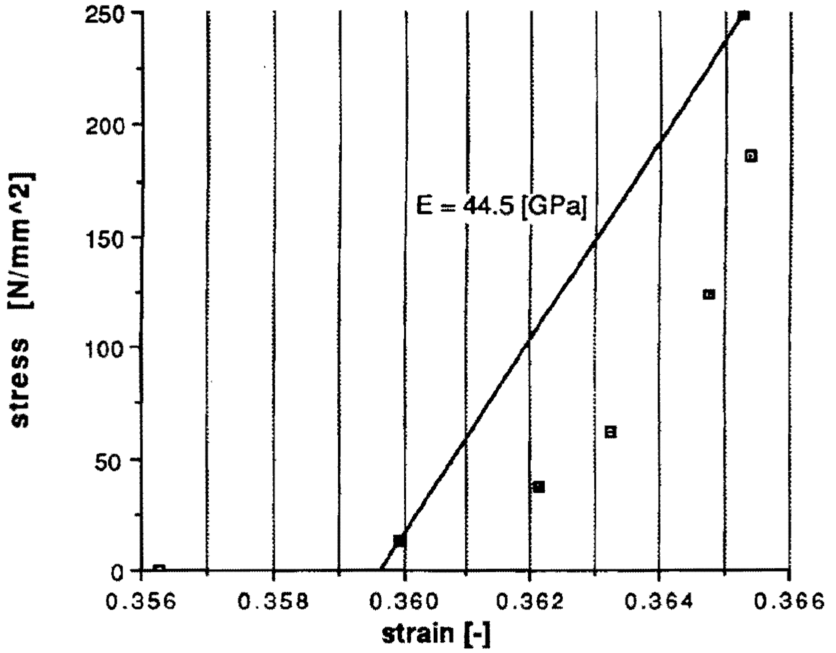


Figure 19

Unloading of 3000 kg (372.848 MPa).

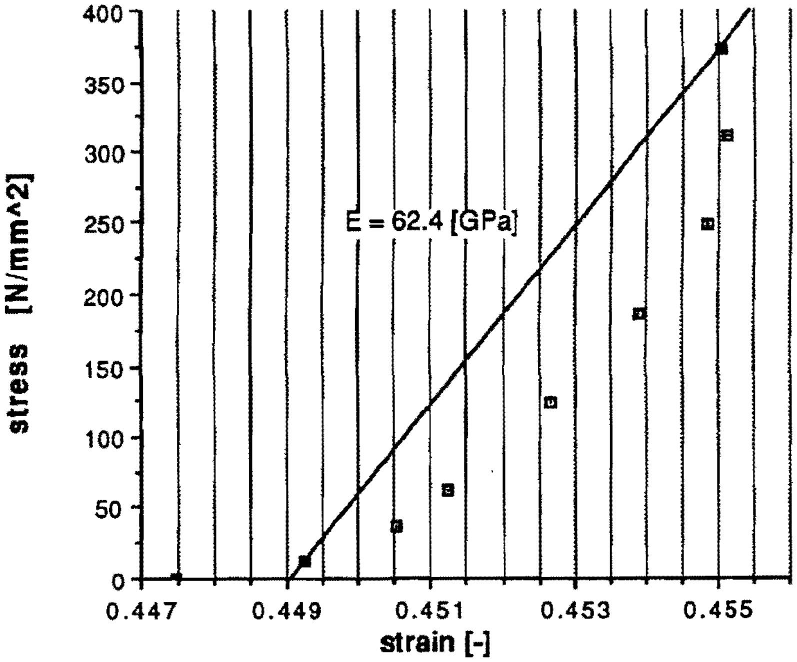


Figure 20

Unloading line of 4000 kg (497.130 MPa).

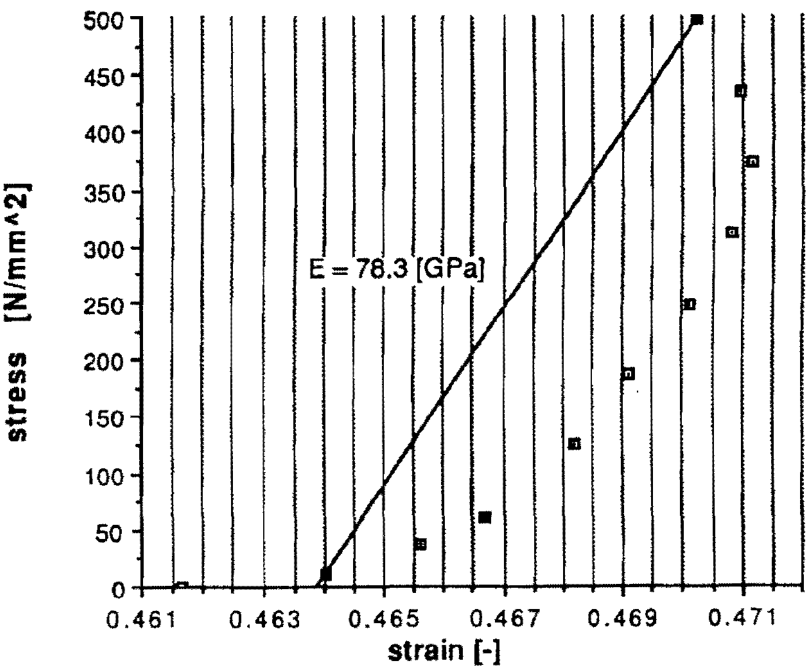


Figure 21

Unloading line of 5000 kg (621.413 MPa).

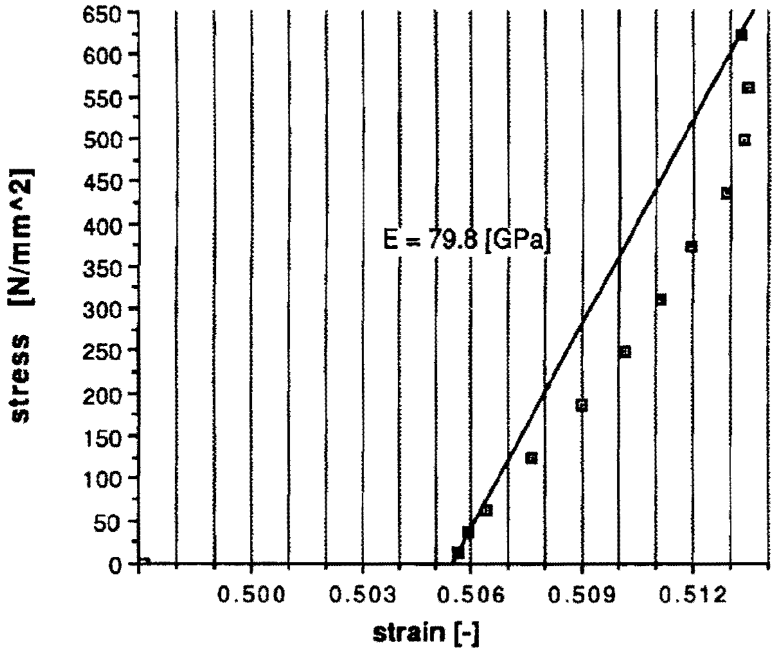


Figure 22

Now we have for 5 different loads (stresses) the matching Young's modulus. We fitted a curve through these points, taken into account the asymptote for E = 83 GPa (see figure 23). This curve can be described with the following equation:

$$\sigma = \frac{6.6078\,E^2 - 636.2437\,E + 6809.5745}{E - 83} \qquad (44)$$

$$12.5 < E < 83$$

The reciproque of this equation is:

$$E = \frac{\sigma + 636.2437}{13.2156} - \sqrt{\frac{\sigma^2 - 921.3022\,\sigma + 224820.8203}{174.6521}} \qquad (45)$$

$$\sigma > 0$$

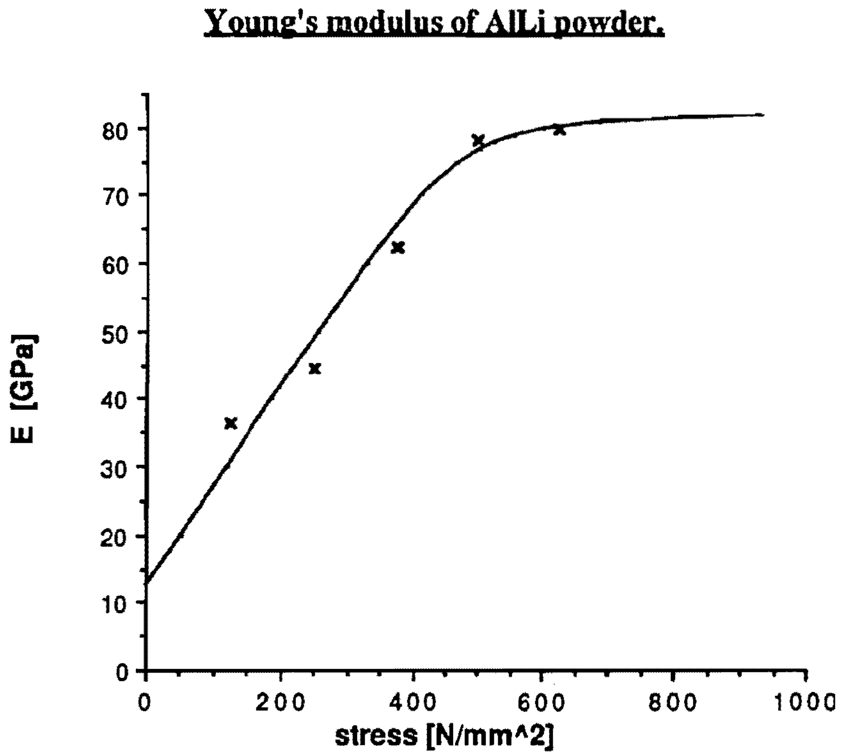


Figure 23

And with the stress - density relation (43) this will become:

$$E = 1080 - 6210\,\rho + 13858\,\rho^2 - 13677\,\rho^3 + 5105\,\rho^4$$

$$- \sqrt{\begin{aligned}&1994443 - 12384556\,\rho + 66198998\,\rho^2 - 199385565\,\rho^3 \\ &+ 372083367\,\rho^4 - 442465076\,\rho^5 + 328537769\,\rho^6 \\ &- 139630672\,\rho^7 + 26056667\,\rho^8\end{aligned}} \qquad 0.54 < \rho < 1 \qquad (46)$$

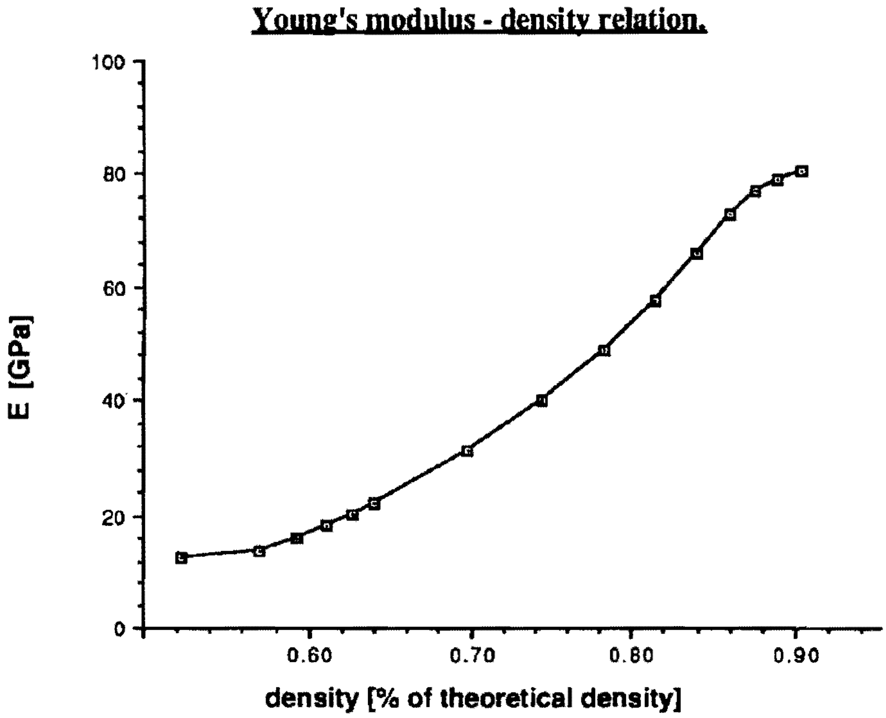


Figure 24

If we look again to the figures 18 to 22, you can notice two deflections from a straight line. One at the top and one at the bottom.
The one at the bottom can be explained as a similarity to pressing onto elastic balls. The biggest displacements will be in the beginning, and when releasing again at the end. In our test we also release some kind of elastic balls, namely the spherical powder particles.
For the second deflection we still don't have a good explanation. It seems that there is still some compaction while lowering the load already. This gave us the idea that there was the possibility of some creep. Therefore we did a test in which we compacted the powder, kept it on the maximum load for 10 minutes, and then unloaded it (see figure 25).Indeed there was some creep, but the same phenomena still appeared. So creep cannot be the reason for this deflection.
Above mentioned made us decide to draw a straight line through the point of end of loading and the point of 12.4 N/mm^2. This line is parallel to the points in the middle section. This is another reason to pick these two points and calculate Young's modulus with that.
This means that the E - ρ relationship is a trend. However a small difference in the magnitude of Young's modulus has only a slight influence upon the simulation program.

"Creep" in the static compaction.

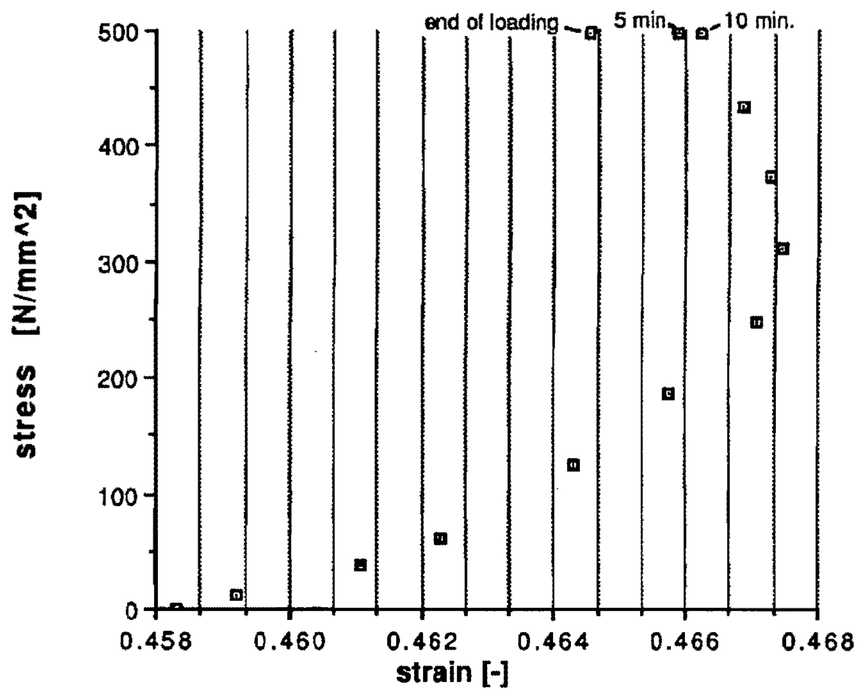


Figure 25

- **More about the constitutive equation.**

If we take a look again at the constitutive equations with the use of a lubricant (41) and (43), and combine these with the equation for plastic-wave-velocity (7), we can build an equation of the plastic-wave-velocity against the stress (see figure 26).
A shock wave will occur when the higher stresses move faster as the lower ones and catch up. If now the derivative of the plastic-wave-velocity against the stress is high, then it is more likely that there will be shock waves, even in a short period of time. So especially the rise of the stress around 25 and 250 N/mm^2 has to be kept low. The shape of the stress wave is very important to avoid shock waves. If you know the shape of the wave, you know the stress growth against time and with figure 26 the speed of each stress level. You know also with which delay the different levels will start to run through the powder. The last information we need to calculate wether there will be shock waves or not, is the distance that the wave has to travel between the two edges of the particle.
It's now possible to predict the appearance of shock waves, if we know the shape of the stress wave and the distance that it has to travel.

Plastic wave-velocity depending on the maximum stress of the wave.

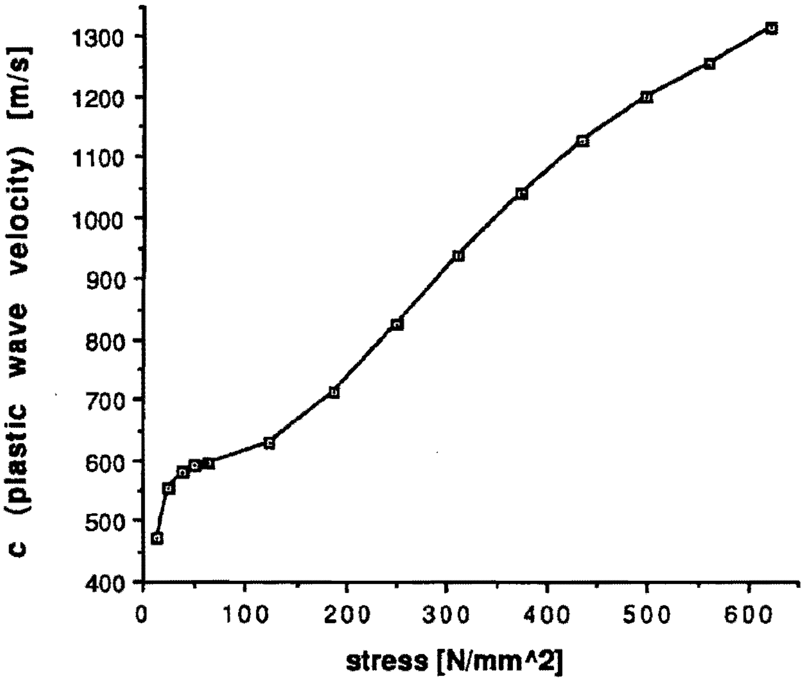


Figure 26

- **Remarks and recommendations.**

First we will give some remarks, followed by some recommendations.

- Because of the elastic area of the SKD-11 it wasn't possible to go beyond a load of 6000 kg.

- Bear in mind that when you are compacting without a lubricant, to keep the friction surface small. Otherwise the ejection force will become a lot bigger than the compaction force.

- We decided to represent the constitutive equation as a polynomial, because it is more suitable for a computer program.

- The density distribution in the specimens is not homogeneous. Due to the friction between the powder and the die wall, is the density in the top slightly higher then at the bottom. There is also a higher density at the sides then in the middle of a cross section. The densities in this paper are the average densities of the specimen [7].

- If after compaction and unloading, the specimen will be loaded again until the same load, you will notice a complementary compaction. The top part, with a higher density, can be seen as a 'solid' punch that compacts the lower part, which has not reached the corresponding density yet.

Therefore the first recommendation is to investigate the density distribution and the influence of complementary compactions on the average relative density. These influences can be put into the simulation program, so that the program will be able to cope with two consecutive stress waves of the same magnitude.

Another recommendation is to look at the widening of the container during compaction. The widening increases the transverse section and by that it decreases the average stress and the relative density. Keep in mind that you can't assume that there is plain stress, because the container is of a significant height. You have to take plain strain as starting-point.

- **Conclusions.**

There has been made a simulation program for the compaction of AlLi- powder, based on the method of characteristics.
We found two constitutive equations of the AlLi-powder. One without the use of a lubricant and one with the use of a lubricant. For the latter we build also a unloading relationship. These can be put into the simulation program to complete it.
The program can be improved by looking at the widening of the container during compaction and the density distribution in the specimens.

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5) Jenkins, I., Wood, J.V., Powder Metallurgy: An Overview, 1991, (p. 130).

6) Metals Handbook, Volume 2, Properties and Selection: Non-ferrous Alloys and Special Purpose Materials, 10th edition, ASM International, October 1990.

7) Metals Handbook, Volume 7, Powder Metallurgy, 9th edition, ASM International, 1984, (p. 286 - 289, p. 296 - 307) and (p. 482 - 483).

8) Annual Book of ASTM Standards 1991, Volume 02.05, Metallic and Inorganic Coatings; Metal Powders, Sintered P/M Structural Parts, B 328 (p. 121 - 122) and B 331 (p. 130 - 131).

- **SYMBOLS.**

A	Area	[m^2]
c	Wave velocity	[m/s]
	Slope of characteristic curve	[m/s]
cl	Lefthand slope	[m/s]
cr	Righthand slope	[m/s]
d	Derivative	[-]
∂	Partial derivative	[-]
E	Young's modulus	[N/m^2]
f	Function	[-]
F	Force	[N]
g	Function	[-]
h	Specimen height	[mm]
h_0	Starting height	[mm]
H_{cs}	Corrected starting height	[mm]
l	Lenght of punches	[mm]
l_0	Starting lenght of punches	[mm]
N	Iteration step	[-]
R	Radius	[m]
t	Time	[s]
Δt	Time-step	[s]
U	Displacement	[m]
V	Particle velocity	[m/s]
X	Position	[m]
δ	Variation	[-]
ε	Strain	[-]
ε_e	Elastic strain	[-]
ε_p	Plastic strain	[-]
μ	Friction factor	[-]
ρ	(relative) Density	[-]
ρ_0	(relative) Starting density	[-]
σ	Stress	[N/m^2]
σ_r	Radial stress	[N/m^2]
Φ	Function for instantaneous plastic response	[N/m^2]
Ψ	Function for non-instantaneous plastic response	[N/m^2 s]