Operational amplifiers

Eindhoven International Institute

Course In Electronic Engineering

Survey Nr 272/4	Course: : **INTRODUCTORY SEMESTER 1992** **- Electronic Engineering -**
Subject	: OPERATIONAL AMPLIFIERS
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OPERATIONAL AMPLIFIERS.

1.0. Introduction.

The operational amplifier is a linear direct-coupled high gain amplifier which also has provisions for external feedback.

Through the external feedback circuitry the response of the amplifier can be controlled, virtually independent of the internal parameters.

Therefore the internal circuitry can usually be ignored. The operational amplifiers are primarily used to accomplish amplification, addition, subtraction, differentation, integration and other mathematical operations, hence the name operational amplifier.

In order to perform these operations the operational amplifiers are used with linear or non-linear external components in the feedback configuration.

Using non-linear external components it is possible to perform non-linear operations like:
logarithm and anti-logarithm conversion, absolute value determination, peak detection, comparators etc.

Figure 1 shows the representation of a conventional operational amplifiers.

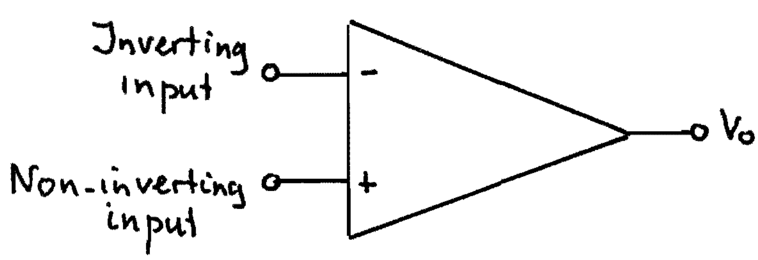


figure 1.1.

Input terminals labelled (-) and (+) are usually called inverting and non-inverting terminals resp.
They are also called differential input terminals because output voltage V_o depends on the difference in voltage between them, that is

$$V_o = A_o\ (V_2 - V_1)$$

where A_o is the open-loop voltage gain of the op. amp.
In some case the non-inverting input is not externally available. This is a single-ended input output op. amp. (figure 1.2a).

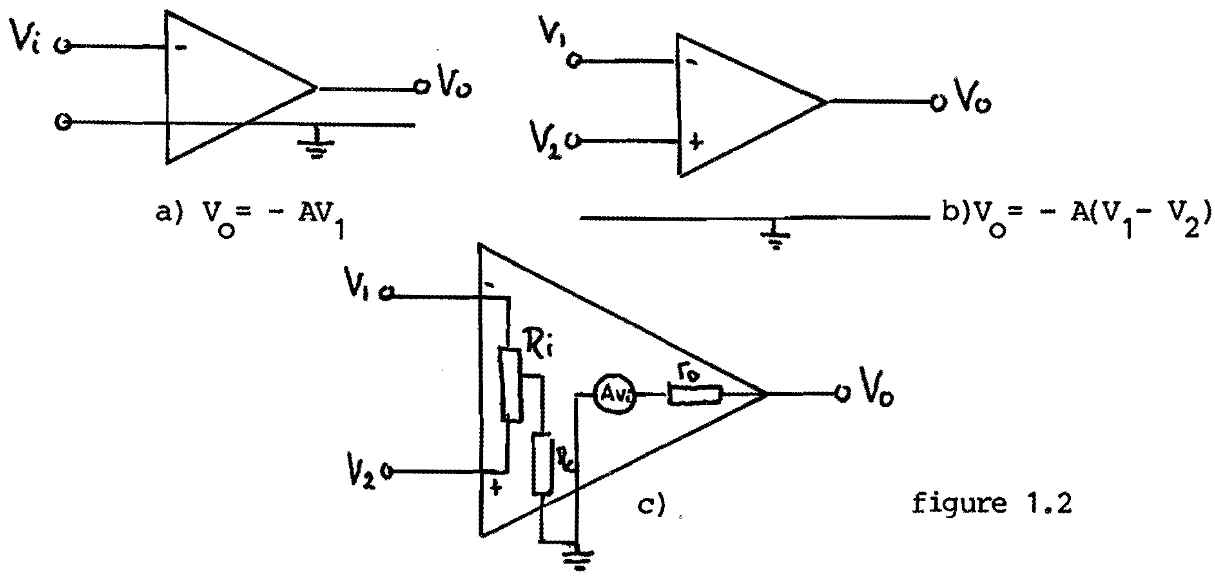


figure 1.2

The double-ended input, single-ended output op. amp. is far more versatile than the former (figure 1.2b).

The equivalent-circuit model of an op. amp. consists of an input resistance R_i connected between the input terminals, a common mode resistance Rc between these terminals and ground. The output circuit consists of a controlled source A_oV_i in series with an output resistance r_o as shown in figure 1.2c.
The open-loop voltage gain A_o of the op. amp. is usually very large, $A_o > 10^5$. The input resistance is much larger than 100 kOhm. The output resistance is about 100 Ohm or less and it may be neglected for many applications.
All these quantities are defined at d.c. operations.

1.1. Ideal operational amplifiers.

The ideal operational amplifier has the following characteristics:

1. The open-loop voltage gain is infinite $A_o = \infty$
2. The input resistance and the common mode input resistance are infinite; $R_i = \infty$, $R_c = \infty$
3. The output resistance is zero: $r_o = 0$
4. The bandwidth is infinite: $BW = \infty$
5. The input and output offset voltage and the input bias currents are zero:

$$V_o = 0 \text{ if } V_i = V_2 - V_1 = 0.$$

6. Insensitivity of the op. amp. to temperature and power supply variations.

These ideal parameters have not been achieved but many of these parameters are sufficiently close to the ideal that one can neglect the difference in many practical applications.

For example, input bias currents are in the range of 5pA for FET input amplifiers, while input resistances are larger than 10^{12} Ohm. Offset voltages are less than 1mV in many cases.

For an ideal op. amp. the open-loop gain A is infinite. This means that with the conditions $V_{min} < V_o < V_{max}$ and the op. amp. working in the linear region the difference input voltage is:

$$V_i = V_2 - V_1 = 0 \quad \text{or } V_2 = V_1$$

Also can be said that when negative feedback is used, the output voltage V_o goes to the voltage which nulls the input voltage $V_2 - V_2$ of the op. amp.

1.1.1. Basic operational amplifier configurations. ($A \neq \infty$)

The static transfer characteristic of an op. amp. is shown in figure 1.3.

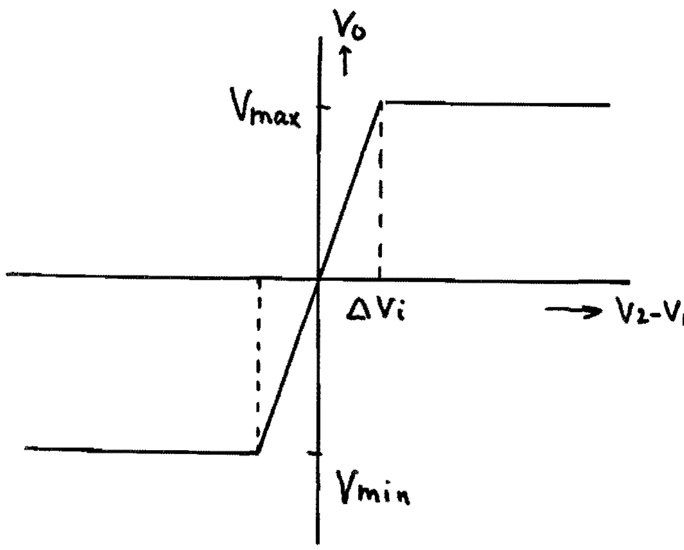


figure 1.3

If the input signal $V_2 - V_1 < \Delta V_i$ the op. amp. is working in the active linear region.

The gain A in this region is given by

$$A = \frac{V_{max}}{\Delta V_i} = \frac{V_{min}}{\Delta V_i}$$

in practice $\Delta V_i < 10^{-4}$ Volt.

Therefore it is only possible to operate the op.amp. in the active linear region if one applies negative feedback.

a) General feedback configuration

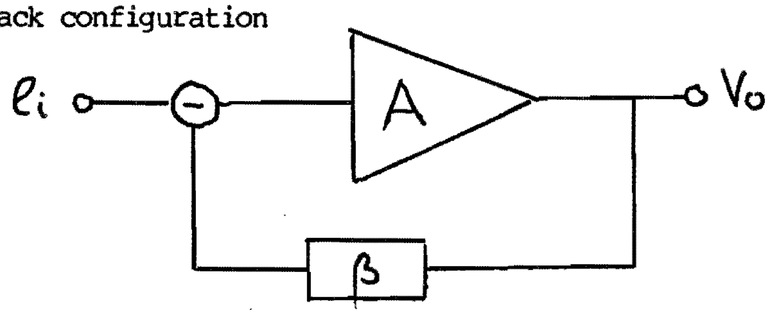


figure 1.4

A part of the output signal (βV_o) is subtracted from the input signal (e_1) and the result is amplified by the amplifier

$$(e_1 - \beta V_o)\, A = V_o$$

or

$$\frac{V_o}{e_1} = \frac{A}{1+\beta A} = \frac{1}{\beta}\,\frac{1}{1+\frac{1}{A\beta}} = A' \qquad 1)$$

The voltage gain of the amplifier with feedback (A'), called the closed-loop gain, can be studied by using a Bode plot of the open-loop gain (A) and the feedback (β). (The use of Bode plots is extensively discussed in chapter 3).

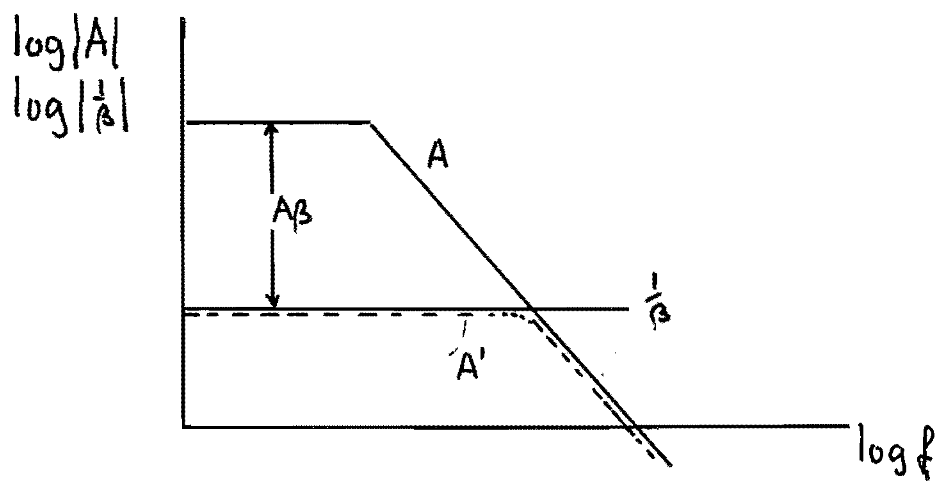


figure 1.5

Analysis of 1) yields the following:

$A' = 1/\beta$ for $A > 1/\beta$

$A' = A$ for $A < 1/\beta$

The Bode plot shows that the intersection of $\log|A|$ and $\log|1/\beta|$ indicates this discontinuity of the voltage gain A'.

b) Non-inverting amplifier

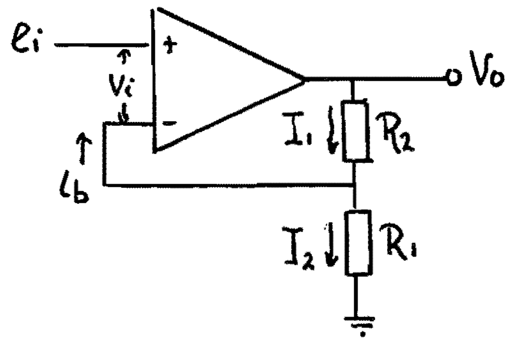


figure 1.6

The input voltage is applied to the (+) input terminal. A fraction of the output voltage is applied to the (-) input terminal through the voltage divider formed by R_1 and R_2.

Since the input resistance of an ideal op. amp. is infinite and the input bias currents are zero, the current flowing into the op. amp. is zero. (I_b = 0).

Thus $I_1 = I_2$

This means that the voltage $V_- = \frac{R_1}{R_1 + R_2} \cdot V_o$

with this approximation we get:

$$e_1 - V_i = V_o \frac{R_1}{R_1 + R_2} \quad ; \quad V_i = \frac{V_o}{A}$$

The closed-loop gain is found to be:

$$\frac{V_o}{e_i} = \frac{R_1 + R_2}{R_1} \qquad \frac{1}{1 + \frac{1}{A}\left(\frac{R_1 + R_2}{R_1}\right)} \qquad 2)$$

Comparison of (2) with (1) shows that with

$$\frac{R_1 + R_2}{R_1} = \frac{1}{\beta}$$

the non-inverting amplifier behaves like the normal feedback amplifier concerning transfer characteristic and stability.

The input resistance of the non-inverting amplifier is

$= \frac{e_i}{I_i}$. Since $I_i = 0$ for an ideal op. amp. the input resistance R_i

is infinite.

c) Inverting-amplifier.

The input signal e_i is applied to the (-) inverting terminal through R_1.
The feedback is arranged through R_2.

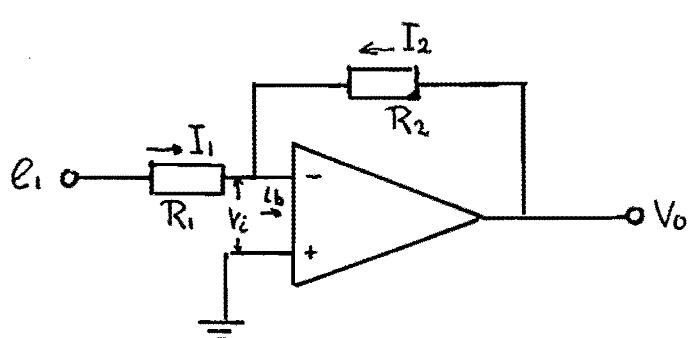


figure 1.7

Applying Kirchhoff's current law at the node (1) we get

$$\frac{e_1 - V_i}{R_1} + \frac{V_o - V_i}{R_2} = I_b$$

$$V_i = -\frac{V_o}{A}$$

Since the input resistance is infinite the current I_b is zero.

Thus $I_1 + I_2 = 0$.

The closed-loop gain is found to be

$$\frac{V_o}{e_1} = -\frac{R_2}{R_1} \cdot \frac{1}{1+\frac{1}{A}\left(\frac{R_1+R_2}{R_1}\right)} \qquad 3)$$

Comparison of (3) with (1) and (2) shows now:

- The error terms $\frac{1}{1+\frac{1}{A}\left(\frac{R_1+R_2}{R_1}\right)}$ are identical

- The closed-loop gain is not $-\frac{R_2+R_1}{R_1} = -\frac{1}{\beta}$

  but $-\frac{R_2}{R_1} = -\frac{1}{\beta}(1-\beta)$

The deviation of the gain is caused by the type of the applied feedback configuration. The feedback loop attenuates also the input signal. This can be calculated by applying the superposition theorem.

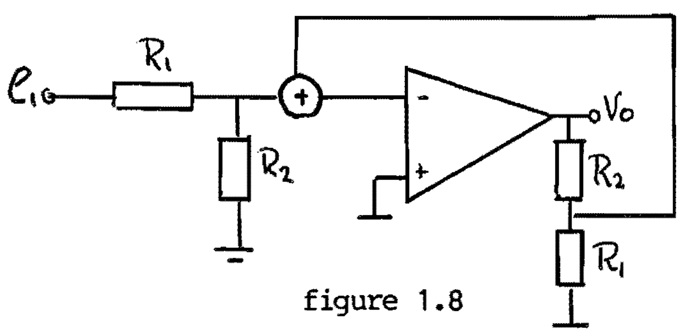


figure 1.8

$$\frac{V_o}{e_1} = -\frac{R_2}{R_1+R_2} \cdot \frac{R_1+R_2}{R_1} \cdot \frac{1}{1+\frac{1}{A}\left(\frac{R_1+R_2}{R_1}\right)} \qquad (4)$$

$$= -(1-\beta) \cdot \frac{1}{\beta} \quad \frac{1}{1+\frac{1}{A\beta}}$$

$$= -\frac{R_2}{R_1} \quad \frac{1}{1+\frac{1}{A}\left(\frac{R_1+R_2}{R_1}\right)}$$

The following may be concluded from 4.

- The stability conditions and error terms of both configurations (Non-inverting / inverting) are identical.
- The transfer characteristics differ and are respectively $\frac{1}{\beta}$ and $-(1-\beta).\frac{1}{\beta}$

The open-loop gain A_o is almost infinite thus $V_i \sim 0$ or $V_- = V_+$.

The (-) input terminal is considered to be internally connected to ground. One can say the amplifier has a virtual ground at its input.

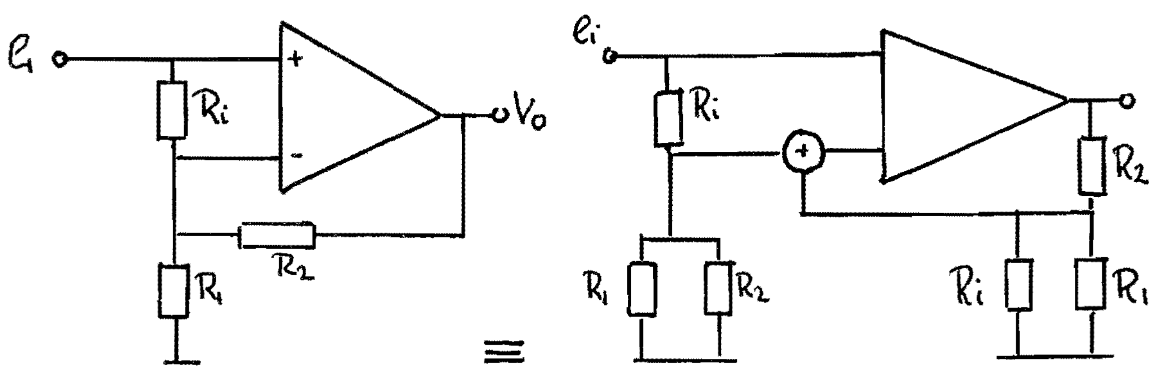
It is also important to observe that the input resistance seen by the signal source e_i is $R_i = \frac{e_i}{I_1} = R_1$

1.2. Non-ideal operational amplifiers ($A \neq \infty$, $R_i \neq \infty$, $R_o \neq 0$)

The non-ideal behaviour of an op. amp. can be represented by inserting the input impedance and output impedance in the feedback circuit as shown

in fig. 1.9 in the case $R_i \neq \infty$, $R_o + 0$

Non-inverting:



inverting:

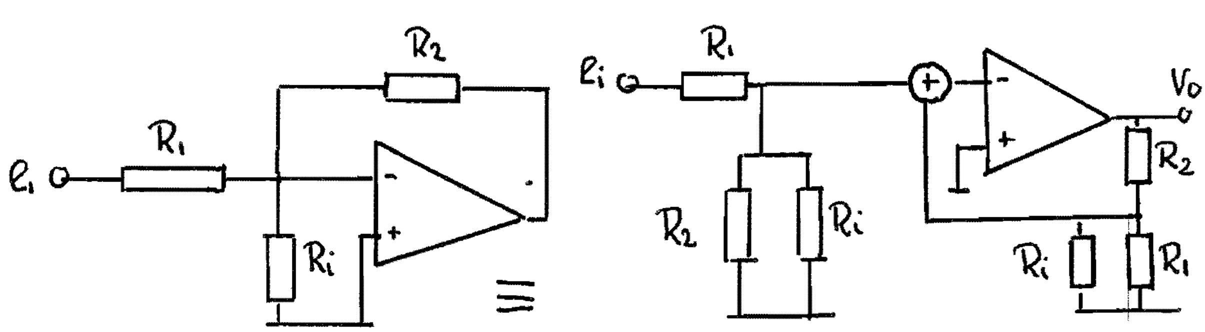


fig 19

The transfer characteristics can be found by using the superposition diagrams

Non-inverting:

$$e_1\left(1-\frac{R_1/\!/R_2}{R_1/\!/R_2+R_i}\right)A - V_0\,\frac{R_i/\!/R_1}{R_i/\!/R_1+R_2}A = V_0$$

With $\beta = \dfrac{R_i/\!/R_1}{R_i/\!/R_1+R_2} = \dfrac{R_iR_1}{R_iR_1+R_iR_2+R_1R_2}$

$$\frac{V_0}{e_i} = \frac{R_i(R_1+R_2)}{R_iR_1+R_iR_2+R_1R_2}\cdot\frac{1}{\beta}\cdot\frac{1}{1+\frac{1}{A\beta}} = \frac{R_1+R_2}{R_1}\cdot\frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_i}\right)}$$

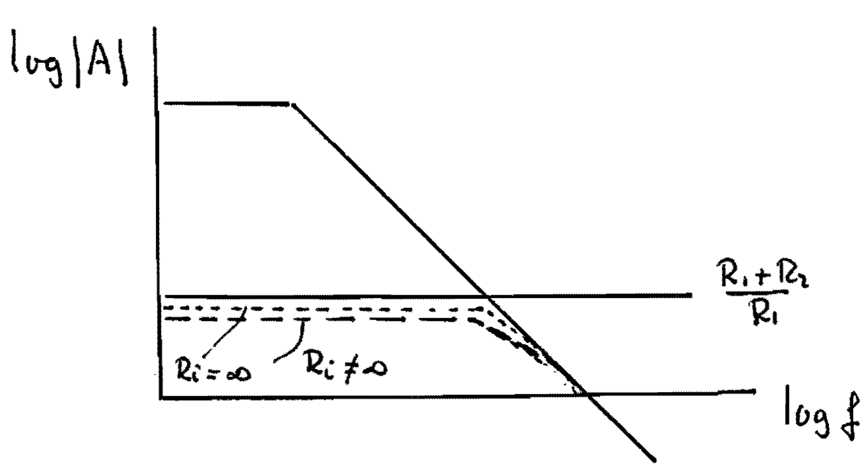
inverting:

$$-e_1\,\frac{R_i/\!/R_2}{R_i/\!/R_2+R_1}A - V_0\,\frac{R_i/\!/R_1}{R_i/\!/R_1+R_2}A = V_0$$

or

$$\frac{V_0}{e_i} = -\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_i}\right)}$$

These calculations show that the transfer characteristics are not influenced by the input impedance R_i. However the error term and the stability conditions are changed by R_i.



1.3 examples of feedback configurations.

a) Differential amplifier (subtractor)

This is a combination of the two previous configurations.
The amplifier has signals applied to both input terminals.

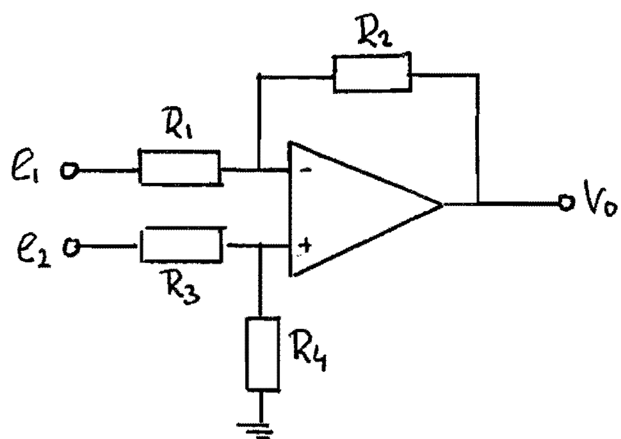


figure 1.10

with the approximation of an ideal op. amp.

$$\left.\begin{array}{l} A_o \sim \infty \\ R_i \sim \infty \end{array}\right\} \quad V_- = V_+ \qquad \text{and } I_b = 0$$

thus

$$V_+ = \frac{R_4}{R_3 + R_4} . e_2$$

$$V_- = \frac{R_2}{R_1 + R_2} . e_1 + \frac{R_1}{R_1 + R_2} . V_o$$

$$\frac{R_4}{R_3 + R_4} e_2 = \frac{R_2}{R_1 + R_2} e_1 + \frac{R_1}{R_1 + R_2} V_o$$

$$V_o = \frac{R_1 + R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} e_2 - \frac{R_2}{R_1} e_1 \qquad (5)$$

For simplicity let $R_1 = R_3$ and $R_2 = R_4$ then the gain of the differential amplifier is found to be

$$A = \frac{V_o}{e_2 - e_1} = \frac{R_2}{R_1} \tag{6}$$

This is the gain of the amplifier for differential mode signals. This configuration tends to reject a signal common to both input terminals.

b) Inverting summing amplifier.

This is a variation of the inverting amplifier.
The output voltage is proportional to the linear sum of the input voltages.
The output voltage can be determined by noting that the feedback causes a virtual ground, $V_i = 0$, and noting also that $I_b = 0$.

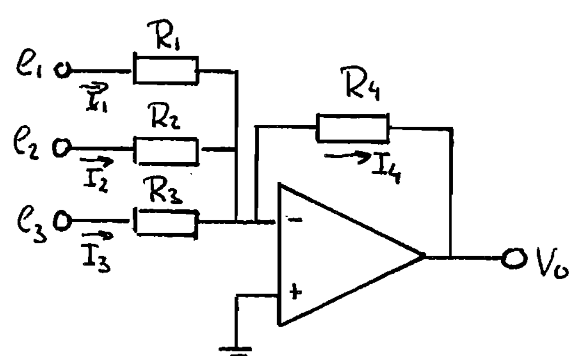


figure 1.11

Applying Kirchhoff's current law at node (1) we get:

$$I_1 + I_2 + I_3 = I_4$$

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} = -\frac{V_o}{R_4}$$

$$V_o = -\left(\frac{R_4}{R_1}e_1 + \frac{R_4}{R_2}e_2 + \frac{R_4}{R_3}e_3\right)$$

Each input is multiplied by a different scaling factor and then added.
The gains are independently selected by R_1, and R_2 and R_3.

Remark: the bandwidth and stability are determined by R_4 and $R_1//R_2//R_3$, see chapter 1.2

c) Integrator.

An ideal integrator produces an output voltage proportional to the integral of the input voltage. The basic integrator is shown in the figure.

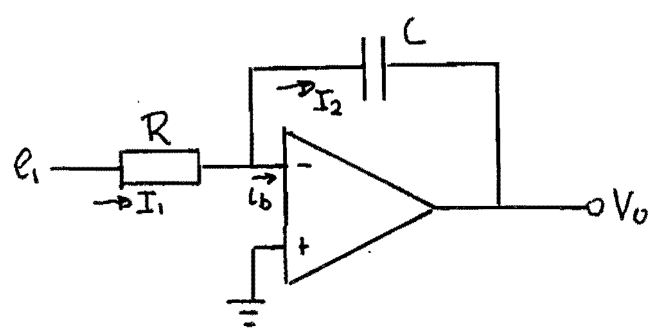


figure 1.12

The input current of the ideal op. amp. is zero, $I_b = 0$.

The feedback through the capacitor C maintains a virtual ground at the inverting input terminal. The voltage across C is equal to the output voltage V_o thus

$$I = \frac{e_i}{R} = -C\frac{dV_o}{dt} \rightarrow V_o(t) = -\frac{1}{RC}\int_0^t e_i\,dt + V_o(o) \qquad 7)$$

The initial output voltage V_o (o) can be set to a desired value.

In the frequency domain one can write:

$$I = \frac{e_i}{R} = -jwcV_o \rightarrow V_o = \frac{-e_i}{jwRC}$$

$$\text{The magnitude of the gain } A_{CL} = \left|\frac{V_o}{e_i}\right| = \frac{1}{\omega RC} \qquad 8)$$

and the argument of the gain

$$\arg\left(\frac{V_o}{e_i}\right) = \varphi = \frac{\pi}{2}$$

These quantities can be plotted in the Bode-plot.

The accuracy and stability are determined by $\beta = \dfrac{j\omega CR}{1+j\omega CR}$

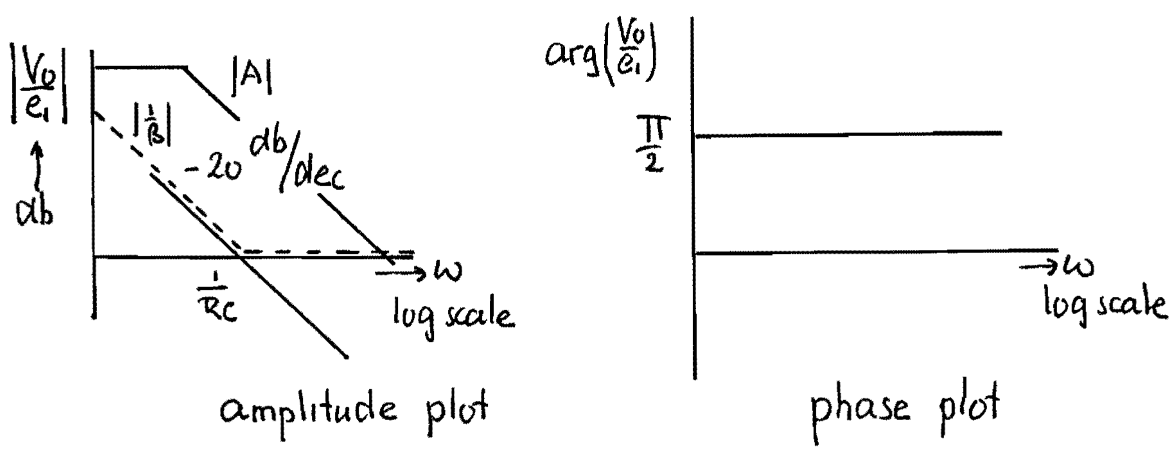


figure 1.13

d) Differentiator.

An ideal differentiator produces an output voltage proportional to the derivative of the input voltage.

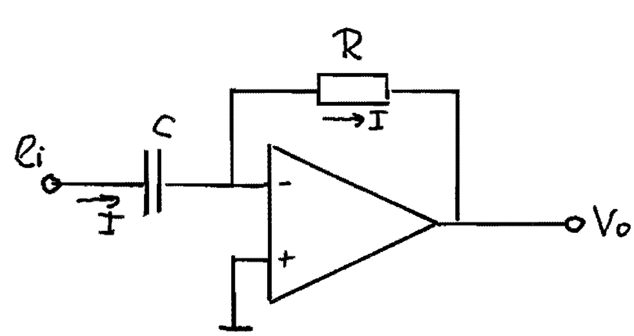


figure 1.14

The open-loop gain of an ideal op. amp. is infinite and the input bias current is zero.
We can write in the time domain:

$$I(t) = C\frac{de_i}{dt} = -\frac{V_o(t)}{R} \rightarrow V_o(t) = -RC\frac{de_i}{dt} = -\tau\frac{de_i}{dt} \qquad 9)$$

In the frequency domain one can write

$$I = jwC\,e_i = -\frac{V_o}{R} \text{ thus } A_{CL} = \left|\frac{V_o}{e_i}\right| = -jwRC = -jw\tau \qquad 10)$$

The Bode-plot of A_{CL} is as follows: $\left(\beta = \frac{1}{1+jwCR}\right)$

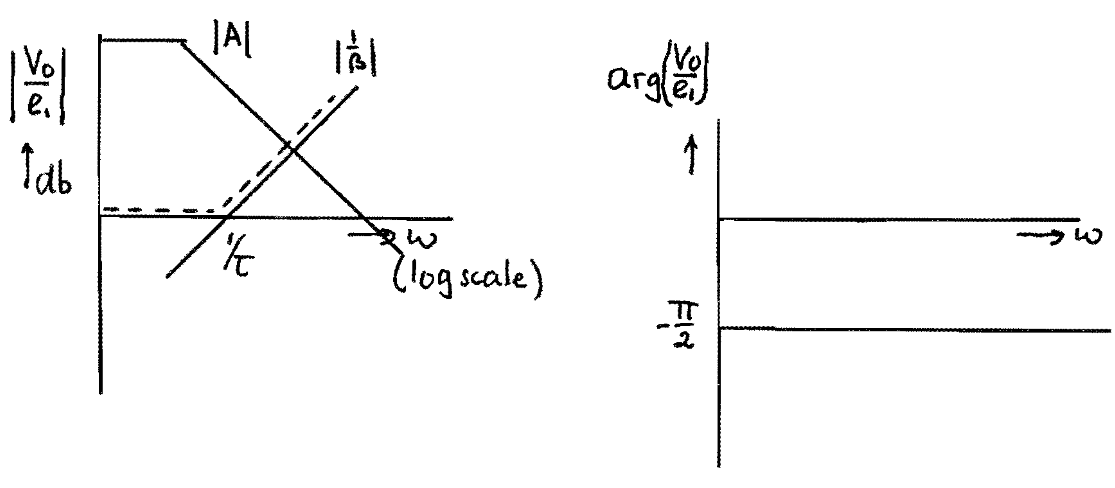


figure 1.15

2.0. OPERATIONAL AMPLIFIER CHARACTERISTICS.

An operational amplifier can be considered being a normal twoport

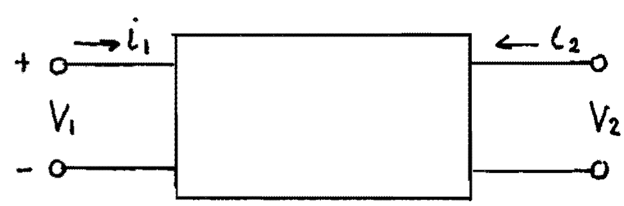


figure 2.1

The behaviour of such a twoport can be described by a set of parameters (z, y, h, g). See G&M(8.4 - 8.6)

e.g.

$$V_1 = Z_{11}i_1 + Z_{12}i_2$$

$$V_2 = Z_{21}i_1 + Z_{22}i_2$$

The parameters can be measured by doing some open- and short-circuit experiments
e.g.

the input impedance: $Z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2 = 0}$

the output impedance: $Z_{22} = \left.\frac{V_2}{i_2}\right|_{i_1 + 0}$

the open loop gain: $\frac{V_2}{V_1} = \left.\frac{Z_{21}}{Z_{11}}\right|_{i_2 = 0}$

The parameters are functions of frequency, temperature, time, supply voltage etc.
In the case of small-signal behaviour of op. amps., the parameters are constants. The parameters are functions of the amplitude in large-signal applications.

2.1. DC Parameters.

2.1.1. Biasing. (GℓM 6.2)

Just a transistor can be considered as the most simple op. amp.

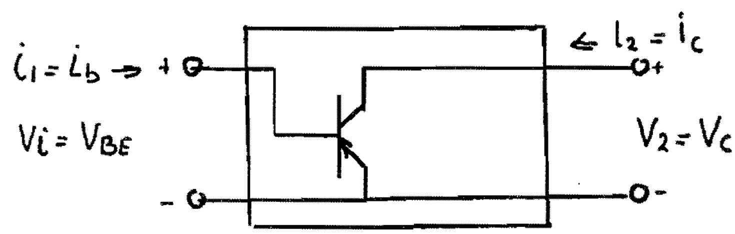


figure 2.2

the large-signal parameter-set for DC are the well-known transistor characteristics.

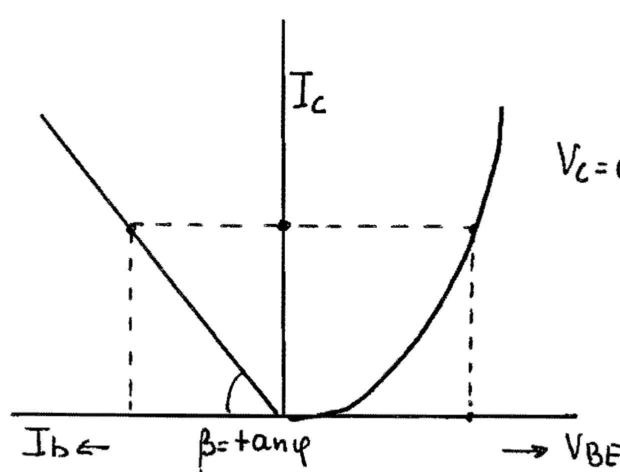


figure 2.3

$$Z_{12} : \quad I_C = I_O \quad (e^{\frac{qV_{BE}}{kT}} \quad -1)$$

$$\frac{-Z_{21}}{Z_{22}} \; : \; \frac{I_C}{I_B} \quad = \beta = \tan \varphi \; . \quad (V_C = 0)$$

The biasing of a transistor can be realized by applying a bias voltage source V_{BE} and a bias current source I_B.

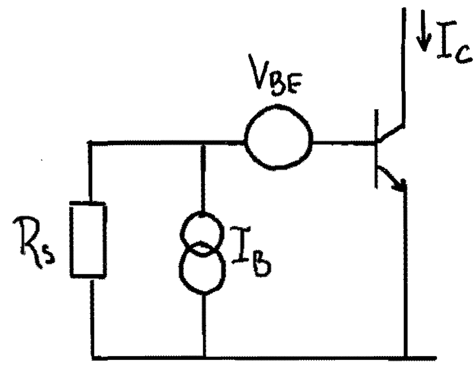


figure 2.4

The same holds for JFET and MOST.

These two sources are necessary in order to make the biasing (I_C = constant) independent of the value of the source impedance R_S.
This can be verified by choosing $R_S=\infty$ and $R_S=0$. If the value R_S is known, the biasing can be performed by just one source e.g.
$V_{BIAS} = V_{BE} + I_b R_s$

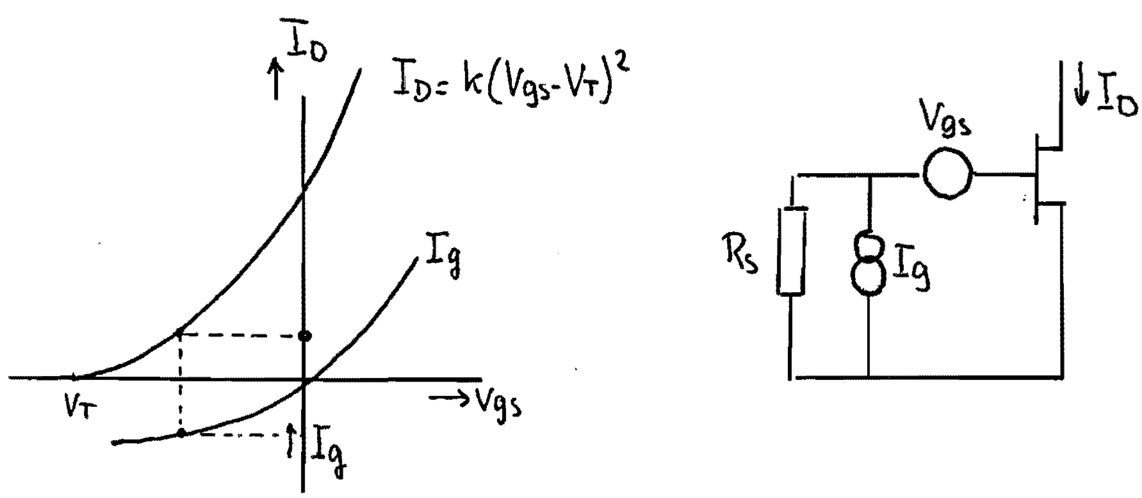
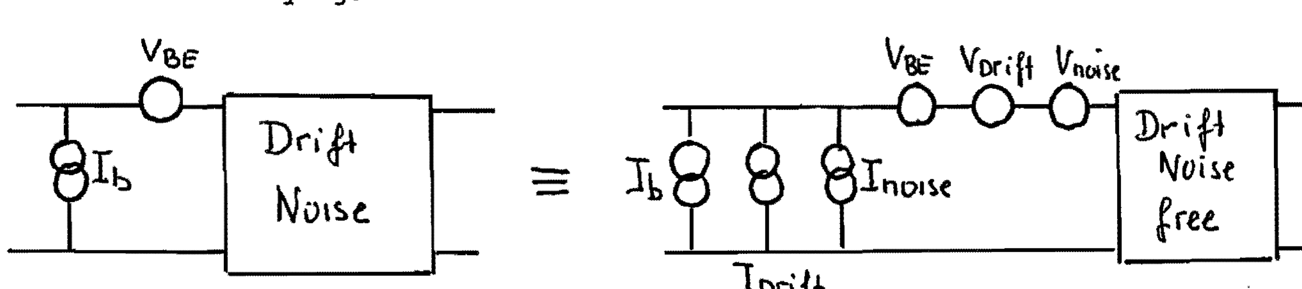


figure 2.5

The small signal behaviour of the transistors can be described by the gradient of the characteristics at the quiescent collector currents I_C and I_D

As discussed, the parameters are functions of temperature time and supply voltage. For one transistor it means that the characteristics are dependent on those effects (Drift etc.).

With constant bias-sources, the output current changes if the characteristics are varying.



The same variations at the output can be generalised by considering stable characteristics and adding extra current- and voltage sources with right-chosen values at the input.

In this way the number of deviations of amplifiers can be represented. Ideal amplifiers, but extra disturbing sources at the input.

2.1.2. Differential Amplifiers.

Most monolithic operational amplifiers contain a differential amplifier as input stage. The biasing is performed by a tail-current current source and two input current sources I_{b1} and I_{b2}.

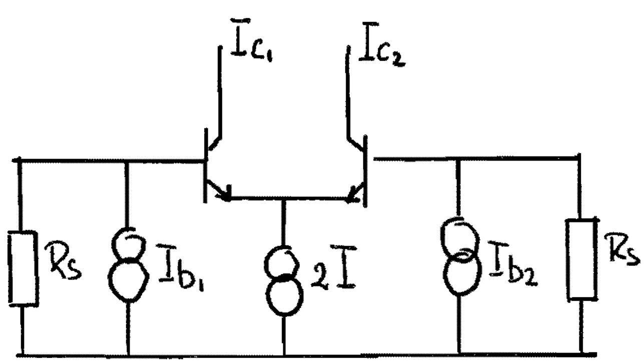
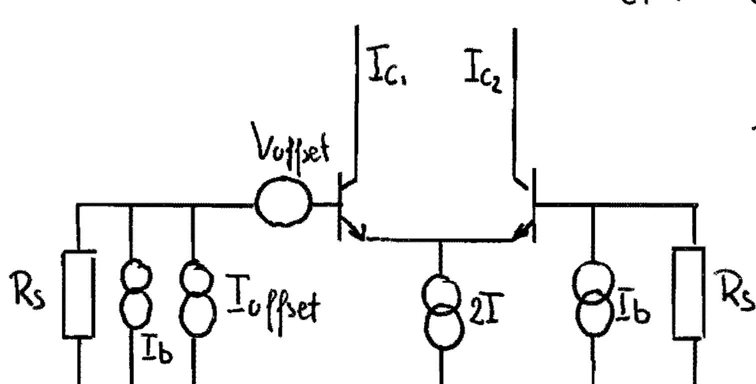


figure 2.7

The tail current generates the bias voltages V_{BE1}, V_{BE2} across the base/emitter of the transistors.
The transistors are biasing each other with those V_{BE} voltage sources.

If the two transistors are equal (βetas, and emitter areas equal). I_{c1} and I_{c2} resp. I_{b1} and I_{b2} will be identical.

However, if there is mismatch, two sources, V_{offset} and I_{offset}, have to be added in order to balance I_{c1} and I_{c2}.



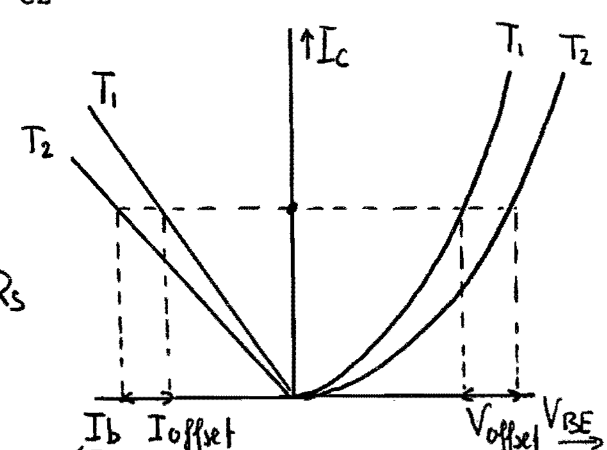


figure 2.8

emitter areas mismatch means $I_{01} \neq I_{02}$

$$I_1 = I_{01}\left(e^{\frac{qV_{BE} + V_{offset}}{KT}} - 1\right) = I_2 = I_{02}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$V_{offset} = \frac{KT}{q} \ln \frac{I_{01}}{I_{02}}$$

$$(I_b + I_{offset})\,\beta_1 = I_b\beta_2 \qquad I_{offset} = \frac{\Delta\beta}{\beta}\, I_b$$

2.1.3. Drift, caused by temperature variations.
- -

As discussed the effect of temperature on the parameters, the characteristics, can be described by adding extra sources at the input.

Bipolar.
The value of voltage sources can be calculated from the collector current equation

$$I_C = I_O\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$\text{with } I_o = CT^n\, e^{\frac{-qV_{gap}}{KT}} \quad , \; n = 2.2 \qquad V_{gap} = 1.2\,V$$

$$\frac{q}{KT}(V_{BE} - V_{gap}) = \ln \frac{I_c}{C} - n \ln T$$

$$-\frac{q}{KT^2}(V_{BE} - V_{gap}) + \frac{q}{KT}\frac{dV_{BE}}{dT} = -\frac{n}{T} \quad (I_c = \text{constant})$$

$$\left(\frac{dV_{BE}}{dT}\right)_{I_c = c} = \frac{V_{BE} - V_{gap}}{T} - \frac{nK}{q}$$

this gives $\underline{V_{drift}} \sim -\dfrac{V_{gap} - V_{BE}}{T} \sim -2mV/^{o}C$

The current drift arises from base current changes, and is approximately 0,5 to 1% per 0C.

$$\text{i.e.}\quad \frac{1}{IB}\left(\frac{dI_B}{dT}\right)_{I_C = c} \approx 0{,}5 - 1\%/^oC$$

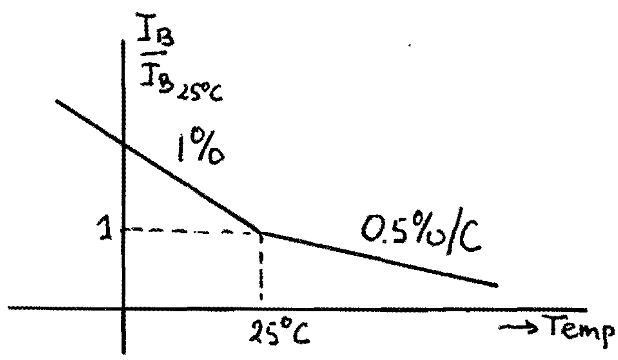


figure 2.9

FET's.

The temperature behaviour of FET's can be described by two compensating drift effects.
As the temperature rises, the resistance of the channel increases because the mobility decreases. However, this rise reduces the tickness of the depletion layer that cuts off the channel. This is the same phenomenon that causes the dV_{BE}/dT effect in transistors, and therefore has a drift of -2mV/°C. It can be expected that these two effects compensate each other at a certain Vgs. Under this bias condition the drain current is temperature independent.

The current drift of Junction FET's has the same temperature sensitivity as the leakage current of a diode i.e. 2-3 time Ig per 10°C.

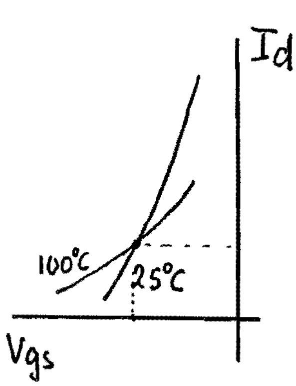


figure 2.10

$$I_o = CT^n e^{\frac{-qV_{gap}}{KT}}$$

$$\frac{dI_o}{dT} = CT^n e^{\frac{-qV_{gap}}{KT}}\left(\frac{n}{T} + \frac{qV_{gap}}{KT2}\right) = \frac{I_o}{T}\cdot\frac{1.2V}{26mV} \quad \text{or} \quad \frac{dI_o}{I_o}\Big/\frac{dT}{T} \sim 50$$

$$\frac{dI_o}{I} = 2\times/6°C$$

<u>Differential amplifiers.</u>

The voltage drift as given by

$$V_{drift} = - \frac{V_{gap} - V_{BE}}{T_1} + \frac{V_{gap} - V_{BE2}}{T_2} = - \frac{V_{offset}}{T} + \frac{V_{gap} - V_{BE}}{T} \; \frac{\Delta T}{T}$$

$$\text{if } T_1 = T_2 : \quad V_{DRIFT} = - \frac{V_{offset}}{T}$$

These relations show that it is better to balance the differential output of a differential amplifier by adjusting collector resistors and to keep the collector currents unequal. There is no voltage drift if $V_{offset} = 0$.

For equal source impedances R_{s1} and R_{s2} the current drift can be considered to be caused by the drift of I_{offset} (1 - 0,5% of I_{offset}/0C).

2.1.4. Balancing of op. amps.

As discussed, the biasing of a differential amplifier can be realised by an offset voltage source, two input bias current sources and offset bias current source. As the input stage of an op. amp. consists of a differential amplifier, the biasing can be represented in the same way.

These sources produce an offset voltage at the output of the op. amp. depending on the feedback configuration.

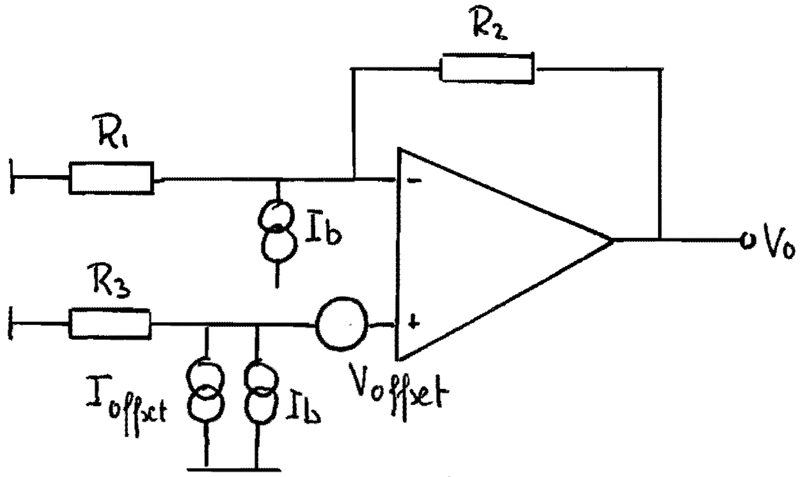


figure 2.11

$$V_0 = \frac{R_1 + R_2}{R_1} V_{offset} + I_b \left(R_2 - R_3 \left(\frac{R_1 + R_2}{R_1} \right)\right) + I_{offset} \; R_3 \cdot \frac{R_1 + R_2}{R_1}$$

The formula shows that the offset, caused by the bias current can be eliminated by choosing

$$R_3 = \frac{R_1R_2}{R_1 + R_3}$$

The source resistance seen by both op. amp. input terminal must be identical I_b offset can be cancelled by giving R_3 a value with a little deviation.

The error caused by the offset voltage can often be nulled with an external potentiometer between special terminals. This is the best method with respect to drift (see remark 2.1.3. differential amplifier).

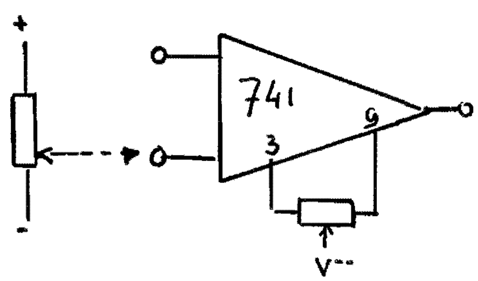


figure 2.12

The output can also be driven to zero by an external voltage, applied to one of the inputs.

Typical values:

V_{offset} : < 5mV,

I_{bias} : bipolar: 10–100nA, I_{offset} 10% I_{bias}

FET : 10pA (at 20°C)

at 100°C : ~100nA. (2 x I_{bias}/6°C)

2.2. NOISE. (See G&M chapter 11)

The noise of an op. amp. results in the lower limit to the size of an electrical signal that can be amplified without significant deterioration in signal quality.
The existence of noise is basically due to the fact that electrical charge is not continuous but is carried in discrete amounts equal to the electron charge.

2.2.1. Types of noise.

a) Shot noise:

This type of noise is caused by minority carriers crossing junction (or electrons striking the cathode). The passage of each carrier across the junction is a purely random event. The external current I is composed of these carriers, a series of random independent pulses; the fluctuation in I is termed, shot noise. If the average value of I has a value I_D the resulting noise current has a mean-square value.

$$\overline{i^2} = 2qI_D\Delta f$$

with $q = 1,6 \times 10^{-19}$C, and Δf the bandwidth in Hz.

b) Thermal Noise:

Thermal Noise is due to the random thermal motion of electrons in conductors as conventional resistors. Since electron drift velocities in a conductor are much less than electron thermal velocities, this type of noise is unaffected by the presence of a direct current.

In a resistor R, thermal noise can be represented by a series voltage generator or by a shunt current generator.

$$\overline{v^2} = 4KTR\,\Delta f$$

$$\overline{i^2} = 4KT\,\frac{1}{R}\,\Delta f$$ at room temperature $4KT = 1.66 \times 10^{-20}$ V.C.

c) Flicker Noise: (1/f noise).

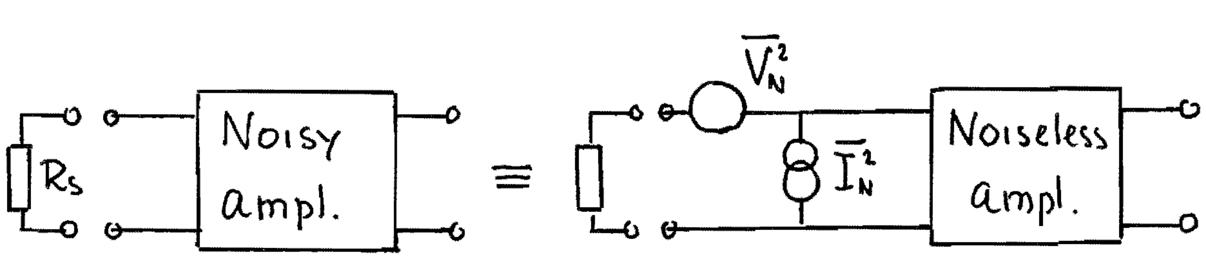
Flicker Noise is found in all active devices, passive elements and always associated with flow of direct current I. The spectral density of this type of noise has a 1/f frequency dependence

$$\overline{i^2} = K_1\,\frac{I^a}{f}\,\Delta f.$$

K_1 is an unknown constant and depends on the perfection of the device, a is a constant in the range 0,5 to 2.

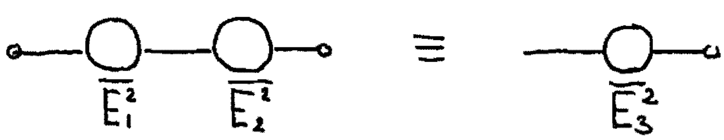
2.2.2. Noise of operational amplifiers (G&M. 11.5)

The noise behaviour of op. amps. can be described in the same way as chosen for the representation of drift. Consider the amplifier noiseless and represent the noise by equivalent input voltage and current generators.



The values of the equivalent noise generators can easily be measured. The value of the voltage generator is found by short circuiting the input, measuring the output noise, and divide this quantity by the gain of the amplifier.
The value of the current generator can be obtained in the same way by open circuitry of the input.
The values of both equivalent input noise generators depend on the design, the chosen transistors (bipolar, MOS) and the biasing of the input stage of an op. amp. They can be calculated by putting the thermal and shot noise sources in an equivalent model of a transistor and doing open and short circuiting experiments (see G&M. 11.5.1.)

The manufacturer gives the values of the equivalent noise sources in his databooks. Sometimes as a function of frequency. At low frequencies, the sources can be considered to be uncorrelated: i.e.



$$\overline{E_3^2} = \overline{(E_1+E_2)^2} = \overline{E_1^2} + 2\overline{E_1E_2} + \overline{E_2^2} = \overline{E_1^2} + \overline{E_2^2}$$

if the sources are uncorrelated

The correlation factor $\rho = \dfrac{\overline{E_1E_2}}{\sqrt{\overline{E_1^2}}\,.\sqrt{\overline{E_2^2}}} = 0$

Typical data of op. amps. are given in the figure below. FET op. amps. already show 1/f noise at higher frequencies.

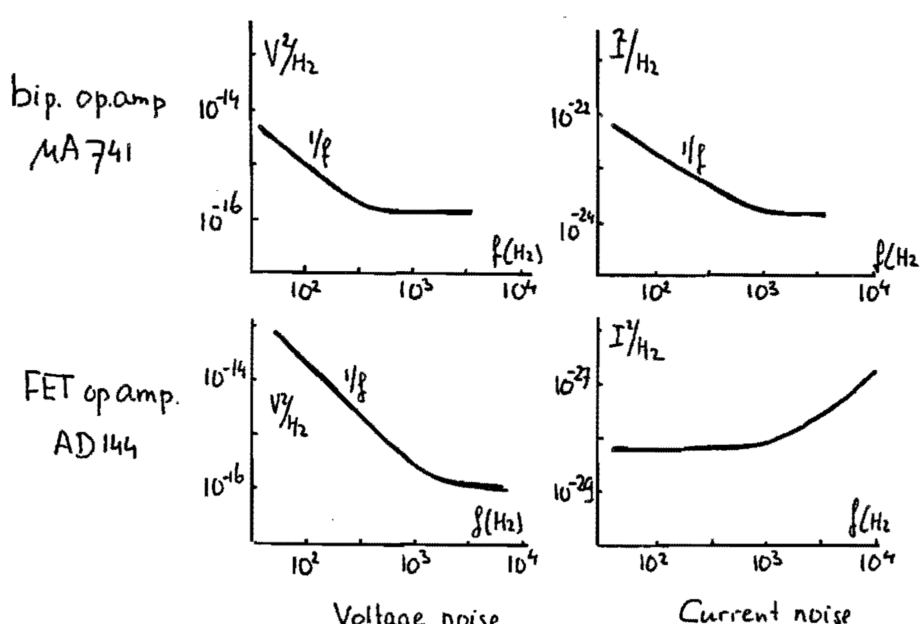


figure 2.14

2.2.3. Noise Figure of an amplifier (G&M. 11.10)

As discussed, the noise behaviour of an amplifier can be described as a noise-free equivalent with two basic noise generators at the input. Such an amplifier will always add noise to an input signal, generated by a signal source with an internal source impedance R_S. The signal/noise ratio at the output of the amplifier will be lower than that of the signal source itself. The deterioration of the signal/noise ratio is indicated by the Noise figure F, usually expressed in decibels.

$$F = \frac{\text{total output noise}}{\text{that part of the output noise due to the source resistance}}$$

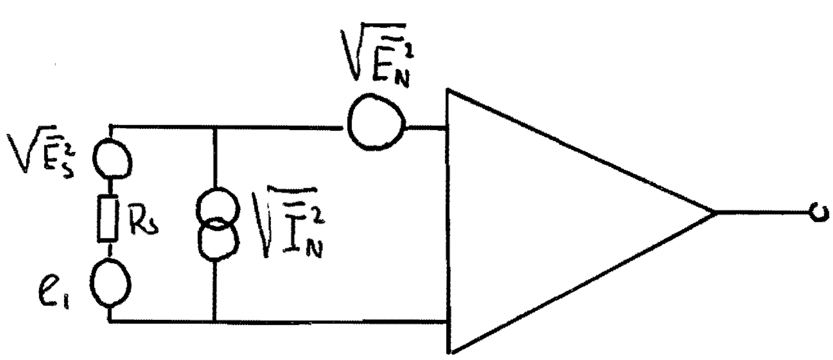


figure 2.15

$$F = \frac{\overline{E_S^2} + \overline{E_N^2} + \overline{I_N^2} R_S^2}{\overline{E_S^2}} = 1 + \frac{\overline{E_N^2}/R_S + \overline{I_N^2} R_S}{4kT\ \Delta f}$$

because $\overline{E_S^2} = 4kTR_S\Delta f$

It is apparent from this relation that F has a minimum as R_S varies. By differentiations with respect to R_S we can calculate the value of R_S giving minimum F

$$\frac{dF}{dR_s} = 0 \qquad F_{min} = 1 + \frac{\sqrt{\overline{E_N^2} \cdot \overline{I_N^2}}}{2kT\Delta f}$$

$$\text{for } R_S = \sqrt{\frac{\overline{E_N^2}}{\overline{I_N^2}}}$$

The noise current source ($\overline{I_n^2}$) of FET op. amps. is lower than that of bipolar amplifiers. Therefore for source resistances of the order of mega-ohms or higher, a FET op. amp. has a lower Noise Figure than a bipolar one.

2.2.4. Noise in Operational Amplifiers with feedback.

All the noise sources, representing the noise of the resistors and the amplifier are indicated in the figure below

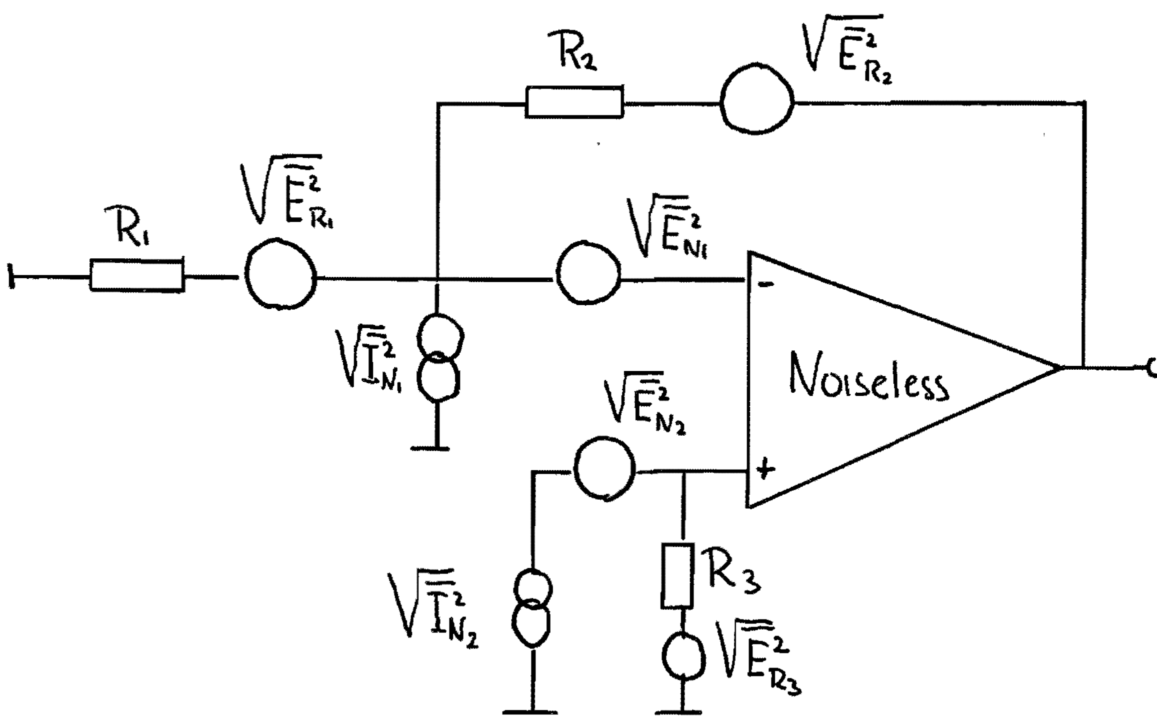


figure 2.16

With uncorrelated sources the output noise is:

$$\sqrt{\overline{V_o^2}} = \sqrt{\left(\overline{E_{R_1}}\frac{R_2}{R_1}\right)^2 + \overline{E_{R_2}^2} + \left(\overline{I_{N_1}}R_2\right)^2 + \left(1+\frac{R_2}{R_1}\right)^2\left(\overline{E_{N_1}^2} + \overline{E_{N_2}^2} + \overline{E_{R_3}^2} + \left(\overline{I_{N_2}}R_3\right)^2\right)}$$

with $\overline{E_R^2} = 4kTR\Delta f$

The noise sources of the op. amp. can be measured using the formula and choosing the right values for R_1, R_2 and R_3.

a) $\sqrt{\overline{E}^2_{N_1} + \overline{E}^2_{N_2}}$

Make the absolute value R_1 and R2 small, $R_3 = 0$.

but $R_2 >> R_1$.

Then the fomula above becomes

$$\sqrt{\overline{V}_0^2} = \left(1 + \frac{R_2}{R_1}\right)\sqrt{\overline{E}^2_{N_1} + \overline{E}^2_{N_2}}$$

These noise sources cannot be measured separately.

b) $\sqrt{\overline{I}^2_{N_1}}$, $\sqrt{\overline{I}^2_{N_2}}$

Choose $R_1 = \infty$, $R_3 = 0$ and R_2 large.

$$\sqrt{\overline{V}_0^2} = \sqrt{\overline{E}^2_{R_2} + (\overline{I}_{N_1} R_2)^2}$$

$\sqrt{\overline{E}^2_{R_2}} = \sqrt{4\,kTR\Delta f}$, proportional to $\sqrt{R_2}$. So this term becomes less important in this formula with very large R_2.

$\sqrt{\overline{I}^2_{N_2}}$ can be measured by making R_1 and R_2 small,

$R_2 >> R_1$ and R_3 very large.

2.3. AC-parameters.

For small a.c. signals the performance is limited by the noise and the frequency response of the op. amp.

For large signals the specifications of slew rate, settling time, full power bandwidth, common-mode rejection and distortion indicate the quality of an op. amp.

2.3.1. Frequency response.

In practice op. amps. are not ideal. Gains are of the size of 10^5 - 10^6 and frequency dependent. Because of internal capacitances the voltage gain decreases at high frequencies. This fall-off is enhanced by the addition an extra, a compensation capacitance, in the circuit to insure that the op. amp. remains stable when feedback is applied.

The frequency characteristic can be expressed by

$$A = \frac{A_o}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

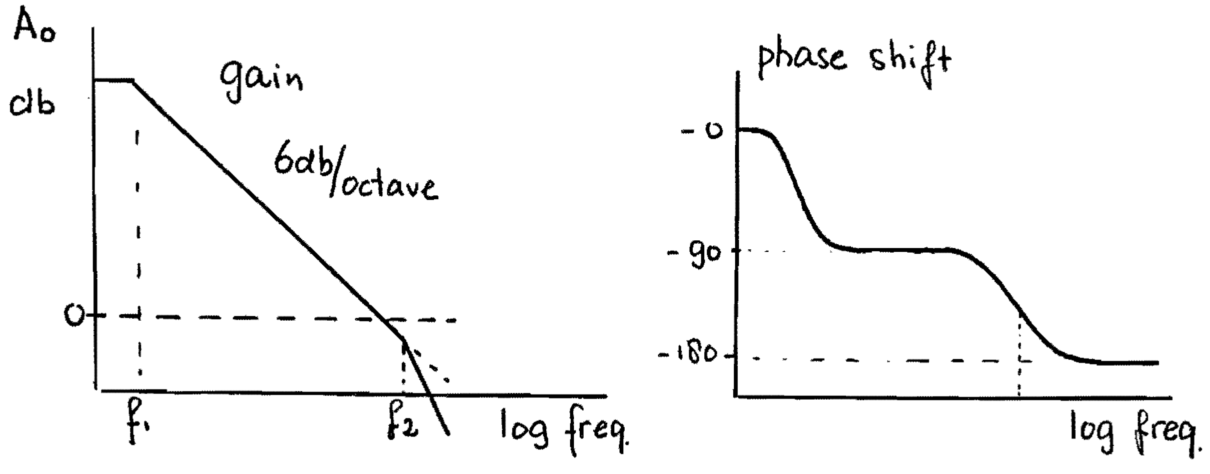


figure 2.17

A_o is the low frequency/DC gain of the op. amp.

The first pole $\tau_1 = \frac{1}{2\pi f_1}$ is determined by the compensation capacitor.

The value is chosen in such a way that at the frequency at which the open-loop voltage gain is equal to unity, the fall-off shows a 6db/octave slope. The phase shift is about 90 - 135°. This frequency indicates the unity-gain bandwidth of an op. amp.

2.3.2. Slew rate. (G&M 9.6)

The best way to understand the large signal behaviour of an op. amp. is to consider the construction of a simple op. amp.

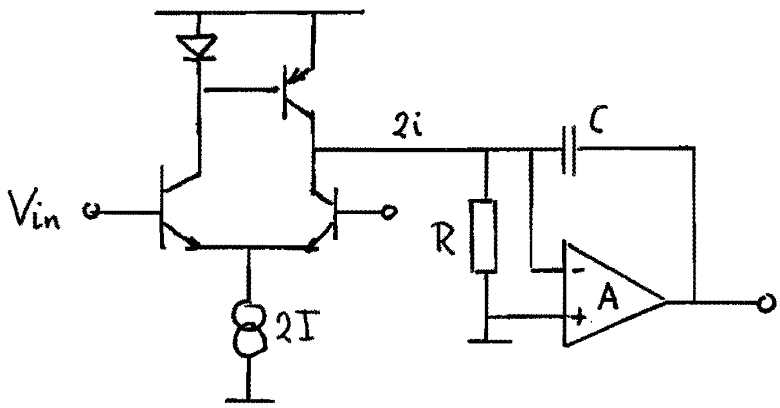


figure 2.18

The operational amplifier consists of a differential input stage with a tail current 2I and an output stage. The amplifier shows a fall-off of 6db/octave because of the feedback capacitor C.

The small signal gain is:

$$A_o = g_m \frac{1}{j\omega C} \cdot \frac{1}{1 + \frac{1}{A}\left(1 + \frac{1}{j\omega RC}\right)} = g_m R \cdot \frac{A}{1 + (A+1)\, j\omega RC}$$

The expression shows:

- $A_o = g_m R$.
- Unity-gain bandwidth: $f_c = \frac{g_m}{2\pi C} \cdot \frac{A}{A+1} = \frac{g_m}{2\pi C}$

- Since using bipolar transistors $g_m = \frac{qI}{kT}$ the unity gain bandwidth of bipolar op. amps. is determined by the tail current and the capacitance C:

$$f_c = \frac{q}{2\pi kT} \cdot \frac{I}{C}$$

Slew rate is defined as the maximum rate of change of the output (dv/dt) with a step input signals.
An input signal, larger than e.g. 1 Volt, overloads the input stage and the output current 2i of that stage equals now $\pm$ 2I: Since the output stage behaves like an integrator the output changes with:

$$\left(\frac{dV_0}{dt}\right)_{max} = \frac{2I}{C}$$

This is the maximum rate of change of the output, the slew rate of the op. amp.

remarks:

- For bipolar op. amps: slew rate = $\frac{2g_m kT}{qC}$

- During slewing of an op. amp. there exists no relations between output and input signal. Therefore feedback does not influence the slew rate.

2.3.3. Full Power Bandwidth.

Full power bandwidth is defined as the maximum frequency at which the full output swing may be obtained without distortion.

Consider a sinusoidal output signal of the form $V_p \sin \omega t$. The maximum rate of change (= ωV_p) must be smaller than the maximum slew rate if no distortion is to occur.

i.e. $\omega V_p \leq \frac{2I}{C}$

example uA741: I = 10 uA, C = 30pf.

- slew rate: $\frac{20.10^{-6}}{30.10^{-12}} = 0{,}6\ V/\mu sec$

- full power bandwidth:

Vp = 10 Volt: $f_{max} = \frac{1}{2\pi.10} \cdot 0.6\ 10^{6} = 10\ kHz$

- unity gain bandwidth : $\frac{g_m}{2\pi C} = \frac{q}{2\pi kT} \cdot \frac{I}{C} = \frac{40.10^{-3}}{2\pi\ 30\ 10^{-12}} = 2\ MHz$

$$\frac{\text{slew rate}}{\text{unity gain bandwidth}} = \frac{2I}{C} \cdot \frac{2\pi C}{g_m} = \frac{4\pi I}{g_m} = \frac{4kT\pi}{q}$$

this ratio is fixed for bipolar transistor $\left(= \frac{4kT\pi}{q}\right)$ but it can be improved by using FET's or by emitter degeneration (thus increasing the ratio I/gm.

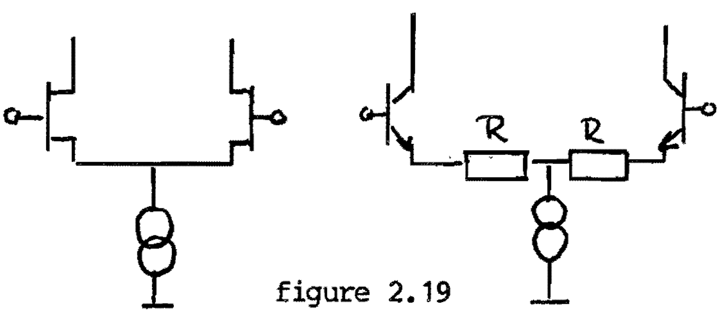
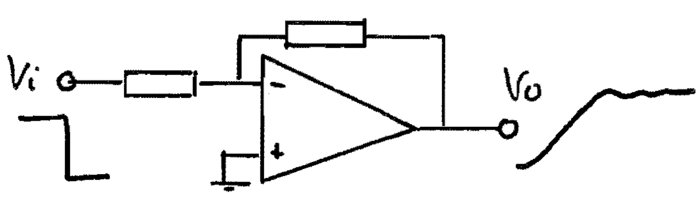


figure 2.19

2.3.4. Settling Time.

Suppose a large step function is applied to an op. amp. working as an inverter



The output signal will have a shape as indicated in the figure:

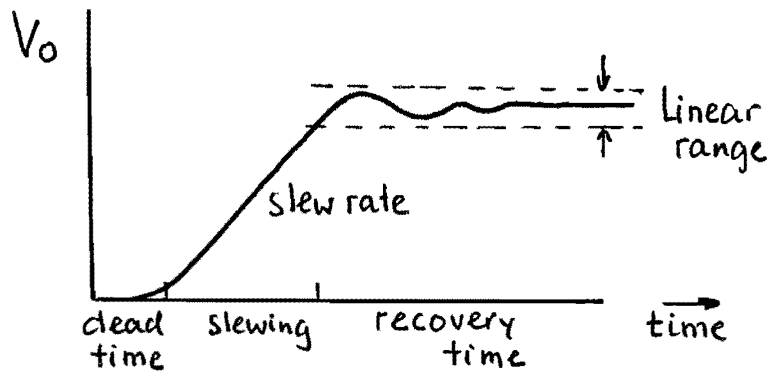


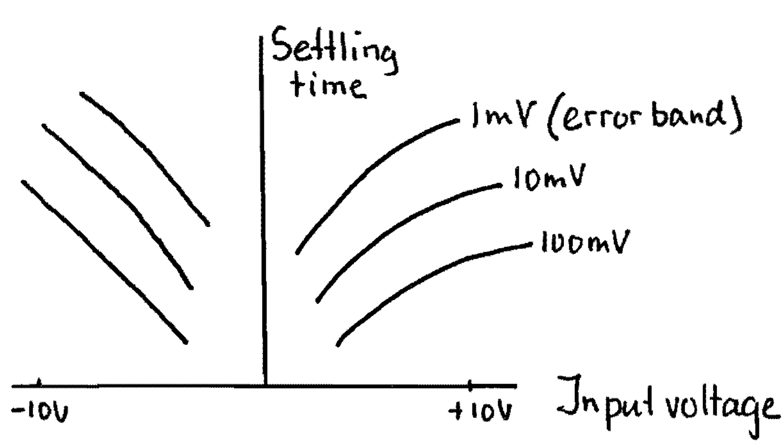
figure 2.21

At first no response at all: the dead-zone.

During the slew period the input stage is overloaded; the output rises with maximum slew rate.
As the output increases, the signal at the -input becomes smaller by the feedback loop. At a certain moment the input stage starts to operate in the linear range and the feedback becomes effective. The op. amp. produces a small-signal response.

The time, necessary to reach the final value, is determined by the acceptable error (e.g. a number of RC-times).

The settling time is defined as the time required to obtain an output signal with an error less than the requirement. The settling time depends on the input step amplitude and the allowable final error



2.3.5. Common Mode Rejection Ratio (CMRR).

Most applications of op. amps. require the amplification of differential voltages often in the presence of fluctuating common-mode voltages. Since the desired signal is usually the differential voltage, the response to the common-mode signal produces an error at the output that is indistinguishable from the signal. The common-mode rejection ratio (also called H) is conventionally defined as the magnitude of the ratio of differential-mode to common-mode gain

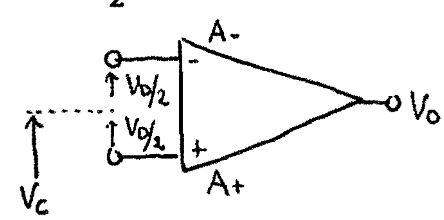
$$H = \frac{A_{dm}}{A_{cm}}$$

The rejection ratio is a factor that indicates how many times a common-mode signal at the input must be larger than a differential signal at the input in order to get the same differential output signal.

This ratio will generally not be infinite because the transfer characteristics of the -input and the +input to the output of an op. amp. are not equal and frequency dependent.
Suppose

$$A_- = \frac{A + \Delta A/2}{1 + j\omega\tau_1} \quad , \quad A_+ = \frac{A - \Delta A/2}{(1+j\omega\tau_1)(1+j\omega\tau_2)} \qquad \text{and}$$

$$V_o = \frac{1}{2} V_D \,(A_- + A_+)$$



With these relations we can find easily

$$A_{diff} = \frac{A}{1+j\omega\tau_1} \left[\frac{1 + \frac{1}{2}\left(1 + \frac{\Delta A}{2A}\right) j\omega\tau_2}{1 + j\omega\tau_2} \right] \sim \frac{A}{1+j\omega\tau_1} \cdot \frac{1 + j\omega\frac{\tau_2}{2}}{1 + j\omega\tau_2}$$

$$A_{common\ mode} = \frac{\Delta A}{1+j\omega\tau_1} \left[\frac{1 + \left(\frac{1}{2} + \frac{A}{\Delta A}\right) j\omega\tau_2}{1 + j\omega\tau_2} \right] \wedge \frac{\Delta A}{1+j\omega\tau_1} \cdot \frac{1 + \frac{A}{\Delta A} j\omega\tau_2}{1+j\omega\tau_2}$$

and for the rejection ratio

$$H = \frac{A}{\Delta A} \cdot \frac{1 + \frac{1}{2}\left(1 + \frac{\Delta A}{2A}\right) j\omega\tau_2}{1 + \left(\frac{1}{2} + \frac{A}{\Delta A}\right) j\omega\tau_2} \qquad \frac{A}{\Delta A} \cdot \frac{1}{1 + \frac{A}{\Delta A} \cdot j\omega\tau_2}$$

This relation shows that the rejection ratio becomes frequency-dependentby the second pole of A^+. The C.M.R.R. at low frequencies equals $\frac{A}{\Delta A}$.

2.3.6. Discrimination factor.

Sometimes an operational amplifier is provided with a differential input as well as with a differential output. A feedback circuit, an attenuator can be constructed in the same way.

In those cases you can also define a discrimination factor F. This factor equals the ratio of the differential gain of the differential signals to the amplification of the common mode signals. This discrimination factor F is important when two amplifying circuits are used in series and the overall CMRR has to be calculated

$$H = \frac{A_{diff \rightarrow diff}}{A_{com \rightarrow diff}} \quad ; \quad F = \frac{A_{diff \rightarrow diff}}{A_{com \rightarrow com}}$$

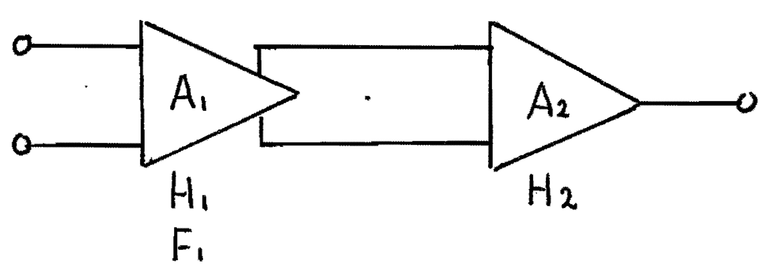


figure 2.23

total $A_{diff} = A_1 \times A_2$

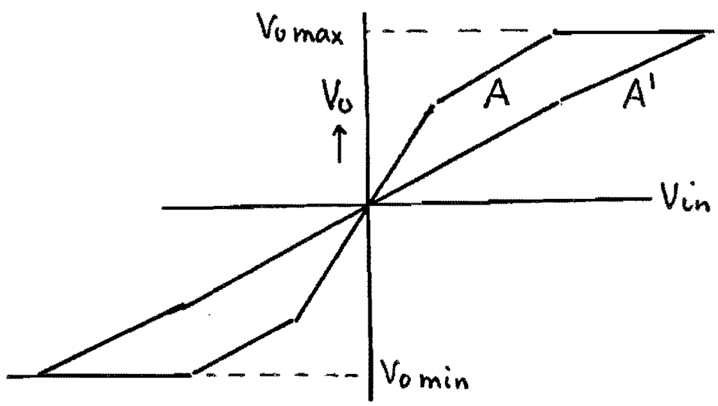
common mode gain: $\frac{A_1}{F_1} \cdot \frac{A_2}{H_2} + \frac{A_1}{H_1} \cdot A_2$

total CMRR: $\dfrac{1}{\frac{1}{F_1 H_2} + \frac{1}{H_2}}$

This relation will be discussed in the chapter: Instrumentation Amplifiers.

2.3.7. Distortion (G&M pg. 469)

Distortion occurs when the transfer characteristic is not linear.



The slope changes since the gain A depends on the amplitude of the signal.
This amplitude sensitivity can be reduced by feedback.

This can be proved by calculating the gain sensitivities of the closed loop relation $\frac{dA'}{A'}$ to the open loop gain: $\frac{dA}{A}$

$$A' = \frac{A}{1+\beta A}$$

$$\frac{dA'}{A'} = \frac{1+\beta A}{A} \cdot \frac{1}{(1+\beta A)^2} \cdot A \cdot \frac{dA}{A}$$

$$\frac{dA'}{A'} = \frac{1}{1+\beta A} \cdot \frac{dA}{A}$$

Indeed: the sensitivity is reduced by the well-known factor $\frac{1}{1+\beta A}$.

The distortion of an amplifier is decreased with the same factor.

3. Negative and positive feedback (G&M chapter 9)

If a fraction of the output voltage is fed back to the input it is called feedback.
A feedback to the inverting input of the op.amp. is called negative feedback and to the non-inverting input positive feedback. A combination of the two is also possible.
If the feedback introduces enough phase shift to the inverting or non-inverting input, feedback may become positive or negative respectively and may cause instability. This effect will be discussed.

The positive feedback is used in wave generators bistable circuits, and comparators.
These applications of positive feedback are discussed later.

3.1. Bode plot

Let us analyse a transfer function or amplification with 3 breakpoints (corner points).

$$A(j\omega) = \frac{A_0}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)(1 + j\omega/\omega_3)}$$

This function A is a complex function.
We can write the complex function in the form:

$$A = a + jb = |A| e^{j\varphi}$$

with $|A|$ is the magnitude
and $\varphi = \arg(z)$ is the phase.

The magnitude can be written:

$$|A| = \frac{A_0}{\{(1 + \omega^2\tau_1^2)(1 + \omega^2\tau_2^2)(1 + \omega^2\tau_3^2)\}^{1/2}}$$

where

$$\tau_1 = \frac{1}{\omega_1}, \quad \tau_2 = \frac{1}{\omega_2}, \quad \tau_3 = \frac{1}{\omega_3}$$

It is common to express the magnitude of the amplification in deciBells.
That is:

$$20\log(A) = 20\log A_0 - 10\log(1 + \omega^2\tau_1^2) - 10\log(1 + \omega^2\tau_2^2) - 10\log(1 + \omega^2\tau_3^2)$$

This can be approximated for high frequencies by:

$$20\log|A| \approx 20\log A_0 - 20\log\omega\tau_1 - 20\log\omega\tau_2 - 20\log\omega\tau_3$$

The Bode plot consists of two plots:

a) magnitude plot $|A|=f(\omega)$

b) phase plot arg $A=g(\omega)$.

The magnitude Bode plot has a double logarithmic scale, i.e. horizontal logarithmic and vertical in deciBells.
With the piece-wise linear approximation we get a curve with straight lines between the breakpoints.
The phase Bode plot has a single logarithmic scale.

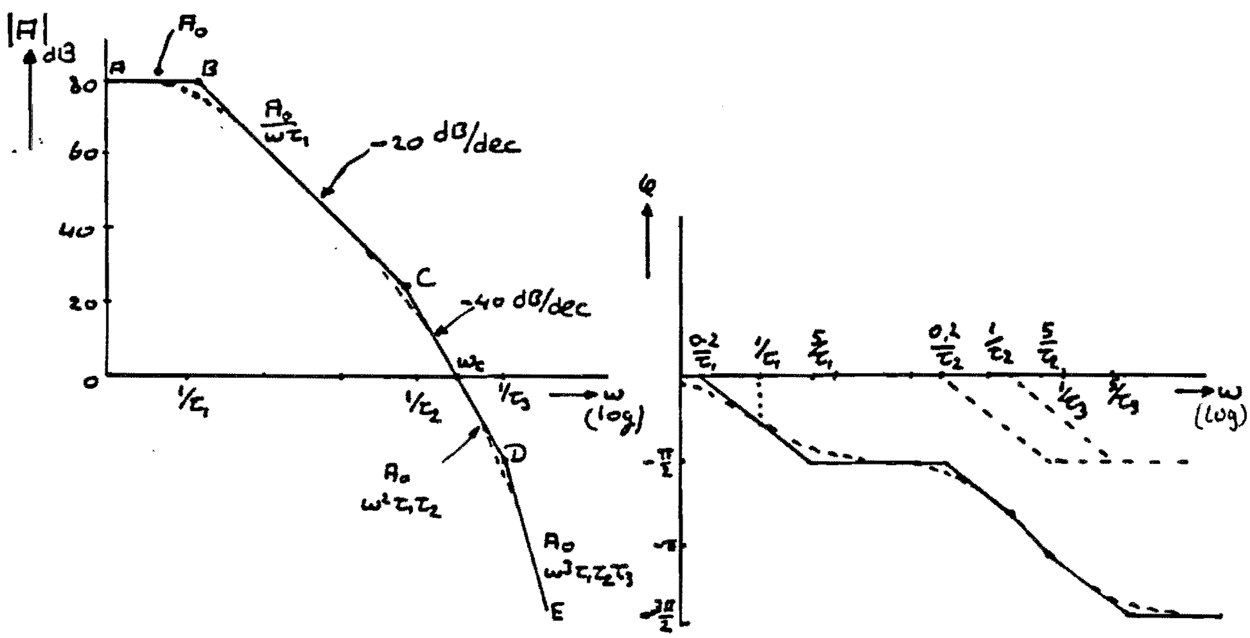


Figure 3.1.

The magnitude of the amplification between A and B is given by A_o.
Between B and C it is given by $\frac{A_o}{\omega\tau_1}$ because

$$20\log|A|=20\log A-20\log\omega\tau_1$$

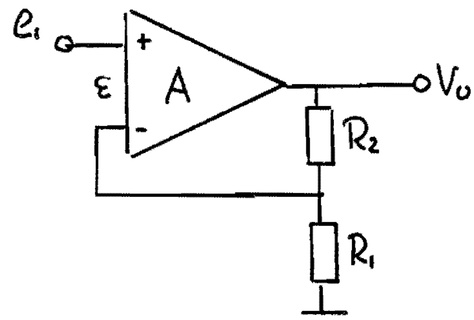
Between C and D the function is given by $|A|=\frac{A_o}{\omega^2\tau_1\tau_2}$,etc.

3.2. Negative feedback

Negative feedback is, in general, used to improve linearity, stability, input or output resistances. In all these cases the improvements are related to the loop gain. The loop gain is defined as the product of the open-loop gain of the op.amp. and the transfer function of the feedback network.
That is, Loop gain $= A_o\beta$.

We will first analyse the gain characteristic of an inverting and a non-inverting amplifier.

3.2.1. Non-inverting amplifier



non-inverting amplifier

Figure 3.2.

Assume that the effects due to the input and output resistances of the op.amp. can be neglected.

We will first calculate the closed loop gain A'.

The transfer function of the feedback $\beta = \frac{R_1}{R_1+R_2}$

$$\varepsilon = e_1 - \beta V_0$$

$$V_o = A\varepsilon = A(e_1 - \beta V_0)$$

Let us assume that A is given by A = $\frac{A_0}{1+j\omega\tau_1}$

We now can express the closed-loop gain in A_o, β and $\omega\tau_1$, that is

$$A' = \frac{A}{1+\beta A} = \frac{\frac{A_0}{1+j\omega\tau_1}}{1+\beta\frac{A_0}{1+j\omega\tau_1}} = \frac{A_0}{1+j\omega\tau_1+\beta A_0} = \frac{A_0}{1+\beta A_0}\cdot\frac{1}{1+j\omega\frac{\tau_1}{1+\beta A_0}}$$

$$A' = A_0'\,\frac{1}{1+j\omega\tau_1/1+\beta A_0} \qquad \text{with} \quad A_0' = \frac{A_0}{1+\beta A_0}$$

The term A_o' is constant, the second term frequency dependent.

The open loop gain is shown in the figure by the solid line.

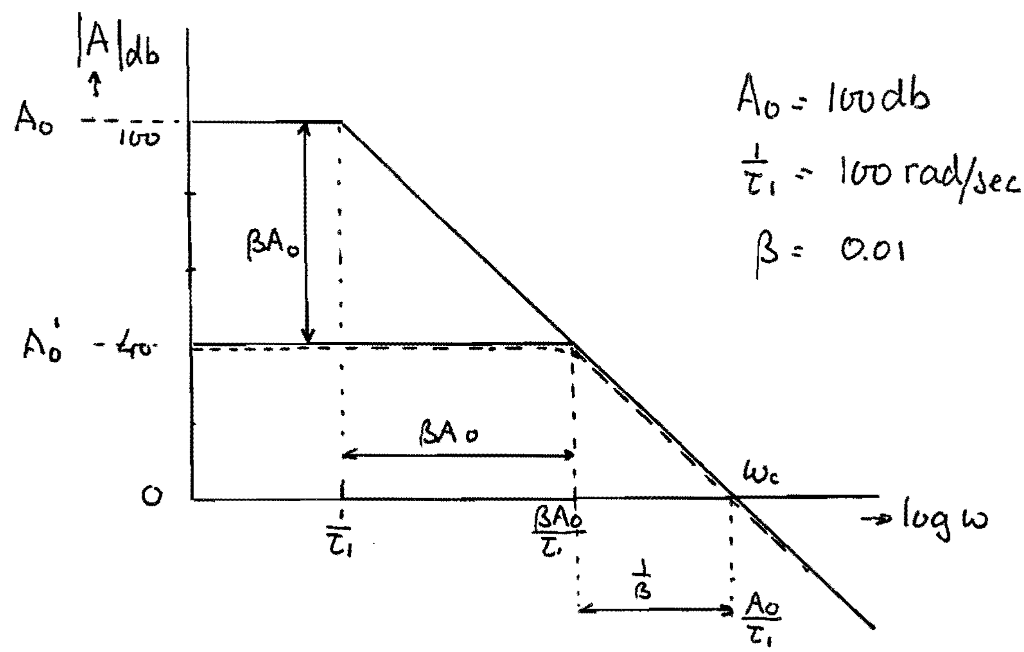


Figure 3.3.

The -3db point is at $\frac{1}{\tau_1}$

Therefore $BW = \frac{1}{2\pi\tau_1}$

Furthermore UGBW $= \frac{\omega_c}{2\pi}$

The closed-loop gain is given by A' $= \frac{A_o}{1+\beta A_o} \cdot \frac{1}{1+j\omega\frac{\tau}{1+\beta A_o}}$

and represented by the dashed-solid line.

a) For low frequencies the term $\frac{1}{1+j\omega\frac{\tau}{1+\beta A_o}} \sim 1$
and $A' = \frac{A_o}{1+\beta A_o} = A_o'$

When we divide two quantities it means in the logarithmic scale just a subtraction of two distances.
Therefore the quantity A_o' is obtained by taking A_o minus the distance Aβ. The distance Aβ corresponds to $1 + \beta A_o$. This quantity is called the loop gain.

Furthermore for very large values of βA_o :

$$A_o' = \frac{A_o}{1+\beta A_o} = \frac{1}{\beta}$$

(at low frequencies).

The accuracy is determined by the loop gain.

b) For high frequencies the term $\dfrac{1}{1+j\omega\frac{\tau_1}{1+\beta A_0}}$ will contribute to A'.

There is a breakpoint in this case at $\dfrac{1}{\frac{\tau_1}{1+\beta A_0}} \approx \dfrac{\beta A_0}{\tau_1}$

The unity gain point is at $\omega_c = \dfrac{A_0}{\tau_1}$

3.2.2. Gain-Bandwidth product.(GBW)

The bandwdith changes as the closed-loop gain varies. The product of the closed-loop gain and -3dB frequency (BW) is constant for a given op.amp. This is called the gain band-width product.

$$GBW = A'_{ideal} \times BW = \text{constant}.$$

Example: The non-inverting amplifier.

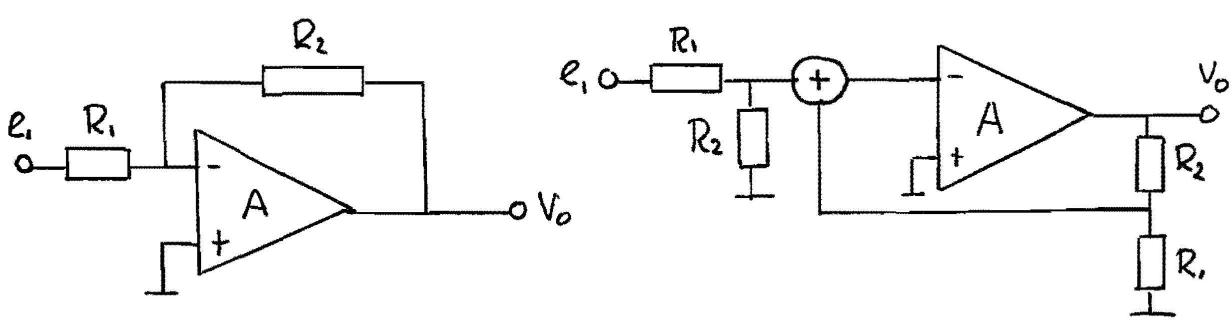
without feedback:

$$\left.\begin{aligned} BW &= \frac{1}{2\pi\tau_1} \\ A_0 & \end{aligned}\right\} GBW = \frac{A_0}{2\pi\tau_1} = \frac{\omega_c}{2\pi} = UGBW$$

with feedback:

$$\left.\begin{aligned} BW &= \frac{1+\beta A_0}{2\pi\tau_1} \\ A_0' &= \frac{A_0}{1+\beta A_0} \end{aligned}\right\} GBW = \frac{1+\beta A_0}{2\pi\tau_1}\cdot\frac{A_0}{1+\beta A_0} = \frac{\omega_c}{2\pi} = UGBW.$$

3.2.3. Inverting amplifier



inverting amplifier replace circuit

Figure 3.4.

The magnitude Bode plot of an inverting amplifier can easily be found in two steps:

a) feedback

It is clear that without attenuation the Bode plot equals the plot belonging to the non-inverting amplifier.
Or

$$A_0' = \frac{A_0}{1+\beta A_0} \sim \frac{1}{\beta}$$

With the breakpoint at $\frac{\beta A_0}{\tau_1}$ and ω_c at $\frac{A_0}{\tau_1}$

See fig. 3.5, solid curve.

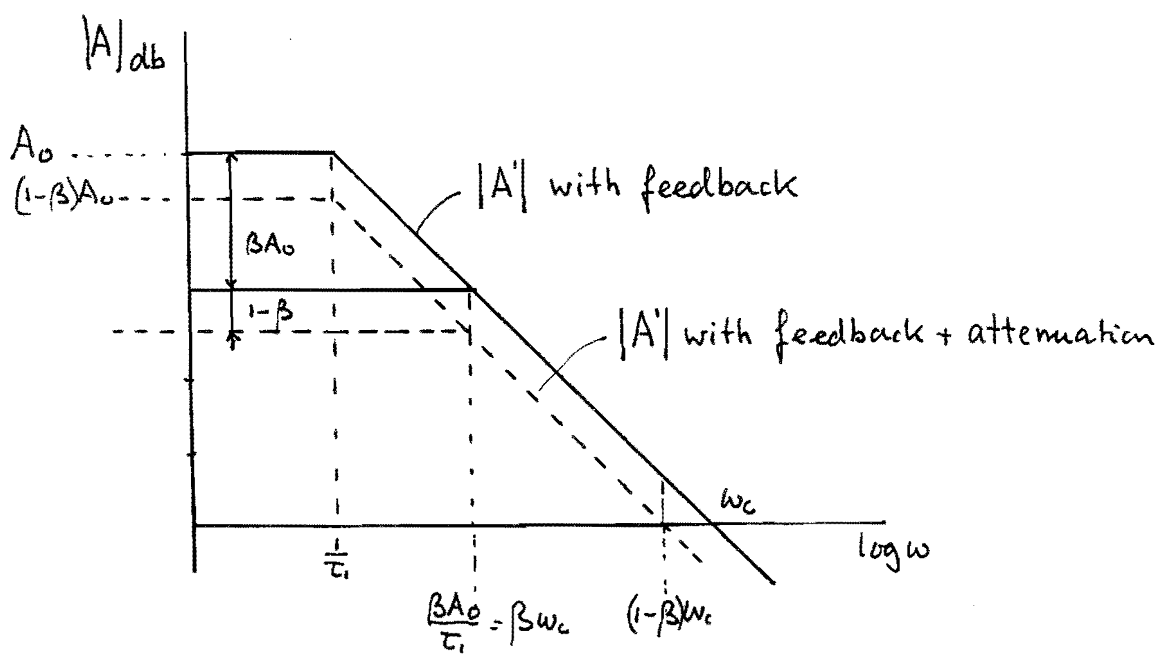


Figure 3.5.

b) Attenuation

The effect of the attenuator 1-β can be represented in the Bode plot by shifting the curve downwards with 1-β, see fig. 3.5, dashed curve.

The gain is reduced by this factor and becomes

$$A_0' = (1-\beta)\,\frac{A_0}{1+\beta A_0} = (1-\beta)\,\frac{1}{\beta}$$

By analysing the Bode plot we observe that the breakpoint remains unchanged $\frac{\beta A_0}{\tau}$ but that ω_c is reduced: $\omega_c = \frac{A_0}{\tau_1}\cdot(1-\beta)$

c) Calculation of Gain bandwidth product.(GBW).

1) Without feedback

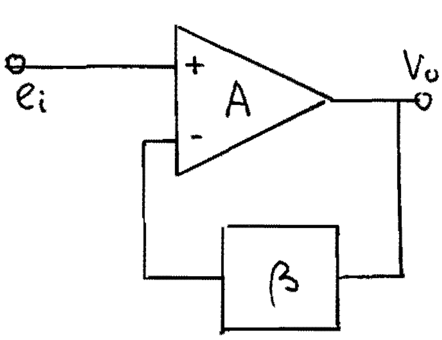
$$\left.\begin{array}{ll} BW = \frac{1}{2\pi\tau_1} & \\ \text{Amplification:} & A_0 \end{array}\right\} GBW = A_0 \frac{1}{2\pi\tau_1} = \frac{\omega_c}{2\pi} = UGBW.$$

2) With feedback.

$$\left.\begin{array}{l} BW = \frac{\beta A_0}{2\pi\tau_1} \\ A_0^{\prime} = \frac{1-\beta}{\beta} \end{array}\right\} GBW = \frac{1-\beta}{\beta} \cdot \frac{\beta A_0}{2\pi\tau_1} = (1-\beta)\frac{\omega_c}{2\pi} = UGBW(1-\beta)$$

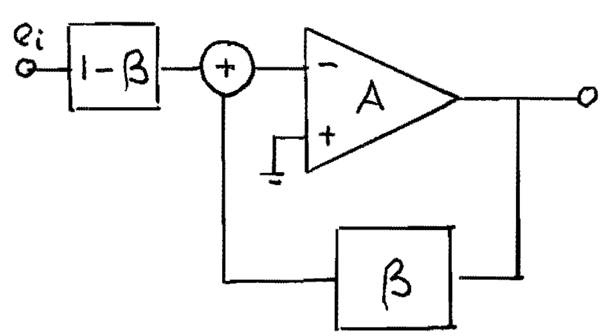
3.2.4. Block diagrams

Block diagrams consist of undirectional operational blocks which represent a transfer function of the variables of interest.
The block diagrams are given for the inverting and non-inverting amplifier configurations in the figure.



non-inverting amplifier

$$A^1 = \frac{A}{1+\beta A}$$



inverting amplifier

$$A^1 = \frac{-(1-\beta)A}{1+\beta A}$$

Example: 741C

$A_o=10^5$, $fc=10^6$Hz,(first order amplification characteristic.)

$\beta = 0.1$

non-inverting amplifier	inverting amplifier
$A_o' = \frac{A_0}{1+\beta A_0} = 10$	$A_o' = \frac{(\beta-1)A}{(1+\beta)A} \sim \frac{\beta-1}{\beta} = -9$
$BW = \beta f_c = 0{,}1MHz$	$BW = \beta f_c = 0{,}1MHz$
$UGBW = f_c = 1MHz$	$UGBW = (1-\beta)f_c = 0{,}9MHz$
loop gain $\beta A_o=10^4$	loop gain $\beta A_o = 10^4$

3.2.5. Voltage gain errors due to the finite value of the C.M.R.R. and the gain

a) Inverting amplifier.

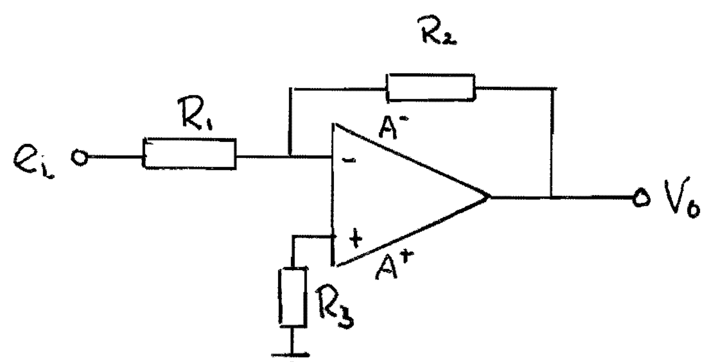


Figure 3.7.

In chapter 2.3.5. it has been shown that the effects of finite gain and finite C.M.R.R. can be calculated by considering the transfer characteristics of the - input and + input to the output (A_- and A_+) unequal and not infinite.

With
$$A_- = A - \frac{\Delta A}{2}$$
$$A_+ = A + \frac{\Delta A}{2}$$

and the C.M.R.R.: $H = \frac{A}{\Delta A}$

The transfer characteristics can be found from the relation:

$$-\left[e_i(1-\beta)+\beta V_0\right]A_- = V_0$$

or

$$A' = \frac{V_0}{e_i} = -(1-\beta)\frac{A^-}{1+A_-\beta} = -(1-\beta)\,\frac{A\left(1-\frac{\Delta A}{2A}\right)}{1+\beta A\left(1-\frac{\Delta A}{2A}\right)}$$

$$= -(1-\beta)\,\frac{A\left(1-\frac{1}{2H}\right)}{1+\beta A\left(1-\frac{1}{2H}\right)}$$

dividing the numerator and the denominator by $\beta A\left(1-\frac{1}{2H}\right)$ gives:

$$A' = -\frac{1-\beta}{\beta}\cdot\frac{1}{1+\frac{1}{\beta A\left(1-\frac{1}{2H}\right)}}$$

In general $\beta A\left(1-\frac{1}{2H}\right) \gg 1$, therefore $A' = -\frac{1-\beta}{\beta}.$

The C.M.R.R. is not important since there is no common mode signal at the inputs.

b) Non-inverting

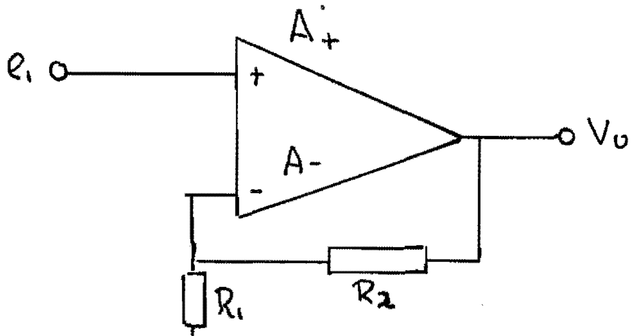


Figure 3.8.

$$e_i A^+ - \beta A^- V_0 = V_0 \quad ; \quad A' = \frac{V_0}{e_i} = \frac{A^+}{1+\beta A^-} =$$

$$\frac{A\left(1+\frac{\Delta A}{2A}\right)}{1+\beta A\left(1-\frac{\Delta A}{2A}\right)} = \frac{A\left(1+\frac{1}{2H}\right)}{1+\beta A\left(1-\frac{1}{2H}\right)} = \frac{1}{\beta}\cdot\frac{1+\frac{1}{2H}}{1-\frac{1}{2H}}\cdot\frac{1}{1+\frac{1}{\beta A\left(1-\frac{1}{2H}\right)}}$$

if $\frac{1}{2H} \ll 1$ and $\beta A \gg 1$ we get:

$$A' \sim \frac{1}{\beta}\left(1+\frac{1}{2H}\right)^2\left(1-\frac{1}{\beta A}\right)$$

$$A' \sim \frac{1}{\beta}\left(1+\frac{1}{H}\right)\left(1-\frac{1}{\beta A}\right)$$

$$A' \sim \frac{1}{\beta}\left(1+\left|\frac{1}{H}\right|+\left|\frac{1}{\beta A}\right|\right)$$

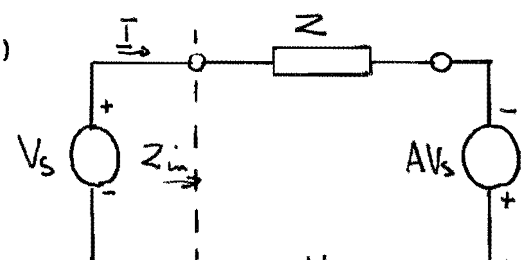
The error due to finite values of H and A

is $\left|\frac{1}{H}\right| + \left|\frac{1}{\beta A}\right|$

3.2.6. Impedance transformer

In general the input and output impedance will change if feedback is applied.
In most of the cases one can catagorize then to one of the following basic circuits.

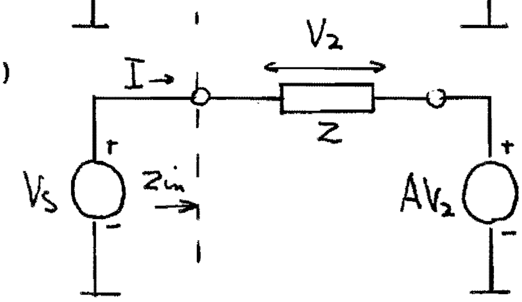
a)



$$Z_{in} = \frac{V_s}{I} = \frac{V_s}{V_z/Z} = \frac{V_s}{\frac{V_s + AV_s}{Z}}$$

$$Z_{in} = \frac{Z}{1+A}$$

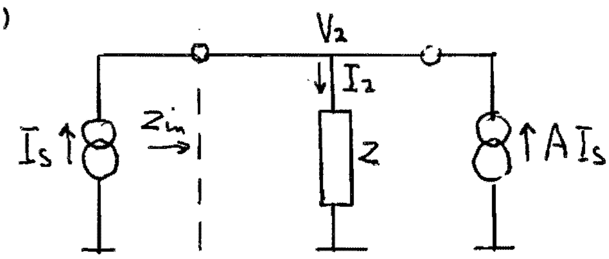
b)



$$Z_{in} = \frac{V_s}{I} = \frac{V_z + AV_z}{I} = \frac{V_z(1+A)}{I}$$

$$Z_{in} = (1+A)Z$$

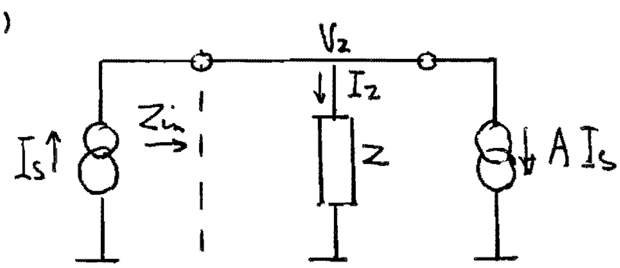
c)



$$Z_{in} = \frac{V_z}{I_s} = \frac{Z I_z}{I_s} = \frac{(I_s + AI_s)Z}{I_s}$$

$$Z_{in} = (1+A)Z$$

d)



$$Z_{in} = \frac{V_z}{I_s} = \frac{V_z}{I_z + AZ_z} = \frac{V_z}{I_z(1+A)}$$

$$Z_{in} = \frac{Z}{1+A}$$

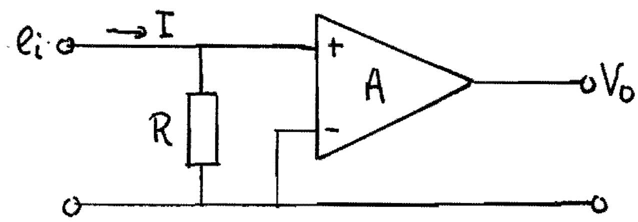
Figure 3.10.

With this knowledge impedance can be made smaller or larger and this technique is very useful in measuring systems.

One is also be able to create infinite impedances. This can be done by taking A = -1 in case a) and d).

Of course there will be a great chance for parasitair oscillation.

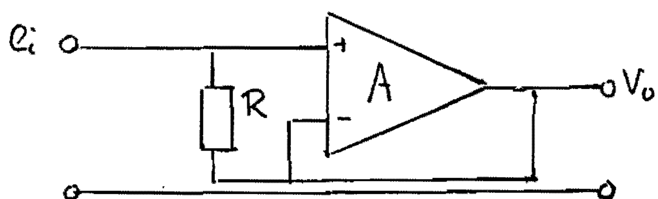
Example 1:



$$\frac{V_o}{e_i} = A$$

$$Z_{in} = \frac{e_i}{I} = R$$

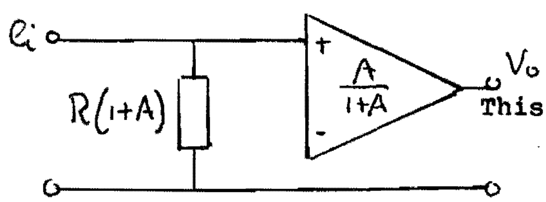
with feedback



$$\frac{V_o}{e_i} = \frac{A}{1+A}$$

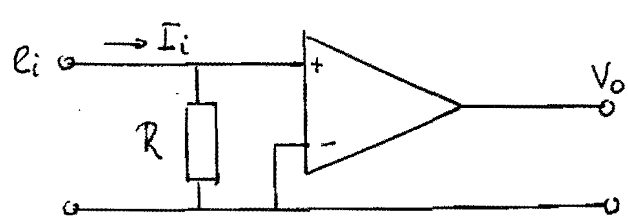
$$Z_{in} = \frac{e_i}{I} = (1+A)R$$

An equivalent circuit can be drawn as follows:



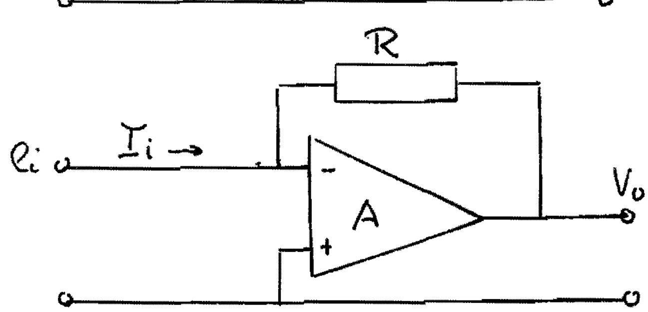
This is a voltage follower.

Example 2:



$$\frac{V_o}{e_i} = A$$

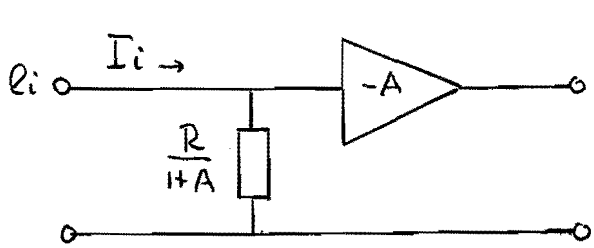
$$Z_{in} = \frac{e_i}{I_i} = R$$



$$\frac{V_o}{e_i} = -A$$

$$Z_{in} = \frac{e_i}{I_i} = \frac{R}{1+A}$$

The input impedance has been reduced by a factor 1+A. The equivalent circuit can be drawn as follows:



3.3. Stability.

Consider the simplified negative feedback configuration shown in the figure:

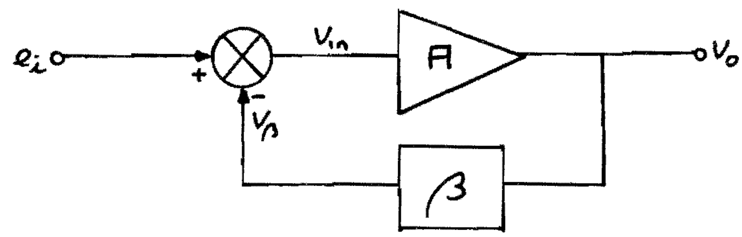


figure 3.11

It is a linear shift-invariant system with one feedback loop.

In this course we will discuss only direct coupled systems i.e. dc can pass directly from input to output. The phase shift at $\omega = 0$ in the loop gain is zero when ideal integrators are not presented in the loop.
From the figure we see:

$$\left.\begin{aligned} V_\beta &= \beta V_o \\ V_{in} &= e_i - V_\beta \\ V_o &= A V_{in} \end{aligned}\right\} \quad A' = \frac{V_o}{e_i} = \frac{A}{1+\beta A}$$

where $A\beta$ = loop gain

if $A\beta \gg 1$, A' can be approximated by $A' = \dfrac{1}{\beta}$

Instability of the amplifier results if the denominator of A' becomes zero. This happens when the open loop $A\beta = -1 = 1\angle 180°$, or $A = \dfrac{1}{\beta}\angle 180°$.

Under this condition A' is infinite indicating that an output results for no input signal and it is characterised by self-sustaining oscillation.
This type of instability can be avoided by reducing the phase shift of the feedback system to less than 180° when the loop gain is greater than or equal to unity.

To examine the effects of feedback on the stability of an amplifier we can use two methods:

1) Nyquist stability criterion.
2) Bode plot.

ad 1)
The Nyquist plot is a polar plot or polar diagram which is obtained by plotting the amplitude and phase of βA when the frequency ω is going from $-\infty$ via 0 to $+\infty$.

The critical point is the point where $\beta A = -1$.
The number of encirclements of the critical point, N, is found by counting circlements which are counterclockwise (c.c.w.) as negative and those which are clockwise (c.w.) as positive when the frequency ω goes $-\infty \rightarrow +\infty$.

The system is stable if the number of encirclements is zero.
Example:

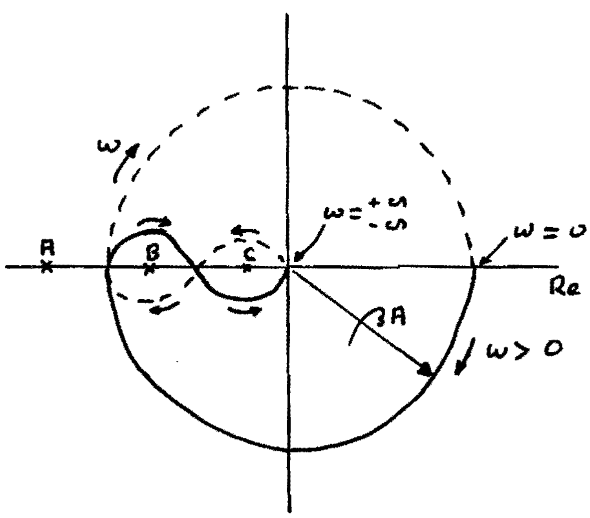


figure 3.12

A is the critical point; N=0; stable system.
B " " " " N=2; unstable system.
C " " " " N=1-1=0; stable system.

For situation C the system is conditional stable because when the dc gain is decreasing the system can become unstable.
In this course no further attention will be paid to the Niquist stability criterion.

ad 2)
The Bode plot of a linear system is much easier to draw than the Nyquist plot.
Especially when the loop gain contains only first-order factors of the shape $(1+j\omega\tau)$ or $j\omega\tau$.
The stability criterion can be reworded as follows:

a single linear feedback system will be stable if at unity loop-gain the phase shift is less than 180° at one frequency.

The following conditions must be fulfilled:

1) DC-feedback is negative (ω=0)
2) $\omega > 0$

Generally the transfer function of op.amps. contains only first-order factors ($1+j\omega\tau$ or $j\omega\tau$).
These are so-called minimum-phase-shift networks.
In these cases the Bode plot can be used to obtain or to improve stability.
The method makes use of two facts:

a) Piece-wise linear magnitude Bode plots with only factor ($1+j\omega\tau$) and $j\omega\tau$ show slopes of n. 20dB/dec or n.6 dB/oct.

b) A slope of n.20 dB/dec. corresponds with a phase shift of n.90°.

The point where the open-loop gain characteristic intersects the line $\frac{1}{\beta}$ is the point where the loop gain magnitude $|A\beta|$= 1 (or 0 dB).

Stability occurs if the phase shift at unity loop gain is less than 180°. We can improve the stability by approaching 90° phase-shift at unity loop gain.

Guidance rules to improve stability:

a) Obtain in the piece-wise linear Bode-plot a -20dB/dec slope near unity loop gain.
b) Increase the "length" of the -20dB/dec. slope near unity loop gain.

Example:

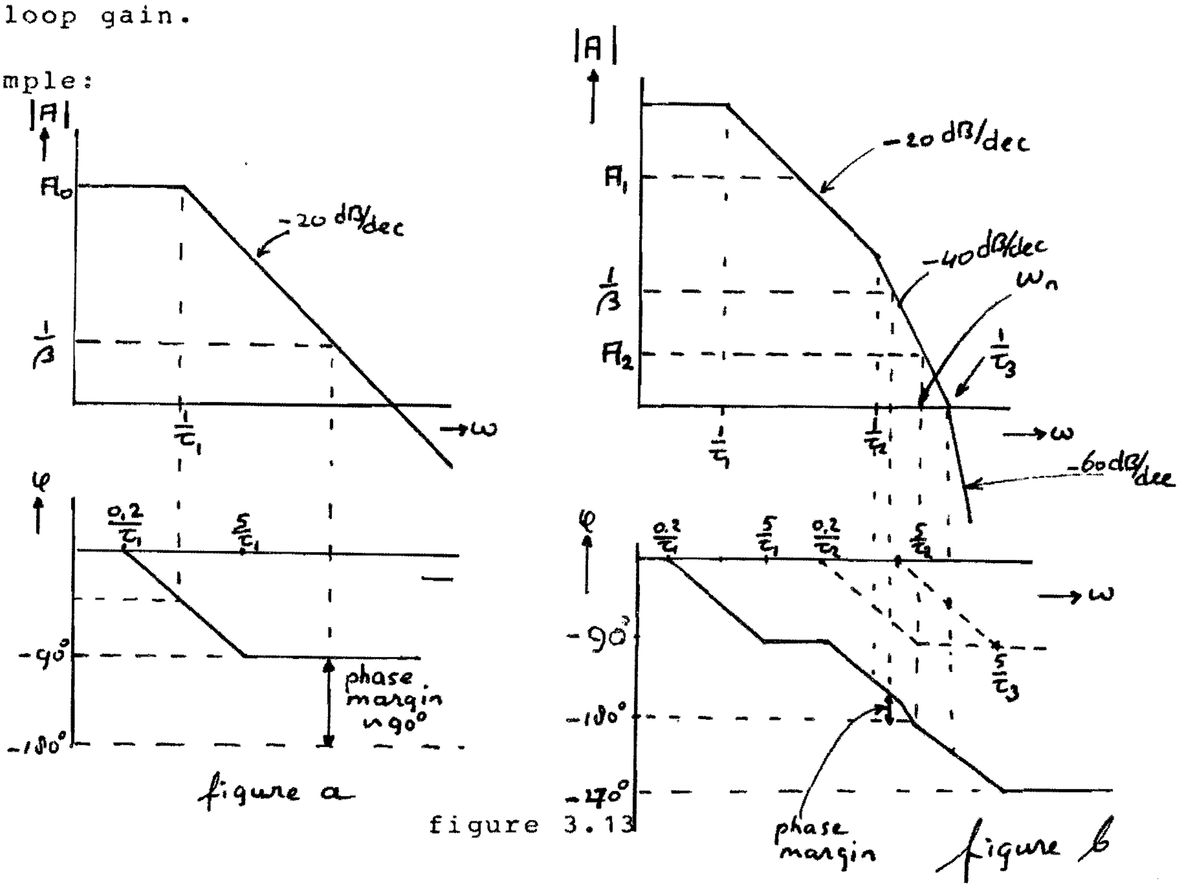


figure 3.13

As seen from figure (3.13a) for a one-pole op.amp. at the pole frequency $\frac{1}{\tau_1}$ the phase shift is -45°. The maximum phase shift is -90°; thus the circuit is always stable.

Now consider the case in figure (3.13b).
For $\left|\frac{1}{\beta}\right| = A_1$ the phase shift of the amplifier is -90° and the amplifier will be stable at this gain level. But for $\left|\frac{1}{\beta}\right| = A_2$ the phase shift is -180° and oscillation will result at a frequency ω_{osc} at intersection of the open loop gain characteristic and the $\frac{1}{\beta}$-line.
The amplifier will be unstable for $|1/\beta| < A_2$.
If the closed-loop gain levels have a value between A_1 and A_2 the phase shift will be between -90° and -180°, which can cause peaking and overshoot.
As $|A\beta|$ is made closer to unity at the frequency where the phase shift of $A\beta$ is -180° the amplifier will have a small margin for stability. The most widely just two quantities for stability analysis are phase margin and gain margin. They are defined as follows:

$$\underline{\text{phase margin}} = 180 - \varphi_{A\beta}$$

where $\varphi_{A\beta}$ is the phase shift of $A\beta$ at frequency where $|A\beta| = 1$. The phase margin must be greater than 0° for stability.
The gain margin is defined to be the value $|A\beta|$ in decibels at the frequency where the phase of $A\beta$ is - 180°.
The gain margin must be less than 0dB for stability. The phase margin for the amplifier having a single pole response is $\sim$-90° (see figure (3.13a)).
The feedback amplifier is shown in fig.b) with the feedback β has a phase margin $\varphi > 0$ which indicates that the system is stable. A typical value for the phase margin is 45°; a value of 60° is also commonly used.

3.4. The influence of feedback upon the amplitude-frequency characteristic.

In section (3.2) we have seen that the closed-loop gain of the amplifier with $R_i \to \infty$ and $r_o \to 0$ is given by

$$A' = \frac{A}{1+\beta A}$$

We can distinguish two cases:

a) The frequency dependency is determined by one time constant so $A = A_0/(1+j\omega\tau_1)$.
Substituting this in equation (above) we have seen that the gain is reduced but the bandwidth is a factor $1+\beta A_0$ larger with feedback (see section 3.2).

b) The frequency dependency is determined by two time constants

$$A = \frac{A_0}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

The closed-loop gain is then (with real β):

$$A' = \frac{A}{1 + \beta A} = \frac{A_0}{(1 + j\omega\tau_1)(1 + j\omega\tau_2) + \beta A_0}$$

$$A' = \frac{A_0}{(1 + \beta A_0) + j\omega(\tau_1 + \tau_2) + (j\omega)^2 \tau_1\tau_2}$$

with no feedback the stepresponse has the so-called S-shape. The **roots** of the denominator of A are always real: $p_1 = -\frac{1}{\tau_1}$ and $p_2 = -\frac{1}{\tau_2}$.

This means using the reverse Laplace transformation we can derive two exponential functions of time.

With feedback the expression for A' can contain real poles or two conjugate complex poles. Therefore the step response can be oscillatory (complex poles) or having the S-shape (real poles). However the denominator of A' cannot become zero. Therefore total oscillation never occurs.

In the following part it will be shown that the quadratic form is very useful for the investigation of stability of circuits with two time constants.

$$A' = \frac{A_0}{1 + \beta A_0 + j\omega(\tau_1 + \tau_2) + (j\omega)^2 \tau_1\tau_2}$$

$$= \frac{A_0}{1 + \beta A_0} \cdot \frac{1}{1 + j\omega \cdot \dfrac{\tau_1 + \tau_2}{\sqrt{(1+\beta A_0)\tau_1\tau_2} \cdot \dfrac{\sqrt{1+\beta A}}{\sqrt{\tau_1\tau_2}}} + (j\omega)^2 \cdot \left(\dfrac{\sqrt{\tau_1\tau_2}}{\sqrt{1+\beta A}}\right)^2}$$

Let $\dfrac{\tau_1 + \tau_2}{2\sqrt{(1+\beta A_0)\tau_1\tau_2}} = \xi \simeq \dfrac{\tau_1 + \tau_2}{2\sqrt{\beta A_0 \tau_1\tau_2}}$ (relative damping)

$$\sqrt{\frac{1 + \beta A_0}{\tau_1\tau_2}} = \omega_n \simeq \sqrt{\frac{\beta A_0}{\tau_1\tau_2}} \quad \text{(natural frequency)}$$

$$\frac{A_0}{1 + \beta A_0} = A_0' \quad \text{(closed-loop dc gain)}$$

so $A' = A_o' \dfrac{1}{1 + 2\frac{j\omega}{\omega_n}\cdot\xi + \left(\frac{j\omega}{\omega_n}\right)^2}$

where $\dfrac{1}{1 + 2\frac{j\omega}{\omega_n}\xi + \left(\frac{j\omega}{\omega_n}\right)^2}$ is called the quadratic factor.

The denominator of the QF is the characteristic equation which determines the response.
We will investigate this expression a little more.
Using the complex frequency S instead of we can write the characteristic equation as follows:

$$1 + 2\cdot\xi\cdot\frac{S}{\omega_n} + \frac{S^2}{\omega_n^2} = 0$$

There are two roots $S_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

From this expression one can see that the poles are conjugate complex if $\xi < 1$
If that is the case one can also prove that the overshoot is equal to $e^{-\pi\xi/(1-\xi^2)^{1/2}}$ (see appendix A). For several values of the relative damping the overshoot has been calculated.

	a	b	c	d
ξ	≥ 1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{4}$
overshoot	0	5%	17%	45%

ad a)
From the condition in the 2nd column we see that there is no overshoot (this means two real poles) if $\xi \geq 1$

$$\text{so } \xi = \frac{1}{2}\,\frac{\tau_1+\tau_2}{\sqrt{(1+\beta A_o)\tau_1\tau_2}} \simeq \frac{\tau_1+\tau_2}{2\sqrt{\beta A_o \tau_1\tau_2}} \geq 1$$

If $\tau_1 \gg \tau_2$ then $\sqrt{\dfrac{(\tau_1+\tau_2)^2}{\tau_1\tau_2}} \simeq \sqrt{\dfrac{\tau_1}{\tau_2} + \dfrac{\tau_2}{\tau_1} + 2} \simeq \sqrt{\dfrac{\tau_1}{\tau_2}}$

The condition becomes $\dfrac{1}{4}\,\dfrac{\tau_1}{\tau_2} \geq \beta A_o$.

and the limit is $\dfrac{\tau_1}{4\tau_2} = \beta A_o$.

In the Bode-plot the factor $\frac{1}{4}$ means -12dB, so $\frac{\tau_1}{4\tau_2}$ is the distance $\frac{\tau_1}{\tau_2}$ minus 12dB.

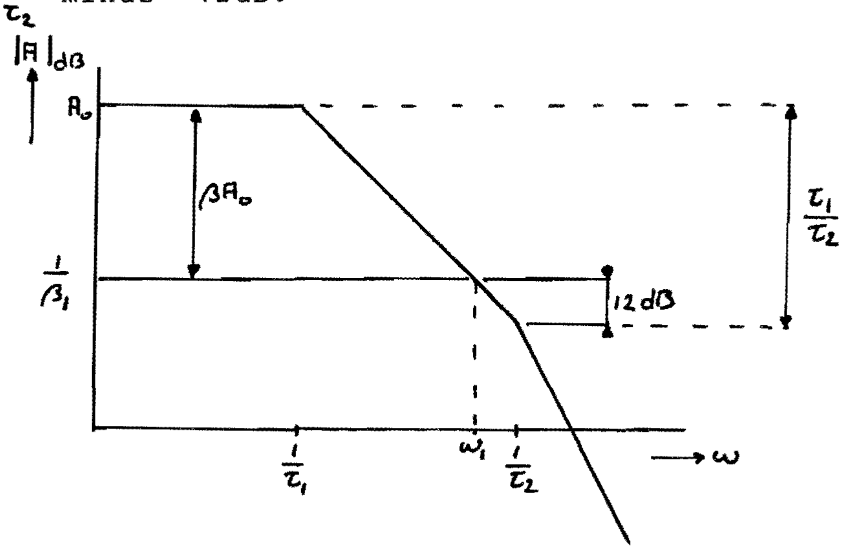


figure 3.14

The phase margin φ with feedback factor β_1, can easily be calculated

$$\left.\begin{array}{l}\text{phase} = 90^\circ + \operatorname{arctg} \omega_1 \tau_2 \\ \omega_1 = \frac{1}{4\tau_2}\end{array}\right\} \quad 90^\circ + \operatorname{arctg} \frac{1}{4} = 104^\circ$$

phase margin $= 180 - 104 = 76^\circ$.

ad b)

Peaking of the amplitude.

Let us analyse the amplitude characteristic of the QF.

$$A' = \frac{A_o'}{1 + 2j\xi\frac{\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2}$$

$$|A'| = \frac{A_o'}{\sqrt{1 - 2(1 - 2\xi^2)\left(\frac{\omega}{\omega_n}\right)^2 + \left(\frac{\omega}{\omega_n}\right)^4}}$$

There is no peaking in $|A'|$ if $-2\cdot(1 - 2\xi^2) \geq 0$

or $\xi \geqslant \frac{1}{\sqrt{2}} \Leftrightarrow \frac{1}{2}\sqrt{\frac{\tau_1}{\beta A_o \tau_2}} \geqslant \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{2}\cdot\frac{\tau_1}{\tau_2} \geqslant \beta A_o$

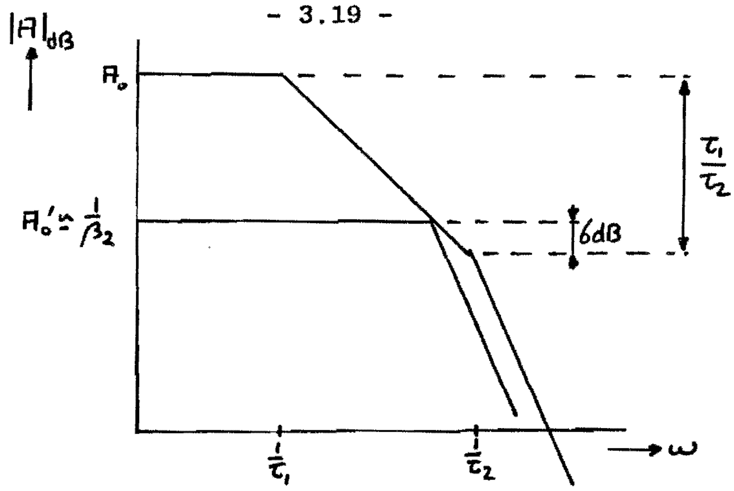


figure 3.15

no peaking if $\beta \leq \beta_2$

Limit value of peaking $\beta_2 A_o = \frac{\tau_1}{2\tau_2}$ (factor 1/2 is -6dB).
Phase margin at this point $\varphi = 180 - (90 + \text{arctg}\, \omega_2 \tau_2)$
with $\omega_2 = \frac{1}{2\tau_2}$ it becomes $\varphi = 180 - 90 - \text{arctg}\, \frac{1}{2} = 180° - 116° = 64°$.

ad c)
If $\xi < \frac{1}{\sqrt{2}}$ or $\frac{\tau_1}{2\tau_2} < \beta A_o$ there is a peak in the amplitude-frequency characteristic.

The peak value of A' occurs if the denominator has his minimum value:

$$\frac{d(\text{denominator})}{d\left(\frac{\omega}{\omega_n}\right)^2} = -2(1-2\xi^2) + 2\frac{\omega^2}{\omega_n^2} = 0$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 - 2\xi^2 \qquad \omega = \omega_p = \omega_n\sqrt{1-2\xi^2}$$

(If $\xi \ll 1$ than $\omega_p \simeq \omega_n$)

The amplitude as a function of the frequency is given by:

$$|A'| = \frac{A_o'}{\sqrt{1 - 2(1-2\xi^2)\left(\frac{\omega}{\omega_n}\right)^2 + \left(\frac{\omega}{\omega_n}\right)^4}}$$

At the peak the frequency $\omega = \omega_p = \omega_n\sqrt{1-2\xi^2}$ which can be substituted into A':

$$A'_{peak} = \frac{A_o'}{\sqrt{1 - 2(1-2\xi^2)^2 + (1-2\xi^2)^2}} = \frac{A_o'}{2\xi\sqrt{1-\xi^2}}$$

For case c) we have the condition $\beta A_o = \frac{\tau_1}{\tau_2}$
so

$$\xi = \frac{1}{2}\frac{\tau_1+\tau_2}{\sqrt{\beta A_o \tau_1 \tau_2}} = \frac{1}{2}\frac{\tau_1+\tau_2}{\tau_1} \simeq \frac{1}{2}$$

Substituting this into the equation of the peak value

$$|A'|\ \text{peak} = \frac{A_o'}{2\cdot\frac{1}{2}\sqrt{1-\left(\frac{1}{2}\right)^2}} = 1{,}15\, A_o'$$

(Factor 1.15 is 1.25dB.)

ad d)

For $\varsigma = 1/4$ $\qquad |A'|_{peak} = \dfrac{1/\beta_4}{2 \cdot 1/4 \sqrt{1-(1/4)^2}} \approx \dfrac{2}{\beta_4}$

this is a peak of 6db.

For $\varsigma < 1/4$ $\qquad |A'|_{peak} \sim \dfrac{1/\beta}{2\varsigma}$

Summary:

As discussed in this chapter and in appendix A, the closed loop frequency characteristic of an op. amp. with two time constants and a real feedback β can be described by

$$A' = \frac{A_0}{1+\beta A_0} \cdot \frac{1}{1 + 2\varsigma \frac{j\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2} \qquad \text{with}$$

if $\tau_1 \gg \tau_2$ $\qquad \varsigma = \frac{1}{2}\sqrt{\frac{\tau_1}{\beta A_0 \tau_2}}$ $\qquad$ (relative damping)

$$\omega_n = \sqrt{\frac{\beta A_0}{\tau_1 \tau_2}} \qquad \text{(natural frequency)}$$

If $\varsigma < \frac{1}{\sqrt{2}}$ there is a peak in the amplitude frequency characteristic at:

$$\omega_p = \omega_n \sqrt{1-2\varsigma^2}$$

$$A'\text{peak} = \frac{A_0}{1+\beta A_0} \; \frac{1}{2\varsigma\sqrt{1-\varsigma^2}}$$

with value $\qquad = \dfrac{1}{\beta} \; \dfrac{1}{2\varsigma\sqrt{1-\varsigma^2}}$

Applying the Laplace transform we found the pulse response: with a resonance frequency

$$\omega_r = \omega_n \sqrt{1 - \xi^2}$$

and an overshoot: $$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

The table gives those values for several loopgains βA_o

loopgain	phase margin				
βA_o	φ	ξ	bode peaking	step response	overshoot
$< \frac{\tau_1}{4\tau_2}$		> 1	no peak 2 breakpoints	S-shape	no
$\frac{\tau_1}{4\tau_2}$	$\sim 75°$	1	no peak 2 breakpoints	critical S-shape	no
$\frac{\tau_1}{2\tau_2}$	$\sim 64°$	$\frac{1}{\sqrt{2}}$	no peaking 1 breakpoint Butterworth	oscillatory	5%
$\frac{\tau_1}{\tau_2}$	$\sim 45°$	1/2	1,25db	oscillatory	17%
$4\frac{\tau_1}{\tau_2}$	$\sim 26°$	1/4	6 db	oscillatory	45%

The relation between these resonances and peaks can easily be verified by measurements if $\zeta < 1/4$.

In that case:

$$\omega_n \approx \omega_p \approx \omega_r$$

$$A'\text{peak} = \frac{1}{\beta}\,\frac{1}{2\zeta}$$

$$\text{overshoot} = e^{-\zeta\pi}$$

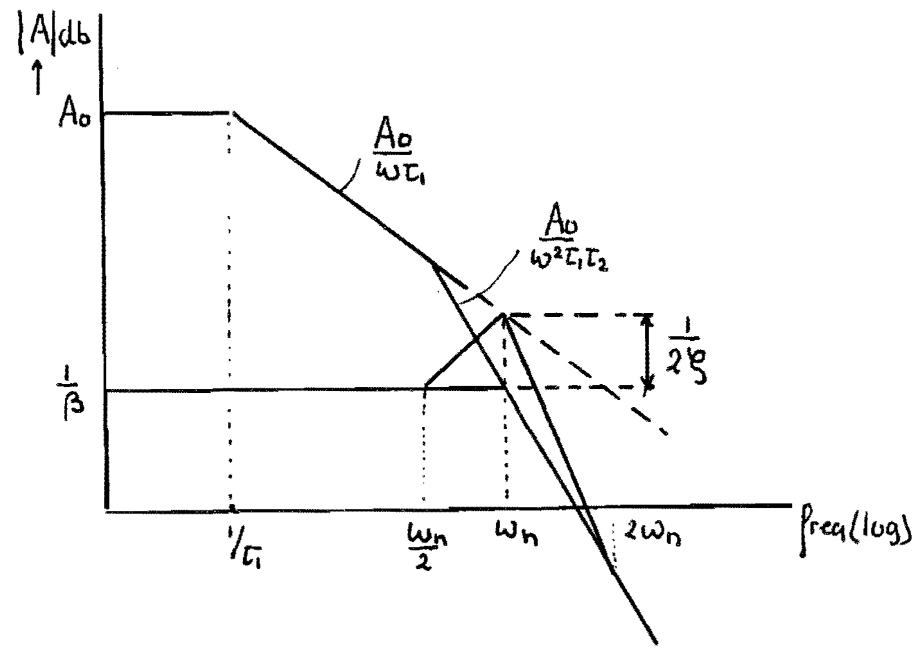


figure 3.16

ζ can be measured by the construction as indicated in the figure. It can easily be proven that the Bode peak touches the production of the line $\frac{A_o}{\omega\tau_1}$ at ω_n. The peak value is $\frac{1}{2\zeta}$ (Db).

The value of ζ, found in this way can be verified by the overshoot of the pulse response.

4. Compensation technique

4.1.

In the previous discussion the feedback element was resistive, which of course does not introduce any phase shift.

Therefore examination of the amplifier phase shift is sufficient to ensure stability. It was shown that the gain A should have a frequency characteristic with a roll-off with a maximum slope of -20dB/dec down to the U.G.B.W.

However, if reactive elements are used in the feedback the overall phase response as well as the closed-loop gain of the amplifier can be modified. This new phase response and the loop-gain of the feedback circuit should be used to determine the condition for stability.
This process is called compensation technique.

Some of the op.amps. are internally compensated. Manufacturers do this by installing a small capacitor, typically 30pF, within the op.amp. during the manufacturing process. This internal compensating capacitor reduces the gain of the op.amp. as frequency increases and therefore prevents oscillations at high frequencies.
Examples for internally compensated amplifiers are the 741, 747 and 766.

Op.amps. without internal compensation are also available. The 101A, 702 and 777 are examples of these. In this case some poles and/or zeros are added to the frequency response of the open loop gain by connecting external elements to the op.amp. The phase shift introduced by β and/or A_o can then be compensated to ensure stability and to limit any peaking in the closed-loop frequency response.

An op.amp. usually contains two gain stages followed by a unity gain buffer.

In fig. 4.1. the gain stages are schematically shown.

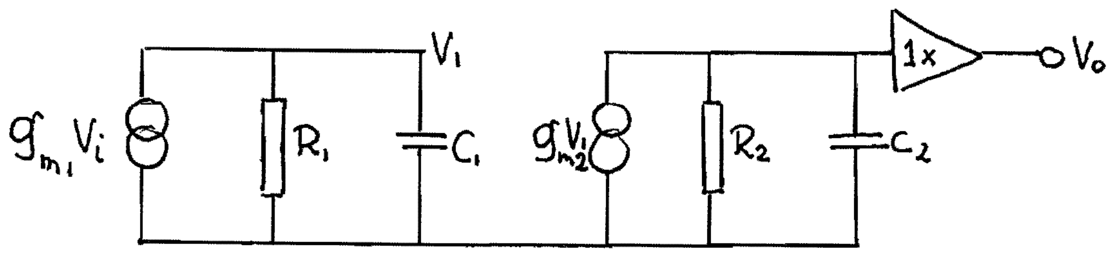


Fig. 4.1. Circuit diagram of op.amp.

The transfer function is given by

$$A = \frac{V_o}{V_i} = \frac{g_{m1}R_1}{1 + j\omega R_1C_1} \cdot \frac{g_{m2}R_2}{1 + j\omega R_2C_2} = \frac{A_{10}}{1 + j\omega\tau_1} \cdot \frac{A_{20}}{1 + j\omega\tau_2}$$

with
$$A_{10} = g_{m1}R_1 , \; A_{20} = g_{m2}R_2 \quad , \; \tau_1 = R_1C_1 \; , \; \tau_2 = R_2C_2$$

In op.amps. the dominant pole is generally determined by R_1C_1.

An identical transfer function can be realized by two op.amps. with feedback applied in series,

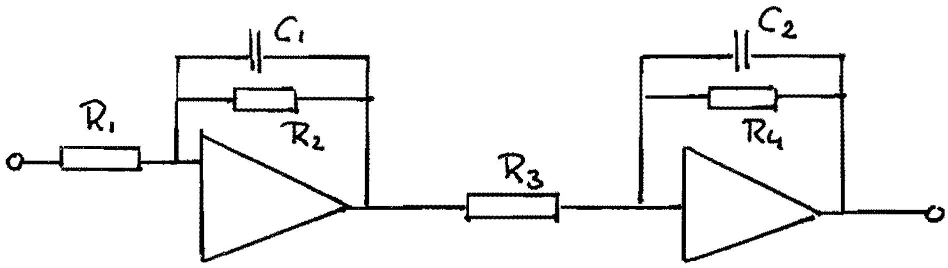
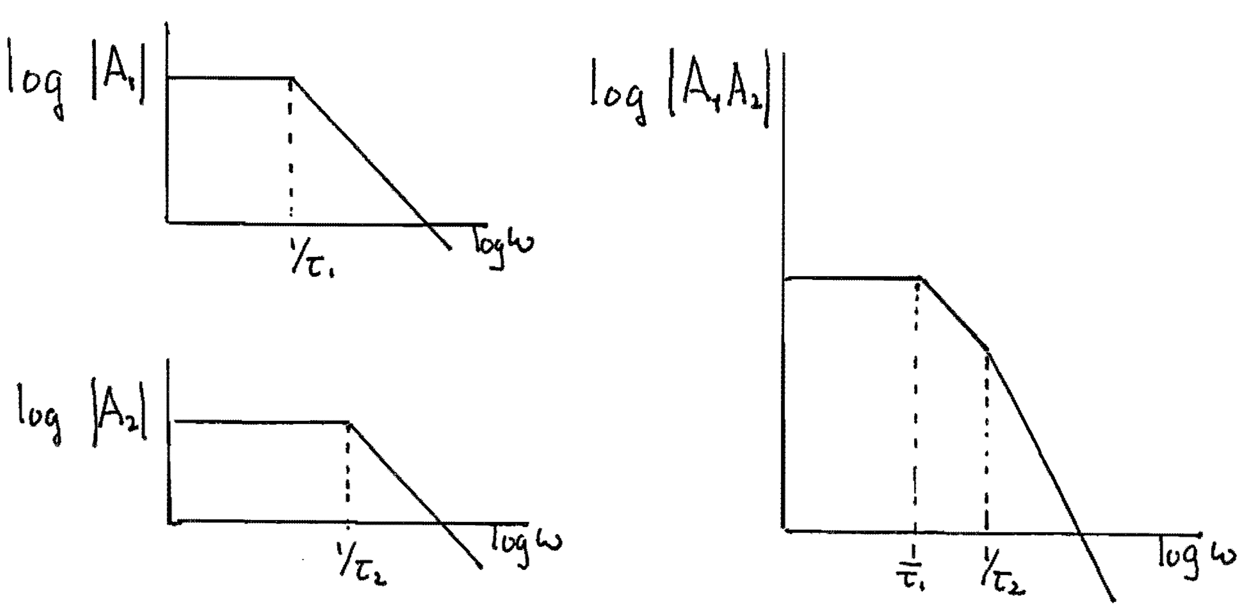


Fig. 4.2. Two op.amps. in series.

with
$$A_{10} = \frac{R_2}{R_1} \quad ; \quad A_{20} = \frac{R_4}{R_3}$$

and
$$\tau_1 = C_1R_2 \quad ; \quad \tau_2 = C_2R_4$$

The Bode plot of the single two stage op.amp. and of the two op.amps in series is illustrated in fig. 4.3.



From above it will be clear that phase compensation can be studied by using one of the configurations.

Various methods of phase compensation exist (G&M pg. 543, 9.4.2.).

1) Connecting a passive RC network (RC-shunt compensation).
2) Using Miller effect, multiplication of a capacitor (Miller compensation, pole-splitting).
3) Using feedforward technique.

4.1.1. RC-shunt compensation

Consider again the amplifier whose gain is shown in fig. 4.3.

Assume that a compensation has to be applied in such a way that no peaking occurs for β = 1 and that the UGBW is as large as possible. In chapter 3 we have seen that this requirement means

$$\frac{\tau_A}{\tau_B} = k\beta A_0 \quad \text{with } k=2$$

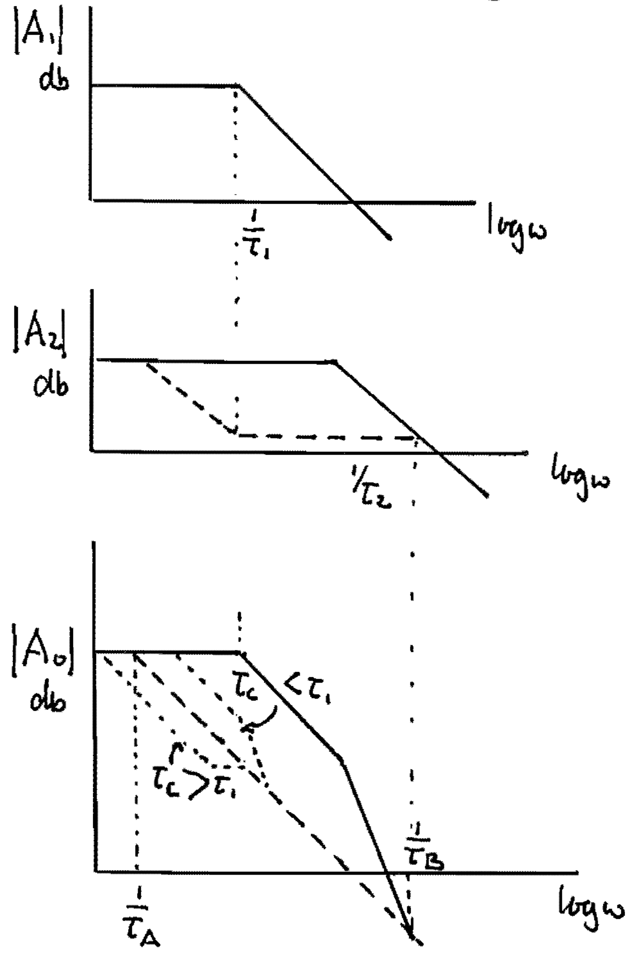


fig 4.3

From the Bode plot (fig. 4.3) it can easily be derived that such a compensation can be obtained by correcting the transfer characteristic of the broader stage.

This can be achieved by changing the original characteristic

$$A_2 = \frac{A_{20}}{1+j\omega\tau_2} \quad \text{into} \quad \frac{A_{20}(1+j\omega\tau_c)}{(1+j\omega\tau_A)(1+j\omega\tau_B)}$$

under the condition

$$\frac{\tau_A}{\tau_B} = k\beta A_1 A_2 \qquad \text{and} \qquad \tau_c = \tau_1$$

Choosing $\tau_c \neq \tau_1$ gives deviations in the Bode plot as indicated by the dashed lines.

Such a correction can be realized by adding a RC-shunt in parallel with C_2 and R_4 as shown.

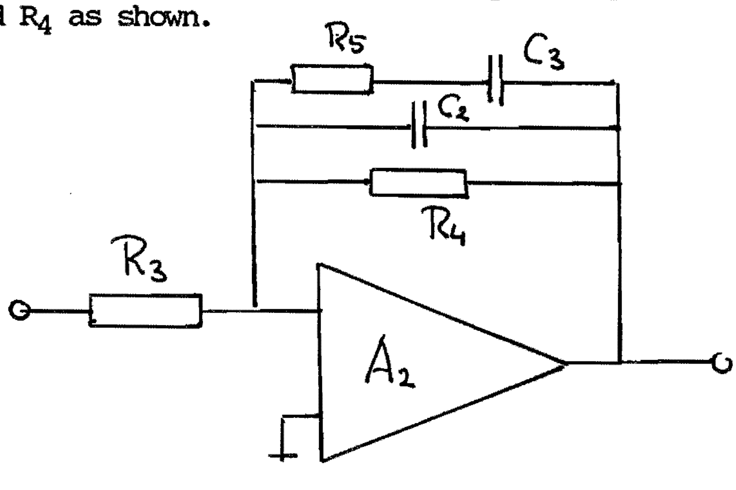


fig 4.4

The transfer function becomes now

$$A_2 = \frac{R_4}{R_3} \cdot \frac{1 + j\omega R_5 C_3}{1 + j\omega\left(R_4(C_2+C_3) + C_3 R_5\right) + j^2\omega^2 C_2 C_3 R_4 R_5}$$

The exact values of the poles τ_A and τ_B can be calculated and are given by

$$\tau_A, \tau_B = \frac{2 R_4 R_5 C_2 C_3}{R_4 C_2 + R_4 C_3 + R_5 C_3 \pm \sqrt{(R_4 C_2 + R_4 C_3 + R_5 C_3)^2 - 4 R_4 R_5 C_2 C_3}}$$

In practice we can approximate these poles by using the relations:

$$\tau_A + \tau_B = R_4(C_2+C_3) + C_3R_5$$
$$\tau_A \cdot \tau_B = C_2C_3R_4R_5$$
$$\text{and:} \quad \tau_A \gg \tau_B$$

This gives

$$\tau_A = R_4(C_2+C_3) + C_3R_5$$
$$\tau_B = \frac{C_2C_3R_4R_5}{R_4(C_2+C_3)+C_3R_5}$$

From the Bode plot (fig. 4.3.) can be derived that

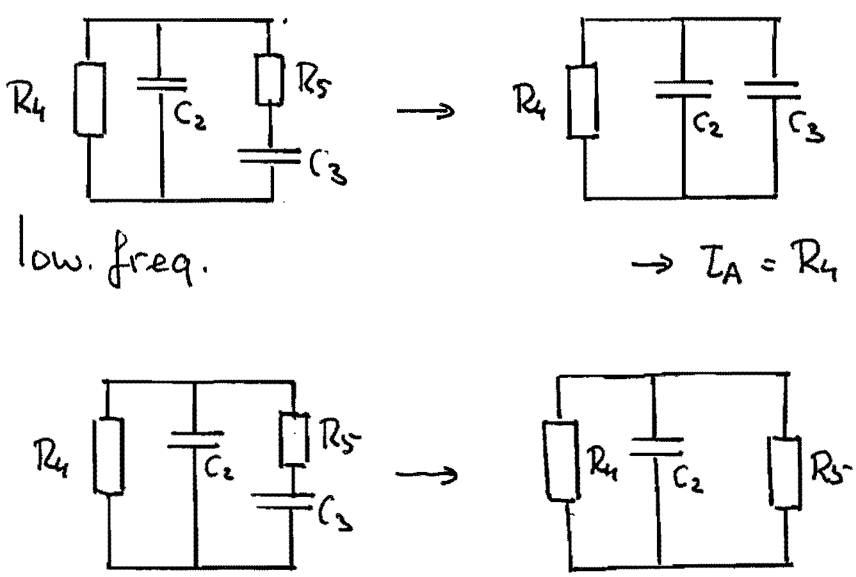
$$\tau_A > \tau_1 \quad \text{or} \quad \tau_A > R_5C_3 \quad \text{and}$$

$$\tau_2 \ll \tau_1 \quad \text{or} \quad R_4C_2 < R_5C_3$$

With these assumptions we obtain for τ_A and τ_B

$$\tau_A = R_4(C_2+C_3) \quad , \quad \tau_B = \frac{R_4R_5}{R_4+R_5} \cdot C_3$$

The same results can be found much easier considering the network and drawing the replacement diagrams for low and high frequencies as shown in fig. 4.5.



$$\rightarrow \tau_A = R_4(C_2+C_3)$$

$$\rightarrow \tau_B = \frac{R_4R_5}{R_4+R_5} \cdot C_2$$

fig 4.5

The values of R_5 and C_3 can now be calculated using the relations

$$\frac{\tau_A}{\tau_B} = k\beta A_{10}A_{20} \quad \text{and} \quad \tau_1 = R_3 C_5$$

The same compensation can be obtained in an op.amp. as shown in fig. 4.1. by shunting R_2C_2 by a series RC-network.

<u>Example</u> Given an op.amp. (fig. 4.1.) with

$$R_1 = 10^6\Omega \ , \ C_1 = 10^{-11}F \ , \ \tau_1 = 10^{-5}\,sec \ , \ g_{m1} = 10^{-3}\,A/V$$
$$R_2 = 10^5\Omega \ , \ C_2 = 10^{-11}F \ , \ \tau_2 = 10^{-6}\,sec \ , \ g_{m2} = 10^{-2}\,A/V$$

Questions:

1. Design a compensation in such a way that no peaking occurs for $\beta = 10^{-1}$ and that the UGBW is kept as large as possible.

2. Idem for $\beta = 1$ with a S-shape stepresponse.

<u>Solutions</u>

1.

$$\tau_A = R_2(C + C_2) \qquad \tau = RC = \tau_1$$

$$\tau_B = \frac{R_2 R}{R_2 + R} C_2$$

$$\frac{\tau_A}{\tau_B} = k\beta A_{10}A_{20} \ ; \ k = 2 \ , \ \beta = 0{,}1 \ , \ A_{10}A_{20} = 10^6$$

gives:

$$\frac{(R_2+R)(C+C_2)}{RC_2} = 2\,10^5$$

substitution of the op.amp. values results in

$$2R^2 - 11R - 10^6 = 0 \qquad \text{or}$$

$$R = \frac{10^3}{\sqrt{2}}\Omega \ , \ C = \sqrt{2}\,10^{-8}F$$

$$\tau_A = R_2(C + C_2) \approx \sqrt{2}\,10^{-3}\,sec \ , \ \frac{1}{\tau_A} = 700\ rad/sec$$

$$\tau_B = \frac{R_2 R}{R_2 + R} C_2 \approx \frac{1}{\sqrt{2}}\,10^{-8}\,sec \ , \ \frac{1}{\tau_B} = 1{,}4 . 10^8\ rad/sec$$

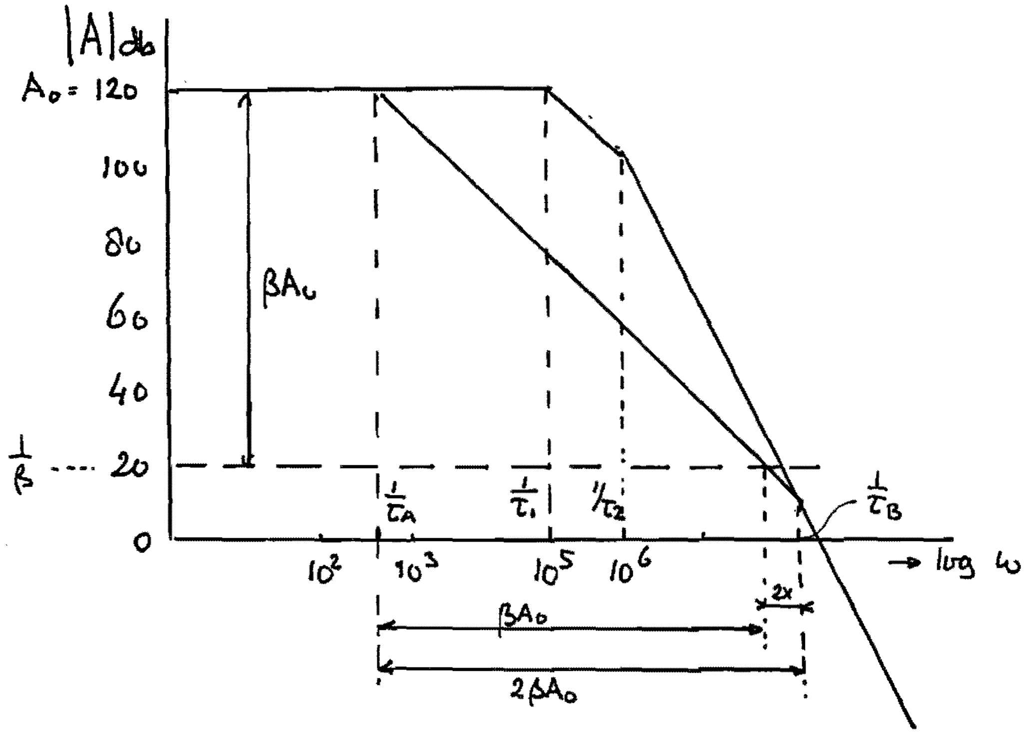


fig 4.6

2) $\frac{\tau_A}{\tau_B} = k\beta A_{10} A_{20}$, $k=4$, $\tau = \tau_1 = RC = 10^{-5}$ sec

Using the same relations as above gives

$$4R^2 - 1.1R - 10^5 = 0$$

or $R = \frac{10^3}{\sqrt{40}}\ \Omega$ and $C = \sqrt{40}\ 10^{-8}$ F

$\tau_A = \sqrt{40}\ 10^{-3}$ sec. , $1/\tau_A = \frac{1}{4}\sqrt{40}\ 10^{2}$ rad/sec

$\tau_B = \frac{1}{4}\sqrt{40}\ 10^{-9}$ sec , $1/\tau_B = \sqrt{40}.\ 10^{8}$ rad/sec

If RC < τ_1 a part of the gain characteristic will have a slope of -40 dB/dec and RC > τ_1 a part will have zero dB.
In all three cases the same UGBW.

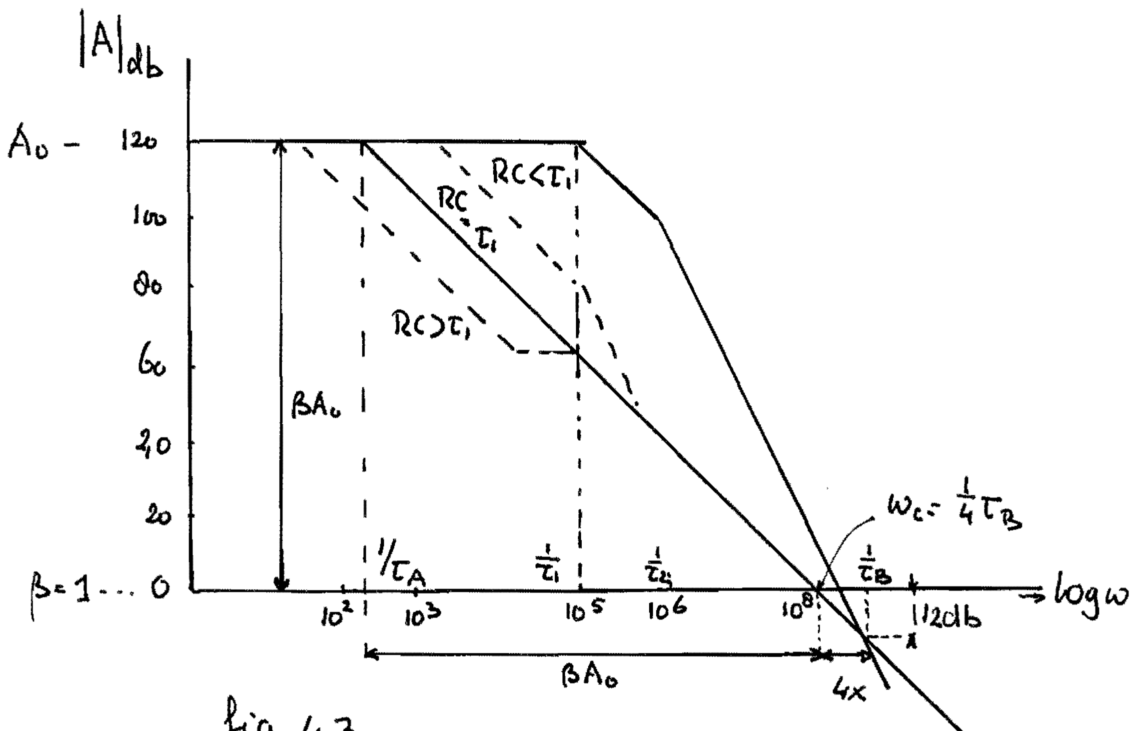


fig 4.7

4.1.2. Miller compensation : (pole splitting) (G&M.9.4.2)

Another compensation of a two stage op.amp.can be achieved by applying feedback around the second stage with a compensation capacitor C (fig. 4.8).

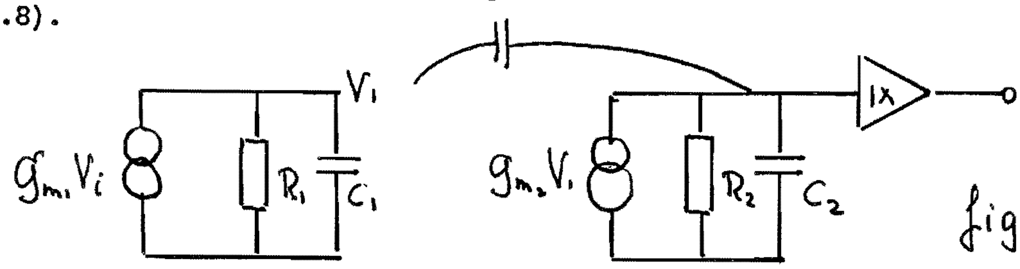


fig 4.8

This capacitor can be small because of the gain of this stage.

The effect of this capacitor can easily be calculated by redrawing fig. 4.8. as an op.amp. configuration.

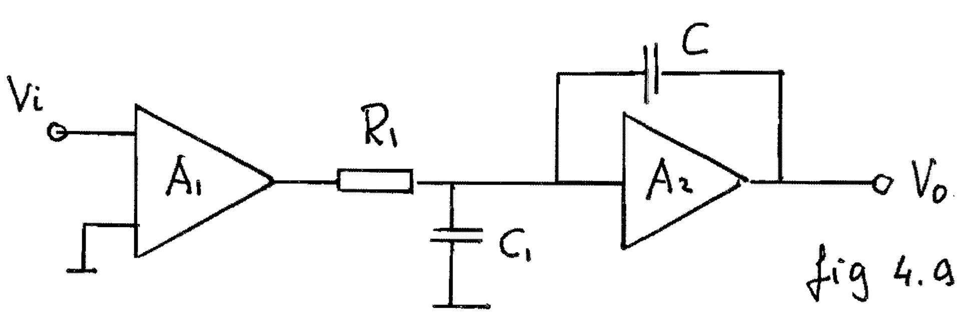
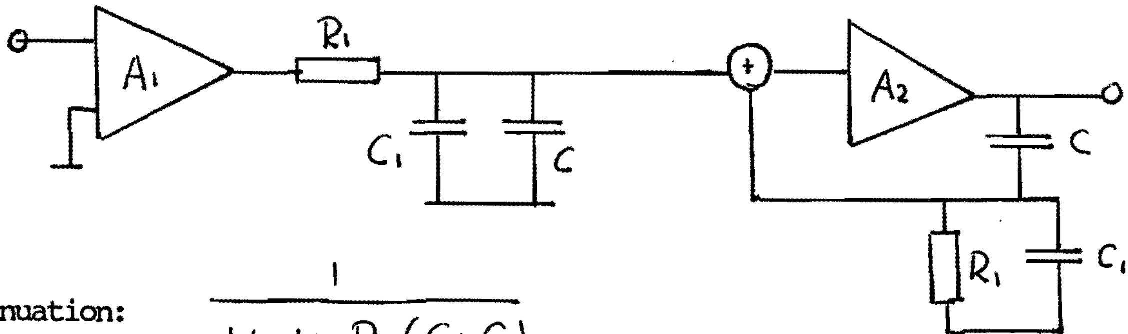


fig 4.9

with. $A_{1_0} = g_{m_1} R_1$ and $A_2 = \dfrac{g_{m_2} R_2}{1 + j\omega C_2 R_2} = \dfrac{g_{m_2} R_2}{1 + j\omega \tau_2}$

Since A_1 has no pole the transfer function of the compensated amplifier can be found by considering the attenuator and the feedback around A_2.



Attenuation: $$\frac{1}{1 + j\omega R_1 (C + C_1)}$$

fig 4.10

Feedback: $$\frac{1}{\beta} = \frac{1 + j\omega R_1 (C + C_1)}{j\omega C R_1}$$

The Bode plots of attenuator and A_2 with feedback and of the complete amplifier are shown in fig. 4.11.

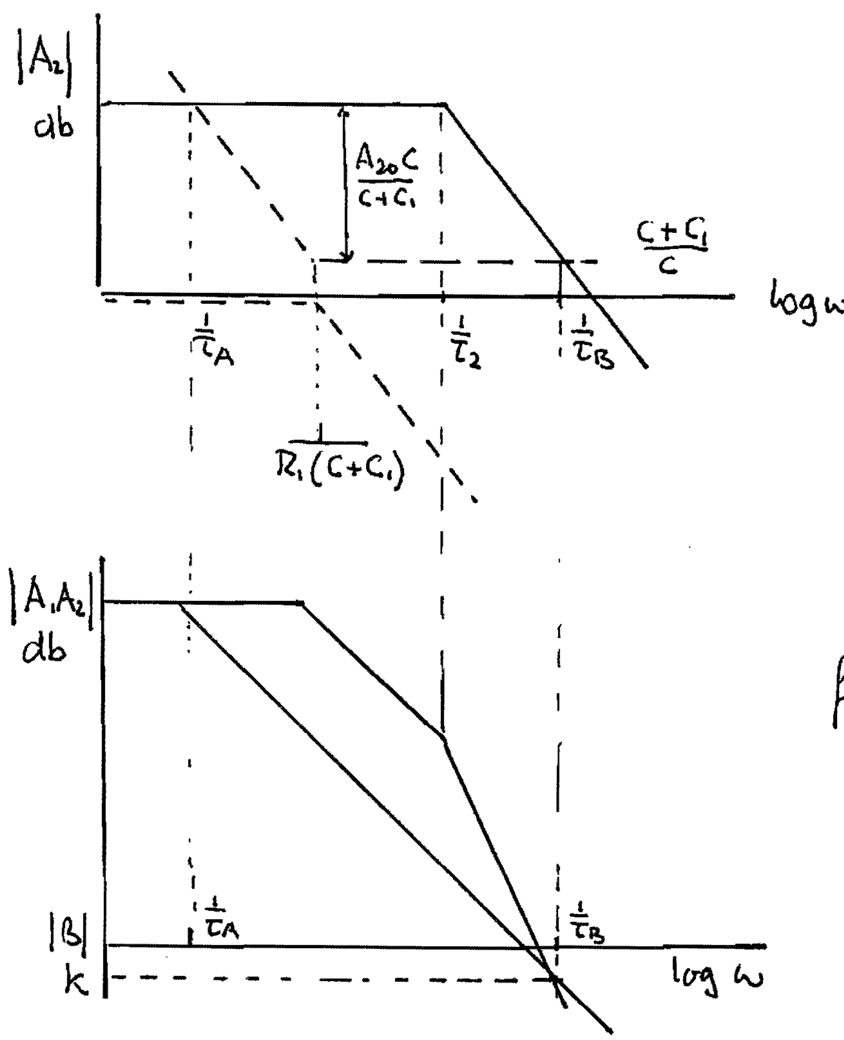


fig 4.11

With $\quad \tau_A = \dfrac{A_{20}C}{C+C_1} \cdot R_1(C+C_1) = A_{20}R_1C$

and $\quad \tau_B = \tau_2 \dfrac{C+C_1}{A_{20}C} = R_2C_2 \dfrac{C+C_1}{A_{20}C}$

The required compensation is obtained if

$$\frac{\tau_A}{\tau_B} = k\beta A_{10}A_{20} = \frac{A_{20}^2 R_1 C^2}{R_2 C_2 (C+C_1)}$$

This equation gives the possibility to calculate C, τ_A and τ_B.

<u>Example</u>

$$g_{m1} = 10^{-3}\,A/V \quad , R_1 = 10^6\,\Omega, \quad C_1 = 10^{-11}\,pf$$
$$A_{20} = 10^3 \qquad , \quad \tau_2 = 10^{-6}\,sec$$

Find for $\beta = 1$ the value of the Miller-capacitor C and the UFBW requiring no peaking in the magnitude Bode plot, or k = 2.

$$\frac{R_1 C^2 A_{20}^2}{R_2 C_2 (C+C_1)} = 2A_{10}A_{20} \qquad \text{or}$$

$$R_1 C^2 A_{20} = 2A_{10} R_2 C_2 (C+C_1)$$

$$C^2 - 2\,10^{-12}C - 2\,10^{-23} = 0$$

$$C = 10^{-12} + \sqrt{10^{-24} + 2\,10^{-23}} = 5.6\;10^{-12}\,F$$

$$\tau_A = A_{20}R_1C = 5.6\;10^{-3}\,sec \quad , \frac{1}{\tau_A} = 179\;rad/sec$$

$$\tau_B = R_2C_2 \cdot \frac{C+C_1}{A_{20}C} = 2.8\,10^{-9}\,sec \quad , \frac{1}{\tau_B} = 3.57\;10^8\,rad/sec$$

$$\omega_c = \frac{1}{2}\frac{1}{\tau_B} = 1{,}79\,10^8\,rad/sec \qquad UGBW = \frac{\omega_c}{2\pi} = 2.85\;10^7\,Hz$$

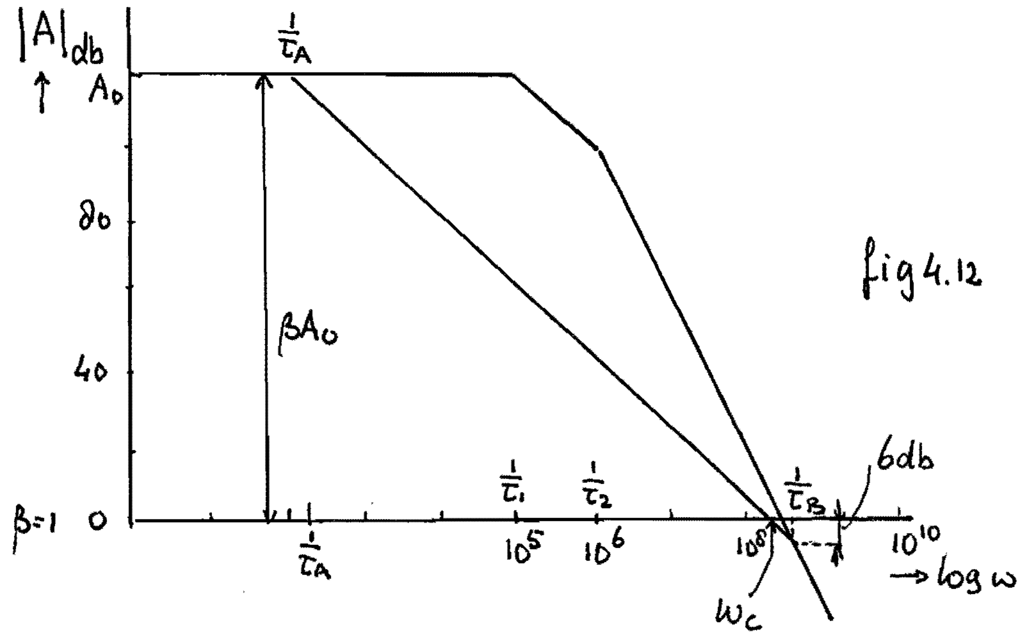


fig 4.12

4.1.3. Feedforward compensation technique

This method of compensation is achieved by applying two modifications:

1) an extra direct forward connection is added around one stage; feed forward
2) the time constant of that stage is increased to τ_A

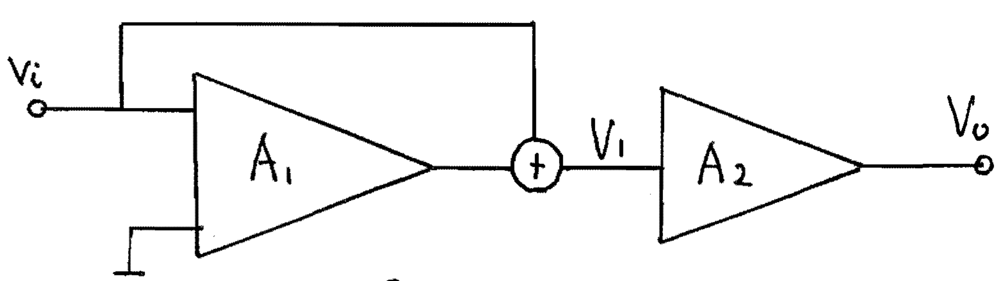


fig 4.13

$$V_1 = A_1 V_i + V_i = \left(\frac{A_{10}}{1+j\omega\tau_1} + 1\right)V_i = \frac{A_{10}+1}{1+j\omega\tau_1}\left(1 + \frac{j\omega\tau_1}{A_{10}+1}\right)V_i$$

$$V_0 = \frac{A_{20}}{1+j\omega\tau_2} V_1$$

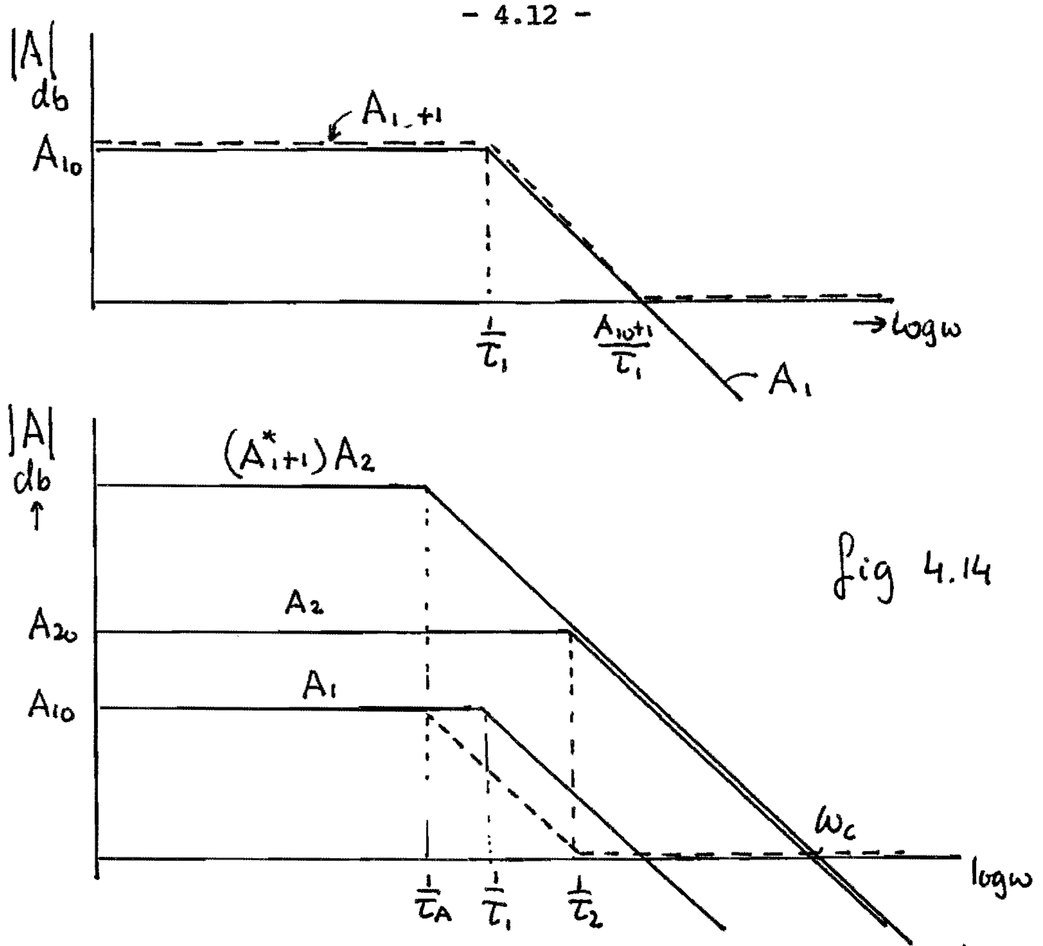


fig 4.14

The feedward loop is dominating for frequencies higher than $\frac{A_{10}+1}{\tau_1}$
for

An ideal roll off is obtained by increasing τ_1 to τ_A
so that

$$\frac{A_{10}+1}{\tau_A} = \frac{1}{\tau_2}$$

<u>Examples</u>

Given $\quad A_{10} = 10^4 \,,\ \tau_1 = 10^{-5}\,\text{sec}$

$$A_{20} = 10^2 \,,\ \tau_2 = 10^{-6}\,\text{sec}$$

Improve the gain characteristic with feedforward technique so that the roll-off equals -20dB/dec.

There are two possibilities.

a) Feedforward around the first stage

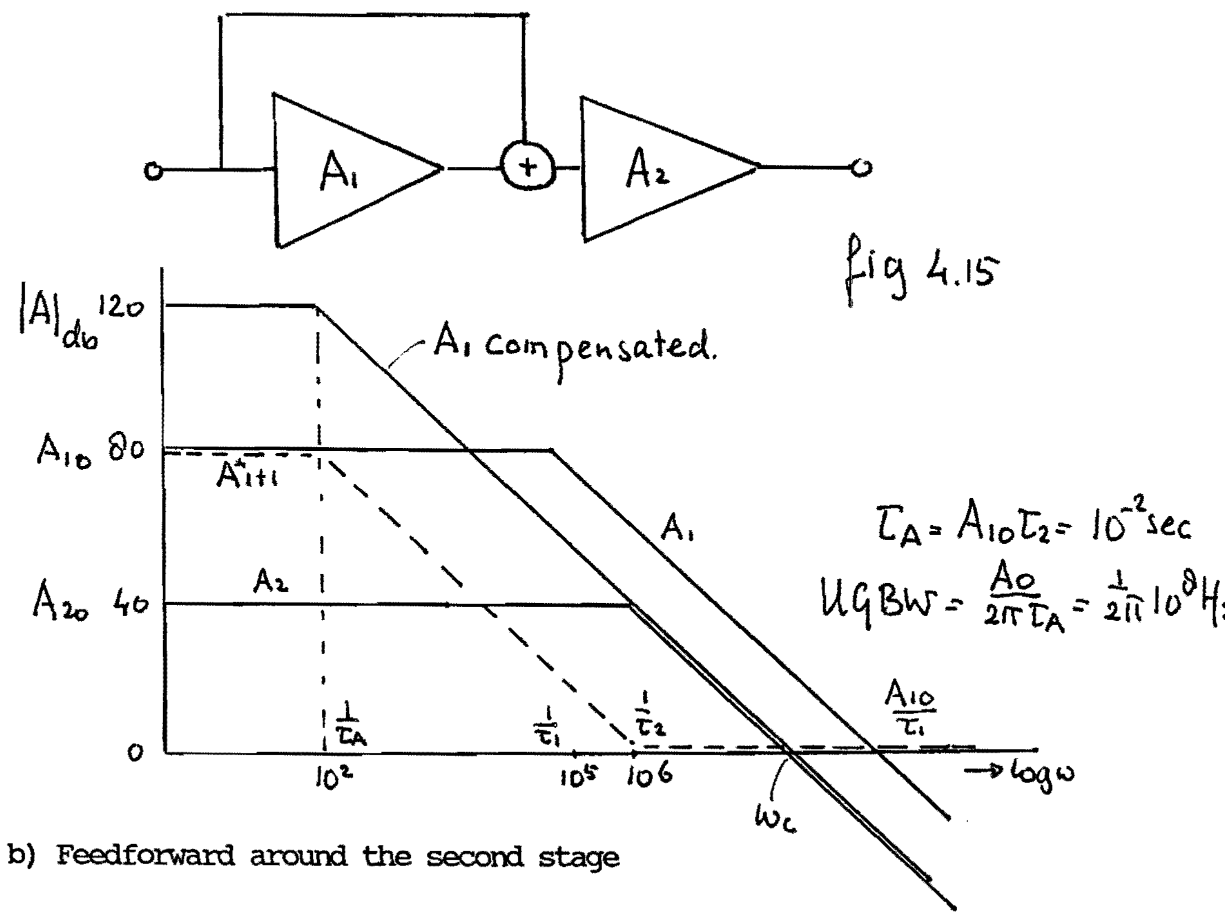


fig 4.15

b) Feedforward around the second stage

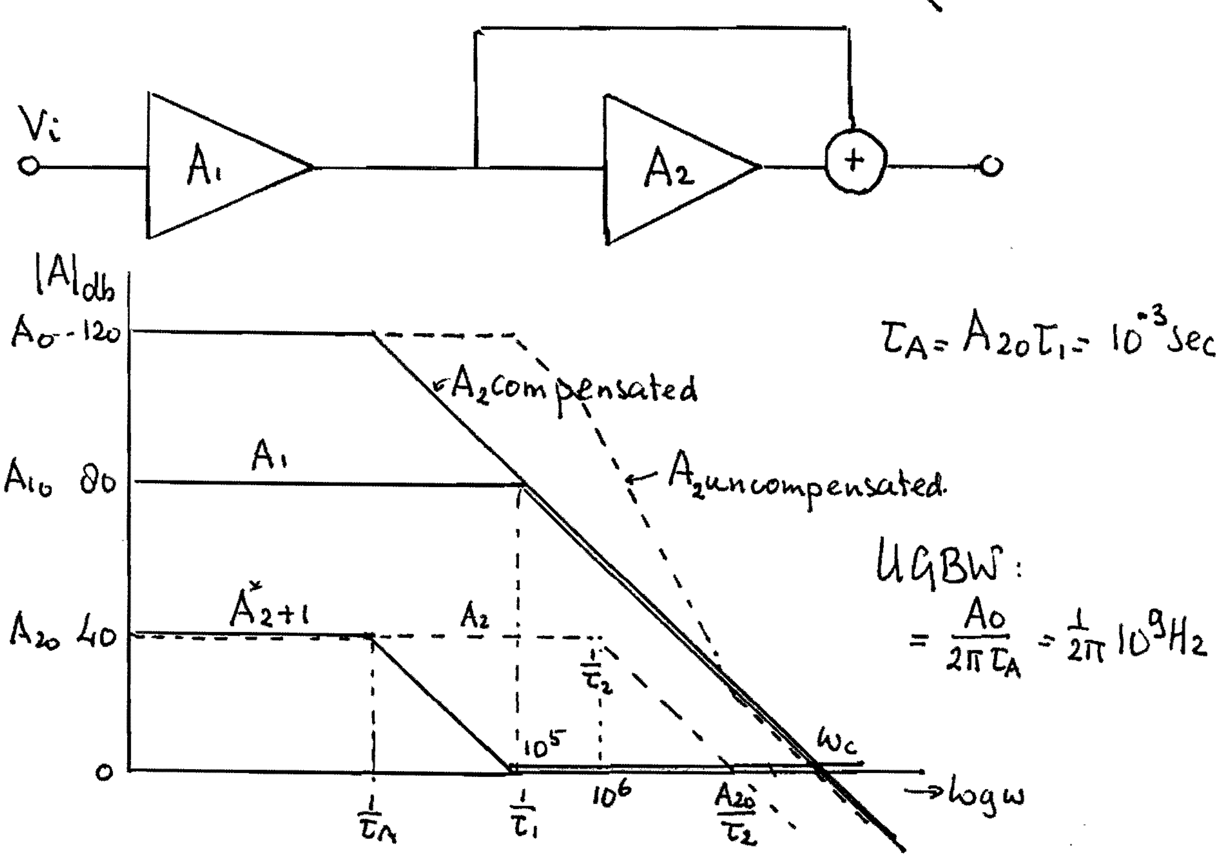


fig 4.16

Conclusions

1) An amplifier configuration which consists of two stages with different UGBW's can be compensated by feedforward in two ways.

2) The UGBW of the compensated system equals the UGBW of the not bypassed stage.

3) For max. UGBW apply feedforward around the narrower stage.

4.2. Stability - Enhancement techniques

The frequency compensation techniques developed in the preceding section are, in general, sufficient to design stable op.amp. circuits. However, when the circuit is wired a designer may be confronted with instability. This may be caused by a bad layout, an insufficient power supply, bypassing input and output capacitances and so on.

LAYOUT AND BYPASSING. During layout the leads to the input and the compensation terminals must be kept short and their placement relative to other wires must be carefully studied. Ground paths should have low resistance. It is always recommended to use a ground plane on printed-circuit boards for a high quality ground. Positive and negative power supplies should be bypassed with good-quality RF capacitors (i.e. 0.1 μF disk ceramic or 1.0 μF tantalytics for every five IC's).

COMPENSATION OF STRAY INPUT CAPACITANCE

At the input of an op.amp. there will always be a few picofarads of stray capacitance plus some wiring capacitance. This is indicated in figure 4.17

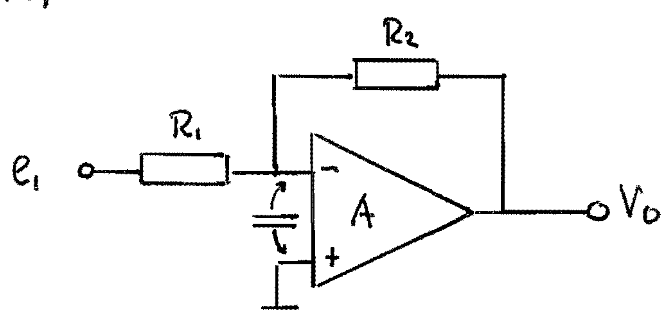


fig 4.17

Let A =

$$A = \frac{A_0}{1 + j\omega\tau}$$

The feedback factor is

$$\beta = \frac{R_1}{R_1+R_2} \frac{1}{1+j\omega R_p C}$$

and the Bode plot

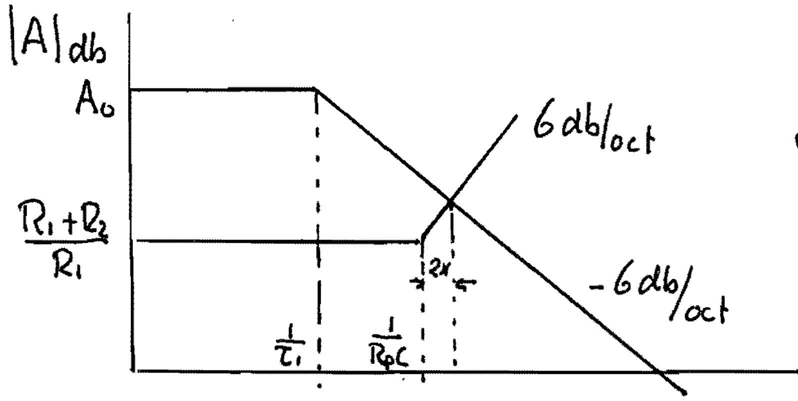


fig 4.18

The Bode plot shows that if R_1 is large (e.g. the input signal is delivered by a currrent source) the frequency $1/R_pC$ may move into the region where it can contribute noticeable phase shift. This can cause oscillation.
A solution is to keep R_2 low but that means a low amplification.
A better solution is to shunt R_2 with a capacitor C_2.

The feedback factor β becomes

$$\beta = \frac{R_1}{R_1+R_2} \frac{1+j\omega R_2 C_2}{1+j\omega R_p (C+C_2)}$$

$$\tau_2 = R_2 C_2$$

The Bode plot is changed by this capacitor as shown in fig. 4.19.

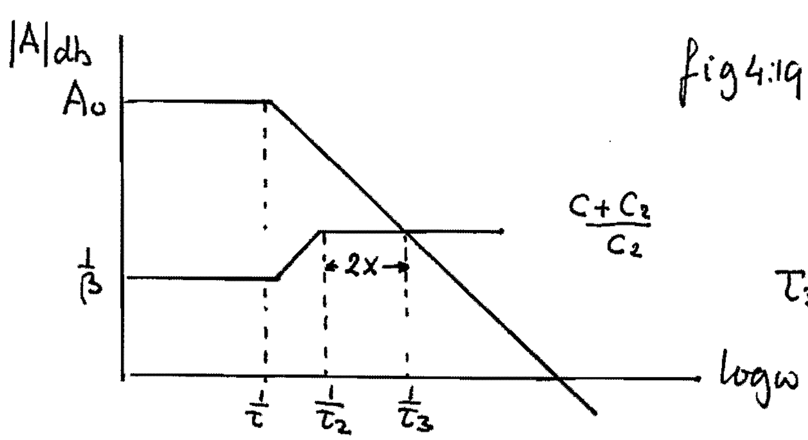


fig 4:19

$$\tau_3 = \frac{\tau_1}{A_0} \frac{C_2+C}{C_2}$$

The stability is improved, the phase will be 65° if $\tau_2 = 2\tau_3$

A disadvantage is that this compensation capacitor causes a drop of the transfer characteristic at higher frequencies.

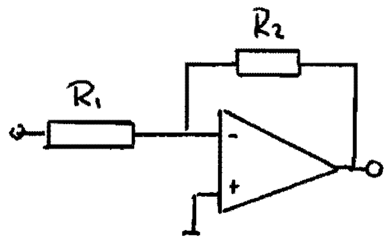
$$\text{transfer function} = \frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega C R_2}$$

5. Applications of operational amplifiers.

Virtually all operational amplifier applications rely on the principles of feedback. By including non-linear elements in the feedback network, operational amplifiers can be used to perform non-linear operations via one or more analog signals. Also positive feedback generates many interesting circuits.
In this section some of those applications of op. amps. will be discussed in more detail.

5.1. Linear applications/Amplifiers.

5.1.1. Inverting/non-inverting Amplifiers.



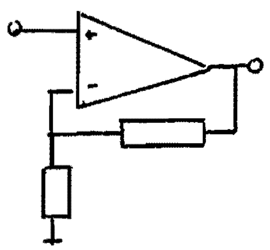


fig. 5.1

Inverting: gain: $-\frac{R_2}{R_1}$

input impedance R_1

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Non-inverting: gain: $1 + \frac{R_2}{R_1} = \frac{1}{\beta}$

input impedance high

5.1.2. Summing Amplifier.

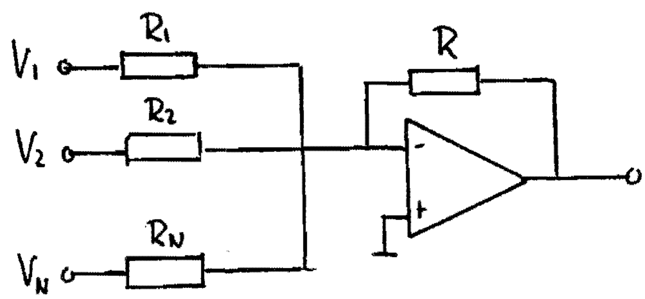


fig.5.2

$$V_{out} = - R_1 \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots \frac{V_N}{R_N} \right]$$

All the inputs are completely isolated from each other because the - input is kept zero by the op. amp. (virtual earth). By changing the adding resistors (R_1--- R_N) the input signals can be added with different amplification factors. The bandwidth of the circuit is determined by the parallel impedance $R_p = R_1//R_2//\dots R_N$.

The feedback equals: $\dfrac{R_p}{R_p + R}$

5.1.3. Integrator.

An integrator circuit complete with the bias sources is shown in the figure:

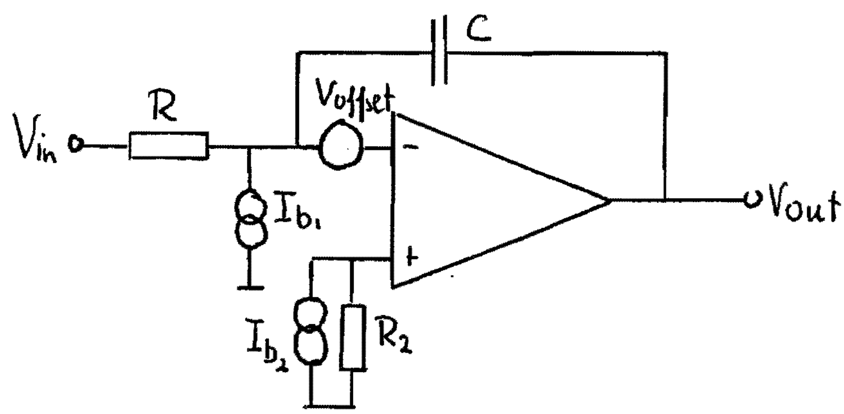


fig.5.3

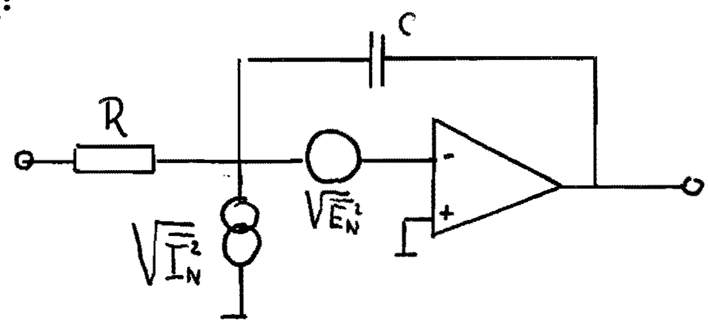
$$V_{out} = \left(V_{offset} + I_{b2}R_2\right)\left(1 + \frac{1}{pCR_1}\right) - I_{b1}\cdot\frac{1}{pC} - V_{in}\cdot\frac{1}{pCR_1}$$

$$= \frac{1}{CR_1}\int_0^t \left(- V_{in} + I_{b2}R_2 - I_{b1}R_1 + V_{offset}\right) dt + V_{offset} + I_{b2}R_2$$

This expression shows that the bias sources are integrated as well. Therefore, it is important to zero the offset voltage and to make $I_{b2}R_2 = I_{b1}R_1$.

The offset term $I_{b2}R_2$ can only be eliminated by making $R_2 = 0$ and by compensating I_{b1} by an external current source.

Integrator Noise:



$$\sqrt{\overline{V^2_{N\,out}}} \approx \sqrt{\overline{E_N^2}\left|1+\frac{1}{j\omega CR}\right|^2 + \overline{I_N^2}\left|\frac{1}{j\omega C}\right|^2}$$

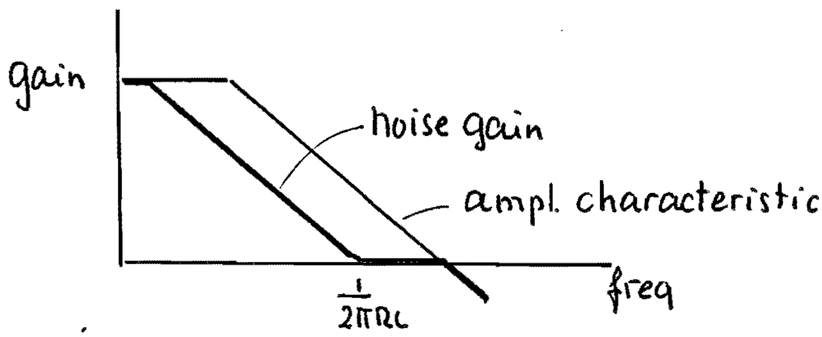


fig. 5.4

The output noise contains frequencies higher than $\frac{1}{2\pi CR}$ because the noise gain remains unity above this frequency. Therefore, it is advisable not to apply a too wideband op. amp. and to insert a lowpass filter behind the integrator if the noise performance is important.

5.1.4. Differentiator.

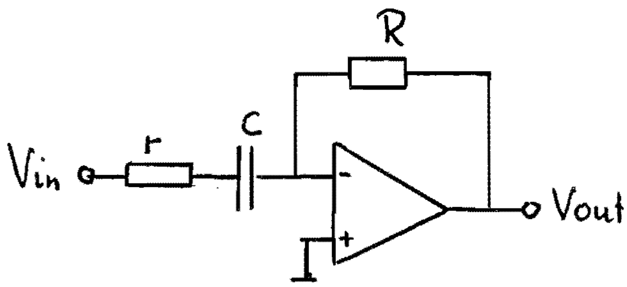


fig. 5.5

A small resistor r is inserted in the circuit to improve the stability and to reduce the high frequency noise, caused by the amplification of the op. amp. noise

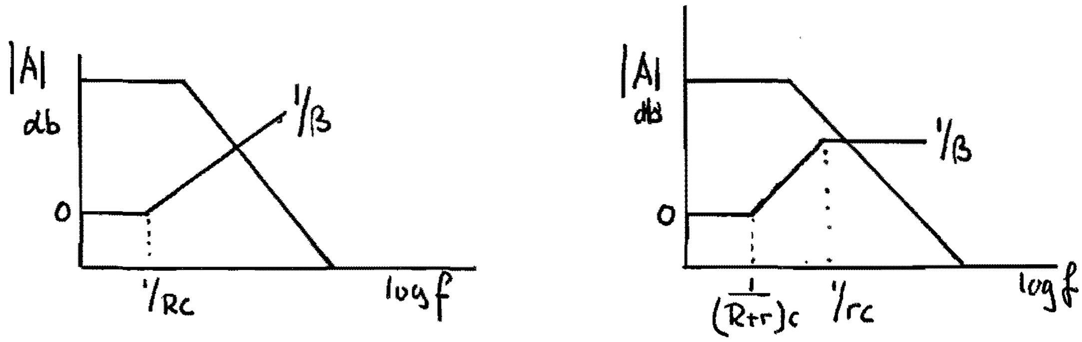


fig. 5.6

without r: $\frac{1}{\beta} = 1 + jwCR$ with r: $\frac{1}{\beta} = \frac{1+jw(R+r)}{1+jwCR}$

- instable - stable

5.1.5. AC Amplifiers.

A common AC-amplifier configuration is shown in the following figure:

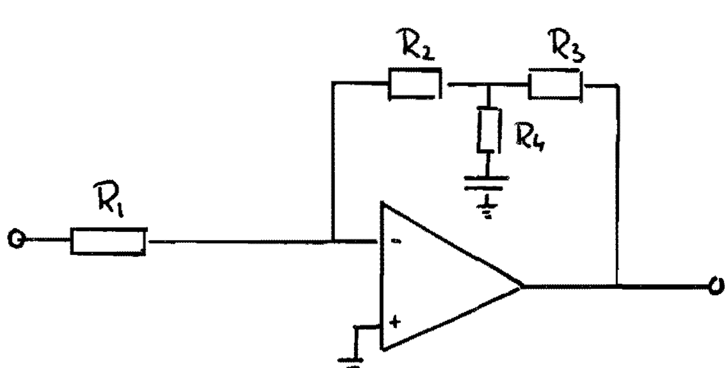
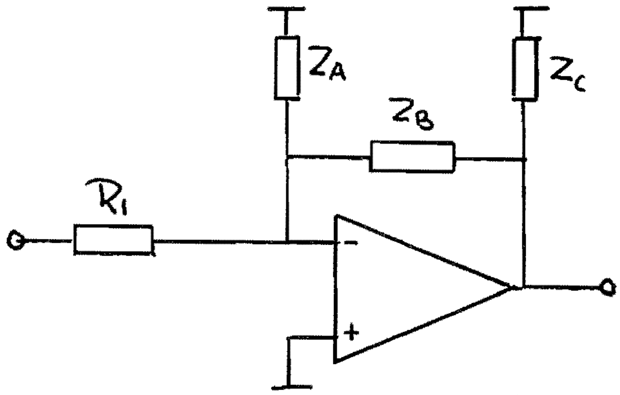


fig. 5.7

The DC-gain equals $-\dfrac{R_2 + R_3}{R_1}$ and can be kept low in order to reduce the DC offset voltage at the output.
The AC-gain can easily be calculated by applying the Y - Δ transformation:



$$Z_A = R_2 + R_4 + \frac{R_2 R_4}{R_3}$$

$$Z_B = R_2 + R_3 + \frac{R_2 R_3}{R_4}$$

$$Z_C = R_3 + R_4 + \frac{R_3 R_4}{R_2}$$

fig. 5.8

Notice that Z_A affects the input impedance but not the gain. Z_C is just a load at the output.
The AC-gain is:

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 + R_3 + \dfrac{R_2 R_3}{R_4}}{R_1}$$

This configuration is useful to obtain a very high AC-gain by making the feedback impedance large (e.g. $R_2 = R_3 = 1M\Omega$, $R_4 = 1K\Omega$ gives $R_B = 10.^9\Omega$)

Note that Z_A cannot be ignored considering the stability:

$$\beta = \frac{R_1 // Z_A}{Z_B + R_1 // Z_A}$$

Noise and drift.

The output noise is also influenced by Z_A.

$$\sqrt{\overline{V^2}_{OUT\ NOISE}} = \sqrt{\overline{I_N^2}\,|Z_B|^2 + \overline{E_N^2}\left|\frac{Z_B + R_1 /\!/ Z_A}{R_1 /\!/ Z_A}\right|^2}$$

This shows clearly that the noise is additionally amplified by this impedance Z_A . The same calculations and conclusions appear with respect to drift when the capacitor C is left out. Thus it is often better to use a real high impedance than to use this configuration.

5.2. Converters.

5.2.1. Current to voltage converter.

A possible configuration of a current to voltage converter is shown in the figure

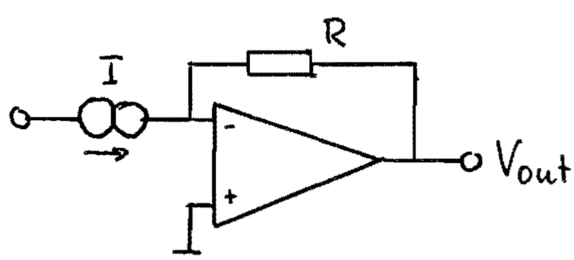


fig. 5.9

The voltage at the -input is kept zero by the op. amp., therefore the current through the parallel parasitic capacitance and resistance of the current source is also zero and the complete current flows through R.

$$V_{out} = - IR$$

Such a circuit is very useful for measuring the current of a photodiode

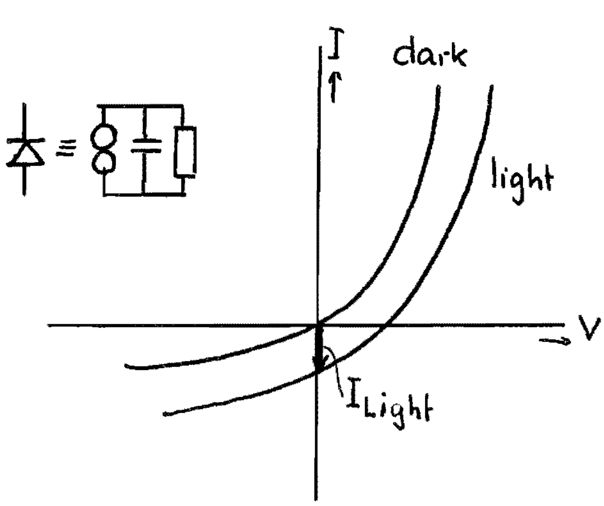


fig. 5.10

photodiode - characteristic:

$$I = I_o \left(e^{ev/KT} - 1\right) - I_{Light}$$

Since V = 0, the current I_L and V_{out} is proportional to the light intensity.

The modulation of light sources can be measured up to high frequencies because the parasitic diode capacitance does not influence the gain. This capacitance can cause instability. It can be compensated by a small capacitor across R.

5.2.2. Voltage to current converters.

a) Floating Load.

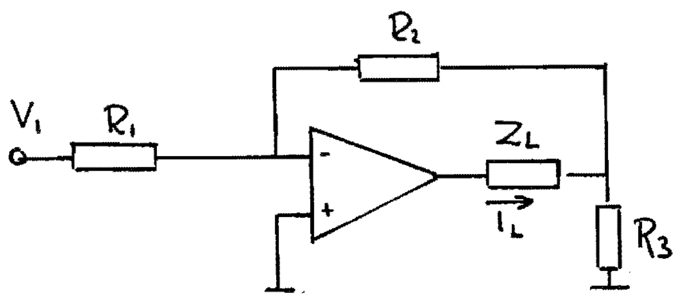


fig. 5.11

If the feedback current is small ($R_3 \ll R_1$)

$$I_L R_3 = -V_1 \frac{R_2}{R_1} \quad \text{or} \quad I_L = -V_1 \frac{R_2}{R_1} \cdot \frac{1}{R_3}$$

This converter is useful when the current through a coil (e.g. deflection coils of a cathode-ray tube) must have a certain waveform.
The inductance of the coil, influences the stability.

b) Non-Floating Load.

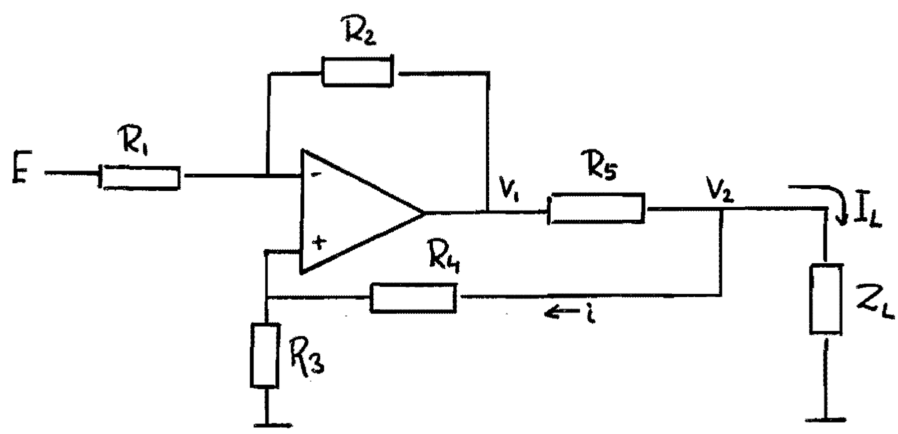


fig.5.12

Assume that the current through the feedback loop $i \ll I_L$

Let $\frac{R_4}{R_3} = a + \frac{\Delta a}{2}$ and $\frac{R_2}{R_1} = a - \frac{\Delta a}{2}$

For stability, the negative feedback (R_2, R_1) should be larger than the positive feedback (R_4, R_3). Hence we require that:

$$\frac{R_4}{R_3} > \frac{R_2}{R_1} \quad \text{for stability}$$

$$\left.\begin{aligned} V_1 &= -E\,\frac{R_2}{R_1} + V_2\,\frac{R_3}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1} \\ V_1 &= I_L\left(R_5 + Z_L\right) \\ V_2 &= I_L Z_L \end{aligned}\right]$$ this gives

$$I_L = \frac{-E\,\frac{R_2}{R_1}}{R_5 + Z_L\left[1 - \frac{R_3}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\right]}$$

Since $\frac{R_3}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1} = \frac{1+a}{1+a}\;\frac{1-\frac{\Delta a}{2(1+a)}}{1+\frac{\Delta a}{2(1+a)}} = 1 - \frac{\Delta a}{1+a}$

The current I_L equals:

$$I_L = \frac{-E\,\frac{R_2}{R_1}}{R_5 + Z_L\,\frac{\Delta a}{1+a}}$$

The circuit satisfies the current source requirement if Δ a is small: The output current I_L is independent of Z_L.

Output impedance with $\Delta a \neq 0$

The output impedance of a current source can be found by open and short circuiting the output

$$Z_{out} = \frac{V_{open}}{I_{short}}$$

$$V_{open} = I_L Z_L \;(Z_L = \infty) = \frac{-Z_L E \cdot \frac{R_2}{R_1}}{Z_L \frac{\Delta a}{1+a} + R_5} = \frac{-E \frac{R_2}{R_1}}{\frac{\Delta a}{1+a}}$$

$$I_{short} = I_L \;(Z_L = 0) = \frac{-E \frac{R_2}{R_1}}{R_5}$$

and thus $Z_{out} = R_5 \cdot \dfrac{1+a}{\Delta a}$

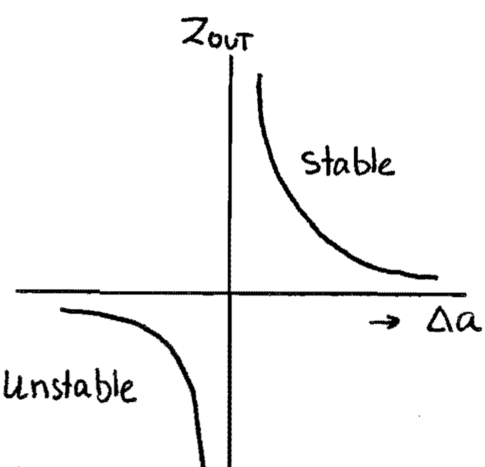


fig. 5.13

The curve shows that the output impedance is negative for Δa negative. In this case DC instability occurs; the output being either V^{++} or V^{--}.

The value $I_L Z_L$ is limited by the maximum output voltage and maximum common mode voltage which can be handled by the op. amp.

Thus

$I_L\,(Z_L + R_5) <$ max. output voltage

$I_L Z_L \cdot \dfrac{R_3}{R_3 + R_4} <$ max. common mode voltage

5.3. Instrumentation Amplifiers.

Instrumentation amplifiers are closed-loop gain blocks with differential inputs and an accurately predictable input to output relationship. They have very high input impedance and common-mode rejection. This makes them ideal for accurately amplifying of low level signals in the presence of large common-mode voltages.
The instrumentation amplifier differs fundamentally from the operational amplifier. It is designed to be used as a close-loop gain block. Operational amplifiers on the other hand, are open-loop devices whose closed-loop performance depends upon the external networks.

Instrumentation amplifiers can be put together by using one or more op. amps. in specific configurations.

5.3.1. One Op. Amp. Instrumentation Amplifier.

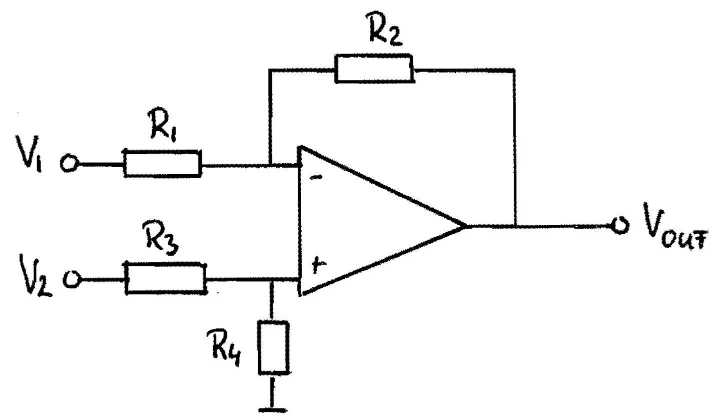


fig. 5.14

Assume: $\frac{R_2}{R_1} = a - \frac{\Delta a}{2}$; $\frac{R_4}{R_3} = a + \frac{\Delta a}{2}$

with this

$$V_{out} = -V_1 \cdot \frac{R_2}{R_1} + V_2 \cdot \frac{R_4}{R_3+R_4} \left(1 + \frac{R_2}{R_1}\right) = (V_2 - V_1)\,\frac{R_2}{R_1} \quad \text{with } \Delta a = 0$$

The differential input impedance of this simple differential amplifier is that of the input resistors R_1 and $R_3 + R_4$, which are generally low.

Also, even though the op. amp. used may have excellent CMRR, the finite matching of the resistors can degrade the overall CMRR.
This can be calculated by using the relations we discussed in chapter 2.3.5 and 2.3.6. The differential amplifier consists of two stages as shown in figure 5.15.

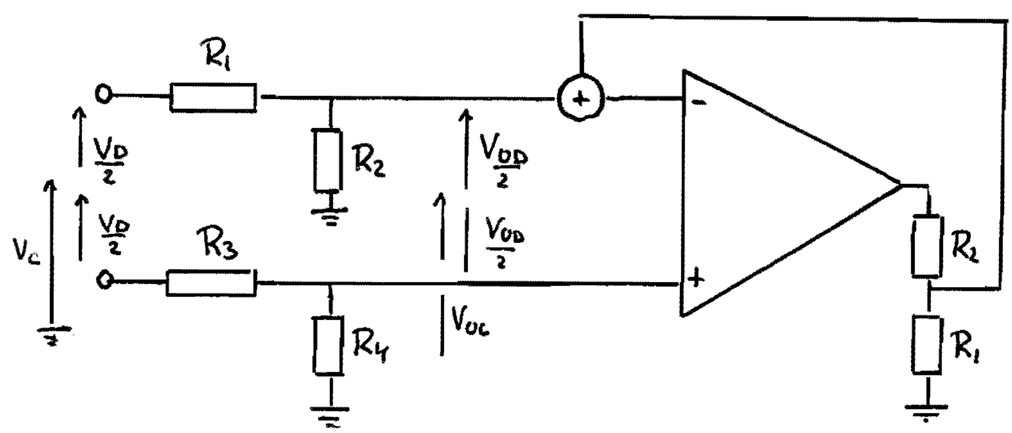


fig. 5.15

The CMRR of the complete circuit is:

$$H_{total} \quad \frac{1}{\frac{1}{F_1 H_2} + \frac{1}{H_1}}$$

so we have to calculculate the values of F_1, H_1, A_1 of the resistor network and H_2 of the amplifier with feedback.

$$A_1 = \frac{\frac{R_2}{R_1 + R_2} + \frac{R_4}{R_3 + R_4}}{2} = \frac{a}{1+a}$$

Common-mode gain: $$\frac{V_C \cdot \frac{R_2}{R_1 + R_2} + V_C \frac{R_4}{R_3 + R_4}}{2V_c} = \frac{a}{1+a}$$

$$F_1 = \frac{A_1}{\text{com. mode gain}} = 1$$

$$H_1 \quad \frac{A_1}{\dfrac{R_2}{R_1 + R_2} - \dfrac{R_4}{R_3 + R_4}} = \frac{\dfrac{a}{1 + a}}{\dfrac{\Delta a}{(1+a)^2}} = \frac{a(1+a)}{\Delta a}$$

In chapter 2.3.5. we have seen that the CMRR of an op. amp. equals:

$$H = \frac{A}{\Delta A}$$

It can easily be shown that the common-mode rejection is not influenced by applying feedback.

So $H_2 = H_{opamp}$

The CMRR of the differential amplifier can now be found:

$$H_{total} = \frac{1}{\dfrac{\Delta a}{a\,(1+a)} + \dfrac{1}{H_2}} = \frac{H_2(1+a)}{H_2\dfrac{\Delta a}{a} + 1 + a}$$

The relations show that even with perfectly balanced resistor network the CMRR cannot exceed the value of H_{opamp}.

The common mode capability of this differential amplifier is limited by the max. allowable common mode input voltage of the op. amp. (V_{Cmax}). So

$$V_{CM_{max.}} = \frac{R_3 + R_4}{R_4} \cdot V_{C_{max}} = \frac{1 + a}{a} \cdot V_{C_{max}}$$

In order to change the gain both resistors R_2 and R_4 have to be varied. This can be rather complicated because the network has to be kept in balance.

The fig. (5.15) shows that the gain is determined by the attenuator as well as by the feedback. It is obvious that gain control can be realised by varying the feedback without influencing the attenuator.

A possible solution for gain control is shown in the figure:

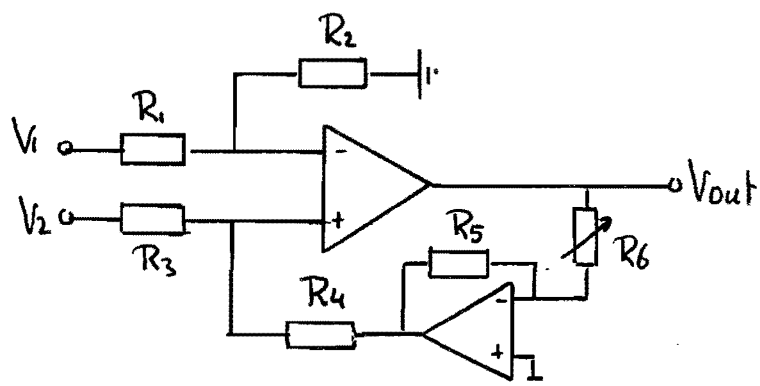


fig. 5.16

The gain in the feedback path is:

$$\beta = \frac{R_5}{R_6} \cdot \frac{R_3}{R_3 + R_4}$$

So the gain of this differential amplifier configuration is:

$$\frac{V_{out}}{V_{dif}} = \frac{R_4}{R_3 + R_4} \cdot \frac{1}{\beta} = \alpha \cdot \frac{R_6}{R_5}$$

It can be controlled by varying R_6.

The input impedances at both inputs are practically equal.

5.3.2. Two op. amp. Instrumentation Amplifiers.

The previous circuits cannot handle very high common mode voltages. One way of overcoming this is to invert one of the inputs.

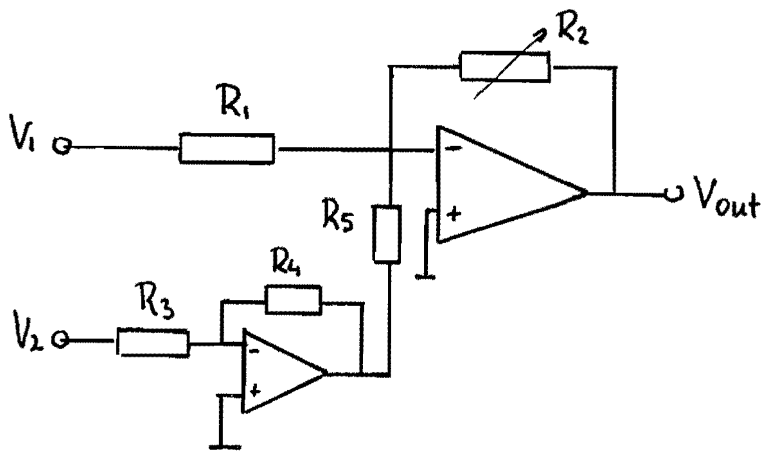


fig. 5.17

$$V_{out} = R_2\left(V_2 \frac{R_4}{R_3R_5} - V_1 \frac{1}{R_1}\right)$$

With: $R_1 = \dfrac{R_3R_5}{R_4}$

$$V_{out} = \frac{R_2}{R_1} (V_2 - V_1)$$

- R_2 can be used for gain control

- The maximum common-mode voltages are limited by the max. output voltage of the op. amps. ($V_{out_{max}}$)

$$V_{cm_{max}} = \frac{R_3}{R_4} \cdot V_{out_{max}}$$

If $R_3 > R_4$ the amplifier can handle relatively high voltages because of the attenuation of the input signals.

- The input impedances are low, R_1 and R_2
- The CMRR can be calculated easily

assume
$$\frac{R_4}{R_5} = a(1 + \delta 1)$$
$$\frac{R_3}{R_1} = a(1 + \delta 2)$$

The common mode gain equals

$$R_2 \left(\frac{R_4}{R_3 R_5} - \frac{1}{R_1} \right) = \frac{R_2}{R_1} (\delta 1 - \delta 2)$$

The differential gain is $\frac{R_2}{R_1}$

So the CMRR = $\frac{1}{\delta 1 - \delta 2}$

A disadvantage of this differential amplifier is the low input impedance.
A way of getting very high input impedances is to use the non-inverting inputs of both amplifiers as shown in the figure.

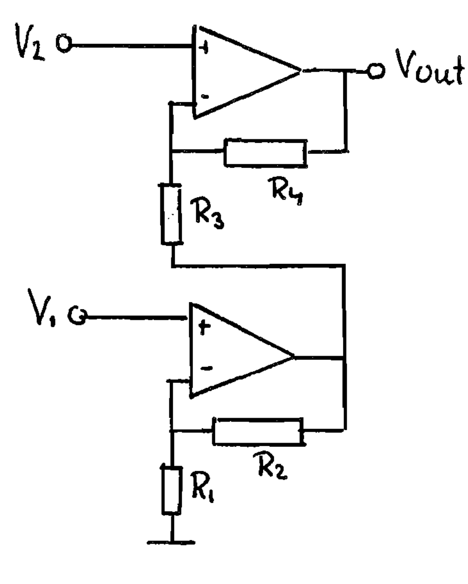


fig. 5.18

$$V_{out} = V_2 \left(1 + \frac{R_4}{R_3} \right) - V_1 \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) .$$

Let $\frac{R_1}{R_2} = a\,(1 + \delta 1)$; $\frac{R_4}{R_3} = a\,(1 + \delta 2)$.

and assume H and A to be infinite, then the differential gain (δ small) equals:

$$\frac{V_{out}}{V_{diff.}} = 1 + a$$

The common-mode gain ($V_1 = V_2$) is

$$\frac{V_{out}}{V_{com.}} = 1 - \frac{R_4 R_2}{R_3 R_1} = 1 - \frac{a(1+\delta 2)}{a(1+\delta 1)} \approx \delta 1 - \delta 2$$

Therefore CMRR equals: $CMRR = \frac{1 + a}{\delta 1 - \delta 2}$

The common-mode voltage capability is

$$V_{CM_{max}} = \frac{R_1}{R_1 + R_2} V_{out_{max}}$$

The main feature of this amplifier is the high input impedance. Disadvantages are the low common mode voltage capability and the difficulty of varying the gain.

These disadvantages can be overcome using three op. amps.

5.3.3. Three op. amp. Instrumentation Amplifier.

A common configuration of an instrumentation amplifier consists of three op. amps. as shown in the figure:

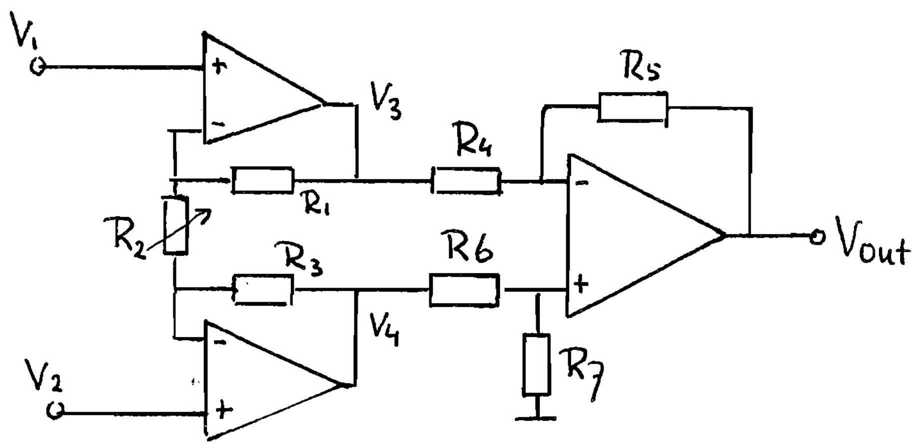


fig. 5.19

The two input op. amps. provide a differential gain

$$V_3 - V_4 = (V_1 - V_2)\ (1 + \frac{R_1}{R_2} + \frac{R_3}{R_2})$$

The output op. amp. is a differential amplifier with a differential gain

$$V_{out} = - \frac{R_5}{R_4}\ (V_3 - V_4) \quad \text{assuming} \quad \frac{R_5}{R_4} = \frac{R_7}{R_6}$$

The total gain of this configuration equals

$$\frac{V_{out}}{V_{diff}} = - \frac{R_5}{R_4}\ (1 + \frac{R_1 + R_3}{R_2})$$

The gain can easily be controlled by varying R_2.

The input followers produce high input impedances. The common-mode voltage capability is limited either by the maximum output voltage or by the maximum common mode voltage of the input op. amps.

Common Mode Rejection Ratio.

As discussed the CMRR of the configuration can be calculated if we know the values H_1, F_1, A_1 and H_2.

Assuming A_1 and $H_1 = 10^5$ to 10^6 and

$$\frac{R_5}{R_4} = a - \frac{\Delta a}{2} ; \quad \frac{R_7}{R_6} = a + \frac{\Delta a}{2}$$

We found already

$$A_1 = \frac{V_3 - V_4}{V_1 - V_2} = 1 + \frac{R_1 + R_3}{R_2}$$

$$H_2 = \frac{a(1+a)}{\Delta a}$$

If $V_1 = V_2$, no current flows through R_1, R_2 and R_3 resulting in for the input stage

$$\frac{\text{com.mode out}}{\text{com.mode in}} = 1 \quad \text{and} \quad \frac{\text{differential out}}{\text{com.mode in}} = 0$$

$$\text{Thus:} \quad F_1 = \frac{A_1}{1} = 1 + \frac{R_1 + R_3}{R_2}$$

The CMRR of the complete instrumentation amplifier can now be calculated.

$$H_{total} = \left(1 + \frac{R_1 + R_3}{R_2}\right) \frac{a(1+a)}{\Delta a}$$

The CMRR can be improved by increasing the gain and thereby the F_1 of the first stage by varying R_2.

5.4. Bridge Amplifiers.

An instrumentation amplifier is ideal for measuring the differential, unbalance signal of a bridge because the common mode voltage (half the bridge voltage) is suppressed by the high common mode rejection of this type of amplifiers.

Op. amp. configurations can be used if elements of the bridge are inserted in the feedback loop.

5.4.1. Small bridge deviations.

For small deviations the following, alternative configuration can be applied:

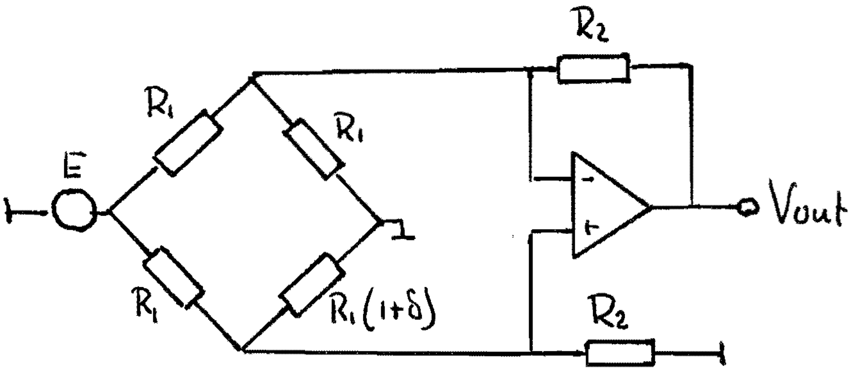


fig. 5.20

$$V_{out} = -\frac{R_2}{R_1} E + \frac{R_1(1+\delta)//R_2}{R_1 + R_1(1+\delta)//R_2} \cdot \frac{2R_2 + R_1}{R_1} .E$$

$$= -\frac{R_2}{R_1} E + \frac{R_2}{R_1} (1 + \delta)(1 - \delta \frac{R_1 + R_2}{R_1 + 2R_2} + \ldots) E$$

$$= E\left(\delta \frac{R_2}{R_1} \cdot \frac{R_2}{R_1 + 2R_2} + a\delta^2 + \ldots \right)$$

Thus the output is proportional to δ if the bridge is close to balance, (δ = small). Large deviations cause non-linearities. This method is advantageous in combination with a strain gauges bridge where anyway the deviation is in the order of 10^{-3}.

A high gain $\left(\sim \frac{R_2}{2R_1}\right)$ can be obtained.

5.4.2. Large bridge deviations.

For a large unbalance of the bridge use can be made of the common mode rejection of an op. amp.

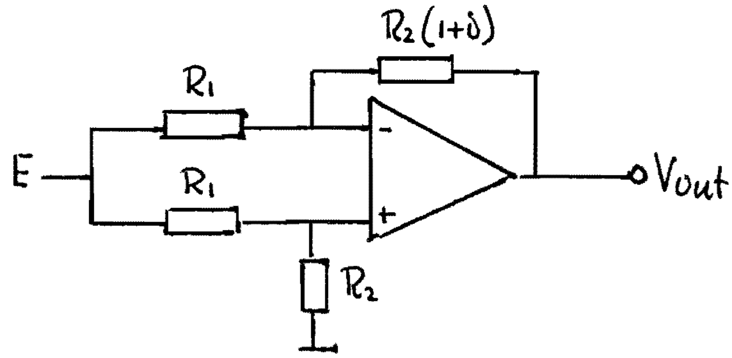


fig. 5.21

As discussed, the rejection is almost completely dependent on the unbalance of the resistors and not on the CRMM of the op. amp.

$$V_{out} = E\left[-\frac{R_2(1+\delta)}{R_1} + \frac{R_2}{R_1+R_2}\cdot\frac{R_1+R_2(1+\delta)}{R_1}\right]$$

$$= -E.\delta.\frac{R_2}{R_1+R_2}$$

The deviation of the bridge can be large, the output remains linear, but the obtainable gain is low.

5.5. Active Filters.

A second order transfer function can be realized by an operational amplifier with multiple feedback.
Successive stages can be cascades to give higher order transfer functions. By selecting the right passive elements for the feedback loop. Butterworth, Chebyshev, Bessel and other filters can be composed.

Example: Second order Low Pass Filter

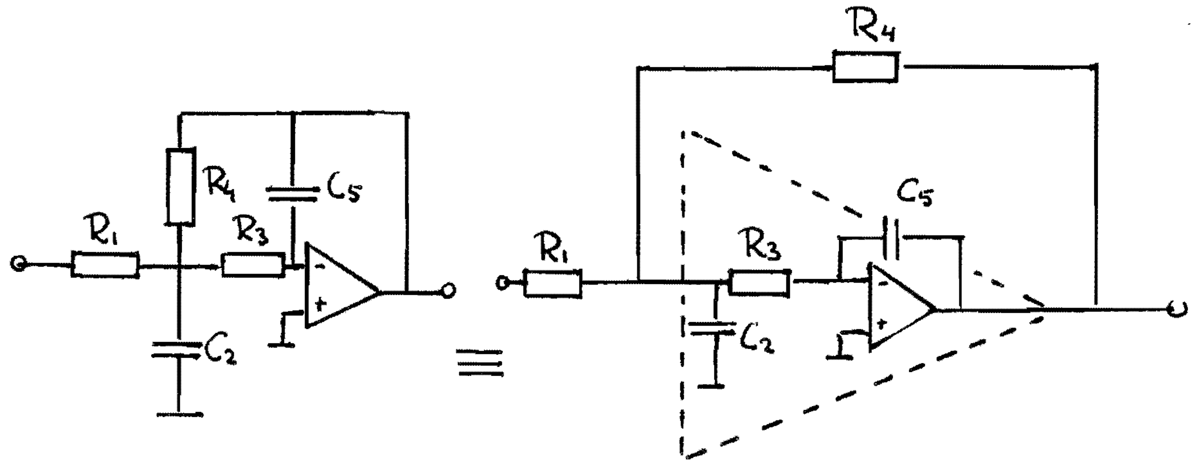


fig. 5.22

As discussed, the transfer function of the inverting amplifier with $R_i = \infty$ is:

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \quad \frac{1}{1+\mu} : \mu = \frac{1}{A}\left(1 + \frac{Z_2}{Z_1} + \frac{Z_2}{R_i}\right)$$

where $A = \frac{1}{jwC_5} \cdot \frac{1}{R_3}$, $R_i = R_3 // \frac{1}{jwC_2}$

$$\frac{V_o}{V_i} = -\frac{R_4}{R_1} \cdot \frac{1}{1 + jwC_5R_3 \left(1 + \frac{R_4}{R_1} + \frac{R_4}{R_3} \left(1 + jwC_2R_3\right)\right)}$$

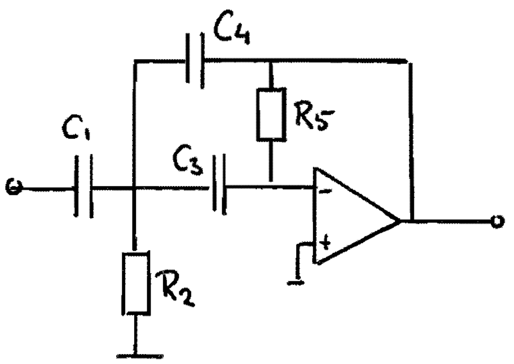
$$= -\frac{R_4}{R_1} \cdot \frac{1}{1 + jwC_5 R_3 \left(1 + \frac{R_4}{R_1} + \frac{R_4}{R_3}\right) + j^2w^2C_2C_5R_3R_4}$$

Hence $w_o = \frac{1}{\sqrt{C_2C_5R_3R_4}}$ and $\zeta = \frac{1}{2}\sqrt{\frac{C_5}{C_2}}\left(\sqrt{\frac{R_3}{R_4}} + \sqrt{\frac{R_4}{R_3}} + \sqrt{\frac{R_3R_4}{R_1^2}}\right)$

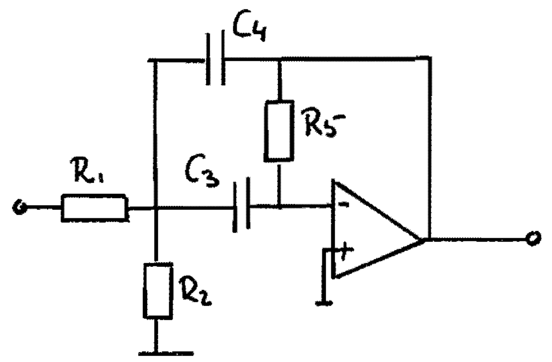
In this way the values of the components for the required filter can be calculated.
The transfer function for the non-inverting input should be investigated in order to check the stability.

These types of filters are useful in low frequency applications because coils can become quite large in this range. So a filter without coils can be desirable.
The operational amplifier provides isolation between the stages so that several stages can be put in series to provide higher order transfer functions.

This low pass filter can easily be transformed into a high pass filter by interchanging the resistors by capacitors and inversely capacitors by resistors. A partly interchange delivers a bandpass filter:



High pass filter



bandpass filter

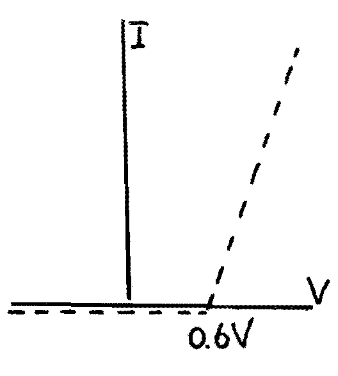
fig. 5.23

5.6. Rectifiers. (G&M, 10.2)

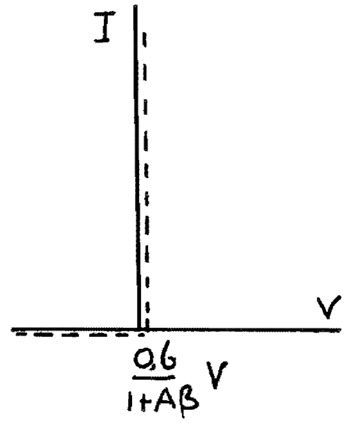
The rectification and detection of signals is normally performed by using diodes. The amplitude of these signals must be larger than a few hundred multivolts since a normal diode cannot handle very low input signals. This drawback of a diode rectifier is caused by the forward drop in the diode of about o,6V, given by the well-known diode characteristic:

$$I = I_o \left(e^{\frac{qV_D}{KT}} - 1 \right).$$

The performance of a rectifier can be greatly enhanced by addition of an op. amp. The diode drop will reduced by $A\beta$.



diode rectifier



diode + op. amp. rectifier

fig. 5.24

5.6.1. Half-wave rectifier.

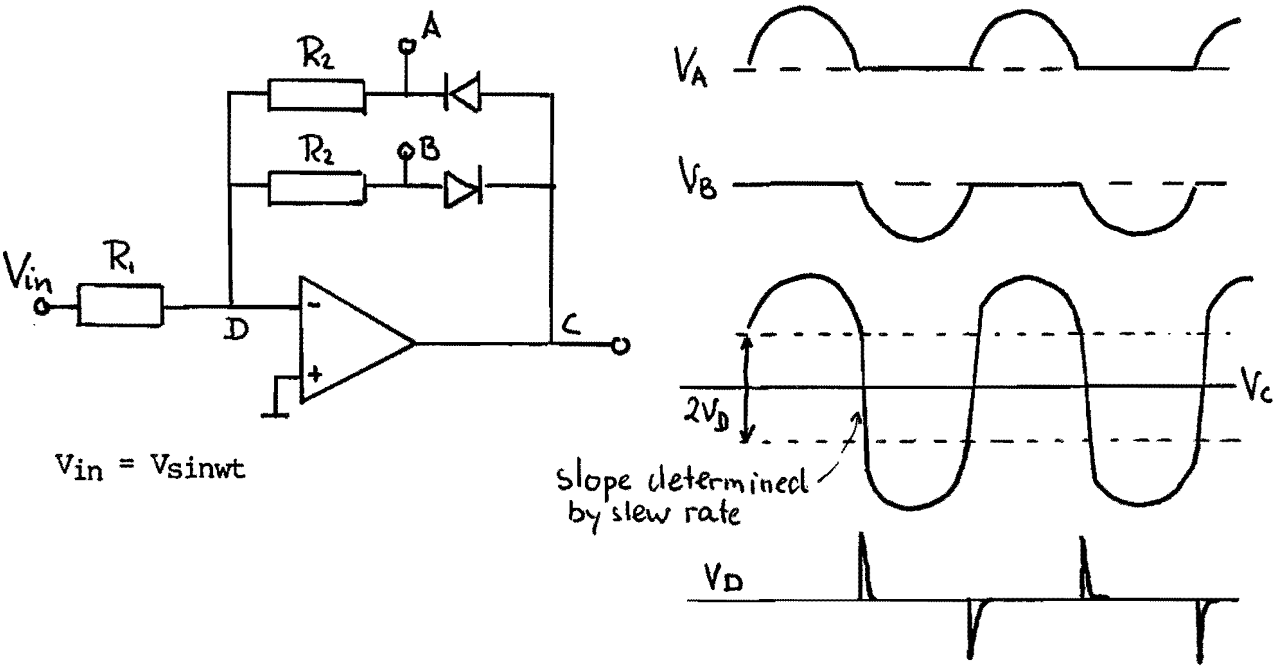


fig. 5.25

5.6.2. Full-wave rectifier.

A full-wave rectifier can be realized by adding a signal with an amplitude 2 Asinwt to the output of the half-wave rectifier as shown in the figure

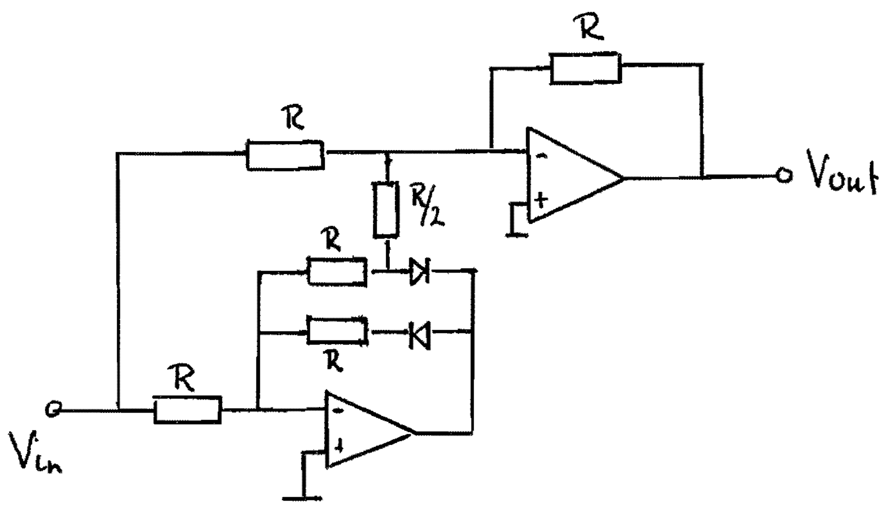


fig. 5.26

If the input is V_{in} = Acoswt and

$$V_{in} = \text{positive:} \quad V_{out} = (- V_{in} \frac{R}{R}) - (\frac{R}{R/2} \cdot (- \frac{R}{R} V_{in}) = V_{in}$$

if

$$V_{in} = \text{negative:} \quad V_{out} = (- V_{in}) \cdot \frac{R}{R} = V_{in}$$

So the output waveform is:

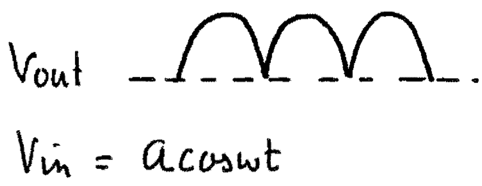


fig. 5.27

An alternative circuit with a high ohmic input and controllable gain is shown in the figure

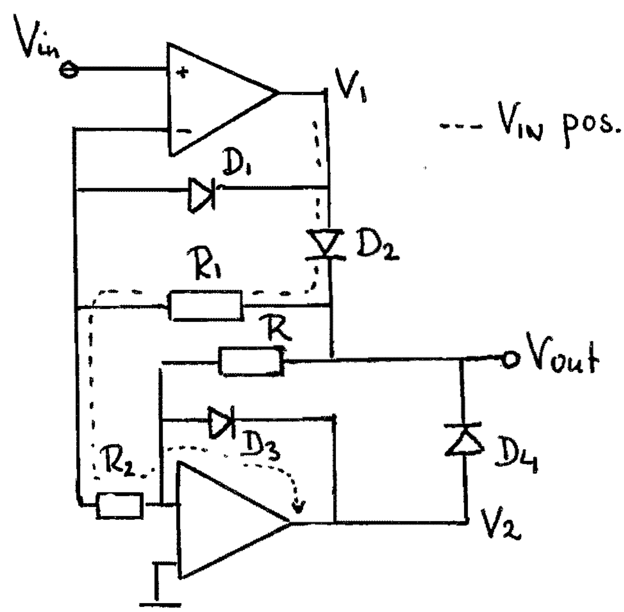


fig. 5.28

If V_{in} is positive:

- point A is kept at earth potentialby op. amp. 2

- D_2, D_3 are conducting, D_1, D_4 are off.

therefore:

$$V_1 = \left(\frac{R_1 + R_2}{R_2}\right) V_{in} + V_{diode}$$

$$V_2 = - V_{diode}$$

$$V_{out} = \frac{R_1 + R_2}{R_2} \cdot V_{in}$$

If V_{in} is negative:

- D_1, D_4 are conducting, D_2, D_3 are off .

$$V_1 = + V_{in} - V_{diode}$$

$$V_2 = - \frac{R}{R_2} + V_{diode}$$

$$V_{out} = - \frac{R}{R_2} \cdot V_{in}$$

Thus full-wave rectification is obtained if $R_1 = R_2 = \frac{1}{2} R$.
Because the parts of the circuit for the positive and negative half cycles are more or less similar, the output is for both half cycles more symmetrical.
Gain control can be inserted into the circuit by replacing R_1 and R_2 by a potentiometer, having a value R and with the tap connected to the - input of the top amplifier.

Assume: $R_1 = (1 - x)\ R$, $R_2 = xR$.

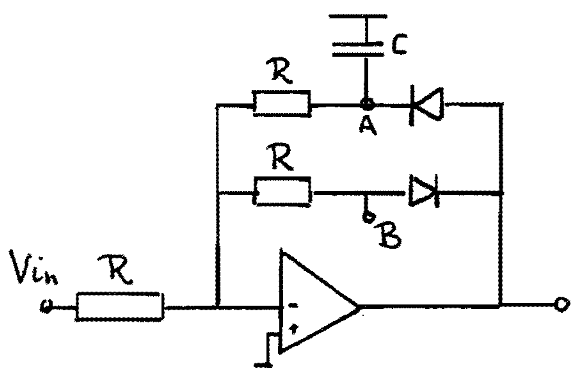
then with

$$V_{in\ positive} : V_{out} = \frac{(1-x)+x}{x}\ V_{in} = \frac{V_{in}}{x} \quad \text{and}$$

$$V_{in\ negative} : V_{out} = -\frac{R}{xR}\ V_{in} = -\frac{V_{in}}{x}$$

5.6.3. Peak-Detector.

A peak-detector can be composed by adding a capacitor to a half wave rectifier either in point A or B



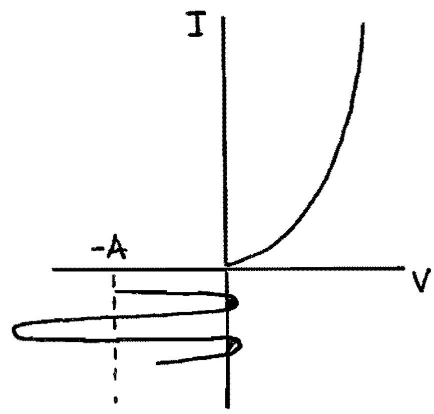


fig. 5.29

The capacitor will be charged till $V_c \sim A$ if V_{in} = Asinwt.

Diode D_1 will be closed except during the amplitude peaks for loading the capacitor. So peak detection on A.

The diode D_1 is almost continuously closed. Therefore the capacitor C can be considered being a voltage source with a voltage V_A = A.

The signal on point B will be:

$$V_B = -\frac{R}{R} V_{in} - V_A \cdot \frac{R}{R}$$

with V_A = A and V_{in} = Asinwt it is clear that V_B is always negative, and clamped at ground. The circuit is also a clamp circuit in B. A disadvantage of this peak detector is that the output A cannot be loaded. The performance can be enhanced by the addition of an amplifier as shown.

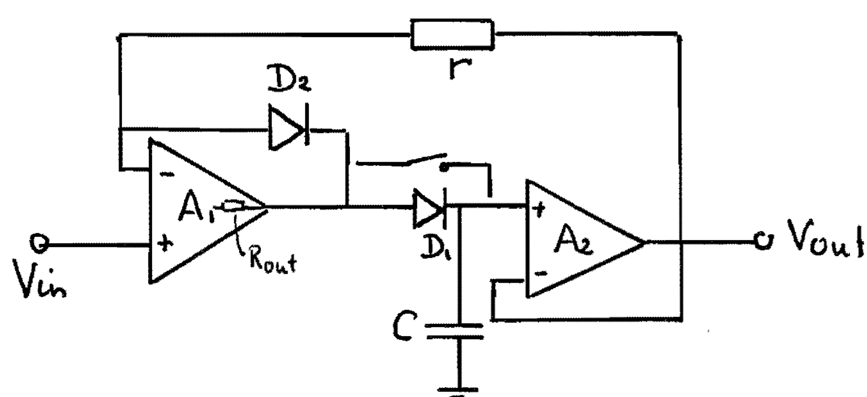


fig. 5.30

This improved peak detector operates as follows:

If V_{in} increases :

- diode 1 conducts; the feedback loop is closed
- The capacitor C will be charged until $V_{out} = V_{in}$
- The voltage on cap C = V_C V_{out} because A_2 operates as a follower.

If V_{in} decreases:

- diode 1 stops conducting; the feedback loop is interrupted
- The output voltage V_{out} is kept by the charge on C. Capacitor is not decharged because the input impedance of follower A_2 is high (FET op.amp., low I gate).
- diode 2 starts to conduct, op.amp. 1 operates as a follower.

Diode 2 prevents that the output of op. amp. 1 is driven in the negative direction until the output stage saturates when the loop is open: Now the output remains on $V_{in} - V_{diode}$: so the jump of the output voltage is limited to $2V_{diode}$ when the loop is again closed by increasing V_{in} and slew rate limiting gives less problems.

Remarks:

- The offset voltage of A_2 is not important: this will be compensated by an extra voltage across C.
- Instability can occur because the loop contains three time-constants in series: op. amp. 1, $R_{out\ op.amp.\ 1}$ with the capacitor C and the follower. Therefore use op. amps. with different time constants (e.g. broadband follower) or put a small resistor in series with the capacitor.

5.6.4. Sample and hold.

The improved peak detector circuit (fig. 5.30) can easily be changed into a sample and hold circuit.
Diode 1 has to be replaced by a switch and diode 2 should be removed.
The output will follow the input signal if the switch is closed.
A sample of this signal will be taken and held by opening the switch.

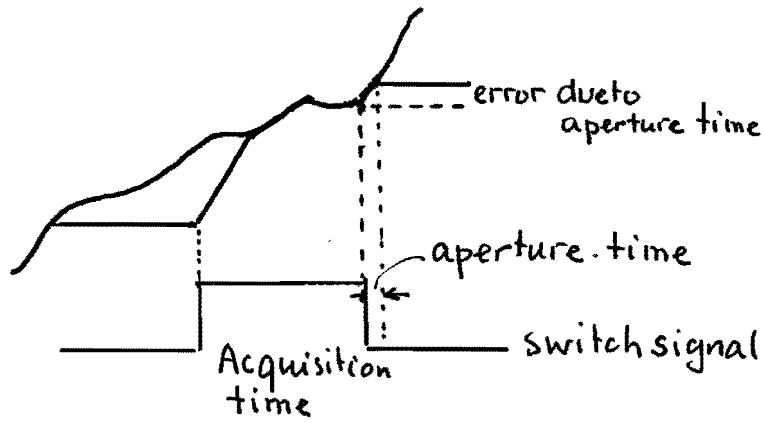


fig. 5.31

Two specifications of a sample and hold are important:

Acquisition time: The time necessary to charge the total capacitor to a full-scale voltage change and remain within a specified error band. It begins at the hold-to-sample transition of the switch.

Aperture time: The time necessary to enter the hold mode after the switch command. This time specifies the uncertainty of the sampling moment.

5.6.5. Floating rectifiers.

Floating rectifiers are applied in voltmeter circuits and can be constructed by using a diode bridge as shown.

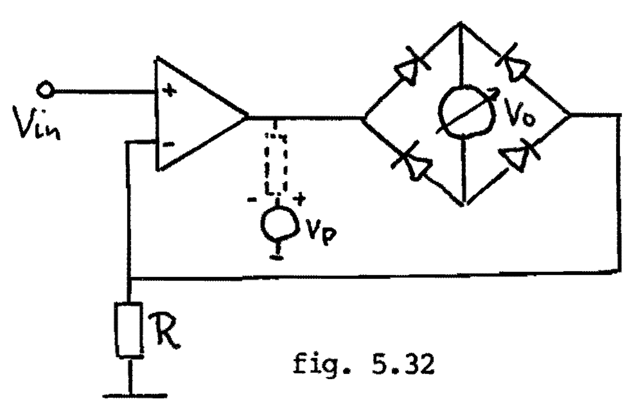


fig. 5.32

The voltmeter indicates a value V_o which is both for A.C. and D.C. proportional with $\frac{V_{in}}{R}$.

For polarity indication a small cheap meter V_p can be used to indicate + or - .

For high-accuracy AC measurements the closed loop gain $A\beta$ should be large.
In order to extend the frequency range, an op. amp. can be used with a 12db/octave part as follows:

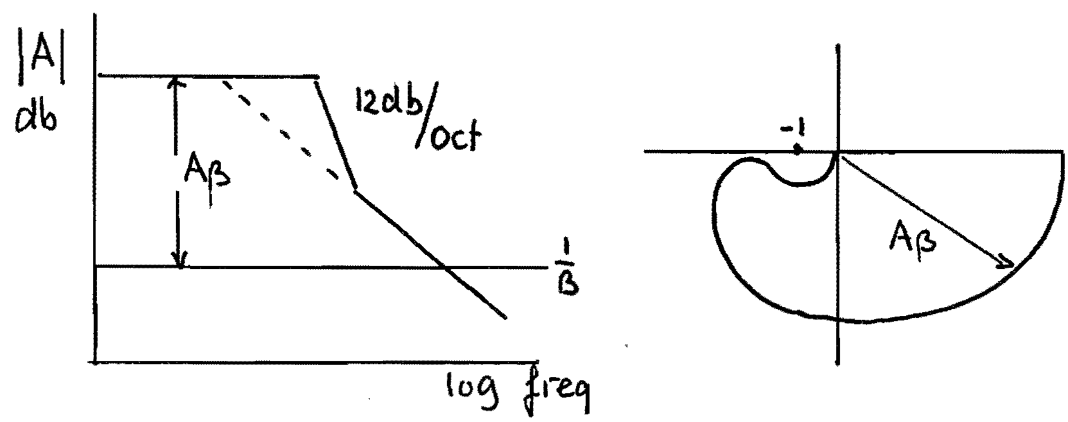


fig. 5.33

For stability reasons the $1/\beta$-line should cut the -6db/oct. part of the $|A\beta|$ curve in the Bode-plot. A drawback of such a circuit is that it can come very close to oscillation if $|A\beta|$ is reduced by some cause.

In the above voltmeter circuit this can happen when large signals are applied and clipping occurs due to the output voltage limits of the op. amp. This clipping can be interpreted as loop gain reduction because the first-harmonic of the output signal becomes smaller.

Also when small signals are applied the circuit becomes more oscillatory. This can be explained by β becoming smaller because the dynamic resistance R_D of the diodes increases.

In the Bode-plot this effects that the 1/β-line moves up.

5.7. Logarithmic Amplifiers.

Log. amplifiers find wide application in instrumentation systems where signals of very large dynamic range must be sensed and recorded. Logarithmic amplifiers can be bought as modules which have an output voltage proportional to the logarithm of the input voltage. The exponential relationship between the collector current and the base-emitter voltage of a bipolar transistor is used to generate this logarthmic characteristic:

$$I_c = I_o \left(e^{\frac{qVBE}{KT}} - 1\right) \quad \text{or}$$

$$V_{BE} = \frac{KT}{q} \ln \frac{I_c}{I_o} \quad ; \quad I_o = \text{leakage current.}$$

This relation is valid over a wide range (10^{-10} -- 10^{-3} A) if $V_{CB} = 0$.

The log. characteristic starts to deviate at 1mA because r_e and r_{bb} become important as compared with $r_o \sim$ 25 ohm.

5.7.1. Log. Amplifier.

The circuit of a log. amplifier is shown in the figure

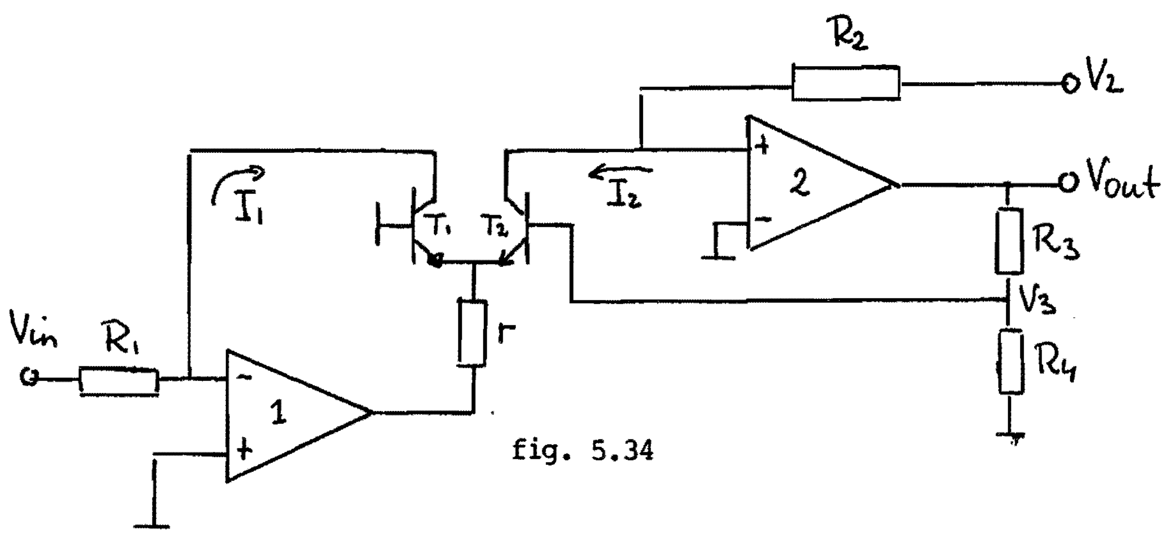


fig. 5.34

We can derive for this circuit if the transistors are identical and at the same temperature:

$$\left.\begin{aligned}
V_{BE_1} &= \frac{KT_1}{q} \ln \frac{V_1}{R_1 I_{01}} \\
V_{BE_2} &= \frac{KT_2}{q} \ln \frac{V_2}{R_2 I_{02}} \\
V_3 &= V_{BE_2} - V_{BE_1} \\
V_{CB_1} &= 0 \ , \quad T_1 = T_2 \\
I_{01} &= I_{02}
\end{aligned}\right\} \quad \text{this gives}$$

$$V_3 = \frac{KT}{q} \ln \frac{V_2}{V_1} \cdot \frac{R_1}{R_2}$$

$$V_{out} = \frac{R_3+R_4}{R_4} \cdot \frac{KT}{q} \ln \frac{V_2}{V_1} \cdot \frac{R_1}{R_2}$$

Remarks:

1. The circuit is a normal log. amplifier with V_2 = constant

   $$V_{out} = K_1 \log K_2 V_{in}$$

2. When V_1 and V_2 are both input signals the circuit delivers the log. ratio.

   $$V_{out} = K_1 \log K_2 \frac{V_2}{V_1}$$

3. The temperature sensitivity of the circuit

$$\frac{dV_{out}}{dT} = V_{out}\ \frac{1}{T} = 0{,}3\%/^{o}C$$ can be compensated by inserting an

NTC resistor in R_3.

4. The feedback factor of the loop around op.amp. 1, without r, varies with the input signal current $I_1 = V_{in}/R_1$

$$\beta = gm.R_1 = \frac{qI_1}{KT}\ R_1$$. the gain of transistor T_1.

The system can become unstable if β becomes larger than unity for large I_1.

Therefore insert r, making $\beta = \frac{R_1}{r}$ independent of I_1.

Amplifier requirements.

1. Log. amplifier for current source signals

requirements: $I_1 >> I_{bias\ op.\ amp.}$

$I_1 << 1mA$

A FET op. amp. with a bias current of 10pA or lower can be used. The offset voltage does not influence the performance.

The input signal current can vary over 6 decades, i.e. from 1nA to 1mA.

2. Log amplifier for voltage source signals

requirements: $V_1 >> V_{offset} + V_{drift}$.

The drift and offset of FET op.amp. amounts 1mV; Thus with a maximum input signal voltage of 10 Volt, 4 decades can be handled.

The choice of a chopper stabilzed amplifier with an offset of 1 μV extends the range to 6 decades.

A FET input op. amp. is apparently used if in the log. amp. specification is stated: I range 10^6 and V range 10^3 .

5.7.2. Antilog Amplifier

The configuration of an antilog amplifier is shown in the figure below. The transistors are again assumed to be identical.

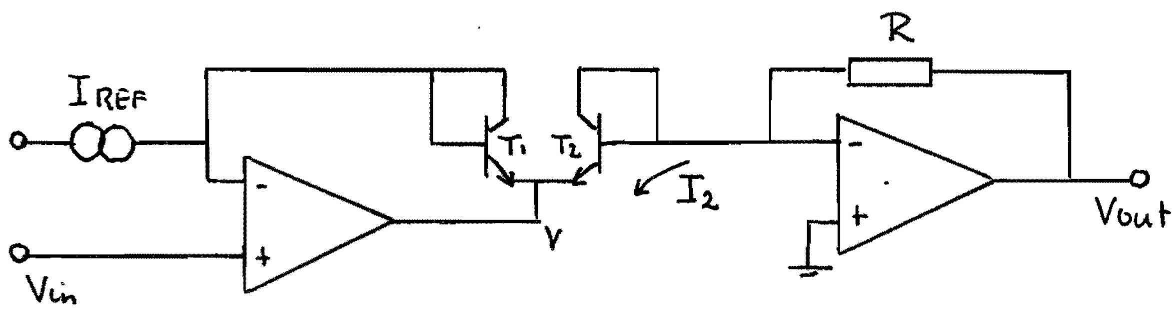


fig. 5.35

Using the same type of relations we can calculate:

$$V = V_{in} - V_{BE1} = -V_{BE2}$$

$$V_{BE1} = \frac{KT_1}{q} \ln \frac{I_{REF}}{I_{01}}$$

$$V_{BE2} = \frac{KT_2}{q} \ln \frac{I_2}{I_{02}}$$

$$= \frac{KT_2}{q} \ln \frac{V_{out}}{R} \cdot \frac{1}{I_{02}}$$

$$V_{in} = \frac{KT}{q} \ln \frac{I_{REF}R}{V_{out}}$$

or

$$V_{out} = I_{REF}R \cdot e^{\frac{-qV_{in}}{KT}}$$

The dynamic range of the antilog amplfier is determined by the realisable range of I_2 . Therefore, a FET op. amp. with a low bias current can be applied as output amplifier: A chopper stabilized amplifier with the inherent low offset is even more favourable.

5.7.3. Amplifier with an exponential gain control

A useful amplifier configuration is illustrated in the figure

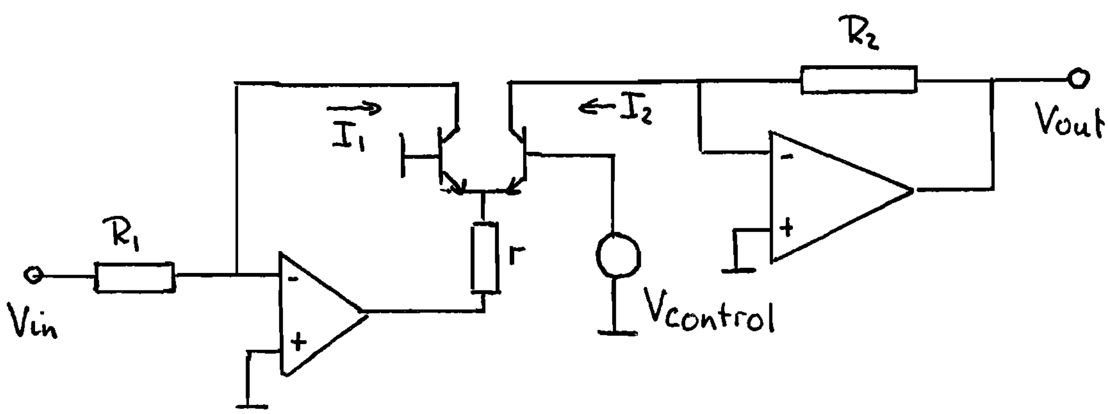


fig. 5.36

Using the logarithmic relations we can calculate the transfer characteristic of this amplifier

$$\left.\begin{aligned} V_{BE1} &= \frac{KT}{q} \ln \frac{V_{in}}{R_1 I_{01}} \\ V_{BE2} &= \frac{KT}{q} \ln \frac{V_{out}}{R_2 I_{01}} \\ V_{control} &= V_{BE1} - V_{BE2} \end{aligned}\right\} \qquad \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1} \cdot e^{\frac{V_{control}}{KT}}$$

This circuit gives the possibility to control the gain over 6 or 7 decades without distortion. The draw back is that the amplifier can only handle positive signals.

5.8. Comparator (Schmitt trigger)

One of the most common applications of operational amplfiers is voltage comparison. This is accomplished with an operational amplifier open-loop gain circuit. Another characteristic provided by comparators is hysteresis This hysteresis can be added to the comparator by applying positive feedback from output to non-inverting input as shown.

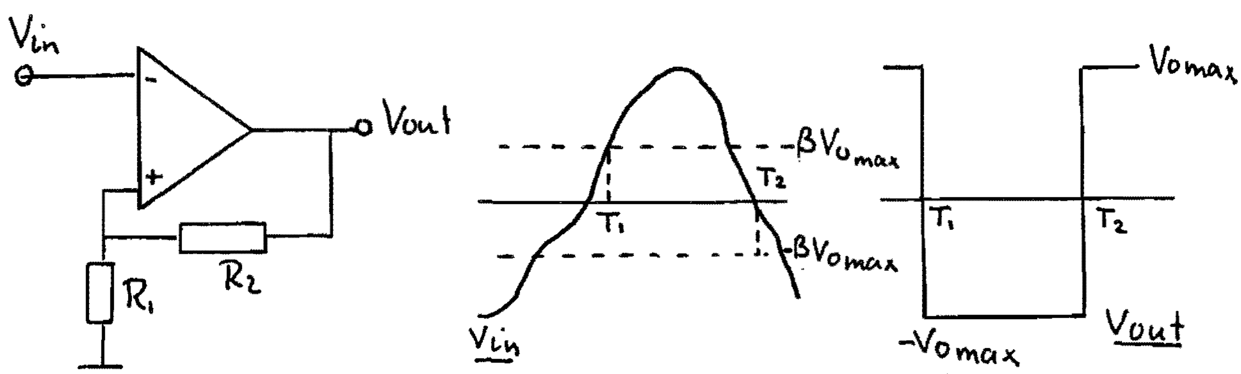


fig. 5.37

When the input signal is larger than βV_0, the output switches to the maximum negative output voltage. The threshold voltage at the + input remains now $-\beta V_0$ until $V_{in} < -\beta V_0$.

The output will flip to the maximum positive voltage. The threshold voltage becomes now $+\beta V_0$. The threshold band for this configuration can be fixed by means of R_1 and R_2 : $\beta = \dfrac{R_1}{R_1 + R_2}$.

The threshold voltage is not well controlled since it is derived from the output saturation voltages of the operational amplifier. Greater accuracy is achieved by clamping the comparator output by zener diodes, diodes etc. as illustrated in the figure.

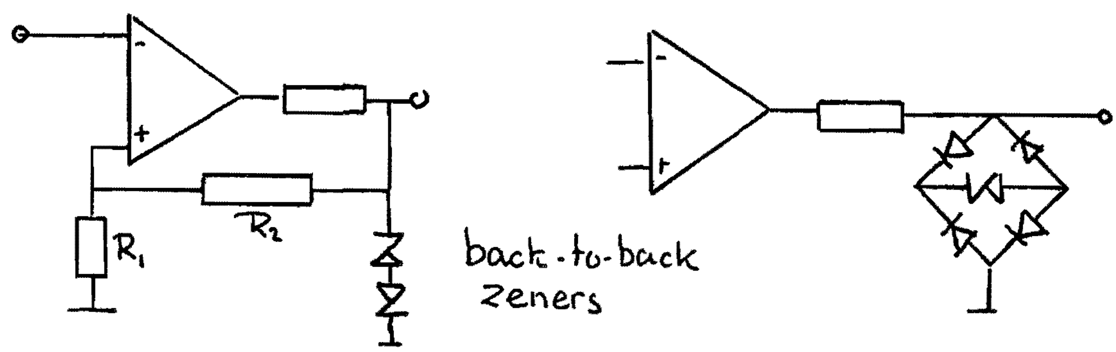


fig. 5.38

The zener/diode bridge will always give a symmetrical output and therefore symmetrical threshold values. Asymmetrical clamping can be realised by using unequal zener diodes.

5.9. Waveform Generators.

5.9.1. Multivibrator: square wave - triangle wave generator

The circuit shown forms a simple square-wave generator. The op. amp. serves the function of comparison. The required regenerative action comes from the positive feedback $\beta = \dfrac{R_1}{R_1 + R_2}$.

Suppose that the output is positive: = $V_{out\ max}$.

The capacitor C is charged via R. When the voltage becomes $\beta V_{out\ max}$, the operational amplifier will flip from saturation in the positive direction to saturation in the negative direction.
Reversal will occur when the capacitor voltage becomes $-\beta V_{out\ max}$.

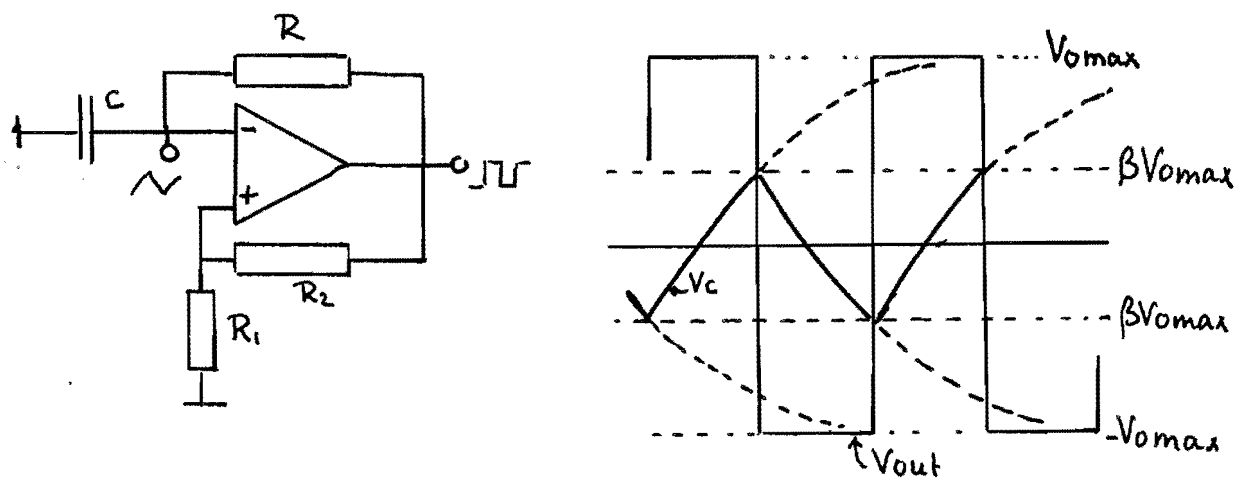


fig. 5.39

The circuit generates a square-wave at the output and a triangle-wave at the -input with an amplitude $\beta V_{out\ max}$. The slopes of the triangle-wave will become more linear by choosing a small β.

The waves can be made asymmetrical by charging the capacitor with different resistors. An other possibility is to clamp the output with unequal zener diodes as shown in fig. 5.38. The triangle-wave becomes a sawtooth signal.

The performance of the circuit can be improved by replacing the RC-combination with an integrator and an inverter. Operational amplifier 1 is applied as a Schmitt trigger. The integrator generates a triangle-wave with linear slopes.

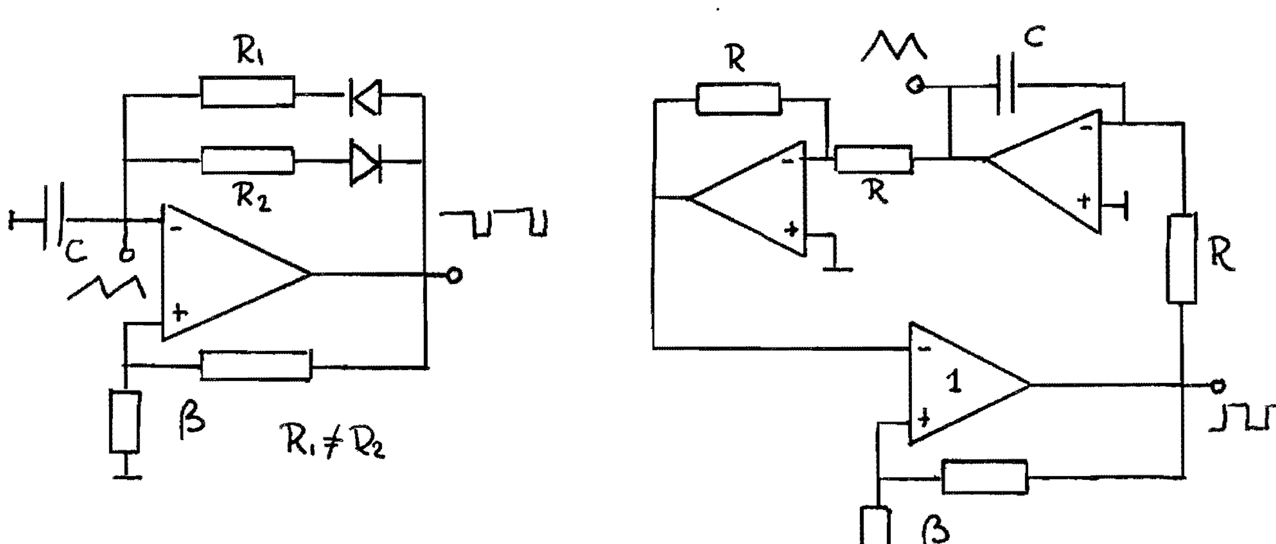


fig. 5.40

5.9.2. Monostable Multivibrator

A relatively simple monostable multivibrator is shown in the figure

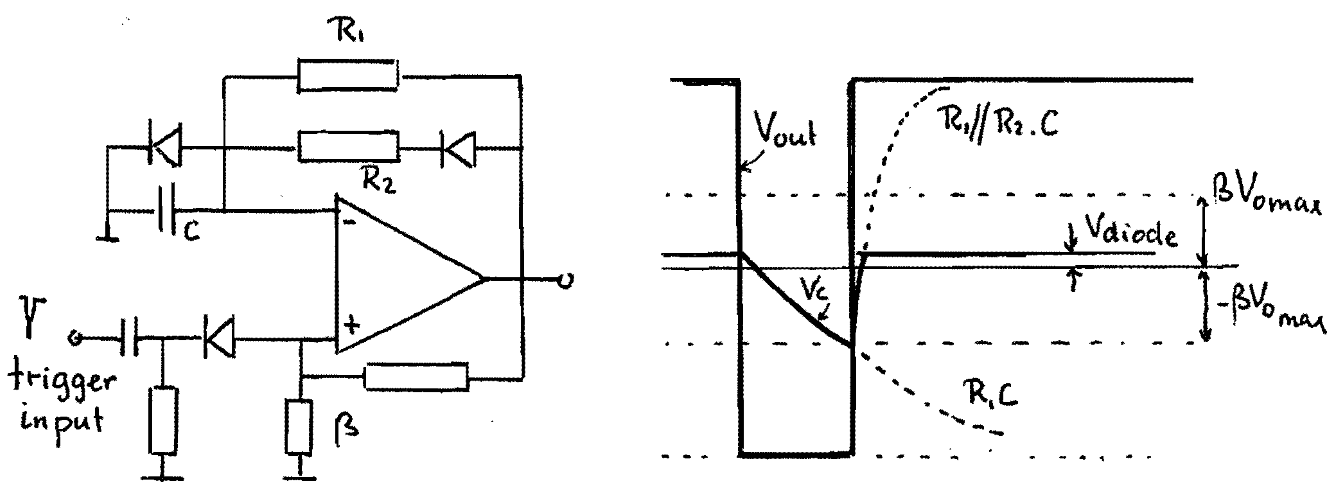


fig. 5.41

In the stable state the output is $+V_{max}$ and the capacitor voltage is clamped at about +0.6V.

A negative trigger of greater than $\beta V_{max} - V_{diode}$ will cause the output to flip negative to $-V_{max}$. The capacitor starts charging through R_1 towards $-V_{max}$. But when V_C is more negative than $-\beta V_{max}$ the output will flip back to $+V_{max}$. This completes the single pulse.
To reset for the next pulse, C_1 is charged through $R_1//R_2$.

Appendix A

Quadratic Form.

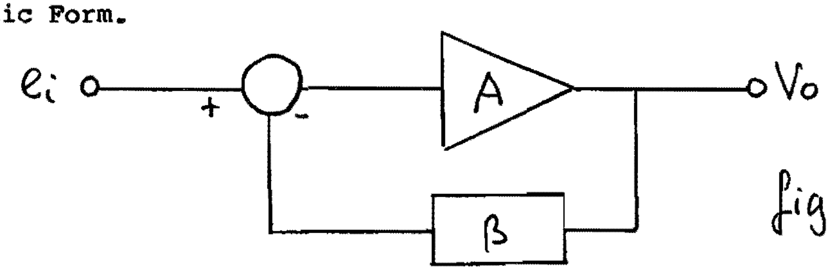


fig A.1

The transfer function of amplifier A is given by

$$A = \frac{A_0}{(1 + j\omega\tau_1)(1 + j\omega\tau_2)}$$

with τ_1 and τ_2 real.

With feedback applied it becomes:

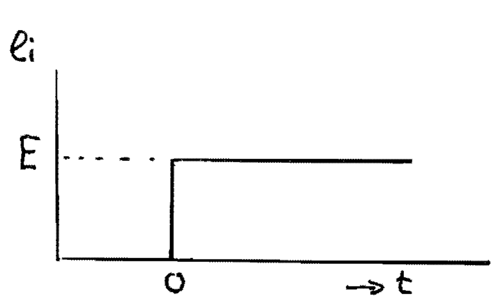
$$A' = \frac{A_0}{(1+\beta A_0)\left(1 + j\omega \frac{\tau_1+\tau_2}{1+\beta A_0} + (j\omega)^2 \frac{\tau_1\tau_2}{1+\beta A_0}\right)}$$

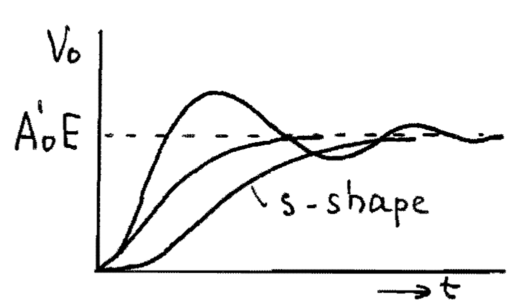
$$A' = \frac{A_0'}{1 + 2\zeta j\frac{\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2} \qquad (1)$$

where the natural frequency $\omega_n = \sqrt{\dfrac{1+\beta A_0}{\tau_1\tau_2}}$

and the relative damping $\zeta = \dfrac{1}{2}\dfrac{\tau_1+\tau_2}{\sqrt{(1+\beta A_0)\tau_1\tau_2}}$

We like to analyse the response of the system if a stepfunction is applied at the input.





Using the Laplace transformation we get:

$$\frac{V_0(s)}{e_i(s)} = \frac{A_0' \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$e_i(s) = \frac{E}{s}$$

$$V_0(s) = \frac{A_0' \omega_n^2 E}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{A_0' \omega_n^2 E}{s(s-p_1)(s-p_2)}$$

$$V_0(s) = A_0' E \left\{ \frac{1}{s} + \frac{\frac{p_2}{p_1 - p_2}}{s - p_1} - \frac{\frac{p_1}{p_1 - p_2}}{s - p_2} \right\}$$

or

$$V_0(t) = A_0' E \left\{ 1 + \frac{p_2}{p_1 - p_2} e^{p_1 t} - \frac{p_1}{p_1 - p_2} e^{p_2 t} \right\} \qquad (2)$$

This expression for the closed loop pulse response shows how it depends on the values of the poles p_1 and p_2. These poles are a function of $A_0\beta$.

This function can be described by using the root-locus technique, that involves calculation of the actual poles and zeros of the amplifier and their movement in the s-plane as the loop-gain magnitude $A_0\beta$ is changed.

The poles are the roots of the equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

this gives :

$$p_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$p_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

Remark: $\zeta\omega_n = \frac{1}{2}\frac{\tau_1 + \tau_2}{\tau_1 \tau_2} \sim \frac{1}{2}\frac{1}{\tau_2} = \text{Constant}$

Pole investigation:

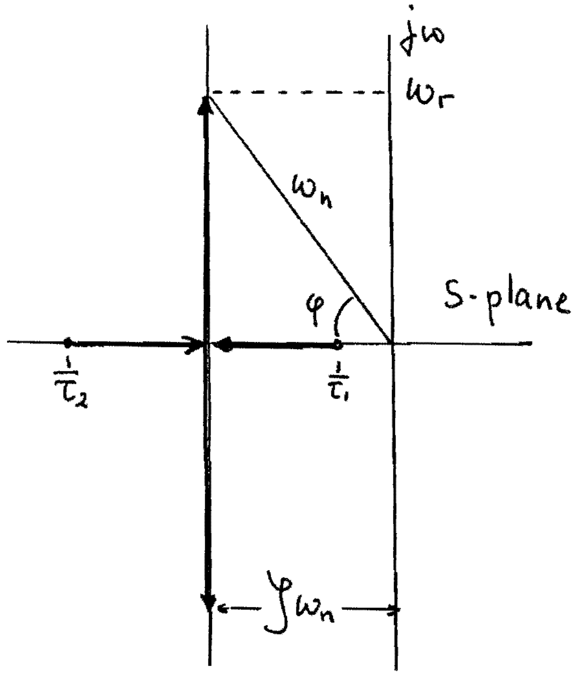
a) $A_0\beta = 0$ poles $-\tau_1, -\tau_2$, open loop

b) $\zeta > 1$ two real poles, the stepresponse has a s-shape,

c) $\zeta = 1$ two identical, real poles,

d) $\zeta < 1$ two complex poles.

The rootlocus of the poles is drawn in fig. A.3.



$$\omega_r = \omega_n \sqrt{\zeta^2 - 1}$$

$$\varphi = \cos^{-1} \zeta$$

$$p_1 - p_2 = 2j\omega_n \sqrt{1 - \zeta^2}$$

$$\frac{p_2}{p_1 - p_2} = -\frac{\zeta}{2j\sqrt{1-\zeta^2}} - \frac{1}{2}$$

$$\frac{p_1}{p_1 - p_2} = -\frac{\zeta}{2j\sqrt{1-\zeta^2}} + \frac{1}{2}$$

Substituting in equation (2) will give:

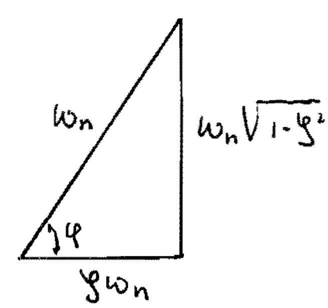
$$v_o(t) = A_0'E\left[1 + e^{-\zeta\omega_n t}\left\{-\frac{\zeta}{2j\sqrt{1-\zeta^2}}\left(e^{+j\omega_n\sqrt{1-\zeta^2}} - e^{j\omega_n\sqrt{1-\zeta^2}}\right) + \right.\right.$$

$$\left.\left. -\frac{1}{2}\left(e^{j\omega_n\sqrt{1-\zeta^2}} + e^{j\omega_n\sqrt{1-\zeta^2}}\right)\right\}\right]$$

$$v_o(t) = A_0'E\left[1 + e^{-\zeta\omega_n t}\left\{-\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_r t - \cos\omega_r t\right\}\right] \quad (4)$$

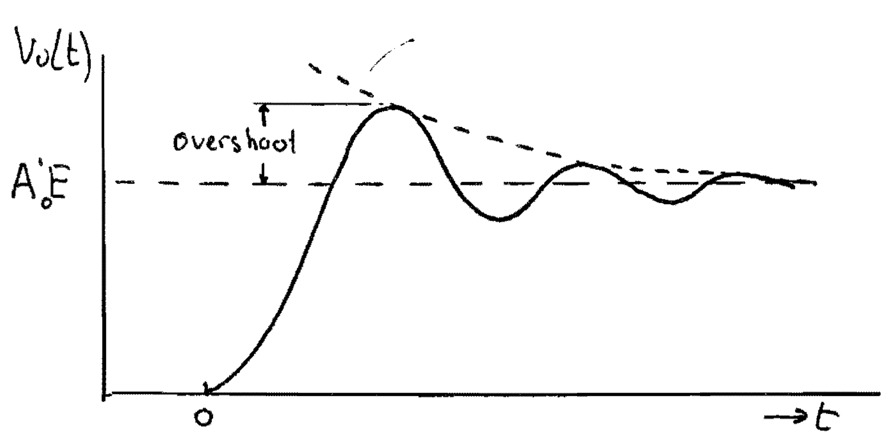
With $\omega_r = \omega_n\sqrt{1-\zeta^2}$: resonance frequency

and $tg\,\varphi = \dfrac{\sqrt{1-\zeta^2}}{\zeta}$, $\sin\varphi = \sqrt{1-\zeta^2}$



Equation (4) becomes:

$$V_o(t) = A_0'E\left\{1 - \frac{e^{-\zeta\omega_n t}}{\sin\varphi}\cdot\sin(\omega_r t + \varphi)\right\}$$



Determination of the overshoot

The maximum occurs if $\frac{dV_0(t)}{dt} = 0$

$$\frac{dV_0}{dt} = 0 \longrightarrow \omega_r t = k\pi \quad , \quad k = 0, 1, 2 \ldots$$

$$t_1 = \frac{\pi}{\omega_r}$$

$$V_0(t) = A_0' E \left(1 + e^{-\frac{\xi \pi \omega_n}{\omega_r}}\right)$$

overshoot $\quad e^{-\frac{\xi \pi \omega_n}{\omega_n(\sqrt{1-\xi^2})}} = e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}$

ξ	overshoot	
$\geqslant 1$	no	S-shape
$\frac{1}{\sqrt{2}}$	5%	oscillatory
$1/2$	17%	"
$1/4$	45%	"

Philips International Institute
1988-02-29

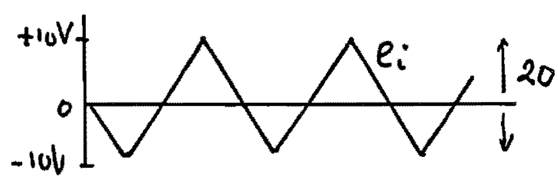
EXAMINATION OPERATIONAL AMPLIFIERS

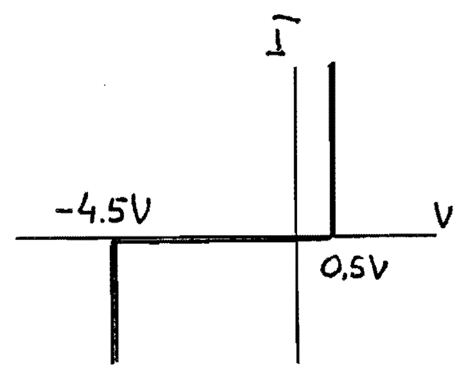
Wednesday, 9th March 1988, 8.45 hours

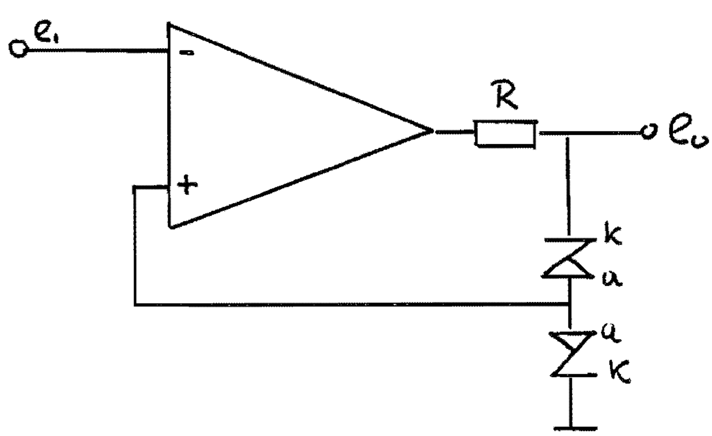
Time available: 3 hours.

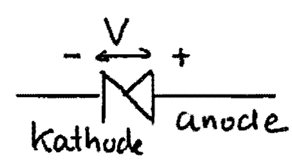
There are 4 problems.

Problem 1







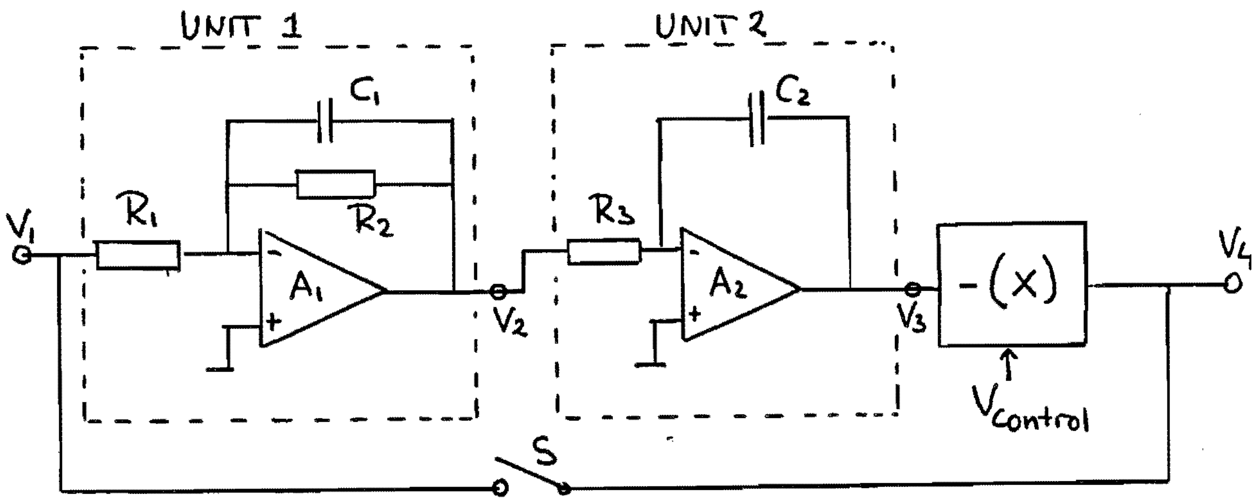


An ideal operational amplifier has a feedback circuit which consists of two zener diodes in series with opposite polarity. The characteristic of the zeners is given in the figure.

a) Draw the output signal e_o when the input signal e_i equals a symmetrical triangular signal with a peak-peak amplitude of 20V.

b) Calculate the ratio of the time intervals between zero-crossings during each period of the output signal e_o .

Problem 2



The amplifiers A_1 and A_2 are ideal ($A = \infty$, UGBW $= \infty$).
The transfer function of the multiplier is given by $V_4 = - V_{control} \times V_3$.

$R_1 = R_3 = 1K\Omega$, $R_2 = 10K\Omega$, $C_1 = \frac{10^6}{2\pi}$ pf, $C_2 = \frac{10^5}{2\pi}$ pf.

a) Calculate the transfer function $\frac{V_4}{V_1}$ and draw the piece-wise linear Bode plot (amplitude and phase) of this function with $V_{control} = 1$ Volt and switch S is open.

b) Switch S_2 is closed; calculate the allowable range of $V_{control}$ for which the system remains stable with a phase margin of at least 45^o.

c) Now the amplifiers A_1 and A_2 are considered not to be ideal. ($A_1 = A_2 = 10^4$. UGBW of A_1 and $A_2 = 10^6$ Hz)

Draw the Bode plots (amplitude and phase) of unit 1 and of unit 2.

Derive from the Bode plots the transfer functions of the units.

Problem 3

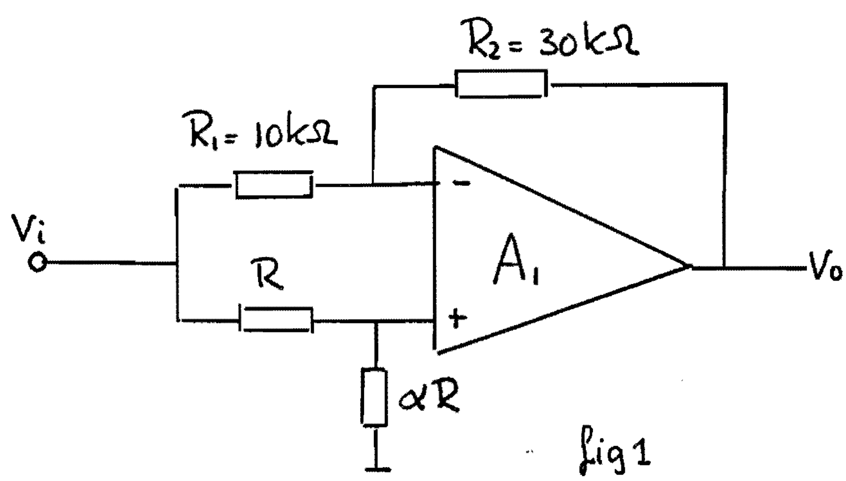


fig1

a) An ideal op. amp. (A_1) is used in a bridge configuration as shown in fig. 1. (Ideal means; $A = \infty$, $R_i = \infty$, $R_0 = 0$); $R_1 = 10K\Omega$, $R_2 = 30K\Omega$.

Calculate the transfer function $\frac{V_o}{V_i}$ as a function of α .

Sketch the DC gain $\frac{V_o}{V_i}$ as a function of α ($0 < \alpha < 10$)

b) The input bias currents and the offset voltage of the op. amp. are respectively given by:

$I_{b-} = I_{b+} = 100nA$

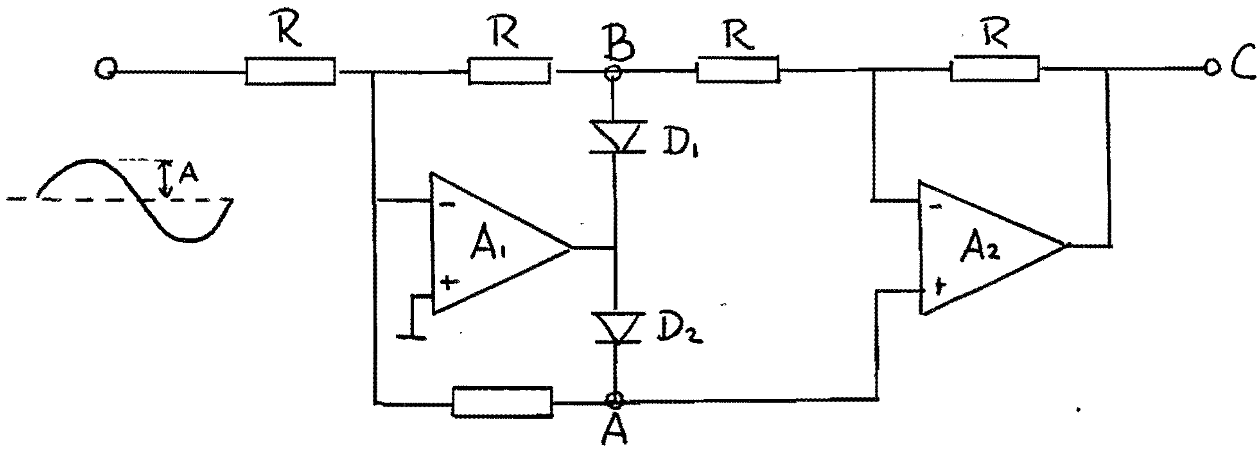
$V_{offset} = 10mV$

Calculate the offset voltage at the output of the op.amp.

Find the value of R (as a function of α) which you prefer in order to reduce the output offset, caused by the input bias currents.

c) Calculate the transfer function $\frac{V_o}{V_i}$ and sketch for a chosen α the magnitude Bode diagram of V_o/V_i of the bridge configuration but now when an operational amplifier is used with $A^o = 10^4$ and a unity gain bandwidth (UGBW) = 4MHz.

Problem 4



The op. amps. A_1 and A_2 are ideal. Draw and calculate the voltages at A, B and C if a complete sinewave $V_{in} = A_{\sin wt}$ is applied at the input.

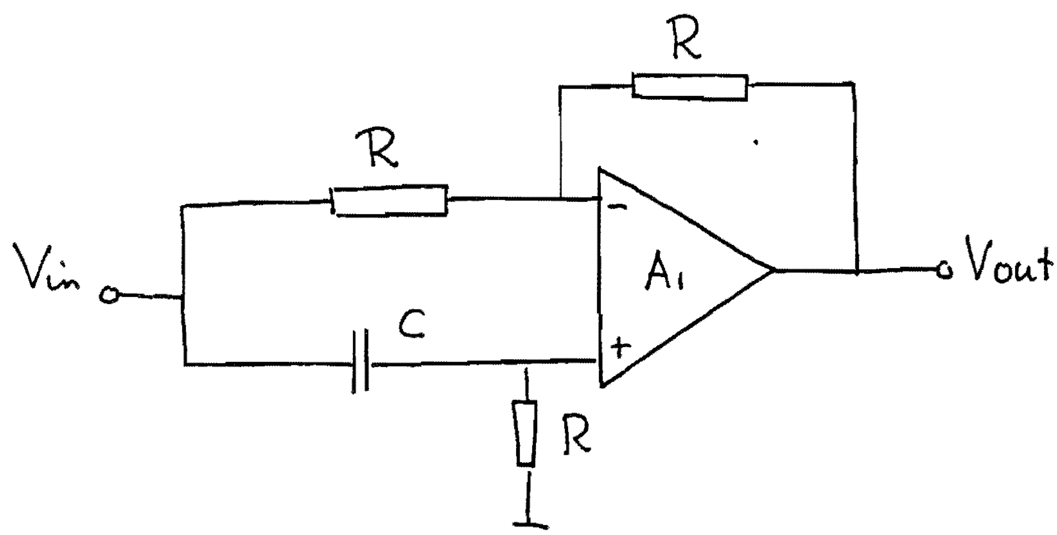
Philips International Institute.
1989-02-16

EXAMINATION "OPERATIONAL AMPLIFIERS"

Monday, 1989-03-06 - 8.45 h

Time available: 3 hours.

Problem 1



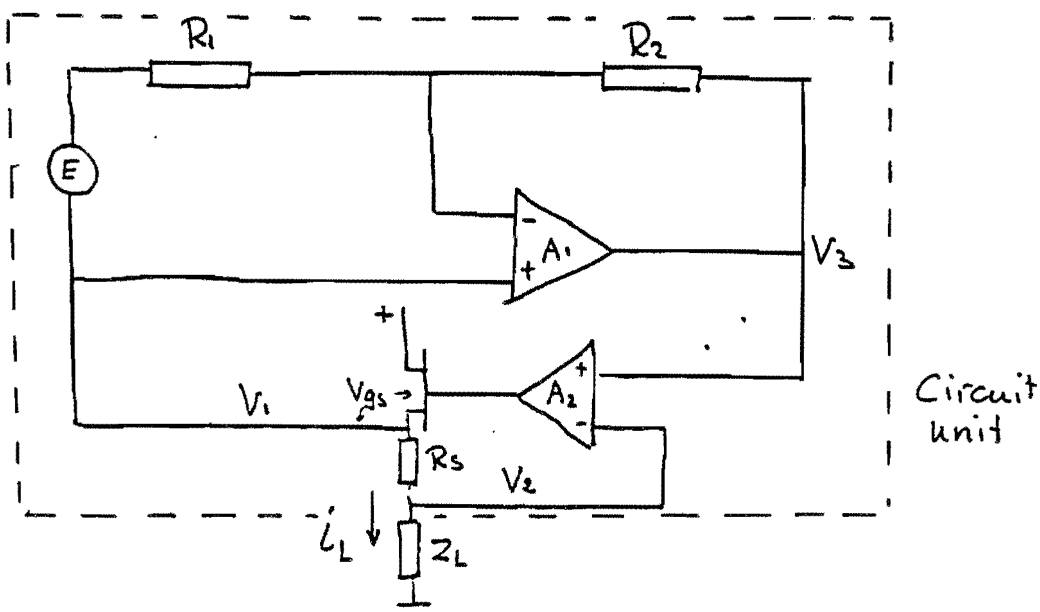
a) Op.amp. A_1 is ideal. Calculate the transfer function of the circuit and draw the Bode plot (amplitude and phase) and the Nyquist plot of this function.

b) The operational amplifier is not ideal ($A_0 = 10^4$, UGBW = 10^6 Hz)

$R = 10\ k\Omega$ and $C = \frac{10^5}{2\pi}$ pf.

Draw the Bode plot (amplitude and phase) of the non-ideal op.amp. with the feedback loop, calculate or derive from the plot the transfer function and draw a Nyquist plot.

Problem 2



a) A_1 and A_2 are ideal opamps (A, UGBW, H = ∞). The circuit unit with the output V_2 is loaded by Z_L.
Calculate the i_L. In what group can this circuit unit be classified? Why?

b) The max. output voltage and the max. common-mode voltage of the opamps equal ± 10 V.
Calculate the max. allowable value of Z_L with E = 1 volt, R_1 = 5 k, R_2 = 10 kΩ and R_s = 1 kΩ and V_{gs} = 0 V (FET).

c) Opamp A_1 remains ideal; opamp A_2 is not ideal in relation to the common-mode rejection ratio H_2 (A_2, $UGBW_2 = \infty$, $H_2 \neq \infty$).
Calculate i_L in this case ($H_2 \neq \infty$) and derive the output impedance of the circuit unit from the results.

Problem 3

A photodiode is connected to an operational amplifier as shown in fig. 1. The replacement diagram of the diode is given in fig. 2.

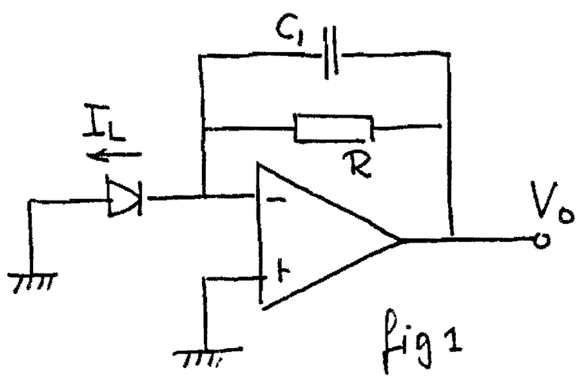


fig 1

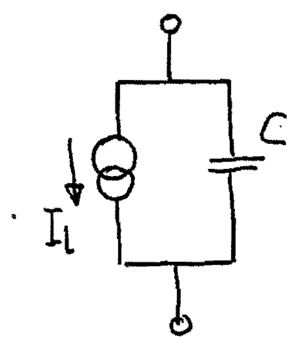


fig 2

The diode current I_L is produced by light. I_L is linearly related to the light intensity L_i according to $I_L = K.L_i$. The diode capacitance, represented by C, equals $\frac{990}{2\pi}$ pF.

The operational amplifier has a D.C. gain of 10^4 and a UGWB = 1 MHz; $R_i = \infty$, $R_o = 0$.

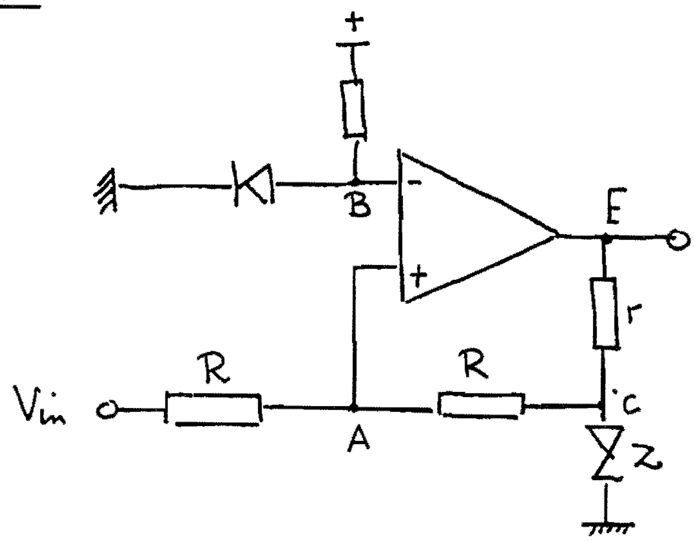
a) In order to keep the system stable the capacitor C_1 has been added to the circuit. Show by using a Bode diagram that this addition improves the stability.

b) Calculate the output noise of the circuit when the noise of the amplifier can be represented by two uncorrelated noise sources, a noise voltage source $\sqrt{\overline{E^2_N}}$ and a noise current source $\sqrt{\overline{I^2_N}}$.
The noise of the resistor R is given by $\sqrt{\overline{E^2_R}} = \sqrt{4\ kTR.\Delta f}$.
Explain that it has sense to make R large in order to improve the signal/noise ratio.

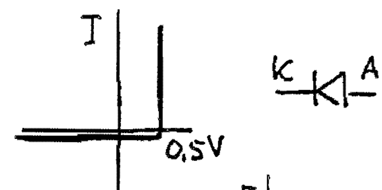
c) Calculate the values of R and C_1 on the following conditions:

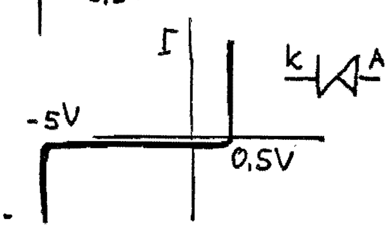
- The circuit is stable with a phase margin of 45°.
- The -3db down frequency of the transfer function $\frac{V_o}{L_i}$ equals 10 kHz.

Problem 4



The characteristics of the diode and the zener are given in the figures.





Resistor $r \ll R$

V_{out} max (E) = ± 10 V

The input signal V_{in} is a ramp function starting at t_0.

$V_{in} = a(t-t_0)$; $V_{in} = 0$ for $t < t_0$; $a = 1$ Volt/sec.

- Draw the signals at the points A, B, C and E as a function of V_{in}.
- Calculate the moment t_1 at which the output (E) changes its polarity.
- What is the amplitude of V_{in} at this moment (t_1).
- Now the slope of the ramp function is inverted. $V_{in} = V_{t_1} - a(t-t_1)$.

  At what input voltage will the output (E) switch its polarity again and when (t_2) ?

- - - - -

Eindhoven International Institute.
1990-02-20

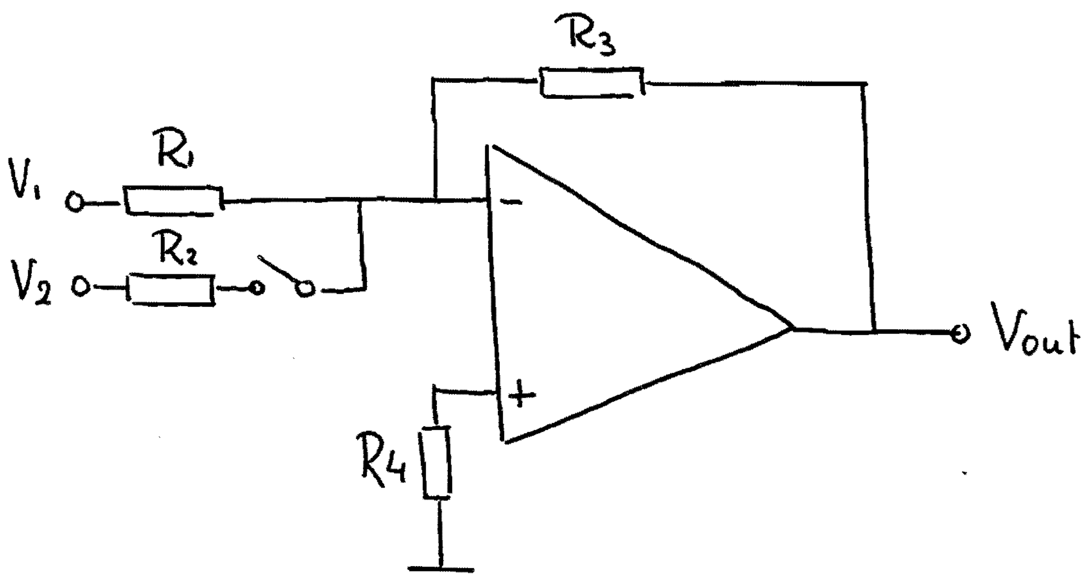
EXAMINATION "OPERATIONAL AMPLIFIERS"

Monday, 1990-03-05 - 8.45 h

There are 4 problems.

Time available: 3 hours.

Problem 1



The operational amplifier has a DC gain of 10^4, a frequency characteristic with two poles, τ_1 and τ_2, at respectively 1 kHz and 100 kHz.

$R_i = \infty$, $R_{out} = 0$, $R_1 = R_2 = 1\ k\Omega$ $R_3 = 100\ k\Omega$

V_1 and V_2 are signal sources with an output impedance of 0 ohm.

Switch S = open

a) Draw the Bode-diagrams (amplitude and phase) of the frequency characteristic of the opamp (without feedback), determine the UGBW.

b) Calculate the feedback and the transfer function V_{out}/V_1.

c) Sketch the feedback $(\frac{1}{\beta})$ and the transfer function $\frac{V_{out}}{V_1}$ in the Bode-diagram of the opamp.

What is the phase margin of the transfer function ?

d) The input bias currents and the offset voltage of the opamp are respectively given by

$I_{b-} = I_{b+} = 100$ nA

$V_{offset} = 100$ µV

Calculate the offset voltage at the output.
What value of R_4 do you prefer ?

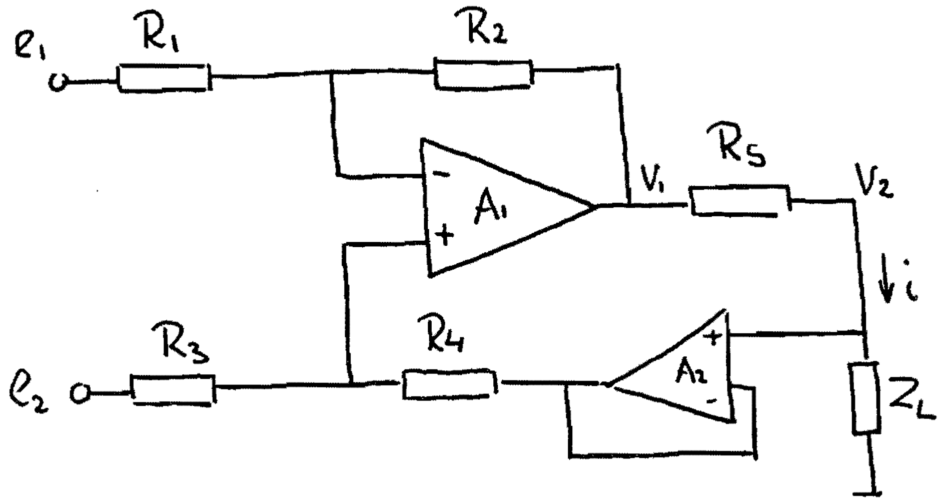
Switch S = closed

e) Calculate again the feedback.

f) Sketch the feedback and the transfer function $\frac{V_{out}}{V_1}$ in the Bode diagram of the opamp.
What is the phase margin ?

g) Calculate the offset voltage at the output with a value of R_4 you have chosen in d).

Problem 2



a) Find the algebraic expression for the current through the load Z_L if $R_1 = R_2 = R_3 = R_4$.
Take A_1, A_2 ideal amplifiers ($A = \infty$, $R_i = \infty$, $R_o = 0$ $i_{input} = 0$).

b) $e_1 = 12$ Volt, $e_2 = 14$ Volt.
The max. output voltage and max. allowable common mode voltage of the amplifiers A_1 and A_2 are 10 Volt.
Calculate R_s if $i = 1$ mA and determine the range of Z_L for which the system operates.

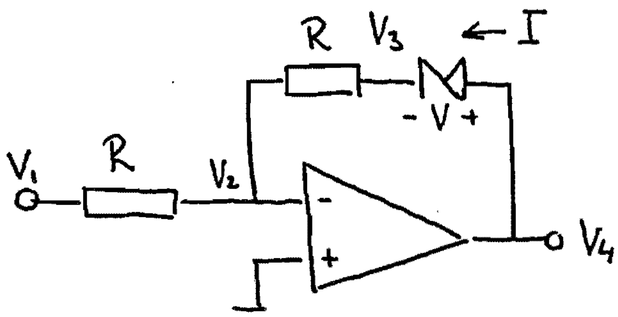
c) The resistors R_1, R_2, R_3 and R_4 are not identical.

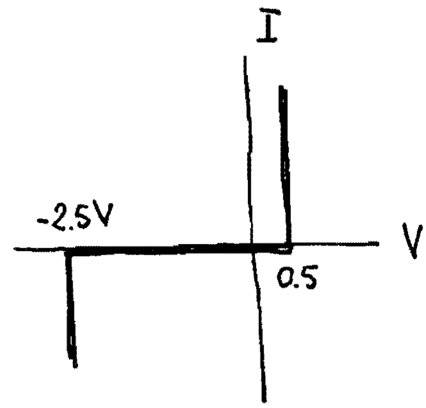
$$\frac{R_2}{R_1} = 1 - \delta \quad , \quad \frac{R_4}{R_3} = 1 + \delta \quad , \quad \delta = \text{small}.$$

Calculate the output impedance of the current source.

Problem 3

An opamp is used in combination with a Zener diode as shown in the figure. The characteristic of the Zener diode is also given.





V_1 = a triangle shaped signal with an amplitude

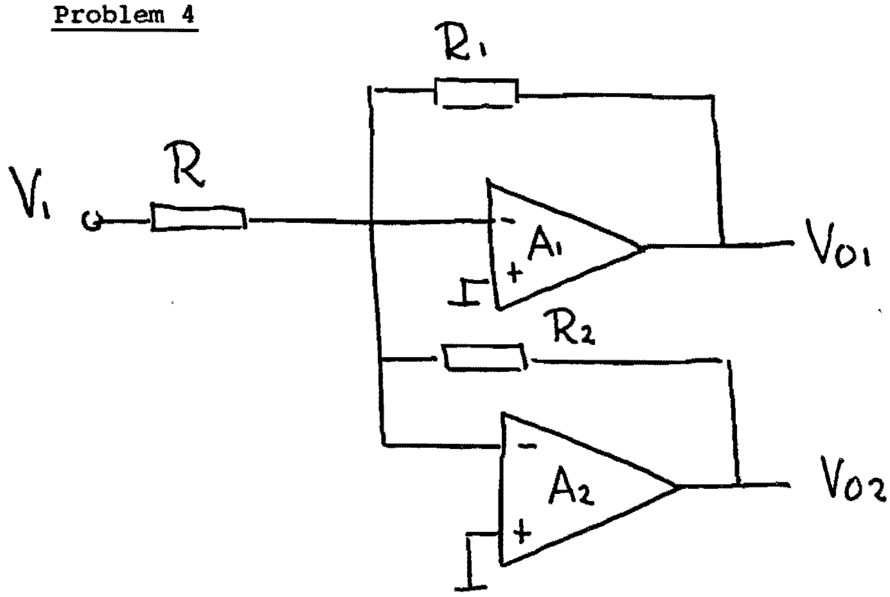
A = 1 Volt and a frequency of 25 kHz.

a) The opamp is ideal.
Draw the input signal V_1 and the signals V_2 , V_3 and V_4.

b) Assuming that the opamp has a slew rate of 1 V/µsec and that it operates linearly only for very small signals between + and - input, draw the output voltage V_4 as a function of time.

c) For the same assumptions as given in question b) draw accurately the signals V_3 and V_2 during the time that V_1 increases from -1 to +1 Volt.

Problem 4



The opamps are ideal except for the gain.
The gains are respectively A_1 and A_2.

Calculate $\frac{V_{01}}{V_1}$ and $\frac{V_{02}}{V_1}$ if the gains A_1 and A_2 are large but different.

- - - - -

Eindhoven International Institute
1991-02-28

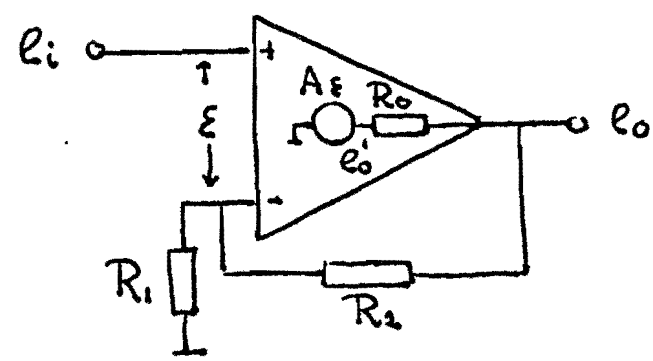
EXAMINATION "OPERATIONAL AMPLIFIERS"

Friday, 1991-03-15 - 8.45 h

There are 4 problems

time available: 3 hours

Problem 1



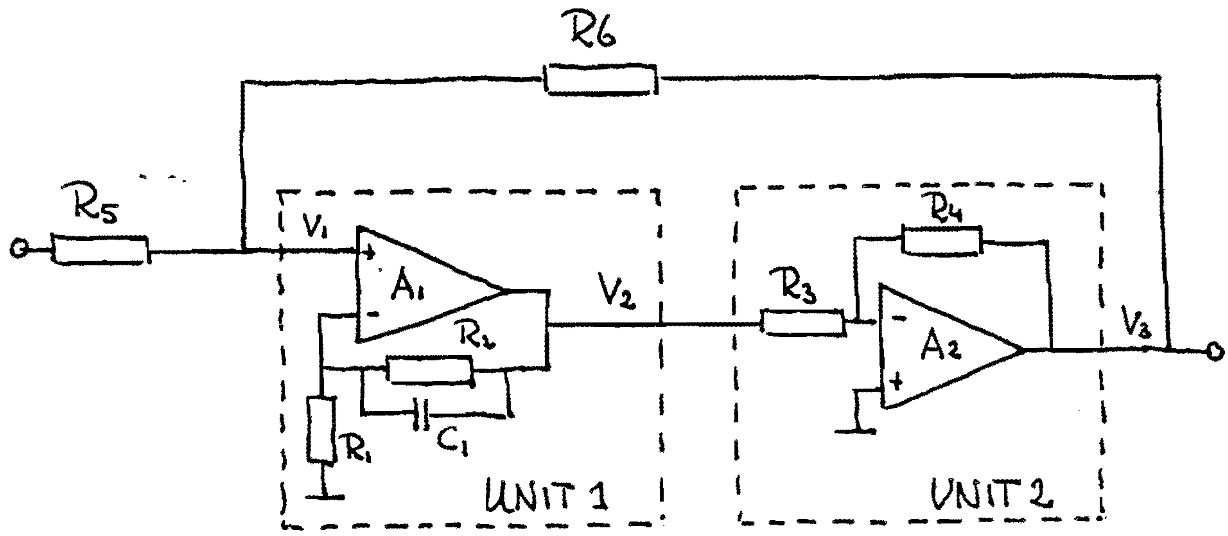
The opamp is ideal except for the gain and the output impedance (gain = A, output impedance = R_o).
The amplifier is applied in a noninverting configuration as shown.

a) Calculate the transferfunction e_o/e_i as a fuction of A, R_o , R_1 and R_2

b) Calculate the output impedance of the noninverting circuit.

c) Calculate the bandwidth (-3 db point) and sketch the Bode plot of the noninverting amplifier if R_1 =R_2 =R_o =1K
A= 10^3 and UGBW = 10^6 Hz

Problem 2



The amplifiers are considered not to be ideal

$A = 10^4$ UGBW $= 10^6$ Hz
$A = 10^4$ UGBW $= 2.10^4$ Hz

$R_1 = 10/99\ K\Omega$, $R_2 = R_3 = R_4 = R_5 = 10\ K\Omega$

$C_1 = 10^5/2\pi$ pf.

a) Calculate the transfer functions of the unit 1 and the unit 2, (V_2/V_1, V_3/V_2), and draw the piece-wise linear Bode plot (amplitude and phase) of the units.

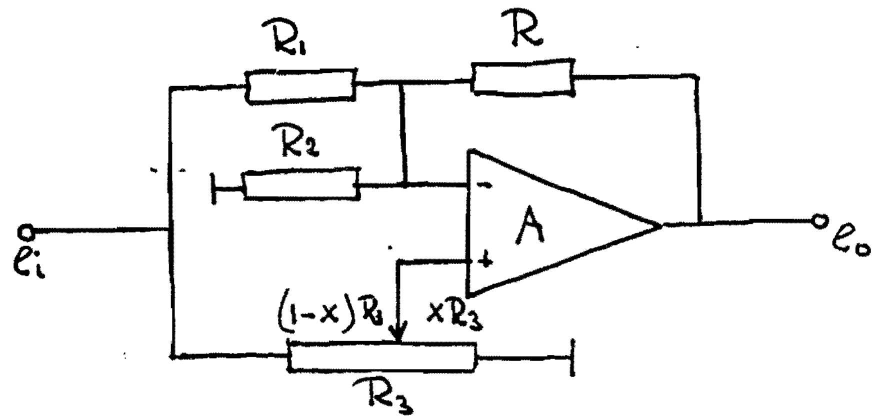
b) Draw the Bode plot and the Nyquist plot of the units 1 and 2 together, (V_3/V_1)
Find the possible values of R_6 for which the closed loop remains stable with a phase margin of 45^o

c) The input bias currents and the offset voltages of the opamps A_1 and A_2 are respectively given by:

A_1 : $Ib+ = Ib- = I_1$ and $V_{offset} = E_1$
A_2 : $Ib+ = Ib- = I_2$ and $V_{offset} = E_2$

Calculate the offset voltage at the output of the complete circuit with feedback.

PROBLEM 3



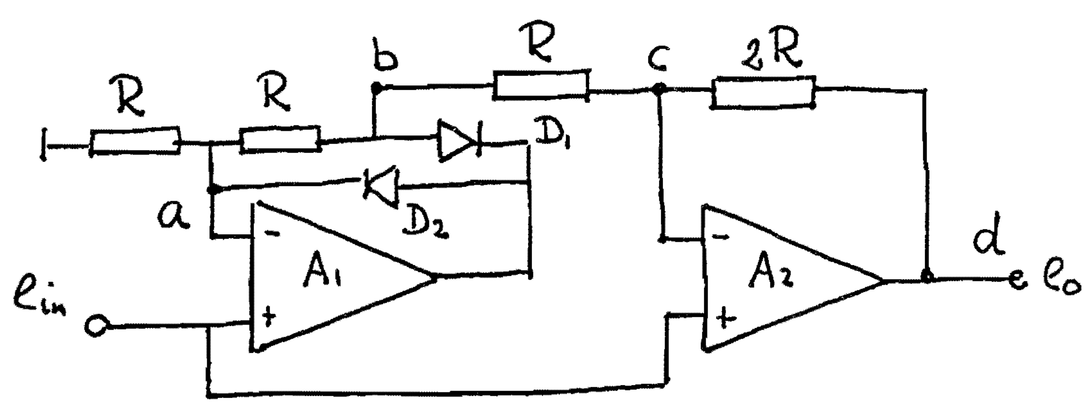
An ideal opamp is used in the configuration as shown.

$R_1 = R/n$, $R_2 = R/(n-1)$

a) Prove that the gain V_o/V_1 can be linearly varied through positive and negative levels with the potentiometer R_3

b) The common mode rejection ratio of the opamp is not infinite ($H= A/\Delta A$). Calculate the effect of the CMRR on the gain as a function of the potentiometersetting X for n=2 .
Discuss your results for X=1 and X=0.

PROBLEM 4



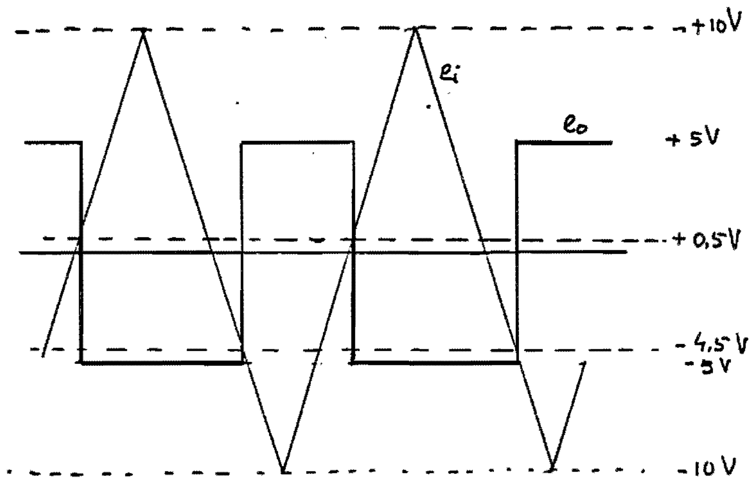
The opamps A_1 and A_2 are ideal. Draw and calculate the voltages at the points a, b, c and d if a complete sinewave $e_{in} = A \sin \omega t$ is applied at the input.

Philips International Institute
1988-03-01

SOLUTIONS EXAMINATION OPERATIONAL AMPLIFIERS

1988-03-09

Solution 1



a) e_o will be $\pm$ 5 Volt because zener1 + zener2 = $\pm$ 5V

e_o becomes - 5V when increasing e_1 passes + 0,5V

e_o becomes + 5V when decreasing e_1 passes - 4,5V

b) e_o remains -5V during a voltage variation of e_1 of 9,5 + 14,5 = 24V.

e_o remains +5V during a voltage variation of e_1 of 5.5 + 10.5 = 16V.

The time-interval ratio = $\frac{24}{16}$ = 3/2 because e_1 is symmetrical around 0 V.

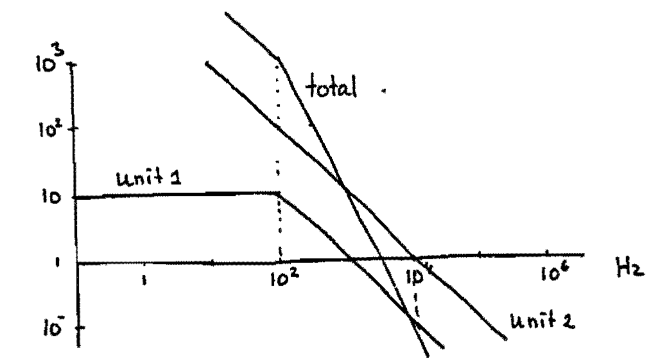
Solution 2

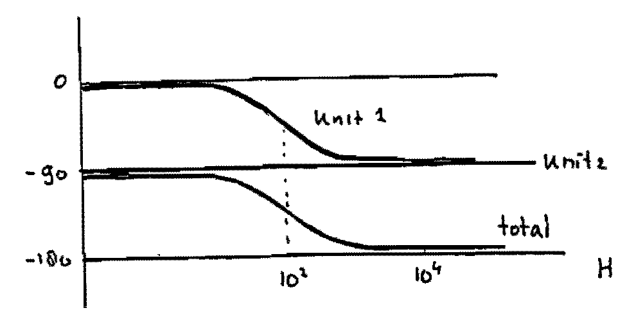
a) The transfer function $\frac{V_4}{V_1} = -\frac{R_2}{R_1} \cdot \frac{1}{1+jwC_1R_2} \cdot \frac{1}{jwC_2R_3} \cdot V_{control}$

$C_1R_2 = \tau_1 = \frac{10^{-2}}{2\pi}$ sec $\quad f_1 = 10^2$ Hz

$C_2R_3 = \tau_2 = \frac{10^{-4}}{2\pi}$ sec $\quad f_2 = 10^4$ Hz

$\frac{R_2}{R_1} = 10$, $V_{control} = 1V$.





Solution 2

b) A phase of 135° is obtained at a gain of $\frac{V_4}{V_1} = 10^3$ and with $V_{control} = 1$.

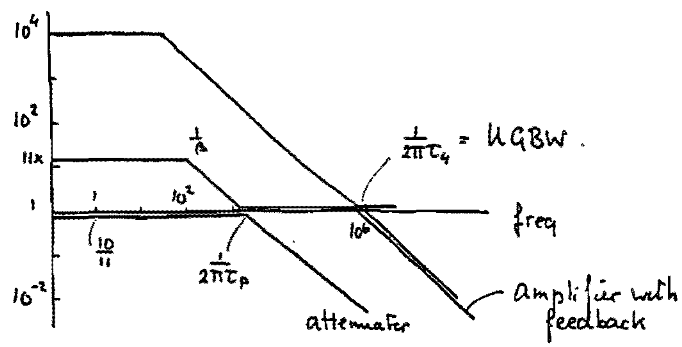
This gain has to be reduced to 1 or lower in order to have a phase margin of 45° with a closed switch.

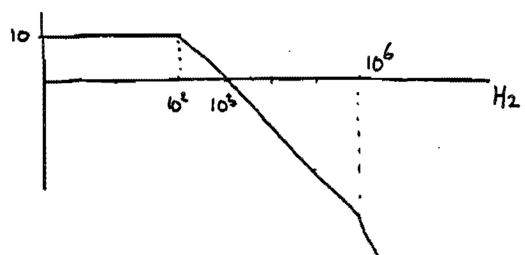
This means $V_{control} < 10^{-3}$V.

c) Unit 1.

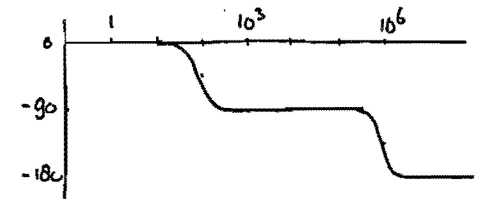
$$\text{Attentuator: } \frac{R_2}{R_1+R_2} \cdot \frac{\frac{1}{JwC_1}}{\frac{R_1R_2}{R_1+R_2}} = \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+jw\tau_p}$$

$$\frac{1}{\beta} : \frac{R_1+R_2}{R_1} \cdot \frac{1+jw\tau_p}{1+jw\tau_1} \quad , \quad \frac{1}{2\pi\tau_p} = 1.1.10^3 \text{ Hz.}$$



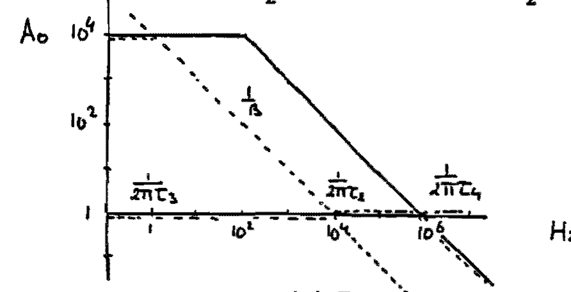


Solution 2



$$\text{transfer function: } -\frac{R_1+R_2}{R_1} \cdot \frac{1+jw\tau_p}{1+jw\tau_1} \cdot \frac{1}{1+jw\tau_4} \times \frac{1}{1+jw\tau_p} \cdot \frac{R_2}{R_1+R_2}$$

Unit 2. Attentuator: $\frac{1}{1+jw\tau_2}$ · , $\frac{1}{\beta} = \frac{jw\tau_2}{1+jw\tau_2}$

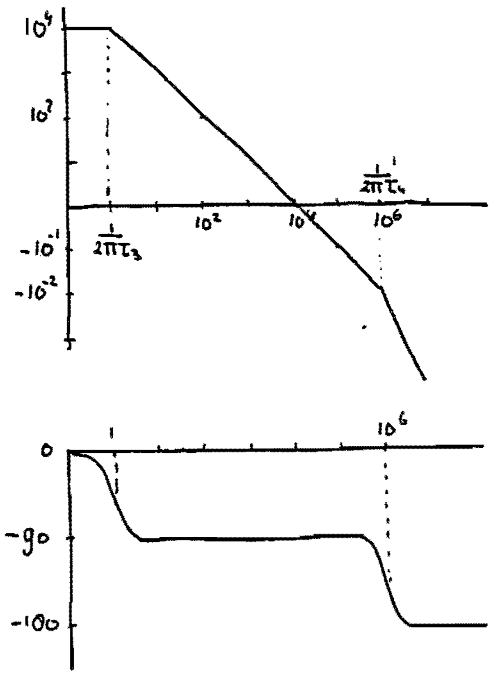


$$\text{transfer function: } \frac{A_o}{1+jw\tau_3} \cdot \frac{1+jw\tau_2}{1+jw\tau_4} \cdot \times \underbrace{\frac{1}{1+jw\tau_2}}_{\text{attenuator}} = \frac{A_o}{1+jw\tau_3} \cdot \frac{1}{1+jw\tau_4}$$

with $\frac{1}{2\pi\tau_3}$ = 1Hz and . $\frac{1}{2\pi\tau_4} = 10^6$ Hz. $A_o = 10^4$.

Solution 2

Bode plot transfer function   Unit 2

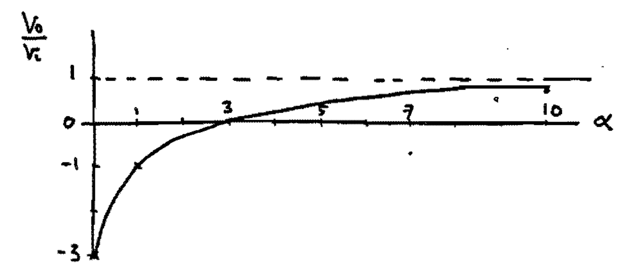


Solution 3

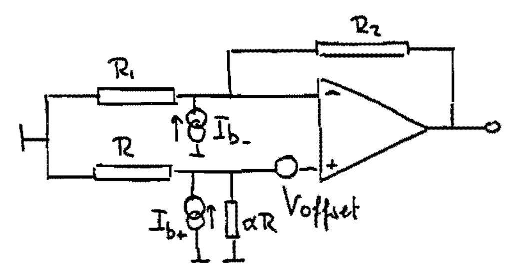
a) $V_{out} = V_i \left(- \frac{R_2}{R_1} + \frac{\alpha R}{(1+\alpha)R} \cdot \frac{R_2 + R_1}{R_1} \right) =$

$$V_i \left(-3 + \frac{\alpha}{1+\alpha} \cdot 4 \right) = \frac{\alpha - 3}{\alpha + 1} \cdot V_i$$

transfer function: $\frac{\alpha - 3}{\alpha + 1}$



b)



$$V_{out} = - I_{b-} R_2 + \frac{\alpha R}{1+\alpha} \cdot I_{b+} \cdot \frac{R_1 + R_2}{R_1} + V_{offset} \cdot \frac{R_1 + R_2}{R_1}$$

$$V_{out} = \left(-3 + \frac{4\alpha}{1+\alpha} R . 10^{-4} + 40 \right) \text{ mV}$$

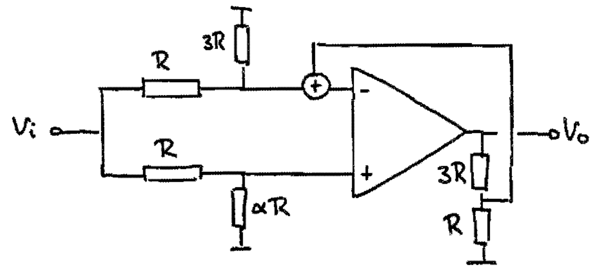
min. input bias current offset: $-3 + \frac{4\alpha}{1+\alpha} R \, 10^{-4} = 0$

or $R = \frac{1+\alpha}{4\alpha} \, 3 \, 10^4 \Omega$

for $\alpha = 3$, $R = 10K\Omega$

Solution 3

c) The configuration can be redrawn as shown:



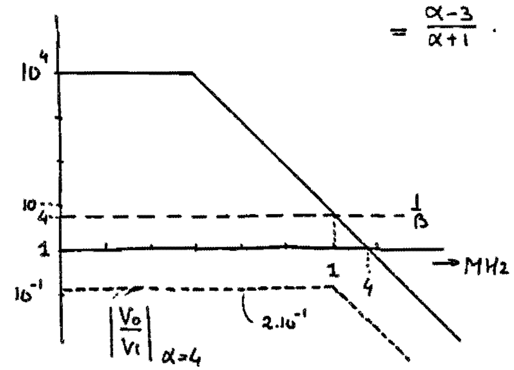
This shows that $\frac{1}{\beta} = 4$ and that the attentuation of the input signal equals:

$$\frac{-3R}{R+3R} + \frac{\alpha R}{R+\alpha R} = \frac{1}{4} \quad \frac{\alpha - 3}{\alpha + 1}$$

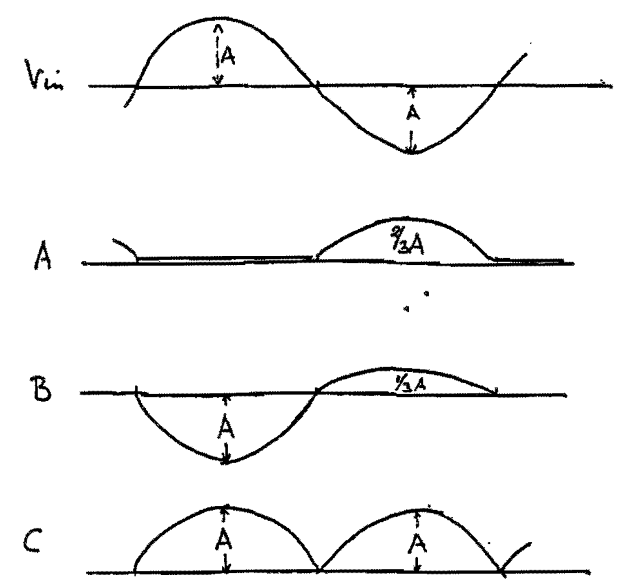
The Bode diagram shows that the complete transfer function $\frac{V_o}{V_i}$ is given by the first order expression:

$$\frac{V_o}{V_i} = \frac{1}{4} \cdot \frac{\alpha - 3}{\alpha + 1} \times 4 \cdot \frac{1}{1 + \frac{jw}{2\pi\, 10^6}}$$

$$= \frac{\alpha - 3}{\alpha + 1} \cdot \frac{1}{1 + \frac{j\omega}{2\pi\, 10^6}}$$



Solution 4



V_{in} positive: D_1 conducts, D_2 open: $B = -V_{in}$, $A = 0$ $\Big\}$ $C = +V_{in}$

V_{in} negative: D_1 open, D_2 conducts: minus input A_2 equals V_A

Input current $\frac{A_{\sin wt}}{R}$ is divided via A and B

Via A: $2/3 \; \frac{V_{in}}{R}$

Via B: $1/3 \; \frac{V_{in}}{R}$

or $A = 2/3 \; V_{in}$

or $B = 1/3 \; V_{in}$

$V_C = 3/2 \cdot V_A$

$V_C = 3/2 \cdot 2/3\, V_{in} = V_{in}$

The circuit is a full-wave rectifier.

SOLUTIONS EXAMINATION "OPERATIONAL AMPLIFIERS" 1989-03-06

Solution 1.

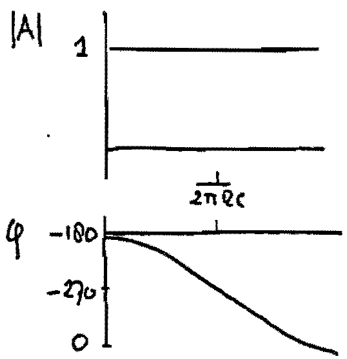
a). $$V_{out} = \left(\frac{R}{R+\frac{1}{j\omega C}} \cdot \frac{2R}{R} - \frac{R}{R} \right) V_{in} =$$

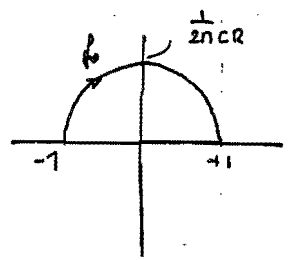
$$-\frac{1-j\omega CR}{1+j\omega CR} = -1\, e^{-j2\varphi}$$

$$\varphi = \text{atn. } \omega CR$$

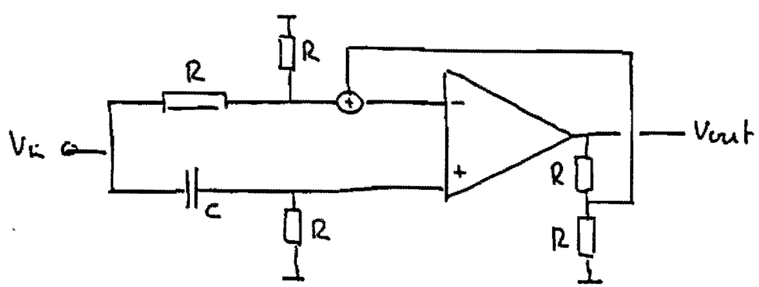
Bode

Nyquist

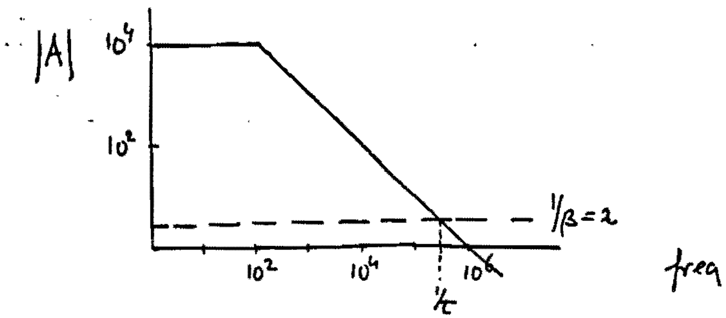




b)



the circuit consists of two attenuators and an opamp with $\frac{1}{\beta} = 2$



the transfer function equals

$$\left(\frac{j\omega CR}{1+j\omega CR} - \frac{R}{R+R} \right) \cdot \frac{R+R}{R} \cdot \frac{1}{1+j\omega\tau}$$

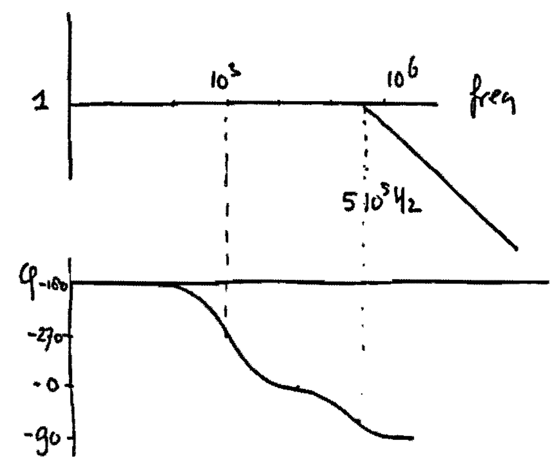
$$\tau = \frac{1}{2\pi\, 5\, 10^5} \text{ sec} = \frac{1}{\pi}\,\mu\text{sec}$$

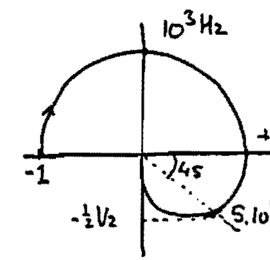
$$= -\frac{1-j\omega CR}{1+j\omega CR} \cdot \frac{1}{1+j\omega\tau}$$

$$CR = \frac{10^{-3}}{2\pi} \rightarrow f = 10^3 \text{ Hz}$$

Bode

Nyquist





Solution 2

a) $V_3 = V_1 \frac{R_1+R_2}{R_1} - (V_1+E)\frac{R_2}{R_1}$

$V_3 = V_2$

$V_1 = V_2 + i_L R_s$

$$\left.\begin{array}{l} V_3 = V_1 \frac{R_1+R_2}{R_1} - (V_1+E)\frac{R_2}{R_1} \\ V_3 = V_2 \\ V_1 = V_2 + i_L R_s \end{array}\right\} \quad i_L = \frac{E\frac{R_2}{R_1}}{R_s}$$

The circuit is a voltage controlled current source because the current i_L is independent of Z_L and controlled by E

b) With $E = 1V$, $R_1 = 5k\Omega$, $R_2 = 10k\Omega$ and $R_s = 1k\Omega$ a) gives
$i_L = 2mA$.

For V_2 and V_3 applies:

$$\left.\begin{array}{l} V_2 < \text{max. com. mode voltage } A_2 \\ V_3 < \text{max output voltage } A_1 \\ V_2 = V_3 \end{array}\right\} \text{ or } V_2 < 10V$$

For V_1 applies

$$\left.\begin{array}{l} V_1 = V_2 + i_L R_s = V_2 + 2 \\ V_1 < \text{max. common mode voltage } A_1 \\ \quad < \text{max output } A_2 \end{array}\right\} V_2 < 8V.$$

Thus Z_L has to be smaller than $4k\Omega$

c) The effect of the finite H can be represented by an offset voltage at the input of A_2 that equals $\pm \frac{i_L Z_L}{H}$

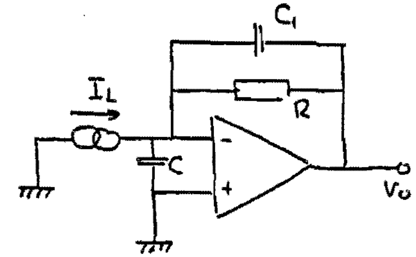
this results in $\quad V_3 = V_2 \pm \frac{i_L Z_L}{H}$

this gives: $\quad i_L = \frac{E\, R_2/R_1}{R_s \pm \frac{Z_L}{H}}$

$$Z_{out} = \frac{V_{open}}{I_{short}} = \frac{\lim\limits_{Z_L\to\infty} i_L Z_L}{\lim\limits_{Z_L \to 0} i_L} \text{ or } \frac{\pm E \frac{R_2}{R_1} \cdot H}{\frac{E\, R_2/R_1}{R_s}} = \pm R_s H$$

In order to keep the circuit stable the CMRR has to be positive

Solution 3



a) **Without C_1**

$\frac{1}{\beta} = 1 + j\omega CR$.

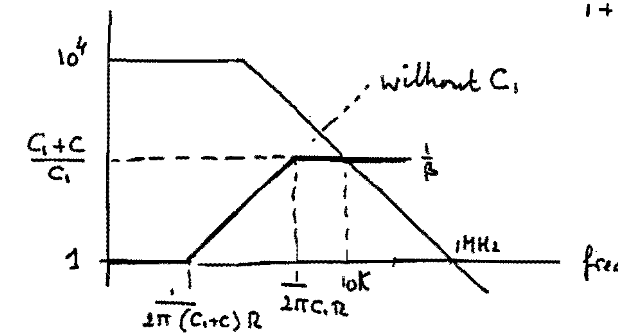
transfer function $K.R.$

With C_1

$\frac{1}{\beta} = \frac{1 + j\omega(C+C_1)R}{1 + j\omega C_1 R}$

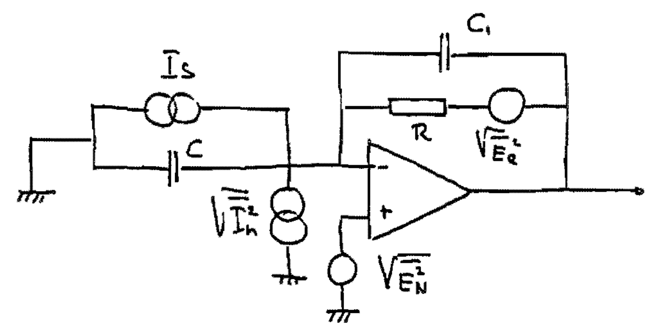
transfer function: $\frac{K.R.}{1 + j\omega C_1 R}$

Bode diagram



With C_1 the intersection slope = 6db/octave → stable

b)



$$\sqrt{\overline{V_{ON}^2}} = \sqrt{\overline{I_N^2}\left|\frac{R}{1+j\omega C_1R_1}\right|^2 + \overline{E_N^2}\left|\frac{1+j\omega(C_1+C)R}{1+j\omega C_1R}\right|^2 + \overline{E_R^2}}$$

$$V_{OS} = \frac{R}{1+j\omega C_1 R}\cdot I_S$$

$$\left(S/N\right)_{power} = \frac{\frac{R^2}{1+\omega^2C_1^2R^2}\,\overline{I_S^2}}{\frac{R^2}{1+\omega^2C_1^2R^2}\cdot\overline{I_N^2} + \frac{1+\omega^2(C_1+C)^2R^2}{1+\omega^2C_1^2R^2}\cdot\overline{E_N^2} + 4kTR\Delta f}$$

By making R large the S/N can be improved when the term $4kTR\Delta f$ dominates the term $\frac{R^2\overline{I_N^2}}{1+\omega^2C_1^2R^2}$

In that case S/N improves with R; otherwise it remains constant.

c). A phase margin of 45° is obtained when the intersection of the $1/\beta$ line and the amplifier gain line occurs at the frequency $\frac{1}{2\pi C_1 R}$

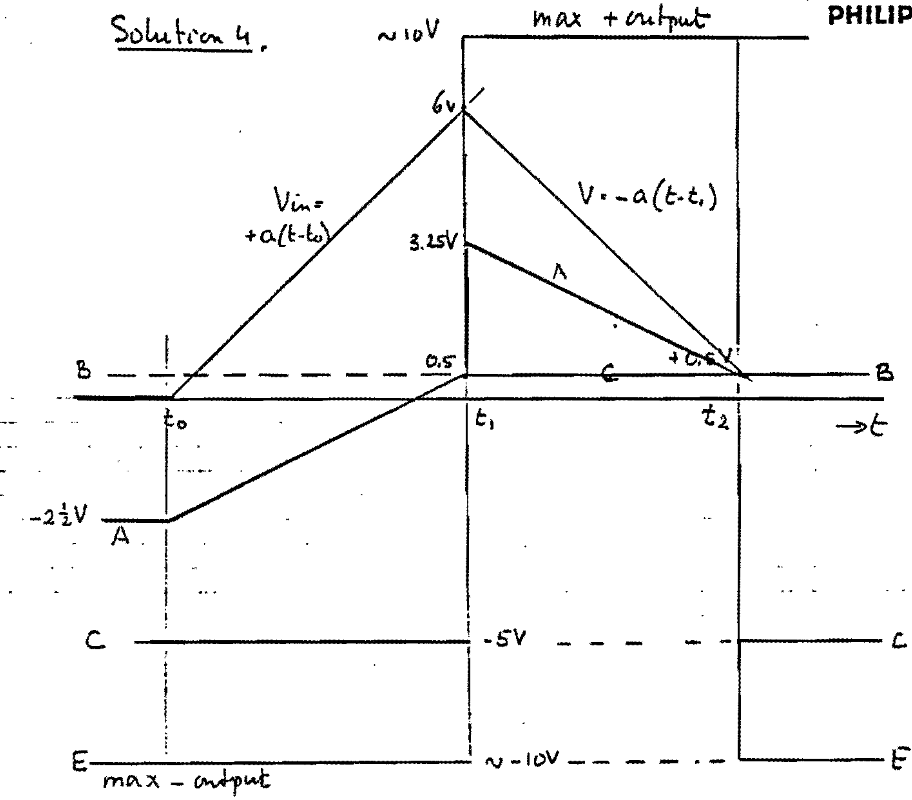
The -3dB frequency of the transfer function equals $\frac{1}{2\pi C_1 R} = 10\,kHz$

Both conditions give.

$$\frac{C_1+C}{C_1} = \frac{U.GBW}{2\pi C_1 R} = \frac{10^6}{10^4} = 10^2. \quad \text{or.}\ C_1 = \frac{C}{99} = \frac{10}{2\pi}\,pf$$

$$C_1R = \frac{1}{2\pi 10^4} \quad \text{or} \quad R = \frac{1}{2\pi.10^4.\frac{10^{-11}}{2\pi}} = 10\,M\Omega$$

Solution 4.



for $t < t_0$: V_A will allways be lower than $V_B = 0{,}5V$ thus the start condition is $V_{in} = 0$

$V_C = -5V$, V_E = max - output

$V_A = \frac{1}{2}V_C = -2\frac{1}{2}V$

for $t > t_0$

$V_A = \frac{1}{2}V_{in} + \frac{1}{2}V_C = \frac{1}{2}a(t-t_0) - 2\frac{1}{2}$

the polarity will change if $V_A = V_B$ or

$\frac{1}{2}a(t-t_0) - 2\frac{1}{2} = 0{,}5$.

or $t - t_0 = 6/a$ sec $= 6$ sec

the amplitude of $V_{in} = a(t-t_0) = 6$ volt.

<u>for $t = t_1$</u>

$V_C = 0{,}5\,V$, $V_{in} = 6$ volt and $V_A = \frac{V_{in}}{2} + \frac{V_c}{2} = 3.25\,V$.

<u>for $t > t_1$</u>

The output will switch its polarity again if

$V_A = V_B = 0{,}5\,V$

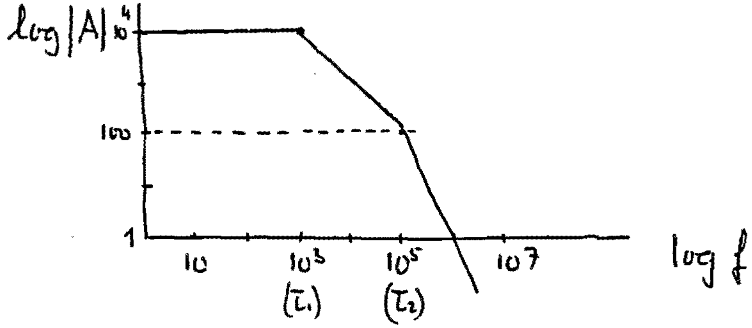
$V_c = 0{,}5$, so if $V_{in} = 0{,}5\,V$

this occurs after $\frac{6-0{,}5}{a} =$ 5,5 sec

Solutions problem 1

PHILIPS

a)



gain at $\omega = \frac{1}{\tau_2}$: $\quad , |A| = \frac{A_0}{\sqrt{(1+\omega^2\tau_1^2)(1+\omega^2\tau_2^2)}}$

$|A| = \frac{A_0}{\sqrt{\left(1+\left(\frac{\tau_1}{\tau_2}\right)^2\right)\underbrace{(1+1)}_{3db}}} \rightarrow |A| = 100$ (piecewise linear)

U.G.BW : $10^5 \times 10 = 10^6$ Hz

b) feedback : $\beta = \frac{R_1}{R_1+R_3}$; $\frac{1}{\beta} = \frac{R_1+R_3}{R_1} = 101$

transfer function $= (1-\beta)\frac{1}{\beta} \cdot \frac{1}{1+\frac{1}{A}\left(1+\frac{R_3}{R_1}\right)} =$

$= (1-\beta)\frac{1}{\beta} \cdot \frac{1}{1+\frac{(1+j\omega\tau_1)(1+j\omega\tau_2)}{A_0}\frac{1}{\beta}} =$

$(1-\beta)\frac{1}{\beta} \cdot \frac{1}{1+1/A_0\beta} \cdot \frac{1}{1+\frac{j\omega(\tau_1+\tau_2)}{1+A_0\beta}+\frac{j^2\omega^2(\tau_1\tau_2)}{1+A_0\beta}}$

with $\tau_1 = 100\tau_2$
and $A_0\beta = 100$ $\rightarrow (1-\beta)\frac{1}{\beta} \cdot \frac{1}{1.01} \cdot \frac{1}{1+j\omega\tau_2+j^2\omega^2\tau_2^2}$

attenuator: $\frac{R_3}{R_1+R_2} = \frac{100}{101}$

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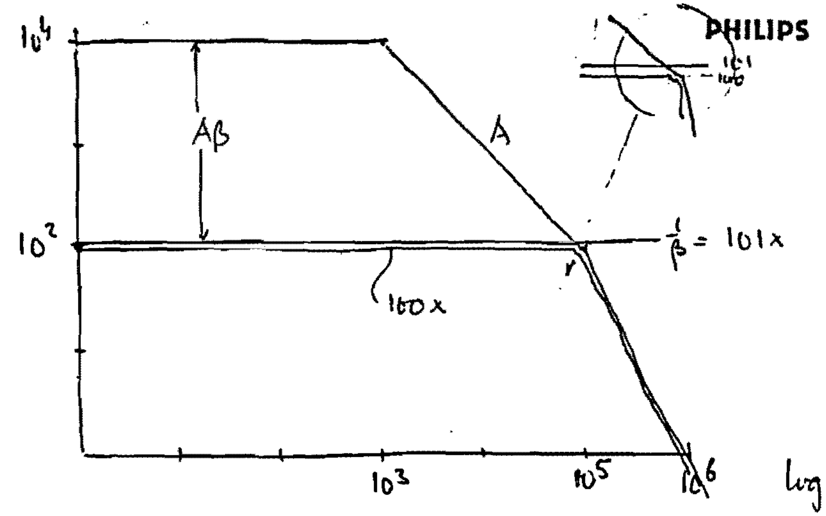
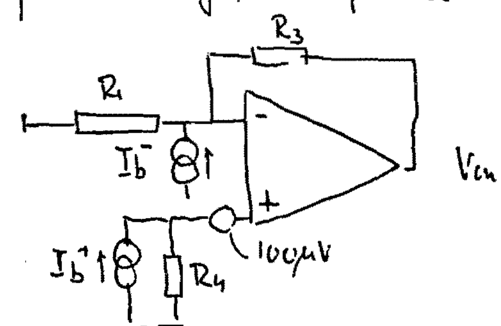
c)



phase margin : $A\beta \sim \frac{\tau_1}{\tau_2} \sim 45°$

d)



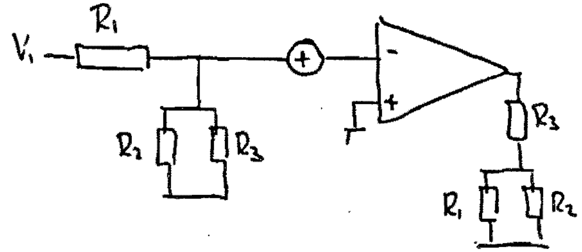
$V_{out} = -I_b^- R_3 + I_b^+ R_4 \cdot \frac{R_1+R_3}{R_1} + V_{offset} \cdot \frac{R_1+R_3}{R_1}$

if $R_4 = \frac{R_1 R_3}{R_1+R_3}$; $V_{out} = V_{offset} \cdot \frac{R_1+R_3}{R_1}$

$= 10{,}1$ mV

$R_4 = \frac{100}{101} \sim 0{,}99$ kΩ

e

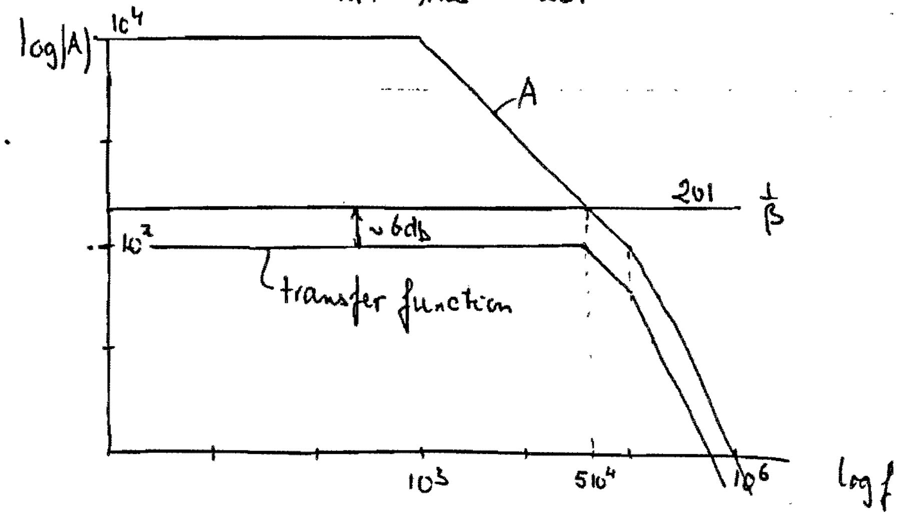


$$\beta = \frac{R_1 /\!/ R_2}{R_3 + R_1 /\!/ R_2} = \frac{0{,}5}{100{,}5} = \frac{1}{201}$$

$$\frac{1}{\beta} = 201$$

attenuator: $\frac{R_2 /\!/ R_3}{R_1 + R_2 /\!/ R_3} = \frac{100}{201} = -6\,\mathrm{db}$

f).



phase margin ~ 90°

g) $V_{out} = -I_b^- R_3 + I_b^+ \cdot R_4 \cdot \frac{R_1 /\!/ R_2 + R_3}{R_1 /\!/ R_2} + V_{offset} \frac{R_1 /\!/ R_2 + R_3}{R_1 /\!/ R_2}$

$= -10 + \frac{20{,}1}{1{,}01} + 20{,}1 = 30\,\mathrm{mV}$

Solutions problem 2

a). $V_1 = -\frac{R_2}{R_1} e_1 + \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} e_2 + \frac{R_3}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} V_2$

$V_2 = i Z_L$

$V_1 - V_2 = i R_s$

$V_1 = -e_1 + e_2 + V_2$

$V_2 = i Z_L$

$V_1 - V_2 = i R_s$

$R_1 = R_2 = R_3 = R_4$

$\rightarrow \quad i = \frac{e_2 - e_1}{R_s}$

b) $i = \frac{e_2 - e_1}{R_s}$: or $\frac{14-12}{R_s} = 10^{-3}$ or $R_s = 2\,\mathrm{k\Omega}$

max output and max com. mode requirements

$|V_1| < 10$, $|V_2| < 10$, $|\text{com. mode voltage } A_1| < 10$

or $V_1 = (Z_L + 2.10^3)\,10^{-3} < 10 \rightarrow Z_L < 8\,\mathrm{k\Omega}$

$V_2 = Z_L\, 10^{-3} < 10 \rightarrow Z_L < 10\,\mathrm{k\Omega}$

$e_2 \frac{R_4}{R_3+R_4} + V_2 \frac{R_3}{R_3+R_4} < 10$ or

$7 + \frac{Z_L\, 10^{-3}}{2} < 10 \rightarrow Z_L < 6\,\mathrm{k\Omega}$

thus: Z_L has to be lower than $6\,\mathrm{k\Omega}$

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c). $i(R_s+Z_L) = -\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}e_2 + \frac{R_3}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\cdot i Z_L$

$$i = \frac{-\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\cdot e_2}{Z_L\left[1-\frac{R_3}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\right]+R_s}$$

$$= \frac{\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}e_2}{Z_L\left[1-\frac{1}{2+\delta}\cdot(2-\delta)\right]+R_s} = \frac{\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\cdot e_2}{Z_L\delta + R_s}$$

$V_2 = i Z_L$

$$V_{open} = \lim_{Z_L\to\infty} V_2 = \lim i Z_L = \frac{-\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}\cdot e_2}{\delta}$$

$$i_{shortc} = \lim_{Z_L\to 0} i = \frac{-\frac{R_2}{R_1}e_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}e_2}{R_s}$$

$$Z_{out} = \frac{V_{open}}{I_{shortc}} = \frac{R_s}{\delta}$$

for stability reasons $\delta > 0$

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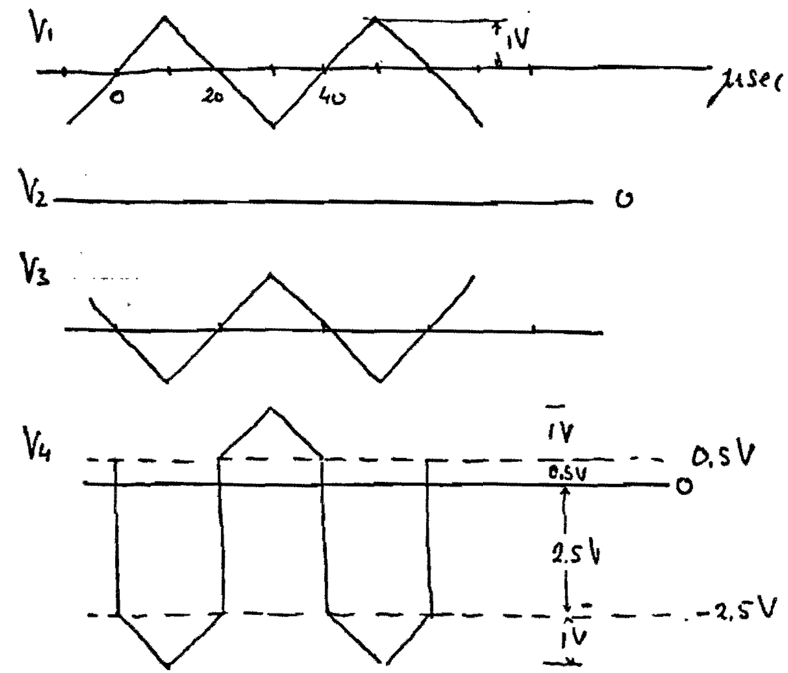
Solutions problem 3

a) V_1 = triangle wave, freq. 25 kHz, T = 40 µsec
A = 1 Volt, slew rate of input signal = 0.1 V/µsec



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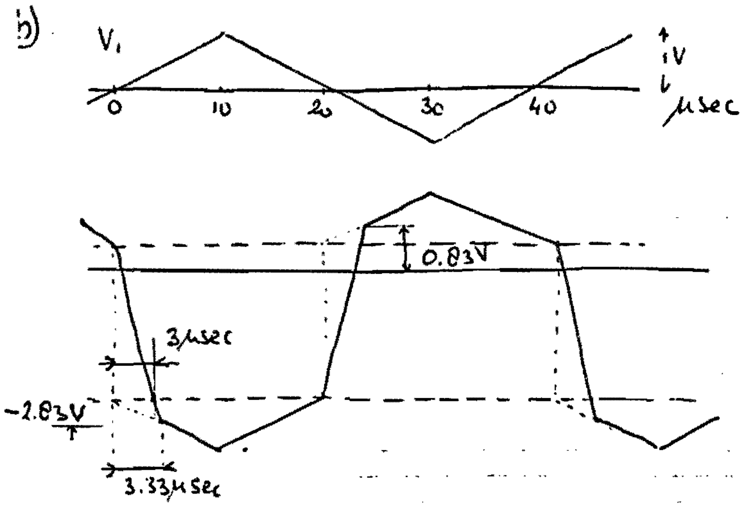
b)



Slew rate = 1 V/μsec → it takes 3μsec to pass the thresholds of the zener (+0,5, -2,5V). The opamp continues to produce an output signal with a slew rate of 1V/μsec up to the intersection with the output signal which would result if no slew-rate limitation would be present.

slew rate opamp = 1V/μsec
a, slope triangle signal = 0,1V/μsec } intersection:

$1t = 3 + at =$

$t = 3 + 0{,}1t \rightarrow t = 3{,}33\ \mu sec$

amplitudes: $0{,}5 + 3.33 \times 0.1 = 0{,}833V$

$-2{,}5 - 3.33 \times 0{,}1 = -2{,}833V$

c) The zener diode is not conducting during the period in which the output V_4 slews between the thresholds of the zener. This means: $V_1 = V_2 = V_3$
At the moment when the slewing output intersects the ideal output (see a), the loop is closed

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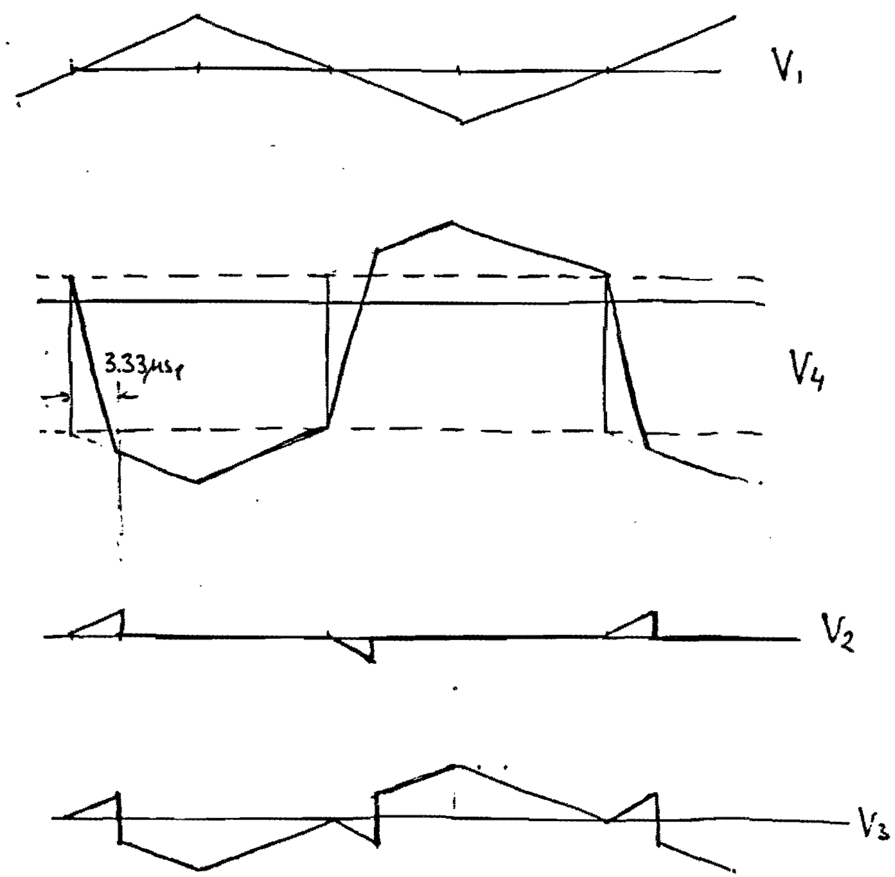
or 3.33 μsec after the zero crossings of the input signal.
after this moment: $V_2 = 0$, $V_3 = V_4 - V_{zener}$



Sol. ww problem 4

$$\left.\begin{aligned} &\frac{(V_i - \varepsilon)}{R} = -\frac{V_{o1} - \varepsilon}{R_1} - \frac{V_{o2} - \varepsilon}{R_2} \\ &\frac{V_{o1}}{A_1} = \frac{V_{o2}}{A_2} \\ &A_1, A_2 \text{ large} \end{aligned}\right\} \rightarrow$$

$$\frac{V_i}{R} = -\frac{V_{o1}}{R_1} - \frac{V_{o2}}{R_2} \quad ; \quad \frac{V_i}{R} = -V_{o1}\left[\frac{1}{R_1} + \frac{A_2}{A_1}\frac{1}{R_2}\right]$$

$$\frac{V_i}{R_1} = -V_{o2}\left[\frac{A_1}{A_2}\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\frac{V_{o1}}{V_i} = -\frac{R_1 R_2}{R}\,\frac{1}{R_2 + R_1 \cdot \frac{A_2}{A_1}}$$

$$\frac{V_{o2}}{V_i} = -\frac{R_1 R_2}{R} \cdot \frac{1}{R_1 + R_2 \cdot \frac{A_1}{A_2}}$$

Solutions Examination "Operational Amplifiers" 1991

problem 1:

a).

$$\left.\begin{array}{l} e_o' = A\left(e_i - \frac{R_1}{R_1+R_2} e_o\right) \\ e_o = e_o' \frac{R_1+R_2}{R_o+R_1+R_2} \end{array}\right\} \rightarrow \frac{e_o}{e_i} = \frac{A}{\frac{R_o+R_1+R_2}{R_1+R_2} + A\frac{R_1}{R_1+R_2}} \rightarrow$$

$$\frac{e_o}{e_i} = \frac{R_1+R_2}{R_1}\cdot \frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_o}{R_1}\right)}$$

b). output impedance $= \frac{V_{open}}{I_{short}}$

$$\left.\begin{array}{l} V_{open} = e_i \frac{R_1+R_2}{R_1}\cdot \frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_o}{R_1}\right)} \\ I_{short} = \frac{Ae_i}{R_o} \end{array}\right\} \rightarrow$$

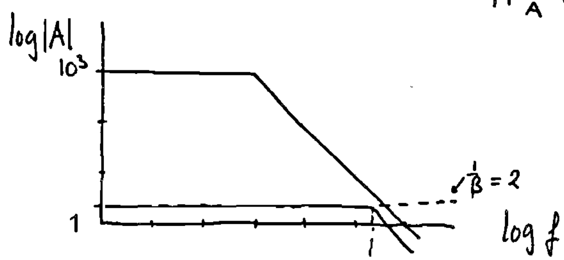
output impedance: $\frac{R_1+R_2}{AR_1}\cdot\frac{R_o}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_o}{R_1}\right)} =$

$$\frac{R_o}{A\beta}\cdot\frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_o}{R_1}\right)}.$$

c)

$$\frac{e_o}{e_i} = \frac{R_1+R_2}{R_1}\cdot\frac{1}{1+\frac{1+j\omega\tau}{A}\left(1+\frac{R_2}{R_1}+\frac{R_o}{R_1}\right)} = 2\cdot\frac{1}{1+\frac{1+j\omega\tau}{A}(3)}$$

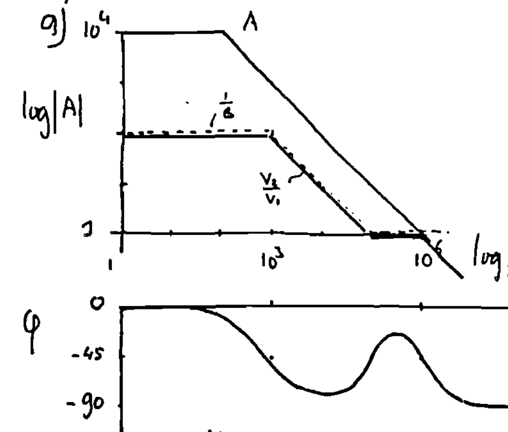
$$= 2\,\frac{1}{1+\frac{3}{A}+j\omega\frac{3\tau}{A}}$$



$$f = \frac{A}{3\tau}\cdot\frac{1}{2\pi} = \frac{10^6}{3}\ \text{Hz}$$

problem 2.

a)



UNIT 1

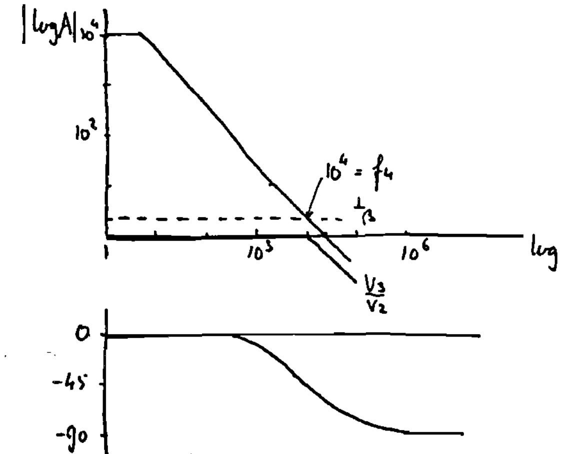
$$\frac{1}{\beta} = \frac{R_1+R_2}{R_1}\,\frac{1+j\omega C_1 R_1/\!/R_2}{1+j\omega C_1 R_2}$$

$\frac{R_1+R_2}{R_1} = 100$, $R_1/\!/R_2 = 100\Omega$

$$\frac{1}{\beta} = 100\cdot\frac{1+j\omega\tau_1/100}{1+j\omega\tau_1}$$

$\tau_1 = \frac{10^{-3}}{2\pi}$ → $f_1 = 10^3$ Hz, $f_2 = 10^5$ Hz, $f_3 = 10^6$ Hz

$$\frac{V_2}{V_1} = 100\cdot\frac{1+j\omega\tau_1/100}{1+j\omega\tau_1}\cdot\frac{1}{1+j\omega\tau_3}$$



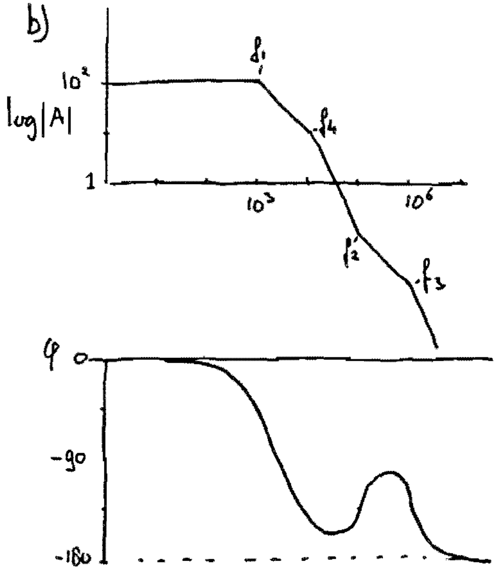
UNIT 2

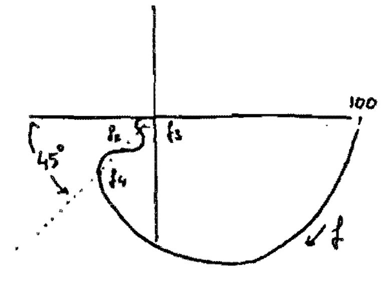
attenuator $= \frac{1}{2}$.

$\frac{1}{\beta} = 2$

$$\frac{V_3}{V_2} = -\frac{1}{1+j\omega\tau_2}$$

$\tau_2 = \frac{10^{-4}}{2\pi}$, $f_4 = 10^4$ Hz b)

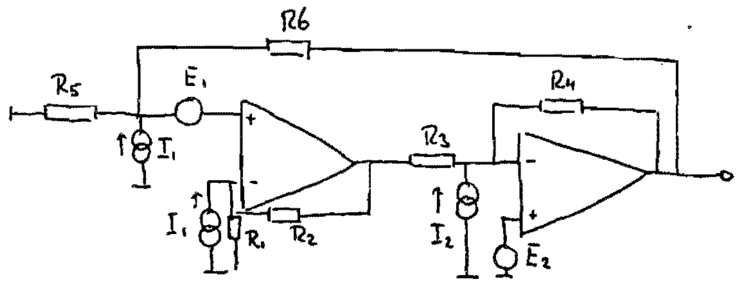




Nyquist plot

Since the feedback loop is passive, $\frac{1}{\beta}$ is always ≥ 1
So $\frac{1}{\beta} = 10$ is the only possible solution giving a phase margin of 45°.. So $\frac{R_5+R_6}{R_5} = 10 \rightarrow R_6 = 90\,k\Omega$

c)



The biasing sources of unit 2 can be replaced by a voltage source E_{R_2}

$$-E_{R_2} \cdot \frac{R_4}{R_3} = -I_2 R_4 + \frac{R_3+R_4}{R_3} E_2$$

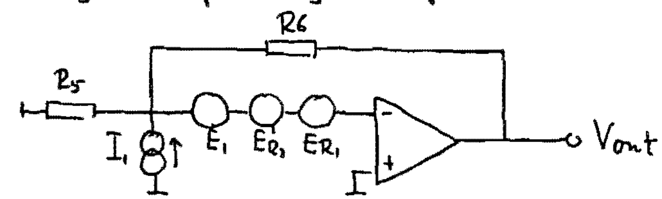
or: $E_{R_2} = I_2 R_3 - \frac{R_3+R_4}{R_4} E_2$

E_{R_2} can be shifted through unit 1 and replaced by $E'_{R_2} = E_{R_2} \cdot \frac{R_1}{R_1+R_2}$. at the + input

The effect of the bias current at the − input of unit 1 can be represented by a voltage source E_{R_1} at the + input:

$$E_{R_1} \cdot \frac{R_1+R_2}{R_1} = -I_1 R_2 \quad \text{or} \quad E_{R_1} = -I_1 \cdot \frac{R_1 R_2}{R_1+R_2}$$

So the output offset voltage can be calculated by using the following diagram.



$$V_{out} = -R_6 I_1 - \frac{R_5+R_6}{R_5}\left[E_1 + E'_{R_2} + E_{R_1}\right]$$

$$= -R_6 I_1 - \frac{R_5+R_6}{R_5}\left[E_1 + \underbrace{\frac{R_1\left(I_2 R_3 - \frac{R_3+R_4}{R_4} E_2\right)}{R_1+R_2}}_{\text{small because } \frac{R_1}{R_1+R_2} \sim 10^{-2}} - I_1 \frac{R_1 R_2}{R_1+R_2}\right]$$

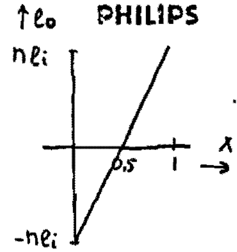
$$V_{out} = -\frac{R_5+R_6}{R_5} E_1 + I_1 \left[\underbrace{\frac{R_5+R_6}{R_5} \cdot \frac{R_1 R_2}{R_1+R_2} - R_6}_{\text{zero if } R_6 /\!/ R_5 = R_1 /\!/ R_2}\right]$$

problem 3

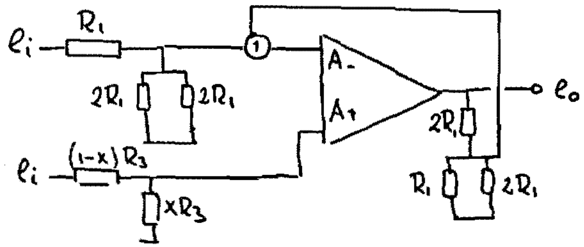
a) $e_0 = e_i\left[-\frac{R}{R_1} + x\left[1+\frac{R}{R_1//R_2}\right]\right] =$

$= e_i\left[-n + x\left[1 + \frac{R}{\frac{R/n \cdot R/n-1}{R/n + R/n-1}}\right]\right] =$

$= e_i\left(-n + x\left[1+2n-1\right]\right) = e_i\left(-n + x\,2N\right)$.



b): for $n=2$ $R = 2R_1$, $R_2 = 2R_1$



$e_0 = -\left(\frac{1}{2}e_i + \frac{1}{4}e_0\right)A^- + x\,e_i A^+$

$\frac{e_0}{e_i} = \frac{\left[-\frac{1}{2}A^- + xA^+\right]}{1+\frac{1}{4}A^-} = \frac{-\frac{1}{2}A + \frac{\Delta A}{4} + xA + \frac{\Delta A}{2}x}{1 + \frac{A}{4} - \frac{\Delta A}{8}} =$

$\frac{A}{4} > 1 \rightarrow \frac{e_0}{e_i} = \frac{4\left(|x-\frac{1}{2}| + \frac{\Delta A}{2A}\left(x+\frac{1}{2}\right)\right)}{1-\frac{\Delta A}{2A}} = 4\left[x - \frac{1}{2} + \frac{\Delta A}{2A}\left(2x\right)\right]$

$= 4x - 2 + \frac{4x}{H}$

Common-mode voltage of amplifier $= xe_i$

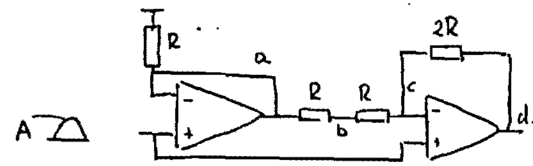
if $x=0$, CM. voltage $=0$, H no influence.

$x=1$ CM voltage $= e_i$ max influence.

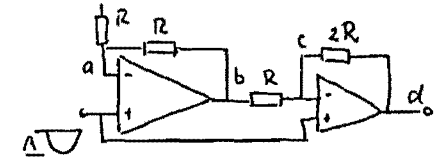
Same results can be found by assuming an error signal $\frac{xe_i}{H}$ at the + input of the opamp

problem 4.

e_{in} positive, D_2 conducting



e_{in} negative, D_1 conducting



voltage at a = A
voltage at c = +input = A →
no current in feedback loop
b, d also A.

voltage at a = −A
at b = −2A
at c = −A
at d: $-2V_b + 3V_{in} =$
$+4A - 3A = A$.

circuit is a full wave rectifier