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Two replenishment strategies for the lost sales inventory model: a comparison

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Two replenishment strategies for the lost sales inventory model: a comparison.

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Abstract

For the lost sales inventory sytem we distinguish two different replenishment strategies.
The simplest strategy is the classical 'fixed reorder level' replenishment rule: every period the inventory position in the system is raised up to a fixed quantity S. For this simple strategy we derive and test heuristics for the determination of the reorder level, given a target service level.

Next we demonstrate that it may be more efficient in a lost sales system to use a different replenishment strategy. This alternative replenishment strategy however is more complex.
Therefore a comparison is made between the two replenishment strategies in a lost sales environment, to get an indication of the price which has to be paid here for simplicity.

Section 1 Introduction.

In an earlier paper [5] we investigated a periodic review lost sales inventory system with a lead time equal to ℓ periods, $\ell \geq 0$. It was found that especially for low service levels the behaviour of a lost sales system differs considerably from the behaviour of a backorder system. For the lost sales system we introduced a new replenishment strategy, the so-called 'dynamic' replenishment strategy. The dynamic replenishment strategy works as follows: every period the replenishment quantity is determined such that the service level of the system after $\ell+1$ periods is equal to the target service level. Although in a backorder environment such a strategy will lead to an inventory position (i.e. the inventory on hand plus on order) after reordering, which is constant in every period, in a lost sales environment this strategy will lead to an inventory position after reordering, which varies every period.

In this paper we will first consider the classical 'order-up-to a fixed reorder level' strategy. This 'fixed reorder level' strategy is well-known from the backorder inventory models in the literature.
The article of Karlin [1] shows that an exact analysis of the lost sales system with a fixed-reorder-level replenishment strategy is hard. Therefore we will look for a heuristic which will enable us to find the appropriate reorder level, which yields a pre-specified service level. In Section 2 two heuristics will be proposed for the determination of this reorder level in a lost sales environment. In Section 3 we briefly describe the best heuristic known for the dynamic replenishment strategy. In [5] we have compared this heuristic with the heuristic which is introduced by Morton [2] and Nahmias [3]. In Section 4 we will compare the best heuristics for the 'fixed reorder level' and the 'dynamic' replenishment strategy. Conclusions are drawn in Section 5.

We will assume that demand is Erlang-η distributed with scale parameter λ and we will use the following notations:

I_t	inventory on hand at the start of period t, before an order arrives
ℓ	leadtime in periods
$Q_{t-\ell}$	quantity ordered at start of period $t-\ell$ (which, by definition, will arrive at the start of period t)
ξ_t	demand during period t
μ	average demand per period, equal to η/λ
α	desired service level, defined as the probability that in a period demand is smaller than or equal to available inventory at the start of the period.
$S_{BO}(\alpha, \lambda, \eta, \ell)$	the reorder-level in a backorder system with target service level α, Erlang-η distributed demand with scaling parameter λ and leadtime ℓ.
$S_{LS}(\alpha, \lambda, \eta, \ell)$	the reorder-level in a lost sales system.
$\beta_{BO}(S, \lambda, \eta, \ell)$	the fraction of demand filled from stock in a backorder system with reorder level S, Erlang-η distributed demand with scaling parameter λ and leadtime ℓ.
$\beta_{LS}(S, \lambda, \eta, \ell)$	the fraction of demand filled from stock in a lost sales system.

For ease of notation we will leave out some of the indices which determine S or β, but only when there is no danger of misinterpretation.

In the reorder cycle of our periodic review inventory model the variables are measured in the following sequence:

1. starting inventory on hand (equal to the inventory on hand at the end of period $t-1$) is I_t
2. the reordered quantity $Q_{t-\ell}$ is received, so the inventory on hand for satisfying demand during period t equals $I_t + Q_{t-\ell}$
3. the reorder quantity Q_t is determined
4. the demand during the period (ξ_t) is met as long as inventory is available; demand which cannot be satisfied is lost.

Section 2 Heuristics for the 'fixed reorder level' replenishment strategy.

In this Section we assume that a replenishment in the lost sales inventory system is determined in the classical way: If the inventory position in period t is below a fixed reorder level S_{LS}, then a quantity Q_t is replenished, with Q_t equal to S_{LS} minus the inventory position. As a result the inventory position after replenishment is equal to S_{LS}. The system is reviewed periodically. The leadtime of the system is ℓ periods. Demand is assumed to be Erlang-η distributed with scale parameter λ. So the probability density function of the demand per period is equal to f(ξ), with:

$$f(\xi) = \frac{\lambda^{\eta}\xi^{\eta-1}e^{-\lambda\xi}}{(\eta-1)!} \tag{2.1}$$

In Section 2.1 a heuristic will be derived for the determination of S_{LS}. This heuristic can be improved further by means of an extension, which makes the heuristic slightly more complicated. This extension is described in Section 2.2.

2.1 A heuristic for the determination of S_{LS}.

In order to find a heuristic for the determination of S_{LS} in the fixed-reorder-level replenishment strategy, the following observations are used:

1. With a dynamic replenishment strategy, the order quantity in period t is determined by the following equation (see [5]):

$$Pr\{\ \xi_{t+\ell}>Q_t\ \cap\ \xi_{t+\ell}+\xi_{t+\ell-1}>Q_t+Q_{t-1}\ \cap\ \cdots \\ \cap\ \sum_{i=0}^{\ell-1}\xi_{t+\ell-i} > \sum_{i=0}^{\ell-1}Q_{t-i}\ \cap\ \sum_{i=0}^{\ell}\xi_{t+\ell-i} > I_t+\sum_{i=0}^{\ell}Q_{t-i}\ \} = 1-\alpha \tag{2.2}$$

which in case of Erlang distributed demand can be written as:

$$1-\alpha = \sum_{i_1=0}^{\eta-1}\frac{(\lambda Q_t)^{i_1}}{i_1!}\sum_{i_2=0}^{2\eta-1-i_1}\frac{(\lambda Q_{t-1})^{i_2}}{i_2!}\ \cdots \\ \cdots\sum_{i_\ell=0}^{\ell\eta-1-\sum_{j=1}^{\ell-1}i_j}\frac{(\lambda Q_{t-\ell+1})^{i_\ell}}{i_\ell!}\sum_{i_{\ell+1}=0}^{(\ell+1)\eta-1-\sum_{j=1}^{\ell}i_j}\frac{\lambda^{i_{\ell+1}}(I_t+Q_{t-\ell})^{i_{\ell+1}}}{i_{\ell+1}!}\,e^{-\lambda\left(I_t+\sum_{i=0}^{\ell}Q_{t-i}\right)} \tag{2.3}$$

2. Several simulations with the dynamic replenishment strategy (see [6]) indicate:
If two systems
a. have an identical service level α,
b. have an identical Erlang factor η and
c. both use equation (2.3) to determine Q_t, then these two systems have the same beta service level (even when their leadtimes are different).

3. It is known, that for a leadtime equal to zero, the performance of the lost sales system is equivalent to the performance of the backorder system.

4. For systems with large leadtimes and small coefficients of variation (that is: large η) the number of calculations needed to determine Q_t from equation (2.3) becomes very large[1].

5. In the long run, the average quantity ordered in a lost sales system is equal to the beta service level[2] times average demand per period ('what goes out has to come in'):

$$E[Q_t] = \beta_{LS}\, E[\xi_t]$$

In our heuristic we assume that demand is Erlang distributed. In order to find an approximation for the fixed reorder level S_{LS} we use formula (2.3), despite the fact that formula (2.3) is derived for the dynamic replenishment strategy. To be more precise: we assume, based on observation 5, that the fixed reorder level S_{LS} can be derived from solving equation (2.3) after substituting

$$Q_t = \beta_{LS}(\ell)\, E[\xi_t] \quad \text{for all } t \tag{2.4}$$

and
$$I_t + \sum_{i=0}^{\ell} Q_{t-i} = S_{LS} \tag{2.5}$$

Next, based on observation 2, we approximate $\beta_{LS}(\ell)$ by $\beta_{LS}(\ell{=}0)$.
From observation 3 we know:

$$\beta_{LS}(\ell{=}0) = \beta_{BO}(\ell{=}0).$$

So we approximate $\beta_{LS}(\ell)$ as follows:

$$\beta_{LS}(\ell) = \beta_{BO}(\ell{=}0) \quad \text{for all } \ell \geq 0 \tag{2.6}$$

From here on $\beta_{BO}(\ell{=}0)$ will be simply abbreviated by β_{BO}.[3] [4]

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1 The exact number of calculations follows directly from equation (2.3)

2 Here, and elsewhere in this Section, the beta service level is defined as the fraction of demand delivered from stock for a system in which the reorder level is based on a service level α. So indirectly β is a function of α.

3 For the sake of clarity we note here that in the equations above we used $\beta_{BO}(\ell{=}0)$ resp. $\beta_{LS}(\ell)$ as an abbreviation for $\beta_{BO}(\, S_{BO}(\alpha,\lambda,\eta,\ell{=}0),\lambda,\eta,\ell{=}0)$ resp. $\beta_{LS}(\, S_{LS}(\alpha,\lambda,\eta,\ell),\lambda,\eta,\ell)$ for any $\ell \geq 0$.

4 Note that equation (2.6) also implies a heuristic for the determination of the reorder-level S which corresponds with a target service-level β^*. Starting with $\alpha_0 = \beta^*$, we determine $S_{LS}(\alpha_0)$. Next we determine the corresponding β_0 using (2.6). If $\beta_0 > \beta^*$ we choose an $\alpha_1 < \alpha_0$, we determine $S_{LS}(\alpha_1)$ and β_1 and so on until the β_i is close enough to β^*. The corresponding $S_{LS}(\alpha_i)$ is the reorder level we looked for.

For $\eta=1$, combining equations (2.3) upto (2.6) gives the following equation:

$$1-\alpha=[(1-\beta_{BO})\sum_{i=0}^{\ell-1}\frac{(\lambda\hat{S}_{LS})^{i}}{i!}+\frac{(\lambda\hat{S}_{LS})^{\ell}}{\ell!}]e^{-\lambda\hat{S}_{LS}} \tag{2.7}$$

$\hat{S}_{LS}$ can be solved from this equation by using an iterative bisection search procedure. (We use the notation $\hat{S}$ here to indicate that we have applied an heuristic for the determination of S_{1}).
The above procedure for the determination of $\hat{S}_{LS}$ is simple and quick for $\eta=1$.

For larger η another approximation is needed. This follows from observation 4. Given the fact that equation (2.3) is relatively easy to solve for a system with η equal to 1 as well as for a system with a leadtime equal to 1 period, it seems natural to use these results for the more general system with any Erlang factor η and any leadtime ℓ. One way to do this is to use the following approximation:

$$\frac{RatioS(\eta,\ell)}{RatioS(\eta,1)}=\frac{RatioS(1,\ell)}{RatioS(1,1)} \tag{2.8}$$

with

$$RatioS(\eta,\ell)=\frac{[S_{BO}(\eta,\ell)-\hat{S}_{LS}(\eta,\ell)]}{\hat{S}_{LS}(\eta,\ell)} \tag{2.9}$$

RatioS(1,1) and RatioS(1,ℓ) can be solved easily using the traditional backorder formula (to find S_{BO}) together with equation (2.7) (to find $\hat{S}_{LS}$). RatioS(η,1) can be solved using the backorder formula and equation (2.10), which follows from combining equations (2.3) upto (2.6).

$$1-\alpha=\sum_{i=0}^{\eta-1}\frac{(\lambda\beta_{BO}\mu)^{i}}{i!}\sum_{j=0}^{2\eta-1-i}\frac{(\lambda\hat{S}_{LS}-\lambda\beta_{BO}\mu)^{j}}{j!}e^{-\lambda\hat{S}_{LS}} \tag{2.10}$$

Once RatioS(η,ℓ) is determined from equation (2.8), $\hat{S}_{LS}(\eta,\ell)$ can be calculated straightforward from equation (2.9).

All combinations of values for the Erlang factor and the leadtime, which are used in formula (2.8), constitute a rectangle (see Figure 1). Therefore the heuristic above is called Recta and the resulting S-level is called S-Recta.

$$\begin{array}{lccc} \eta & x. & \cdots\cdots & .x \\ & \cdot & & \cdot \\ & \cdot & & \cdot \\ \eta=1 & x. & \cdots\cdots & .x \\ & \ell=1 & & \ell \end{array}$$

Figure 1. *The various combinations of values for the Erlang factor* η *and the leadtime* ℓ, *which are used in Recta.*

In order to test this heuristic, the S-level which corresponds with the target service level α has to be found first. To achieve this an iterative simulation search procedure is used, where in the final iteration step the lost sales system was simulated 500,000 periods. Determining the S-level in this way has the advantage that the quality of different heuristics can be evaluated quickly (without additional simulations). Table 1 gives a subset of the S-levels which were determined in this way, together with the 95%-confidence interval on the service level α. Also the S-levels which resulted from the heuristic Recta are reported there[5].

target service level	lead time	Erlang factor	95%-confidence interval on α (sim.)	β (sim.)	S-level (sim.)	S-Recta
75%	1	1	+/- 0.14%	75.0	936.4	934.8
75%	1	16	+/- 0.13%	94.8	862.0	872.9
75%	16	1	+/- 0.18%	74.9	5720.2	5715.7
75%	16	16	+/- 0.24%	93.6	6552.0	6745.4

Table 1. A subset of the simulation results.

The heuristic has been tested for each combination of the following parameter set:
α = 75%, 85% and 95%,
Leadtime = 1, 2, 4 and 16 periods,
Erlang factor = 1, 2, 4 and 16.

The average (absolute) relative error in the estimation of S[6] over these 48 parameter settings is equal to 1.5%. The maximum relative error is equal to 3.8%. This maximum was achieved in case α=75%, Leadtime=16 and Erlang factor=4. Table 2 shows the average relative error per parameter. It shows that the quality of the heuristic decreases if the leadtime increases and/or if alpha decreases.

α			Erlang factor (η)				Leadtime (ℓ)			
0.75	0.85	0.95	1	2	4	16	1	2	4	16
1.82	1.53	1.00	0.20	1.73	2.18	1.69	0.84	1.25	1.62	2.09

Table 2. The average relative error of $\hat{S}_{LS}$ per parameter, using Recta.

5 In the four scenario's of Table 1 the average demand per period was kept constant (μ = 400). So Table 1 shows that in a lost sales environment the S-level may increase if the coefficient of variation ($=\frac{1}{\sqrt{\eta}}$) decreases. This is due to an increase of the beta service level.

6 This is equal to $\frac{|\hat{S}_{LS} - S_{sim}|}{S_{sim}}$ where S_{sim} is the S-level, which corresponds with the target service level and which was found by means of the iterative search procedure using simulation.

2.2 A possible extension of Recta.

A good way to improve the heuristic is to improve the estimator for $S(\eta,\ell=1)$. To show this potential, we created another estimator for $S(\eta,\ell)$. Again we used formula (2.9), but now we used the S-levels for $\ell=1$ and/or $\eta=1$ which were found in the simulations, as estimators for $S_{LS}(\eta, \ell=1)$ and $S_{LS}(\eta=1, \ell)$. We then used formulas (2.7) and (2.8) again to determine $\hat{S}_{LS}(\eta,\ell)$. This resulted in an average and maximum error of 0.1% and 0.6% (measured of course only over the 27 simulations in which neither the Erlang factor nor the leadtime was equal to 1). This shows that the concept of Recta, that is: estimating the reorder level for a system with any value for η and ℓ by relating it to the reorder levels of the corresponding system with η and/or ℓ equal to one, is very good.

In general of course the simulation results for $S(\eta,\ell)$ with η or ℓ equal to one are unknown. Since we know the concept of Recta is very good, we would like to improve the determination of S-Recta by finding a better estimator for $S_{LS}(\eta,\ell)$ with η or ℓ equal to one. Table 2 shows that formula (2.7) already is a good estimator for $S_{LS}(\eta=1,\ell)$. So the major challenge is to find a better estimator for $S_{LS}(\eta,\ell=1)$.

In order to find a better estimator for $S_{LS}(\eta,\ell=1)$, the following observations are made:

1. It is known, that for a leadtime equal to zero, the performance of the lost sales system is equivalent to the performance of the backorder system.

2. From equation (2.2) it is clear that in general the following equation holds:
$S_{LS} \leq S_{BO}$

3. In a limited number of simulations it was observed that, if the leadtime got large, the fixed reorder level S which corresponds with a target service level α can be approximated in a lost sales environment by the S-level from the corresponding backorder system multiplied by the beta service level:

$$\hat{S}_{LS} \approx \beta_{LS}\, S_{BO} \quad \text{if } \ell \to \infty$$

Based on these observations we assume:

$$S_{BO}(\eta,\ell=1) \geq S_{LS}(\eta,\ell=1) \geq \beta_{LS}\, S_{BO}(\eta,\ell=1)$$

Hence it is plausible to write $S_{LS}(\eta,\ell=1)$ as an interpolation between these two boundaries:

$$S_{LS}(\eta,\ell=1) = ip\, S_{BO}(\eta,\ell=1) + (1-ip)\, \beta_{LS}(\eta)\, S_{BO}(\eta,\ell=1), \quad 0 \leq ip \leq 1 \tag{2.11}$$

Next we assume that the interpolation factor ip is independent of η.
Then the easiest way to determine ip is to solve equation (2.11), for the case with $\eta=1$, after substituting $S_{LS}(\eta,\ell=1)$ by $\hat{S}_{LS}(\eta=1,\ell=1)$ (which can be solved from formula (2.7)) and substituting $\beta_{LS}(\eta=1)$ by α (since from the simulations we observed that α and β are equal in case $\eta=1$). So we have:

$$ip = \left[\frac{\hat{S}_{LS}(\eta=1,\ell=1)}{S_{BO}(\eta=1,\ell=1)} - \alpha\right] / \,[1-\alpha] \tag{2.12}$$

Now $\hat{S}_{LS}(\eta, \ell=1)$ can be solved from combining equations (2.11) and (2.12).

The heuristic described here is an extension of Recta, using interpolation to find a better value for $\hat{S}_{LS}(\eta, \ell=1)$. Therefore this heuristic is called Recta-Interpol. Based on the logic of Recta-Interpol new S-levels are calulated for the 48 parameter settings mentioned in Section 2.1. The results are summarised in Table 3. The average resp. maximum relative error in the estimation of S_{LS} using Recta-Interpol is 1.0% resp. 2.4%.

α			Erlang factor (η)				Leadtime (ℓ)			
0.75	0.85	0.95	1	2	4	16	1	2	4	16
1.18	1.08	0.68	0.20	1.11	1.45	1.16	0.56	0.84	1.10	1.42

Table 3. The average relative error of $\hat{S}_{LS}$ per parameter, using Recta-Interpol.

Recta-Interpol has the disadvantage, that it is slightly more complex than Recta. On the other hand, Recta-Interpol clearly outperforms Recta[7].

[7] Yet, it should also be mentioned that the simulation results showed that observation 3 does not hold for all systems. For the system with $\eta=1$, $\ell=16$ and $\alpha=95\%$ for example the beta service level is equal to 95%, so $\beta_{LS}*S_{BO}$ is equal to 9235, whereas S_{LS} is approximately equal to 8632; a difference of 7%. Note that -despite errors like these- the heuristic for estimation of the reorder level in a lost sales system showed a maximum relative error of only 2.4%.

Section 3 Heuristics for the 'dynamic' replenishment strategies

In the previous Section we focussed on a fixed reorder level replenishment strategy. The advantage of such a replenishment strategy is its simplicity. This simplicity however might lead to loss of performance. That's why we would like to compare the performance of such a simple strategy with a more advanced replenishment strategy. In the next Section this comparison is made between the fixed reorder level and the 'dynamic' replenishment strategy.

The exact dynamic replenishment strategy, that is: solving each period Q_t from equation (2.2), requires a lot of CPU-time for systems with a large Erlang factor and a large leadtime. Therefore in [5] heuristics for the dynamic replenishment strategy are introduced and compared with a heuristic, which is proposed by Morton [2] and Nahmias [3]. It is concluded there that for the determination of the replenishment quantity Q_t it is best to use the '2-moments' heuristic as long as the target service level is below 90% and to use the '2-terms' heuristic otherwise. These two heuristics will be described here briefly.

<u>The '2-moments' heuristic</u>

This heuristic is based on the characteristic lost sales equation, which also led to (2.2). This equation states that in a periodic review lost sales system the inventory in period t will be equal to:

$$I_{t+1} = (I_t + Q_{t-\ell} - \xi_t)^+ \qquad (3.1),$$

where $x^+ = \max(0,x)$.

Our objective is to determine Q_t from

$$\alpha = Pr(I_{t+\ell} + Q_t - \xi_{t+\ell} > 0) \qquad (3.2)$$

We proceed as follows. First of all we assume that I_{t+i}, $i \geq 1$, and ξ_t are distributed according to a mixture of two Erlang distributions. We note here that a mixture of two Erlang distributions can be fitted to any pair of the first two moments of a random variable (see [7]). Starting from the given values of I_t and $Q_{t-\ell}$ we can compute the first two moments of I_{t+1} from (3.1). Next we compute the first two moments of I_{t+2} from (3.1). We note that the computations involved are elementary. Thus we continue until we have found approximations for the first two moments of $I_{t+\ell}$. Then we compute Q_t from (3.2) using a bisection scheme.

<u>The '2-terms' heuristic</u>

This heuristic is derived directly from (2.2) by only considering the first and last term and neglecting all other terms in the probability in formula (2.2). This yields the following equation, from which Q_t can be solved:

$$Pr\{ \xi_{t+\ell} > Q_t \;\cap\; \sum_{i=0}^{\ell} \xi_{t+\ell-i} > I_t + \sum_{i=0}^{\ell} Q_{t-i} \} = 1-\alpha \qquad (3.3)$$

Section 4 Comparison of the fixed reorder level and the dynamic replenishment strategies.

To compare the replenishment strategies we first simulated a lost sales sytem with the following parameters:

Service level = α = 70%,
Erlang Factor = η = 1,
Leadtime = ℓ = 4,
Average demand per period = μ = 400 (corresponding with λ=0.0025).

For these parameters we compared the fixed reorder level with the dynamic replenishment strategy. For the latter we considered both the exact replenishment strategy and the '2-moments' heuristic. For the determination of the fixed-reorder level we used the Recta-Interpol heuristic. During the simulations we measured a number of variables: the alpha and beta service level, the average and standard deviation of the replenishment quantity Q_t, the average inventory on hand (equal to the sum of the inventory on hand before demand and the inventory on hand after demand, divided by two) and the average inventory position. The simulation results are based on 10,000 periods. The results are reported in Table 4. The variable which shows clearly the difference in the strategies is the standard deviation of the order quantity Q_t.

	Fixed Heuristic	Dynamic Heuristic	Exact
α	70.05	69.85	70.04
β	69.38	69.06	69.32
average Q_t	281	279	280
standard deviation Q_t	249	52	51
average inventory on hand	495	428	433
average inventory position	1757	1686	1695

Table 4. The simulation results with target service level α=70%, η=1 and ℓ=4.

Apparently with the fixed-reorder level replenishment strategy the order quantity Q_t has a (relatively) very large standard deviation. This is not surprising if we note that the 'dynamic' heuristic limits its order quantity as soon as a large demand occurs. This behaviour can be seen if we plot both the demand in period $t-1$ and the quantity ordered in period t, which were observed in the simulation during 18 periods[8].
This is done for Recta-Interpol (denoted here by 'fixed-s') and the '2-moments' heuristic. See Figure 2. The order quantity based on Recta-Interpol in period t is equal to the demand in period $t-1$ in all periods, except in periods 16 and 17 due to lost sales in these two periods. Clearly the ordering pattern with the 'dynamic' heuristic is much smoother than with the 'fixed-s' heuristic.

[8] This time period is selected randomly.

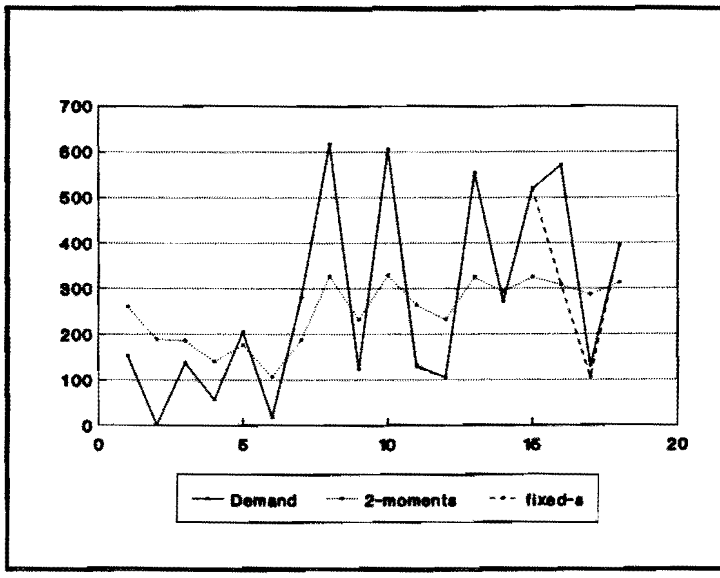

Figure 2. *The ordering behaviour of a 'fixed reorder level' versus a 'dynamic' replenishment strategy.*

The only other difference we noted between the two basic strategies is the fact that when using the fixed reorder level the average inventory on hand is higher than with the dynamic strategies. In this case the difference was approximately 15%. The relative difference in the inventory position (equal to the inventory on hand plus on order) is smaller; in this case it was approximately 4%.

To investigate whether this difference in inventory is incidental or structural, we set up another experiment. We simulated 112 systems with the following parameters:
Service level = 50%, 60%, 70%, 80%, 90%, 95% and 99%,
Erlang factor = 1, 2, 3 and 4,
Leadtime = 1, 2, 3 and 4.
Each of these systems was simulated for 100,000 periods. We measured the service level and the average inventory on hand, using a heuristic for the fixed reorder level resp. the dynamic replenishment strategy. Table 5 shows the mean (absolute) deviation from the target service level, measured over 16 systems (with η=1,..,4 and ℓ=1,..,4) as well as the maximum deviation over these 16 systems. For the sake of reference we also show the results obtained with the well-known heuristic of Morton/Nahmias.

Morton/Nahmias	50%	60%	70%	80%	90%	95%	99%
Mean Deviation	12.77	12.03	8.96	5.93	2.81	1.27	0.19
Maen Absolute Deviation	12.77	12.03	8.96	5.93	2.81	1.27	0.19
Maximum Deviation (in %)	16.84	15.32	12.13	8.41	4.02	1.85	0.30

2-moments	50%	60%	70%	80%	90%	95%	99%
Mean Deviation	0.16	0.09	0.25	-0.75	-1.31	-1.45	-0.89
Mean Absolute Deviation	0.22	0.17	0.35	0.75	1.31	1.45	0.89
Maximum Deviation (in %)	0.77	0.62	-0.74	-1.17	-1.77	-1.79	-1.29

Recta-Interpol	50%	60%	70%	80%	90%	95%	99%
Mean Deviation	0.10	0.65	0.89	0.77	0.49	0.22	0.03
Mean Absolute Deviation	0.26	0.66	0.90	0.81	0.50	0.23	0.04
Maximum Deviation (in %)	0.68	1.59	1.97	1.79	1.38	0.50	0.09

Dynamic replenishment heuristic	50%	60%	70%	80%	90%	95%	99%
Mean Deviation	0.16	0.09	0.25	-0.75	0.75	0.30	0.03
Mean Absolute Deviation	0.22	0.17	0.35	0.75	0.77	0.30	0.04
Maximum Deviation (in %)	0.77	0.62	-0.74	-1.17	1.76	0.62	0.13

Table 5. The deviation from the target service level for the heuristic of Morton/Nahmias Recta-Interpol and the dynamic replenishment heuristic.

It shows that both replenishment strategies are quite capable of achieving the target service level. From the simulations we also derived Figures 3 and 4, which show the inventory on hand, which is needed to obtain a service level α for a system with ($\eta=1,\ell=1$) resp. ($\eta=4,\ell=4$). It shows that in both situations the fixed reorder level replenishment strategy needs more inventory on hand than the dynamic replenishment strategy. The exact difference depends on the target service level, but in most situations it is within 10%. We found similar results for systems with other parameter settings.
The largest difference was found for the system with $\alpha=60\%$, $\eta=1$ and $\ell=4$. There the difference was equal to 17%.

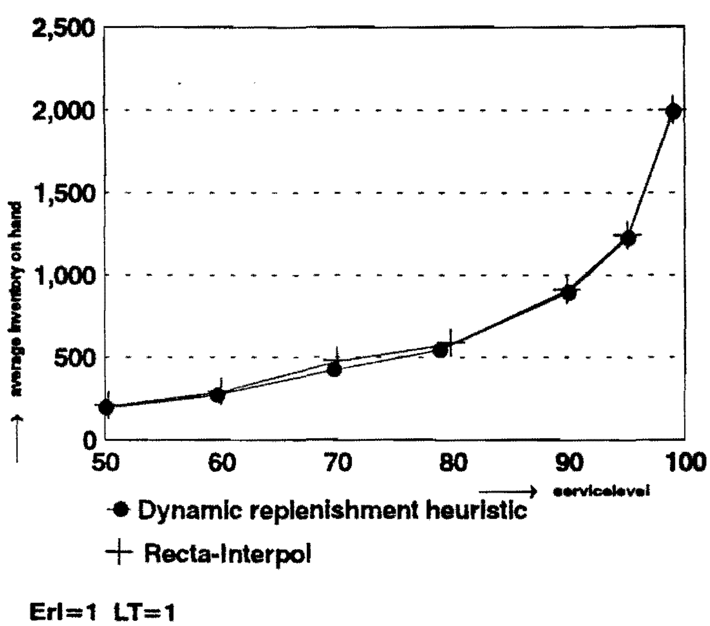

Figure 3. The inventory on hand for Recta-Interpol resp. the dynamic replenishment heuristic in case $\eta=1$ and $\ell=1$.

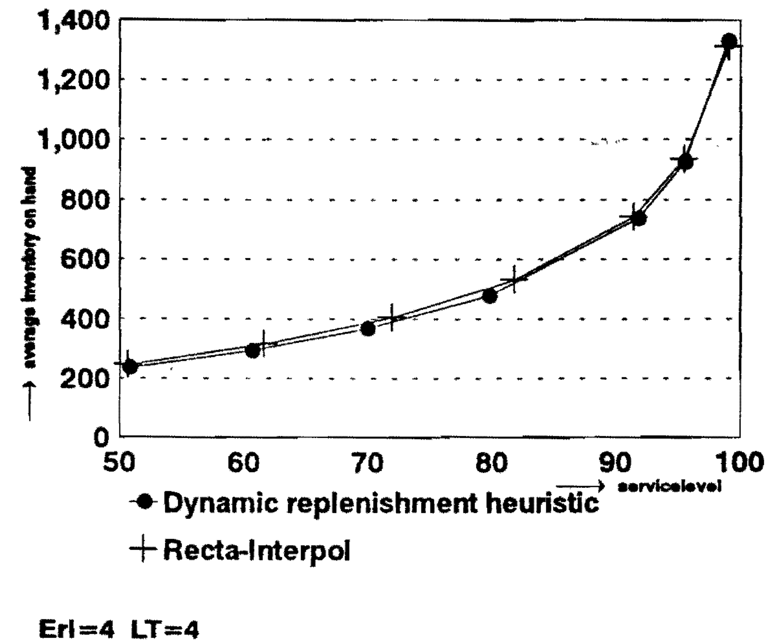

Figure 4. *The inventory on hand for Recta-Interpol resp. the dynamic replenishment heuristic in case* $\eta=4$ *and* $\ell=4$.

Section 5 Summary

In this paper two replenishment strategies for a lost sales environment have been studied: the 'fixed reorder level' replenishment strategy and the 'dynamic' replenishment strategy.

The fixed reorder level replenishment strategy is the simplest strategy. In Section 2 two heuristics are presented for this strategy, which enable a planner to determine the reorder level rather accurately: the average error of the best heuristic is 1.0%.

This fixed reorder level replenishment strategy has been compared with a more sophisticated replenishment strategy: the dynamic replenishment strategy. This replenishment strategy aims for the same service level every period, which in a lost sales environment leads to an inventory position which is not constant over time.

Comparing the fixed reorder level and the dynamic replenishment strategy, it appears that with the dynamic replenishment strategy:

* the ordering pattern is relatively smooth. This is especially beneficial to the supplier and it may lead e.g. to a reduction in price or increased supply-reliability.
* less inventory is needed to obtain a given service level.

We would like to stress here that, although the dynamic replenishment strategy is computationally more complex, this strategy is straightforward to implement within concepts like DRP and MRP, since all data required are available.

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