A defInition of effective shear strain and its application in analysis of flow turning

A DEFINITION OF EFFECTIVE SHEAR STRAIN AND ITS APPLICATION IN ANALYSIS OF FLOW TURNING

by

P.C. Veenstra

Eindhoven, University of Technology, The Netherlands

Paper to be presented to the General Assembly of the C.I.R.P., Torino, 1970.

SUMMARY

It is shown that on a geometrical base a definition of effective deformation (effective strain) in the case of pure shear strain can be formulated.
The relevancy of the definition is investigated in the proces of flow-turning of thin blanks.

ZUSAMMENFASSUNG

An Hand einer geometrischen Betrachtung wird die Möglichkeit einer mathematischen Erfassung der Vergleichsformänderung im Falle einer reinen Schubbeanspruchung gezeigt. Die Bedeutung der entwickelten Definition wird am Verfahren des Projizierdrückens hervorgehoben.

RÉSUMÉ

Il est démontré que basé sur la géometrie une définition de la déformation effective, au cas où il sera en question d'effort de cisaillement pur, peut être formulée.
L'importance de la définition est recherchée au procédé de tournage de cisaillement de lames.

In plasticity mechanics the large strain δ is derived from the incremental strain $d\delta$ by using the definition

$$\delta = \int_0^{\delta} d\delta = \int_{l_o}^{l} \frac{dl}{l} = \ln \frac{l}{l_o} \qquad (1)$$

being the natural or logarithmic strain corresponding with uniform elongation of a material from the original length l_o to the value l.

Denoting the principal strains by $\{\delta_1, \delta_2, \delta_3\}$ and assuming a straight path of stress the effective deformation is given by

$$\bar{\delta} = \sqrt{\frac{2}{3} \left(\delta_1^{\,2} + \delta_2^{\,2} + \delta_3^{\,2}\right)} \qquad (2)$$

As to the large shear strain no analogue operation is at disposal. It however is useful to describe a scalar quantity like the effective deformation in terms of shear strain as many technological processes can be approximated being a state of pure shear. It is remarked that in this case the condition of a straight path of stress is fulfilled.

In Fig. 1 an element $A_oB_oC_oD_o$ is shown being deformed by the large shear strain γ into the element A_oB_oCD. In consequence of this the sub-element A_oE_o defined by its position with respect to the co-ordinate system is strained to the length A_oE in its new position θ.

As a case of pure shear is considered which implicates constancy of surface area of the element it holds

$$D_oE_o = DE$$

$$D_oE = D_oE_o + D_oD \qquad (3)$$

and hence

$$\tan\theta = \tan\zeta + \tan\gamma$$

The natural strain of the element A_oE_o amounts according to eq. 1 to

$$\delta = \ln \frac{A_oE}{A_oE_o} = \ln \frac{\cos\xi}{\cos\Theta}$$

from which by means of eq. 3 is derived

$$\delta = \frac{1}{2} \ln \frac{1 + \tan^2\Theta}{1 + (\tan\Theta - \tan\gamma)^2} \qquad (4)$$

This strain shows extreme values as a function of Θ. Calculating $\frac{d\delta}{d\Theta} = 0$ it follows as a condition

$$\tan^2\Theta - \tan\gamma \tan\Theta - 1 = 0$$

from which is solved

$$\tan\Theta_{1,2} = \frac{1}{2} \tan\gamma \pm \sqrt{1 + \frac{\tan^2\gamma}{4}} \qquad (5)$$

indicating the directions of extreme strain in the case of pure shear showing the shear strain γ.
Obviously these directions prove to be orthogonal as it holds

$$\tan\Theta_1 \tan\Theta_2 = - 1$$

Thus the extreme values of strain corresponding to the directions Θ_1 and Θ_2 can be considered being the principal strains δ_1 and δ_2, resp.
Substituting eq. 5 into eq. 4 renders the values of principal strain

$$\delta_{1,2} = \pm \ln \left(\frac{1}{2} \tan\gamma + \sqrt{1 + \frac{\tan^2\gamma}{4}}\right) \qquad (6)$$

The physical interpretation of the principal strains connected with

large shear strain is given in fig. 2.
According to eq. 2 and using the condition of invariancy of volume

$$\delta_1 + \delta_2 + \delta_3 = 0$$

and hence $\delta_3 = 0$, it follows

$$\bar{\delta} = \frac{2}{\sqrt{3}} \ln \left(\tfrac{1}{2} \tan\gamma + \sqrt{1 + \frac{\tan^2\gamma}{4}}\right) \qquad (7)$$

which in fact represents the analogue of eq. 2 in the case of large shear strain.

2. APPLICATION IN FLOW TURNING

As shown in Fig. 3 a blánk of the thickness h_o is being flow turned applying a feed of s mm/rev. in order to produce a cone with an apex angle 2α. The mechanical properties of the material are given by Nadai's relation

$$\bar{\sigma} = C\,\bar{\delta}^n \qquad (8)$$

The problem is to determine the torque M required and its corresponding tangential force F_t when flow turning on the radius r.
It is assumed that flow turning is a process of pure shear defined by the shear strain γ and hence

$$\tan\gamma = \tan(\pi/2 - \alpha) = \cot\alpha \qquad (9)$$

When considering an incremental rotation $d\phi$ of the cone when the roller acts on the radius r the work done by the torque amounts to

$$dW = M\,d\phi = F_t\,r\,d\phi \qquad (10)$$

The volume of material incrementally deformed is

$$dV = S\,h_o\,r\,\tan\alpha\,d\phi \tag{11}$$

The specific work dissipated in plastic deformation is given by

$$dA = \int_0^{\bar{\delta}} \bar{\sigma}\,d\,\bar{\delta} = \frac{C}{n+1}\,\bar{\delta}^{\,n+1} \tag{12}$$

when applying eq. 8.
Assuming that losses in friction in the roller are negligible compared to the energy of deformation it holds

$$dW = dA\,dV$$

Applying eqs. 10, 11, 12 it follows

$$F_t = \frac{C}{n+1}\,s\,h_o\,\tan\alpha\,\{\bar{\delta}\}^{n+1} \tag{13}$$

Finally substituting eq. 7 the tangential force can be expressed like

$$F_t = \frac{2^{n+1}\,C}{3^{\frac{n+1}{2}}\,(n+1)}\,s\,h_o\,\tan\alpha\left[\ln\left(\tfrac{1}{2}\cot\alpha + \sqrt{1 + \tfrac{1}{4}\cot^2\alpha}\right)\right]^{n+1} \tag{14}$$

3. EXPERIMENTAL RESULTS

Experiments have been directed towards the verification of the defining equation 7 in a state of large shear strain. As to this has been chosen $\alpha = 15^o$ which renders an effective deformation $\bar{\delta} = 1.58$ and $\alpha = 30^o$ to which corresponds $\bar{\delta} = 0.92$.
Using Aluminium 2S { $C = 145\ N/mm^2$, $n = 0.05$ } the torque applied has been measured as a function of the place and hence as a function of r. From this the tangential force can be calculated. Fig. 4 shows the results.

Also the influence of the strain-hardening exponent has been investigated by means of flow-turning several materials as listed below

	C N/mm^2	n
Aluminium S2	145	0.05
Brass Ms 72	789	0.42
El. Copper	417	0.06
Steel SP-O	611	0.17

The experimental results compared with eq. 14 are shown in Fig. 5.

It is remarked that comparison between theory and experiment is based on the material properties C and n which have been determined in a quasi-static tensile test allowing only for a limited range of strain even when Bridgman's correction is applied. Thus in fact on the one hand the stress/strain curve has been extrapolated into the region of large effective strain as occurring in flow turning whilst on the other hand eq. 14 does not account for effects of strain rate.
However it may be concluded that for the purpose of technical approximation the idea of effective deformation of large shear strain proves to be useful.

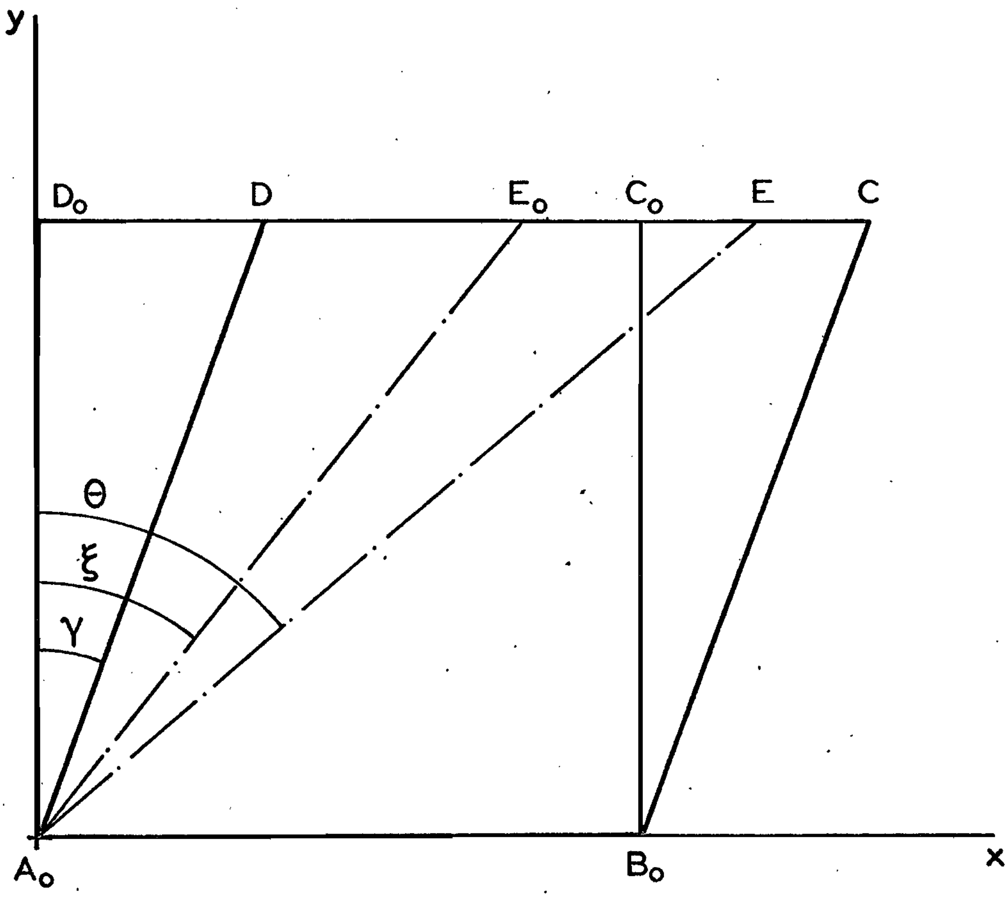

Fig. 1. Definition of natural strain in pure shear

$$\delta = \ln \frac{A_oE}{A_oE_o}$$

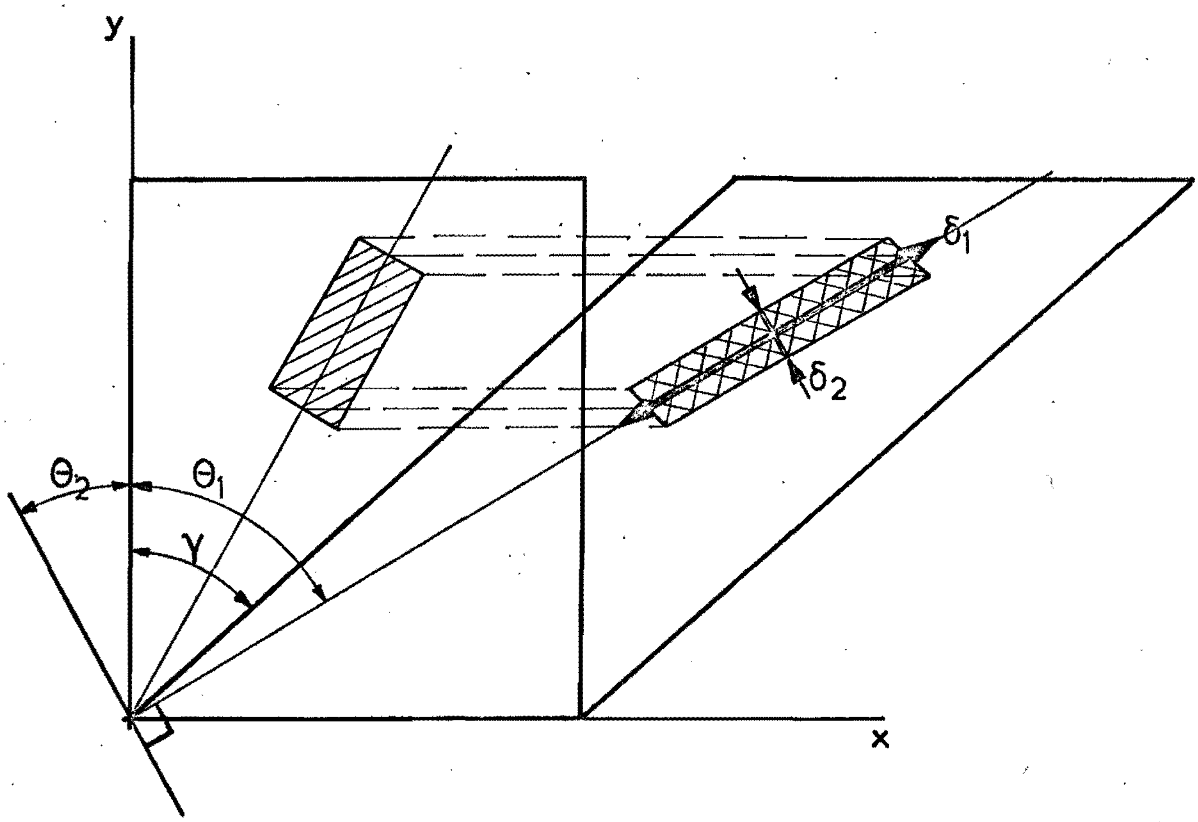

Fig. 2. Physical interpretation of the principal strains corresponding to the shear strain γ

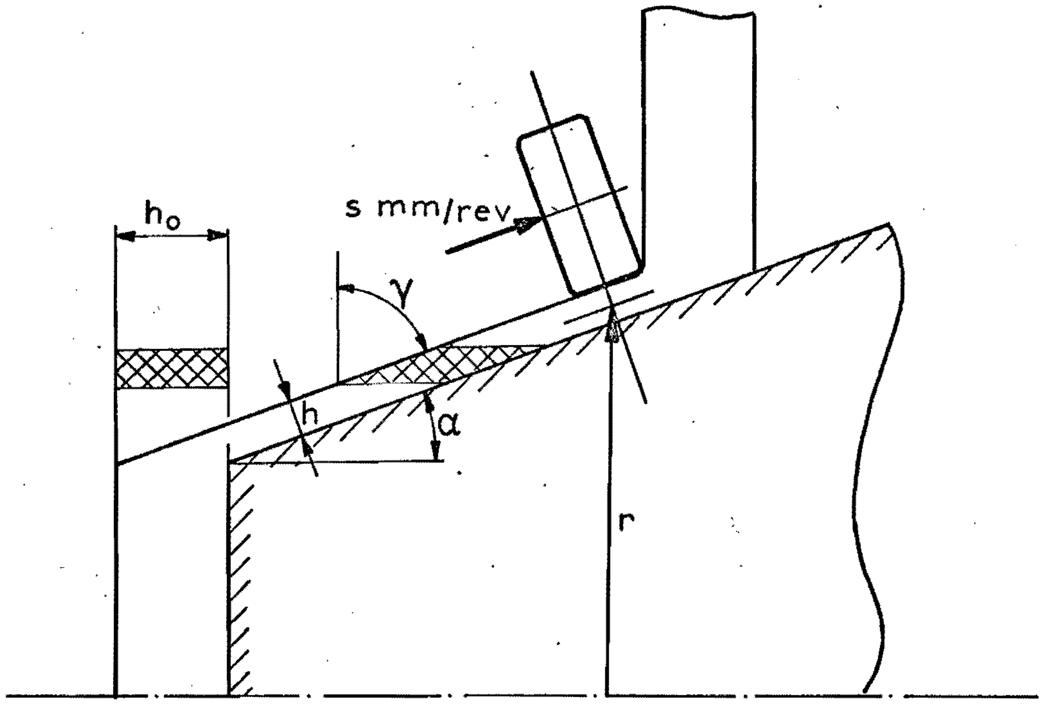

Fig. 3. Principle of Flow Turning as a process of pure shear

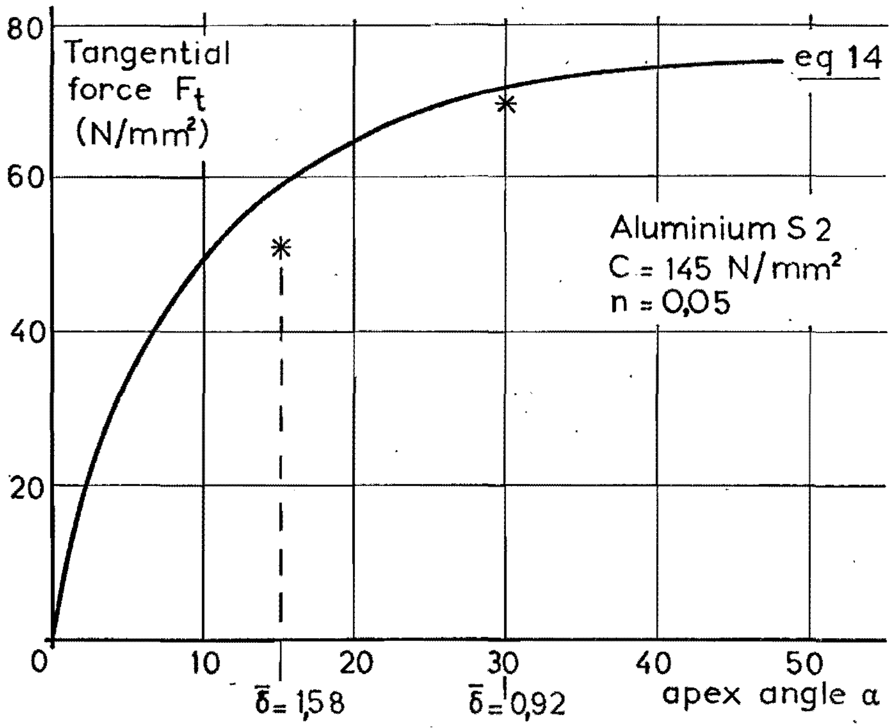

Fig. 4. Comparison between experimental values and theoretical prediction of tangential force

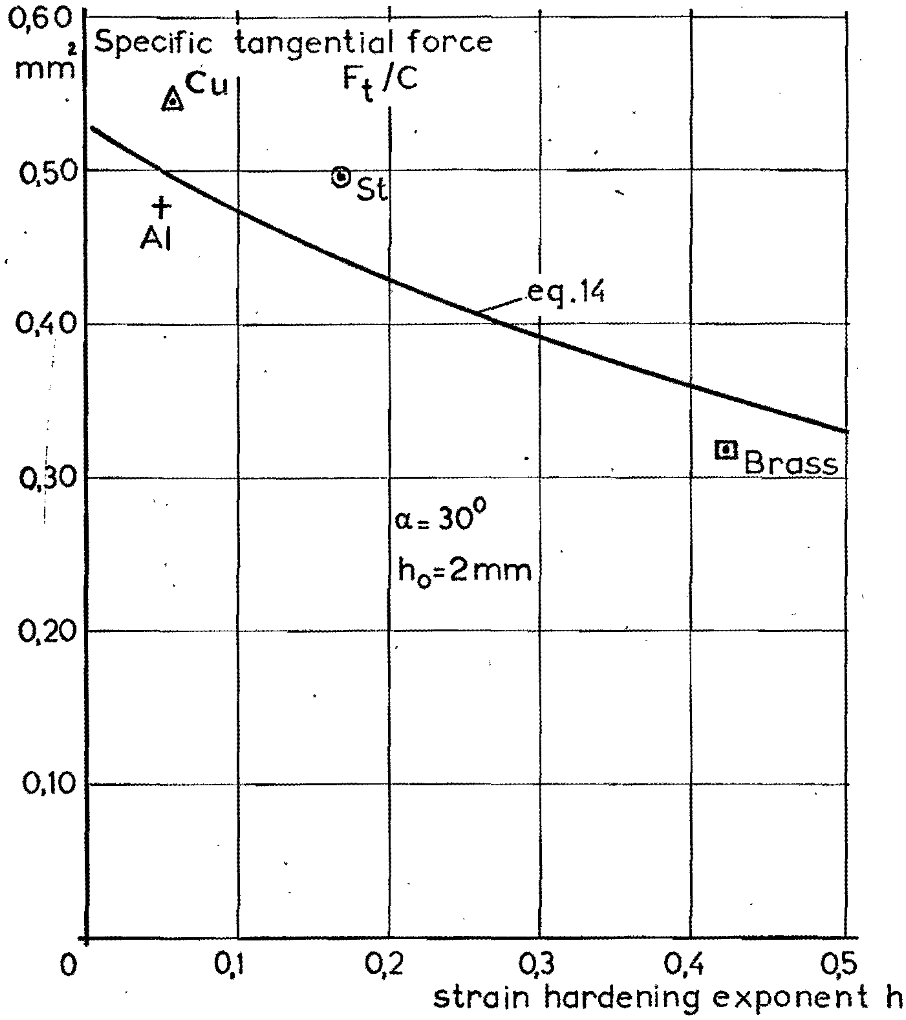

Fig. 5. The influence of the strain-hardening exponent on the specific tangential force compared with theoretical prediction.