

## Alternative preview reconstruction

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# Alternative Preview Reconstruction

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Practical Assignment Report

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# **Alternative Preview Reconstruction**

**An overview of smoothers and unknown input observers to reconstruct the suspension state and road surface required to control actively suspended cargo vehicles with preview**

**A.F.A. Serrarens**

**15th September 1995**

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# Summary

To synthesize the control forces of an active suspension system for cargo vehicles rather accurate information about the road surface is required. This information can be gathered either by look-ahead sensors or *dynamic preview*. This last method is considered in this report.

The measurements of the dynamics of the suspension system can be used to derive which road surface caused these dynamics. This dynamic preview reconstruction of the road is utilized at the front wheels of the tractor. The available information about the road surface between the front and the rear wheels can then be used to derive a feedforward control force at the rear wheels of the tractor. In past studies this way of controlling vehicles is called *active suspension with preview*.

Straightforward observers, such as a Kalman filter or Luenberger observer are not capable to reconstruct the road and suspension state without drift or bias. A modified version of the Luenberger observer, i.e. augmented with a high-pass integrator, makes it possible to reconstruct without drift. Still, a bias (phase lag) due to the extra dynamic properties of the high-pass integrator occurs in the reconstruction of the road and suspension state.

Other methods to obtain the *preview* information (road surface) from simple measurements are investigated.

An observer based on smoothing can reconstruct the road and state without drift. The algorithm is based on the minimization of the integral of an error index. Continuously forward filtering and backward smoothing of the measured quantities makes it possible to obtain accurate and smooth reconstructed signals. The method is tested with a simple two-DOF simulation and observer model. The influence of the measurement noise is limited to slowly oscillating reconstructed values. The noise itself can hardly be recognized in the smooth reconstructed values. Parameter errors in the damping coefficient have the most influence on the performance of the observer. The method is quite accurate but very expensive in practice, because of the considerable memory consumption and required CPU-time. Nevertheless, with some computational adjustments, like advanced interpolation techniques, it might be possible to implement this method in real systems successfully.

The system under consideration can be classified in the group of systems with unknown inputs. To observe these systems the use of *unknown input observers* is also considered in this survey. These observers first reconstruct a reduced-order unknown-input-free system and afterwards the original state and road can be derived from this. The unknown-input-



free systems are defined by useful transformations of state quantities to a form without unknown inputs. A disadvantage of the method is the requirement of the derivatives of the measurements. The linear modelling makes it possible to write these derivatives in explicit formulas. Two unknown input observers are investigated, i.e. a straightforward *Unknown Input Observer* (UIO) and a *Closed-Loop State and Input Observer* (CSIO). The closed loop observer differs from the straightforward unknown input observer in the fact that the estimated state is fed back in the reconstruction of the reduced order unknown-input-free system.

The performance of the UIO is tested with the simple two-DOF model. Again, drift does not occur. The reconstructed road gives rather a noisy sight, caused by the noisy measurements. Parameter errors in the chassis mass have the most influence on the performance of the UIO. The quality of the reconstructed values for incidental roads is less than for continuously varying roads.

Finally the performance of the CSIO is tested with a controlled four-DOF model. The control strategy is obtained from earlier studies on this subject. To use the CSIO in the four-DOF controlled model the addition of a straightforward LQE observer is required. The performance of the controlled system with the use of preview information from the CSIO/LQE observer is satisfactory. The influence of measurement noise is more or less the same as with the use of an UIO.

The unknown input observers are promising but require further investigation to the robustness and the possibilities to implement them in more extended vehicle models or even in real vehicles.

# Chapter 1

## Introduction

This report offers an extension of the Ph.D. thesis of Huisman (1994): “A Controller and Observer for Active Suspensions with Preview”. Among other things he designed a Luenberger observer with a high pass integrator to reconstruct the state and unknown road input that are needed to control an active vehicle suspension of cargo vehicles. An overview of other observers—specifically based on smoothing and unknown input observing—will be investigated in this report. Section 1.1 presents some feedback to Huisman’s work. In Section 1.2 the objectives of this research are given. The outline of this report is described in Section 1.3 .

### 1.1 Review and Preview

Synthesis of control forces for active or semi-active suspensions is a difficult problem because these control forces have to satisfy several conflicting objectives. Moreover, the main problem of designing a control law, that can manage many different sample functions instead of having an average optimality, is the lack of sufficient information about the road input. In fact there is a need for knowledge of the road surface while driving along that road. In that case information of the road can be utilized by the controller to prepare the active or semi-active system for the oncoming input. Under these conditions the required control force can be synthesized in a much more efficient and effective way.

Suspension systems using information about future disturbance are referred to as (semi-) active vehicle suspensions with preview. The idea of preview was first proposed by Bender, 1968 but has received an increasing interest in the last few years with the advent of sonar and laser sensors that have the potential to make this control scheme implementable.

There are two different ways of previewing:

**look-ahead preview:** The road information is gathered from look-ahead sensors at the front of the vehicle. In this way the front and rear suspensions are prepared for the oncoming road input.

**dynamic preview:** The road irregularities at the front wheels are reconstructed from

measurements of the dynamics of the front suspension system. Such measurements can be chosen as the suspension deflection and the chassis acceleration. Assuming that the reconstructed road surface at the rear suspension is the same as at the front suspension, the rear suspension can be prepared for the oncoming road input.

The main problem of using sonar or elektro-magnetic look-ahead sensing is that every single irregularity will be detected. So, also a heap of leaves or mounds of snow will be detected as serious obstacles. Other things won't be detected at all, like a road pit filled with water. Another drawback of this method is that compensation for the vehicle body motions is required because the sensors are measuring the relative motions of the body and the ground. Furthermore, differences between of the scanned and tracked road profiles during turning may result when long preview distances are used, that are required by the theory. This way of previewing is discussed by Foag and Grübel [7], Hać and Youn [11] and many others.

The second method of previewing also has some important drawbacks. This indirect way of previewing only holds for straight tracking of the rear and front wheels. When the track of the rear wheels is different from that of the front wheels (turning, 'twin-mounted' rear wheels, reversing, moving obstacles) there is no guarantee for a good performance. Dynamic previewing has been of interest by many people: Frühauf et al. [8], Abdel Hady [1], Louam et al. [18], Hać and Youn [12], Sharp et al. [26]. Youn [32] makes a comparison between the two preview methods.

Huisman [14] describes the development of an *active* suspension at the rear wheels of the tractor of a tractor-semitrailer combination. The suspension control uses knowledge of the road surface between the front and the rear wheels which has to be reconstructed from measurements at the front suspension system. As mentioned above this way of using measurements for control purpose is called dynamic previewing. The resulting controller scheme has both a feedback and a feedforward part.

## 1.2 Research Objectives

The main objective of this report is to improve the reconstruction of the state and the unknown road input at the front suspension. Huisman derived a reconstruction method, based on a *Luenberger observer* [19]. For the determination of a suitable setting of the observer *Kalman filter* theory [15] is used. This observer has proved to give satisfactory results for stochastic road surfaces. But most attention is paid to deterministic road surfaces. That's why a Luenberger observer is used here, since this observer is especially appropriate for deterministic structures.

One of the measurements is the chassis acceleration. When the chassis displacements is to be derived from this measurement, double integration of the chassis acceleration is required. Unfortunately, for deterministic road surfaces and measurement noise, this causes 'drift' in the reconstructed state. Huisman uses a 'High-Pass Integrator' instead of a pure double integrator to meet this disadvantage. The result is seen as 'measurement' of the

chassis displacement. As shown in the thesis, this bounds the drift in the reconstructed state.

The derived reconstructor in [14] is based on the *filter* theory because only information at time  $t$  is of interest. In this report it is suggested that a reconstructor, based on *smoother* theory, might bring up more improvements.<sup>1</sup> Smoothing in principle is based on the idea that an estimate of state variables at a certain time is composed of information (measurements, state information) over a whole time interval. In the case considered here one could choose the preview interval  $[t, t + t_p]$  for this.

Another way to reconstruct the state and unknown road input is the use of so called *Unknown Input Observers*. In general, these observers define pseudo-state equations in which the unknown road input is removed. This state will be reconstructed and from this, estimate of the whole state and unknown input can be derived.

### 1.3 Outline

To observe the state and preview information (road surface) suitable models for the dynamic suspension system and the road are required. In Chapter 2 these models are presented along with the roads (deterministic and stochastic) that are used to simulate and to evaluate the performance of the observers.

Smoothers derived from the minimization of the integral of a squared error criterium are essentially based on current time filtering with backwards smoothing from the current time up to the initial time. In Chapter 3 such a smoother is derived and investigated for its performance. Such kind of smoothing is known to be very memory-consuming, especially when it is used in continuous-time structures. Nevertheless, it should give better results because at every time  $\tau$  within the interval  $[t, t + t_p]$ , the state is reconstructed backwards again. When boundary conditions can be put on the forward and backward reconstruction, the drift can be limited, or even expelled.

The reconstruction problem, as it is stated here, can be seen as the reconstruction of the state of a system with unknown input. It is necessary to reconstruct this unknown input. Many people have been investigating this problem. Chapter 4 discusses two of those *Unknown Input Observers*.

The combination of the observer and the controller in a more extended model gives the answer to the final performance of the observer in this application. In Chapter 5 such a combination is made with one of the unknown input observers of Chapter 4. Unlike in [14] the performance of the observer is emphasized instead of the performance of the observer based control.

After all these theoretical, and numerical/experimental (simulations) overviews conclusions and recommendations with respect to the goal of this report is described in 1.2, are given in Chapter 6.

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<sup>1</sup>An observer based on the smoother theory is not more profitable in words of expended CPU-time and memory requirements.

# Chapter 2

## Models of suspension system and road surfaces

One cannot derive an analytical controller or observer strategy without using models of the complete system (vehicle dynamics and unknown roadinput). In Section 2.1 models for the dynamic system are discussed and in Section 2.2 the subject will be models for road surfaces.

### 2.1 Models of the vehicle dynamics

Like in Huisman's thesis the starting-point will be *two-dimensional* vehicle models. The benefits of the active suspension are supposed to be gained especially in the longitudinal and vertical directions. When, after all models of the full vehicle dynamics are used problems may arise especially in the roll and yaw movements of the vehicle. These have to be modelled properly, because even worse dynamics could arise. If, for example, the vehicle is turning the tractor-weight will gravitate towards one side of the complete vehicle suspension. This will cause a certain suspension deflection, but this is not because of a road irregularity. However, the reconstructor might still observe a road irregularity.

For now it's only necessary to investigate simple models. In this research two models are used for testing the reconstruction of the state and the road. To give a little insight in the modelling of the vehicle dynamics the complete six-DOF model of Huisman is presented in figure 2.1 .

This model is, a strong simplification of the reality even with the nonlinearities that are introduced. The model suggests that the cabin, the chassis and the engine are one rigid 2-D body. This also holds for the semitrailer. The axles are represented by point masses. The nonlinear model differs only from the linear in the characteristics of the tire, the air spring at the rear wheels of the tractor and the dampers.

An enlargement of the models used here would be the extension to a 3-D model. In that case also the heave-pitch and heave-roll and even the heave-yaw movements have to be in consideration. A brief study of some of these dynamics can be found in [30]. An

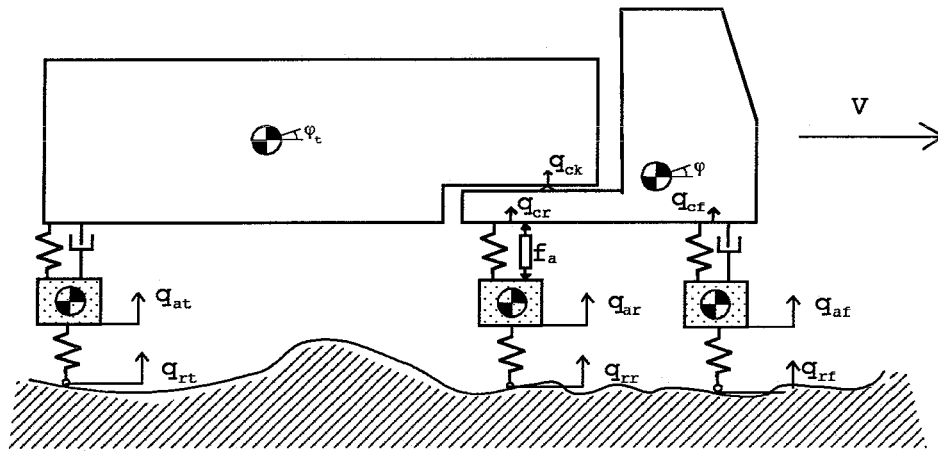


Figure 2.1: *Complete 2-D six-DOF model of cargo vehicle*

overview of vehicle dynamics can be found in [9].

## Models to evaluate the observer

The task of the observer is to reconstruct the road and the state of the system from simple measurements. The road irregularities are reconstructed from the measurements at the front suspension system. The results are used to control the rear wheels (preview). In the 2-D description of the complete vehicle it is sufficient at this moment to choose a quarter-vehicle model to test the observer. Figure 2.2 presents this model.

A quarter of the whole mass of the vehicle is modelled as a point mass. This is also done for the axle. A more realistic observer model would be a 2-D four-DOF vehicle model. Figure 2.3 presents this model. It can be seen that the weight of the semitrailer is modelled by a point mass. This four-DOF model differs not much from the linear six-DOF model.

This four-DOF model will be used as controller, observer and simulation model in Chapter 5 to evaluate the combination of an observer and the controller with preview proposed in [14].

## 2.2 Road surface models

The input excitation of the vehicle is assumed to be the apparent vertical roadway motion, caused by the vehicle's forward speed along a road having an irregular profile. The road

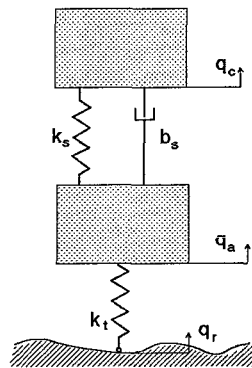


Figure 2.2: 1-D two-DOF model of quarter tractor

profile types can be divided in:

- deterministic road surfaces: the height at every position along the road is known and therefore can be described by *analytical functions*.
- stochastic road surfaces: the height at every position along the road is not known and will be described by *stochastic quantities*.

Another classification could be incidental versus continuously varying road inputs. This classification will strongly be correlated to the performances of several observers as will be shown in the following chapters.

## Deterministic road surfaces

Examples of approximately deterministic road surfaces are incidental road surface irregularities like traffic humps, bricks, railway crossings, pot-holes and kerbstones. In the simulations these irregularities are modelled by functions like a step, rounded pulse, sinusoidal and hole/bump symmetric pulses. The step and rounded pulse are described in [14]. The hole/bump pulses can be described by two point-symmetric bell-shaped functions. An overview of these functions can be found in figure 2.4. Deterministic roads are of special interest in this investigation since they are the most difficult to handle in a straightforward controller or observer based on a performance criterium. And because these deterministic road *incidents* cannot be coped with by a simple LQG scheme that generates controllers for *overall* performances.

## Stochastic road surfaces

As mentioned above stochastic roads can be described by stochastic quantities. Such quantities are often formulated in terms of the probability density function and the power

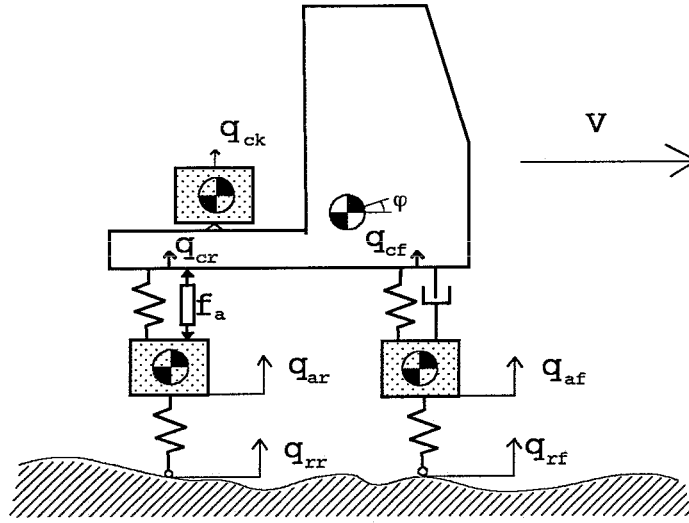


Figure 2.3: 2-D four-DOF model of cargo vehicle

spectral density ([25] and [4]). The probability density functions treat the stochastic road irregularities as a stationary *Gaussian* random process with zero mean. This can be characterized by a single sided power spectral density of the form

$$\Phi(\omega) = \frac{av\sigma^2/\pi}{a^2v^2 + \omega^2} \quad (2.1)$$

where  $\sigma^2$  is the variance of road irregularities,  $a$  is a coefficient depending on the shape of road irregularities and  $v$  the vehicle forward speed. In fact this is a roughness representation of the road and from this the excitation in the observer model can be obtained in the time domain by using a shape filter of the form

$$\dot{q}(t) = -av(t)q(t) + \xi(t) \quad (2.2)$$

where  $\xi(t)$  is a zero-mean white noise process with

$$E[\xi(t)\xi(t - \tau)] = 2av\sigma^2\delta(\tau) \quad (2.3)$$

in which  $E[\cdot]$  denotes the expected value of  $[\cdot]$ . In figure 2.5 examples of (2.1) and the solution of (2.2) are presented in plots.

The observer models which take the road input as an extra state variable will mostly use the model (2.2) for the road. Afterwards this road can be explicitly derived from the reconstructed state of the suspension.

We now have enough tools to do simulations and test the performances of the observers to be discussed in the following chapters.



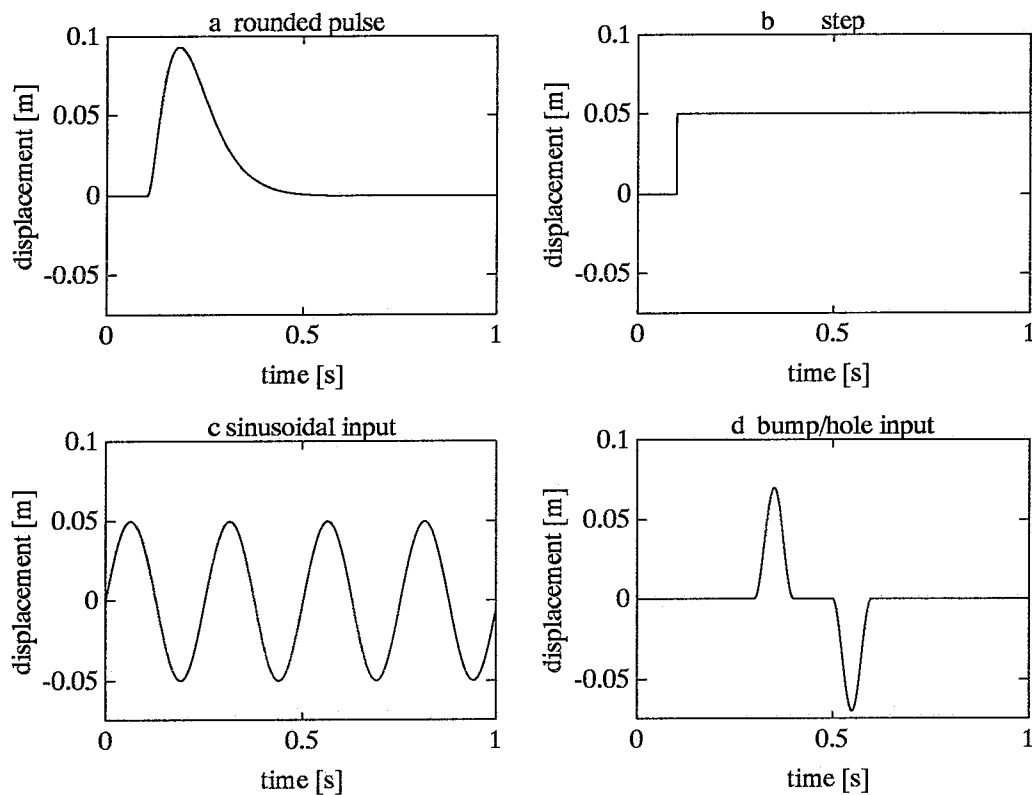


Figure 2.4: *Examples of deterministic road inputs*

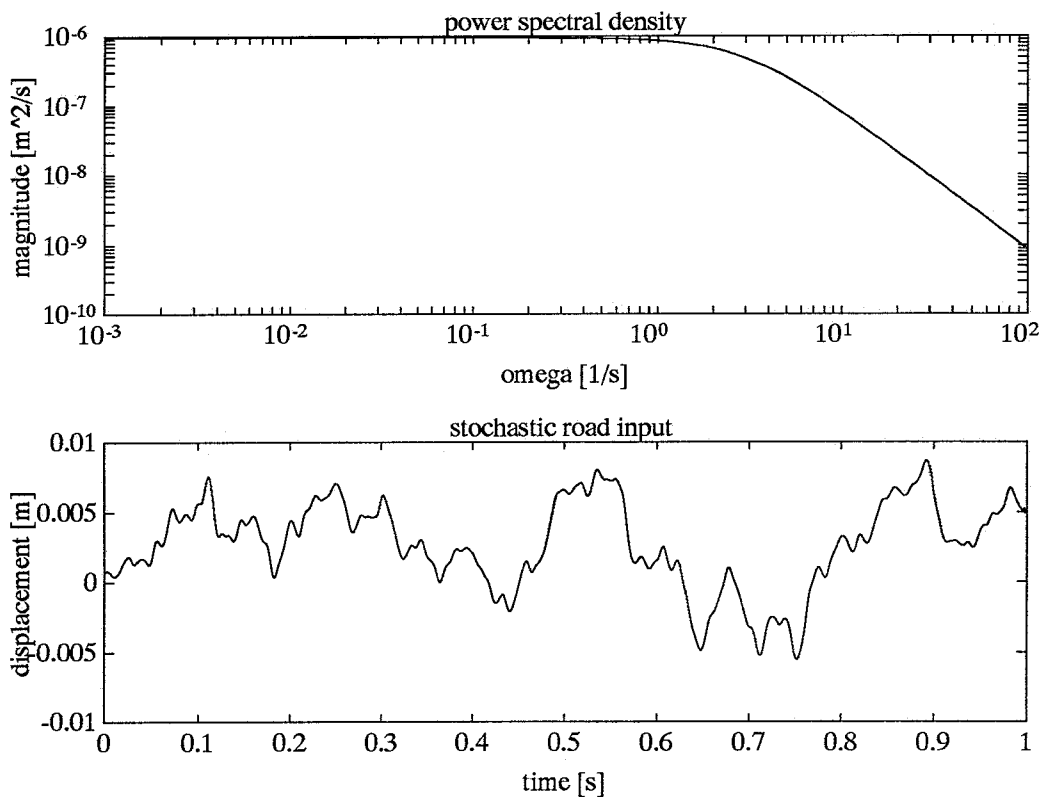


Figure 2.5: *Characteristics of stochastic road inputs*

# Chapter 3

## Continuous Smoothing

In this Chapter the general theory of continuous smoothing is used in a useful way for the application in consideration, i.e. the smoothing will only take place within time sets  $[t, t + t_p]$ . This is useful because after every preview interval the reconstructed information must be available to the controller. The outline is as follows. At first, in Section 3.1 a derivation of the smoother is presented. In Section 3.2 the performance of the smoother with respect to the two-DOF model is presented by means of numerical simulations. Conclusions are drawn in Section 3.3.

### 3.1 Observer Design

The theory presented in this chapter can also be found in [28] in which the method is derived for a linear time invariant system without unknown inputs. The design of the continuous fixed interval smoother presented here is in fact the result of the minimization of the integral of a quadratic error index over the time interval  $[t, t + t_p]$ . The error between ‘reality’ and ‘model’ is an additive error to a linear time invariant system. Consider the following linear time-invariant system which could describe the ‘reality’ of the structure of figure 2.1

$$\dot{x}(\tau) = s(\tau) + \xi_1(\tau) ; \quad x(t) = x_t \quad (3.1)$$

in which

$$s(\tau) = Ax(\tau) + Bu(\tau) + Er(\tau) + \xi_2(\tau). \quad (3.2)$$

Moreover, consider the output equations

$$y(\tau) = Cx(\tau) + Du(\tau) + Fr(\tau) + \zeta(\tau) \quad (3.3)$$

$x \in R^n$  is the state,  $u \in R^m$  is the known input,  $r \in R^p$  is the unknown input,  $y \in R^k$  the model output vector,  $\xi_1 \in R^n$  unmodelled nonlinearities etc.,  $\xi_2 \in R^n$  modelling errors and disturbances and  $\zeta$  denotes the measurement noise and errors. If  $u$  and measurements  $m$  of the output  $y$  are supposed to be known over the interval  $\tau \in [t, t + t_p]$  and  $x_t$  is

also known then the following quadratic error index can be minimized in order to find an optimal estimate of state  $x(\tau)$ .

$$J(t) = \frac{1}{2} \int_t^{t+t_p} [\zeta^T(\tau)V\zeta(\tau) + \xi_1^T(\tau)W_1\xi_1(\tau) + \xi_2^T(\tau)W_2\xi_2(\tau)] d\tau + \frac{1}{2} [x(t) - x_t]^T S [x(t) - x_t] \quad (3.4)$$

With  $\lambda \equiv W_1\xi_1$  the optimal estimate  $\hat{x}(\tau)$  which minimizes (4.4) is

$$\hat{x}(\tau) = P\lambda(\tau) + \mathcal{X}(\tau) \quad (3.5)$$

$$\hat{r}(\tau) = E_r\lambda(\tau) + K_r(m(\tau) - C\hat{x}(\tau) - Du(\tau)) \quad (3.6)$$

in which  $\lambda(\tau)$  and  $\mathcal{X}(\tau)$  are the solutions of the linear differential equations:

$$\dot{\mathcal{X}}(\tau) = A_s\mathcal{X}(\tau) + B_u u(\tau) + M_r m(\tau) - PC_r(m(\tau) - Du(\tau))$$

$$\dot{\lambda}(\tau) = -A_s^T\lambda(\tau) + C_r C\mathcal{X}(\tau) + C_r(m(\tau) - Du(\tau))$$

with boundary conditions  $\mathcal{X}(t) = x_t - P\lambda(t)$  and  $\lambda(t + t_p) = 0_n$ . The description of the occurring matrices can be found in Appendix A.

For the implementation of this smoother the observer model is designed as follows. Corresponding to the model of figure 2.2 the state  $x$  can be defined as  $x = [q_c \ \dot{q}_c \ q_a \ \dot{q}_a]^T$ . The model in figure 2.2 is passive, which implies that  $u=0$ . The design of a realizable smoother (4.5),(4.6) requires the matrices  $E$  and  $F$  to have rank= $p$ . The unknown input vector  $r$  is a one-dimensional vector—the unknown road input  $q_r$ . If the two outputs  $y = \begin{bmatrix} q_c - q_a \\ \ddot{q}_c \end{bmatrix}$  are used as estimates for the measurements of the suspension deflection and chassis acceleration, then the road input  $q_r$  will not be a part of the output model (4.3). Mathematically this means that  $F$  is a zero matrix  $\in R^{k \times p}$ ,  $\text{rank}(F) = 0 \neq p$ . Hence, the availability of these two measurements is not sufficient to result in a realizable smoother. An extra measurement, the axle acceleration, results in a realizable smoother, since then the road  $q_r$  is a part in the description of the output vector  $y$ . This implies that  $\text{rank}(F) = p$ . The output vector will be redefined as  $y = [q_c - q_a \ \ddot{q}_c \ q_r - q_a \ \ddot{q}_a]^T$  in which  $q_c - q_a$  denotes the suspension deflection,  $\ddot{q}_c$  is the chassis acceleration,  $q_r - q_a$  is the tire deflection and  $\ddot{q}_a$  is the axle acceleration. The third output  $q_r - q_a$  cannot be corresponded to a measured quantity, but is derived by reformulating the other three measurements.

In the model under consideration the axle acceleration can be formulated by

$$\ddot{q}_a = \frac{b_s}{m_a}(\dot{q}_c - \dot{q}_a) + \frac{k_s}{m_a}(q_c - q_a) + \frac{k_t}{m_a}(q_r - q_a) \quad (3.7)$$

This implies that

$$q_r - q_a = \frac{m_a}{k_t}\ddot{q}_a - \frac{b_s}{k_t}(\dot{q}_c - \dot{q}_a) - \frac{k_s}{k_t}(q_c - q_a) \quad (3.8)$$

The chassis acceleration is formulated as

$$\ddot{q}_c = -\frac{b_s}{m_c}(\dot{q}_c - \dot{q}_a) - \frac{k_s}{m_c}(q_c - q_a) \quad (3.9)$$

If the output vector  $y$  models the measurement vector  $m$ , i.e.

$$m = \begin{bmatrix} \text{susp. defl.} \\ \text{chass. acc.} \\ \text{tire defl.} \\ \text{axle acc.} \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} \quad (3.10)$$

then

$$m_3 = \frac{m_c}{k_t}m_2 + \frac{m_a}{k_t}m_4 \quad (3.11)$$

The matrix  $F$  can now be formulated as  $F = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{k_t}{m_a} \end{bmatrix}$ , with  $\text{rank}(F) = p = 1$ .

The last elements of the smoother are the weighting matrices  $W_1, W_2$  and  $V$ .  $W_1$  and  $W_2$  can be chosen quite arbitrarily. In numerical simulations (see Section 4.2) this can be done in a trial and error sense. The weight matrix  $V$  denotes the confidence one can put in the output model and measurements. If it is assumed that the measurement  $m$  satisfies

$$m = y + v \quad (3.12)$$

and  $v$  is assumed to be a white noise process then  $\nu_m$  and  $V_{mm}$  are defined as

$$\nu_m = v - E(v) \quad (3.13)$$

$$E[\nu_m(\tau)\nu_m^T(\rho)] = V_{mm}\delta(\tau - \rho) \quad (3.14)$$

High values of  $V_{mm}$  denote high variances in the measurements. Therefore these measurements deserve less confidence. The opposite also counts for low values of  $V_{mm}$ . A justified choice of  $V$  will be  $V = V_{mm}^{-1}$ , which mathematically expresses the words above.

In the next section all aspects of this section will be used to implement the smoother in the passive model of figure 2.2.

## 3.2 Numerical Simulations

In this section a survey of several sample functions for the road input (deterministic and stochastic) to be reconstructed by the continuous smoother is presented. Moreover, the effect of parameter errors will also be presented by varying the system parameters respectively for a sinusoidal road input.

The implementation of the smoother algorithm is rather simple. Within a time span  $[0, t_f]$  the smoothing takes place in blocks of  $\tau \in [t, t + t_p]$ . These blocks have to be smoothed

after every time sample  $dt$  when a new measurement  $m$  has become available. The initial boundary condition  $\mathcal{X}(t + dt) = x(t + dt) - P\lambda(t + dt)$  of a new block  $[t + dt, t + t_p + dt]$  is derived from the corresponding estimate  $\hat{x}(t + dt)$  and  $\lambda(t + dt)$  of the previous block. The boundary condition of the backwards integration of the  $\lambda$ -equation is always a zero vector. These aspects are illustrated in figure 3.1.

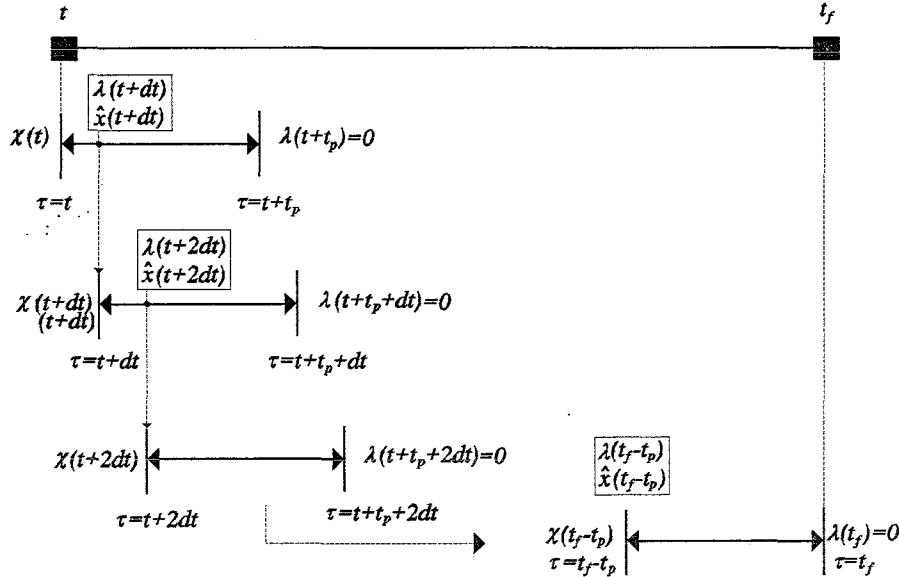


Figure 3.1: Processing of boundary conditions in the smoother algorithm

It is assumed that the vehicle speed is 90 km/h, which implies that the preview time  $t_p$  is 0.13 seconds. Furthermore, the measurements are corrupted by a significant, but realistic noise level. The rounded pulse in figure 3.2 can be reconstructed by the continuous smoother with no drift within the time span of 3 seconds. Note the influence of the measurement noise by the slowly oscillating quantities after the pulse has taken place. The tuning of the weighting matrices has been done with this sample function.

Figure 3.3 shows the reconstruction of a stochastic sample function for the road input along with the corresponding suspension state. The same weighting is used as in the rounded pulse function. As can be seen, also this sample function seems no problem to be reconstructed by the smoother.

Figure 3.4 shows the reconstruction of the step sample function as road input along with the suspension state. The weighting is not adjusted, which results in a less smooth reconstruction of this sample function. Although this sample function looks simple, it is rather difficult for this and other observers to reconstruct the step sample function. Probably this is caused by the limited persistency of excitation in the step which is in fact concentrated in just one point. With some further tuning of the smoother also this sample function can be reconstructed with better results.

Numerical simulations show that if longer preview times are used (lower vehicel speed) the quality of the reconstructed signals is better. This is not very surprising, because the

Table 3.1: *RMS values for the differences between the actual performance and the performance with parameter errors as percentage of the actual performance. The actual performance corresponds to 100%.*

quantity	no error	$m_a$	$m_c$	$b_s$	$k_t$	$k_s$
$q_c$	100	134	29	30	115	68
$\dot{q}_c$	100	63	11	8	62	19
$q_a$	100	80	52	52	91	54
$\dot{q}_a$	100	106	23	19	103	35
road	100	52	52	30	37	43

smoothing takes place over a larger time interval. This means that the state at every time instant is reconstructed from much more measured information.

Finally, in figure 3.5 a sinusoidal road input is reconstructed with the actual value of the suspension damping  $b_s$  and a reconstruction with an error in this parameter up to 150% of its actual value. Note the phase lag in the reconstructed road with this parameter variation.

This sample function is also used to test the performance of the smoother when parameter errors in the other model parameters are introduced. The results are presented in table 3.1 as RMS-values scaled with the RMS-values corresponding to the actual parameters. The values of the parameters in the observer model are changed one after the other to 150% of their value in the simulation model. The error in  $b_s$  seems to give the highest performance reduction.

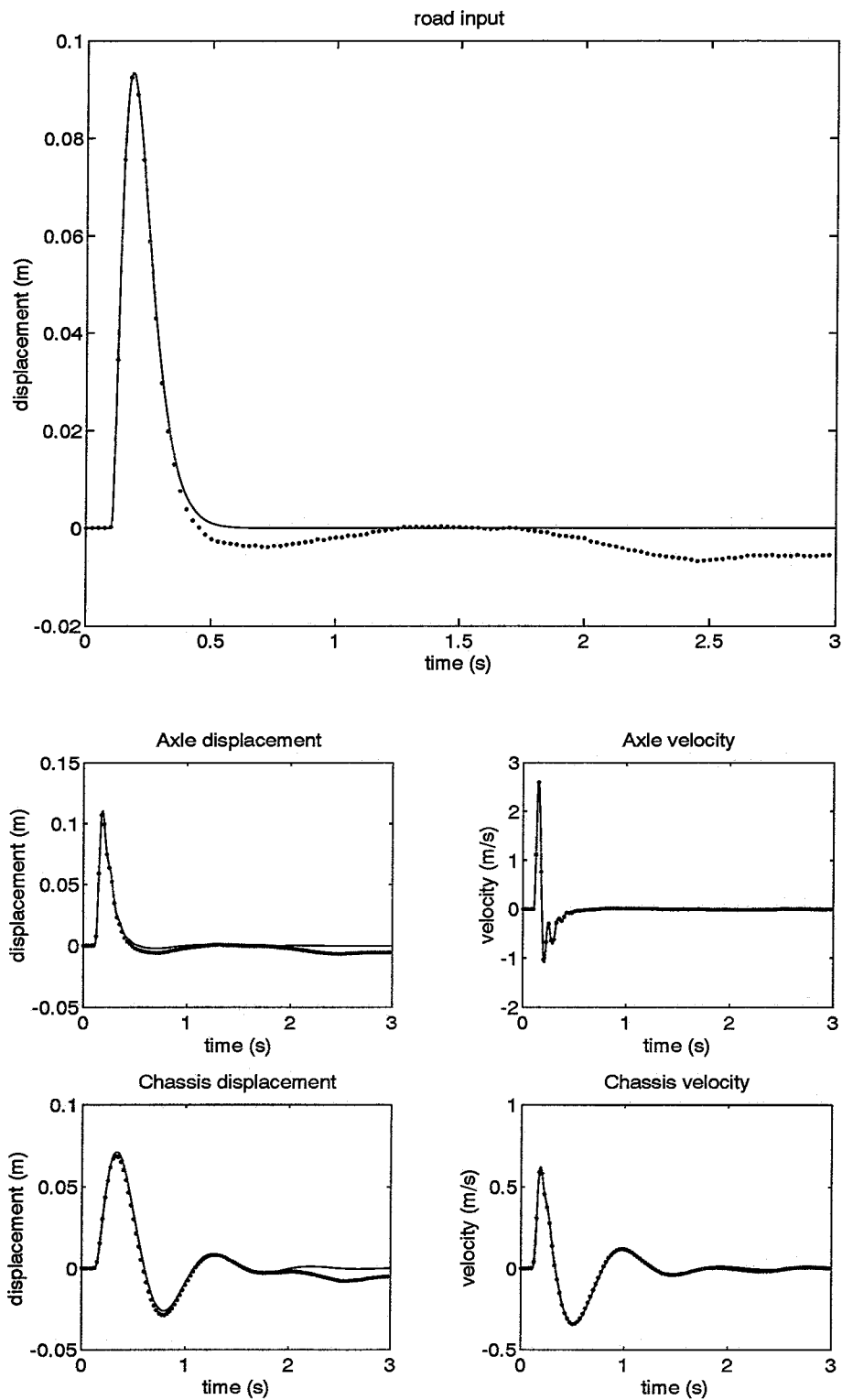


Figure 3.2: *Reconstruction of the road input and the suspension state for a rounded pulse sample function.* — actual values; ··· reconstructed values

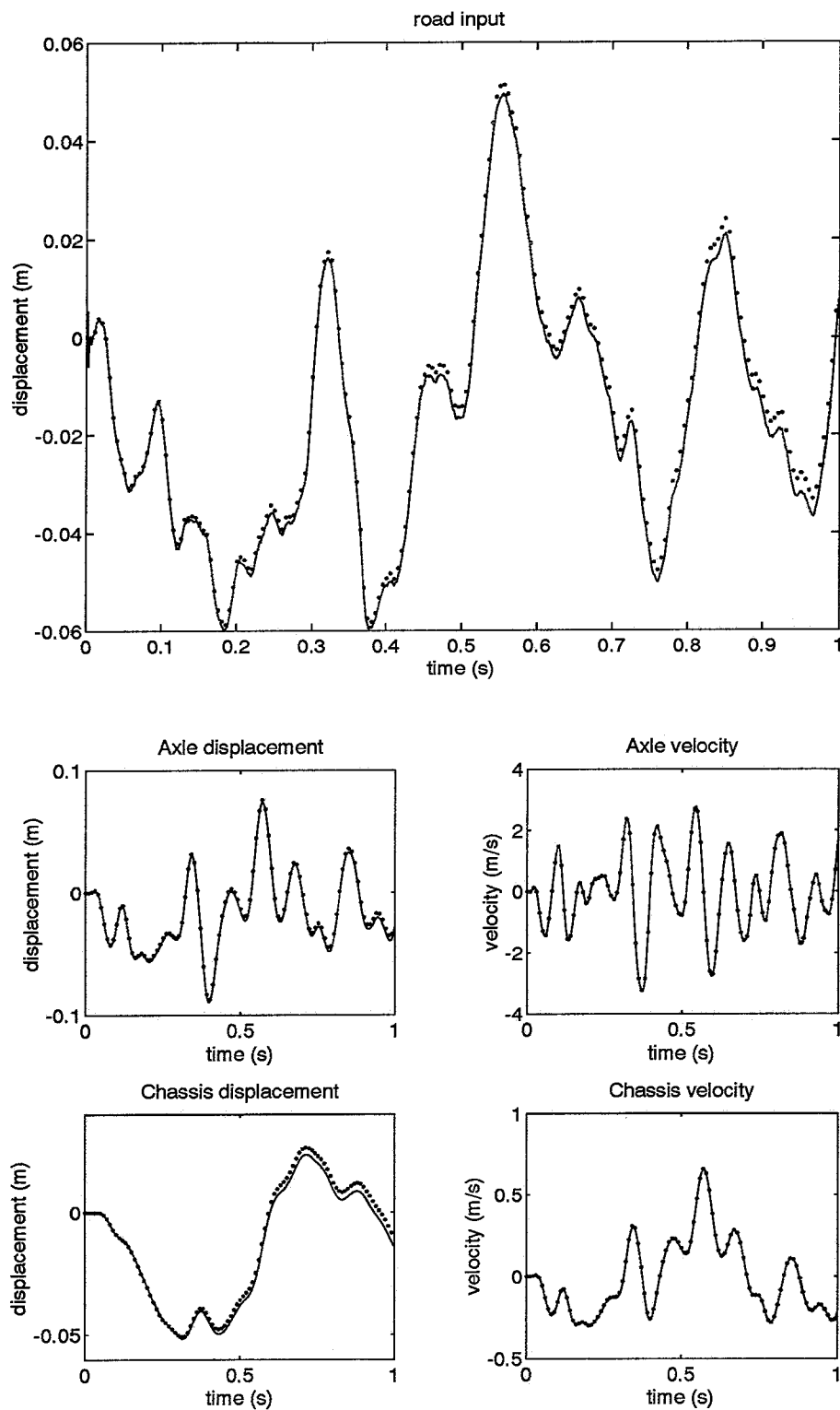


Figure 3.3: *Reconstruction of the road input and the suspension state for a stochastic sample function. — actual values; ··· reconstructed values*



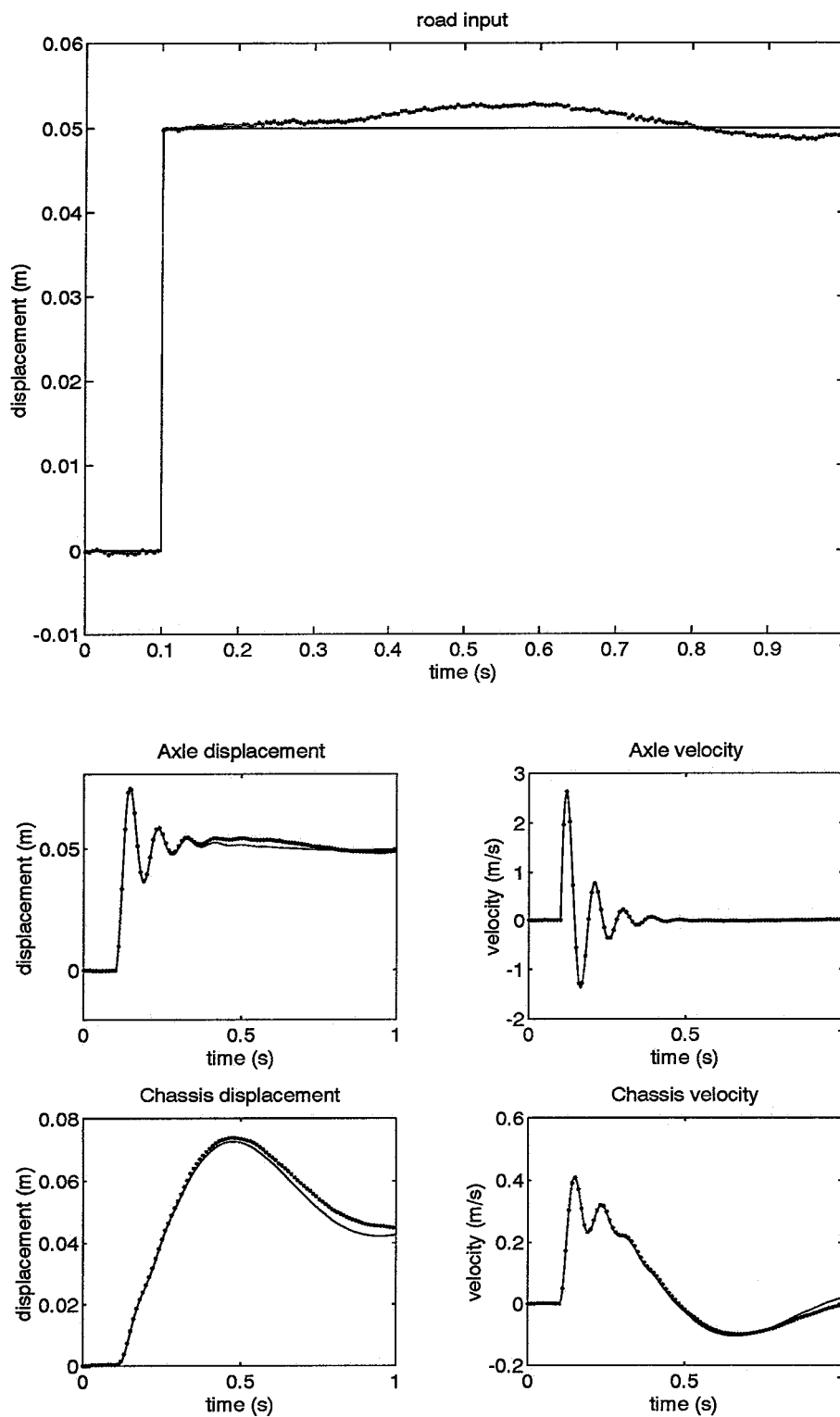


Figure 3.4: *Reconstruction of the road input and suspension state for a step sample function.* — actual values; ··· reconstructed values

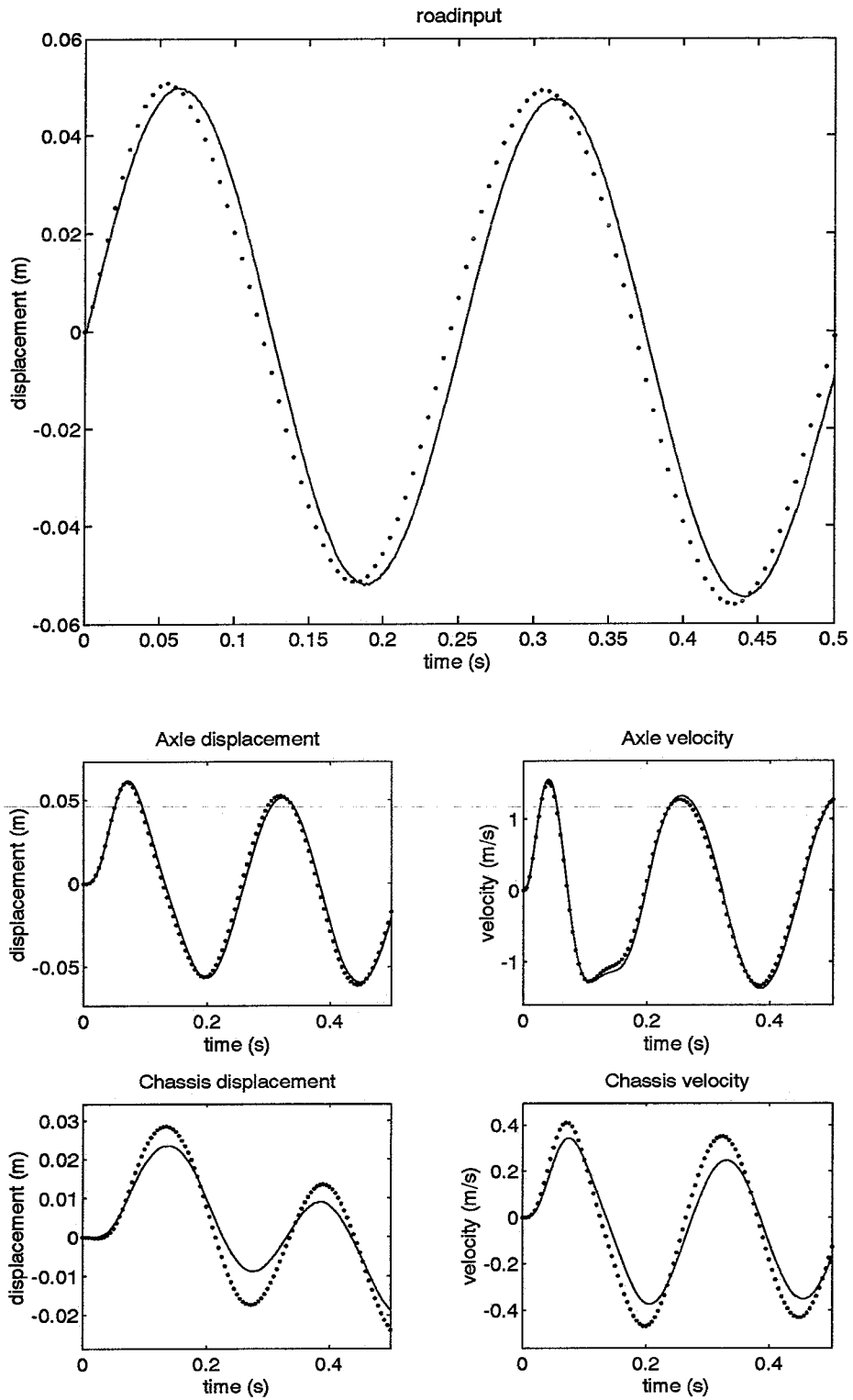


Figure 3.5: Influence of a parameter error in:  $\dots b_s$ ; — actual value

### 3.3 Conclusions

In this chapter a continuous time smoother is derived to reconstruct the suspension state and unknown road input from measurements of the suspension deflection, the chassis acceleration and axle acceleration. The reconstruction is based on the simple two-DOF model of figure 2.2. From this chapter the following conclusions can be drawn:

- The use of an extra measurement, i.e. the axle acceleration makes it possible to design a realizable smoother, which minimizes an error index.
- Appropriate choice of intensity matrices for the measurement noise together with a trial and error choice of the weighting matrices for the process noises in the error index results in a satisfying performance
- The method is very memory consuming and requires a lot of CPU-time. This is a big disadvantage in real time applications.
- The overall performance of this smoother observer is satisfying and gives a good basis for the observer based controlling for which it is intended.
- Further investigation has to be done in the performance of this smoother in a controlled vehicle model. For this application some adjustments are required, because at every time instant the road and state information at the front has to be available at the rear side. This means that forwards and backwards smoothing should be done in nearly no time. Because this is not possible, adjustments have to be made like the use of interpolation techniques.

# Chapter 4

## Unknown Input Observers

The observer problem, as already stated in Chapter 1, is trying to reconstruct the state of a system with an unknown input (road surface). Also this unknown input has to be reconstructed. In literature many people have dealt with this problem. In section 4.1 a small introduction to this kind of observers is given. In section 4.2 a reduced order observer for the inknown-input-free system is derived following the conventional Luenberger observer design. Simulations show the performance of this observer. In section 4.3 a closed loop version of a state and unknown input observer is derived. Finally, in section 4.4 conclusions are drawn.

### 4.1 Introduction

The appearances of unknown inputs in dynamic systems are very divers. In practice there are many situations with plant disturbances, or inaccessible inputs. Also actuator failures can be modelled as unknown inputs to the system. If one looks at the problem in a very broad sense, these items can be seen as the whole collection of unmodelled dynamics of a plant. These unmodelled dynamics are defined as unknown inputs in the description of the system.

The unknown input in the system discussed here is the road input. In this chapter the application of *unknown input observers* is investigated. The work of Hou and Müller [13], to be discussed in Section 4.2, is just one of the many publications on this subject in literature (see also: [3], [6], [16], [17], [22], [29] and [31]). Hou and Müller designed a straightforward reduced-order observer for linear systems with unknown inputs which is in fact a modification of the full-order Luenberger observer. Also the conditions for the existence of the observer are presented.

In [13] no assumptions are made about the measurement noise or process noise. Also these noises can be seen as unknown inputs. Park and Stein [23] designed a closed-loop state and unknown input observer. The feedback of the reconstructed state in this closed-loop observer might be useful to cope with the process and measurement noise. With this method also a few unknown or time varying parameters in the system can be identified,

which can be very useful in the considered case (e.g. varying cargo loads, varying tire stiffness and so on). Model based observing can then be extended to a more adaptive manner to obtain maximal performance even with time varying system parameters. The derivation of the identification method of this observer can be found in [23].

## 4.2 Observing the state and road with an UIO

Consider the linear time-invariant system

$$\begin{aligned}\dot{x} &= Ax + Bu + Er \\ y &= Cx\end{aligned}\quad (4.1)$$

where  $x \in R^n$  is the suspension state  $u \in R^p$  is the known input,  $r \in R^q$  is the unknown road input and  $y \in R^m$ ,  $m > q$  is the output of the systems. This output  $y$  models the available measurements  $m_x$ . These measurements are the suspension deflection and the chassis acceleration. Contrary to the preceding chapter the definition of the state vector  $x$  here is:  $x = [q_c - q_a, \dot{q}_c, q_c, \dot{q}_a]^T$ . This definition makes it possible to relate the measurement of the suspension deflection directly to the first state. The measurement of the suspension deflection is afflicted with noise. To reconstruct the suspension deflection no state integration is required, because we have direct knowledge of this quantity by the measurement. If  $q_c$  and  $q_a$  are inserted in the state as separate values then only the difference  $q_c - q_a$  can be compared with the measurement of the suspension deflection. The values of  $q_c$  and  $q_a$  itself cannot be compared with a measured quantity. Because of this, drift etc. mostly occurs in straightforward observing, since then the measurement noise is integrated too.

Note that the measurement and process noise are not considered here, which makes the method less general (see [5] in which these assumptions do are made). Nevertheless, when the filter gain is chosen appropriate in the following this won't reduce the performance of the observer when in simulations process and measurement noise are introduced.

For the following the reader is referred to Appendix B. The state  $x$  is transformed by a transformation matrix  $T$  to the pseudo-state  $\bar{x}$ .

$$x = T\bar{x} = T \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

with  $\bar{x}_1 \in R^{n-q}$  and  $\bar{x}_2 \in R^q$ . Matrix  $T \in R^{n \times n}$  is defined such that  $\bar{x}_1$  is not influenced by the unknown road input  $r$ . This transformation implies a reduced order system for  $\bar{x}_1$  (Appendix B):

$$\begin{aligned}\dot{\bar{x}}_1 &= \tilde{A}_1 \bar{x}_1 + \tilde{B}_1 u + G_1 y \\ \bar{y} &= \tilde{C}_1 \bar{x}_1\end{aligned}\quad (4.2)$$

This system is called the *unknown-input-free system*. If the estimate of  $\bar{x}_1$  is defined as  $\hat{\bar{x}}_1 \equiv w$  then a reduced order observer for the unknown-input-free system can be designed

following Luenberger observer design as

$$\dot{w} = (\tilde{A}_1 - L\tilde{C}_1)w + \tilde{B}_1u + L^*m_x, \quad w \in R^{n-q} \quad (4.3)$$

The theory of reduced order observers is extensively described in O'Reilly [24]. From this observer the final estimation of  $x$  can be derived as

$$\hat{x} = T \begin{bmatrix} w \\ U_1m_x - U_1CNw \end{bmatrix} \quad (4.4)$$

with  $\hat{x} \rightarrow x$  as  $t \rightarrow \infty$ . The estimate  $\hat{r}$  of the unknown input  $r$  can be determined by the following expression

$$\hat{r} = U_1\dot{m}_x + G_3w + G_4m_x + G_5u \quad (4.5)$$

The existence conditions for observer (4.3)–(4.5) can be found in Appendix B.

The design of the observer under consideration requires the choice of the Luenberger observer gain  $L$  and a proper choice of  $U^{-1} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$  (see Appendix B). The design of  $L$  can be done by pole-placement or the use of Kalman filter theory. In the latter case assumptions must be made about the process and measurement noise in system (4.3) to obtain an optimal filter gain. Here pole-placement is used, since then it is quite easy to derive a gain  $L$  and rather good results are obtained as will be shown in the numerical simulations further on in this section.

A disadvantage of this method is that derivatives of the measurements have to be available to reconstruct the unknown input  $r$ . Since these derivatives are not available they have to be determined by numerical algorithms. When this is done in a straightforward manner, measurement noise will significantly deteriorate the derivatives. Low-pass filters on both the measurements and the 'reconstructed' derivatives can compensate this. On the other hand an extra measurement, i.e. the axle acceleration, makes it possible to write the time derivatives of the measurements in an explicit formula.

Consider the following three system outputs

$$\begin{aligned} y_1 &= q_c - q_a \\ y_2 &= \ddot{q}_c = -\frac{k_s}{m_c}(q_c - q_a) - \frac{b_s}{m_c}(\dot{q}_c - \dot{q}_a) \\ y_3 &= \ddot{q}_a = \frac{k_s}{m_a}(q_c - q_a) + \frac{b_s}{m_a}(\dot{q}_c - \dot{q}_a) + \frac{k_t}{m_a}(q_r - q_a) \end{aligned} \quad (4.6)$$

From the second and third relation it is seen that

$$\dot{y}_1 = -\frac{k_s}{b_s}y_1 - \frac{m_c}{b_s}y_2 \quad (4.7)$$

$$\dot{y}_2 = \frac{k_s^2}{m_c b_s}y_1 + \left(\frac{k_s}{b_s} - \frac{b_s}{m_c}\right)y_2 + \frac{b_s}{m_c}y_3 \quad (4.8)$$

If  $m_x$  and the measurement  $m_{\ddot{q}_a}$  of the axle acceleration are available then  $\dot{m}_x$  can be determined as

$$\begin{aligned} \dot{m}_x &= M \begin{bmatrix} m_x \\ m_{\ddot{q}_a} \end{bmatrix} \\ M &= \begin{bmatrix} -\frac{k_s}{b_s} & -\frac{m_c}{b_s} & 0 \\ \frac{k_s^2}{b_s m_c} & \frac{k_s}{b_s} - \frac{b_s}{m_c} & \frac{b_s}{m_c} \end{bmatrix} \end{aligned} \quad (4.9)$$

Note that the axle acceleration is not a component of the output vector  $y$ , but only defined as a measurement value.

## Numerical simulations

For several sample road inputs simulations have been done. The results are given as the road input, the stroke (suspension deflection), the chassis velocity and the axle velocity. In all simulations the measurement is corrupted with a significant noise level. In figure 4.1 the rounded pulse is used as sample function for the road input. As can be seen the reconstruction is fairly good and within 5 seconds simulation time drift or bias does not occur. The stochastic road seems no problem for this method neither, as can be seen in figure 4.2. The step function in figure 4.3 gives more problems. For this function, with its low persistency of excitation after the step, it's difficult for the UIO to reconstruct this road input. With other pole placements it is possible to reconstruct this road sample function better, but then again drift in the road and chassis displacement does occur after some time. In figure 4.4 an other representative deterministic road input—the bump/hole sample function—is reconstructed. Finally, the parameter error sensitivity of this UIO is tested. The parameters in the observer model are changed to 150% of the corresponding value in the simulation model. The road input is a sinusoidal sample function. The results are given in table 4.1. In figure 4.5 the result of the parameter error in  $m_c$  is given in a plot since this parameter had the most influence on the error in the reconstructed values. The application of this observer in a real system can result in a worse performance when the chassis mass varies. Varying cargo loads is an example of a varying chassis mass.

Table 4.1: *RMS values for the differences between the actual performance and the performance with parameter errors as percentage of the actual performance. The actual performance corresponds to 100%.*

quantity	no error	$m_a$	$m_c$	$b_s$	$k_t$	$k_s$
$\dot{q}_a$	100	338	46	70	338	338
$\dot{q}_c$	100	25	21	51	26	33
road	100	58	26	42	69	58

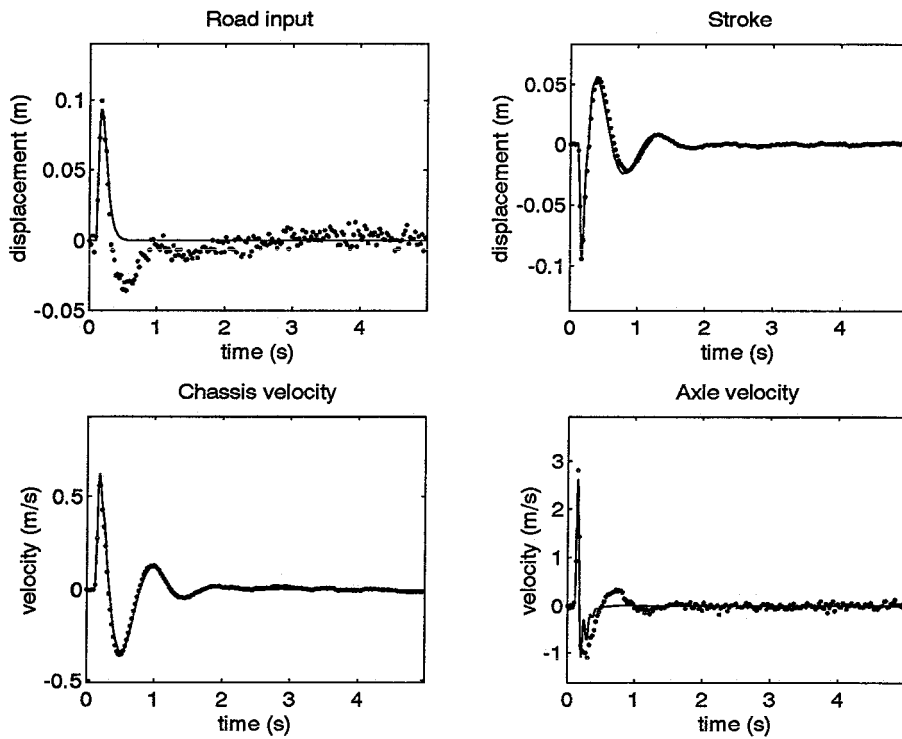


Figure 4.1: *Reconstruction of the road input and the suspension state for a rounded pulse sample function.* — actual values;  $\cdots$  reconstructed values

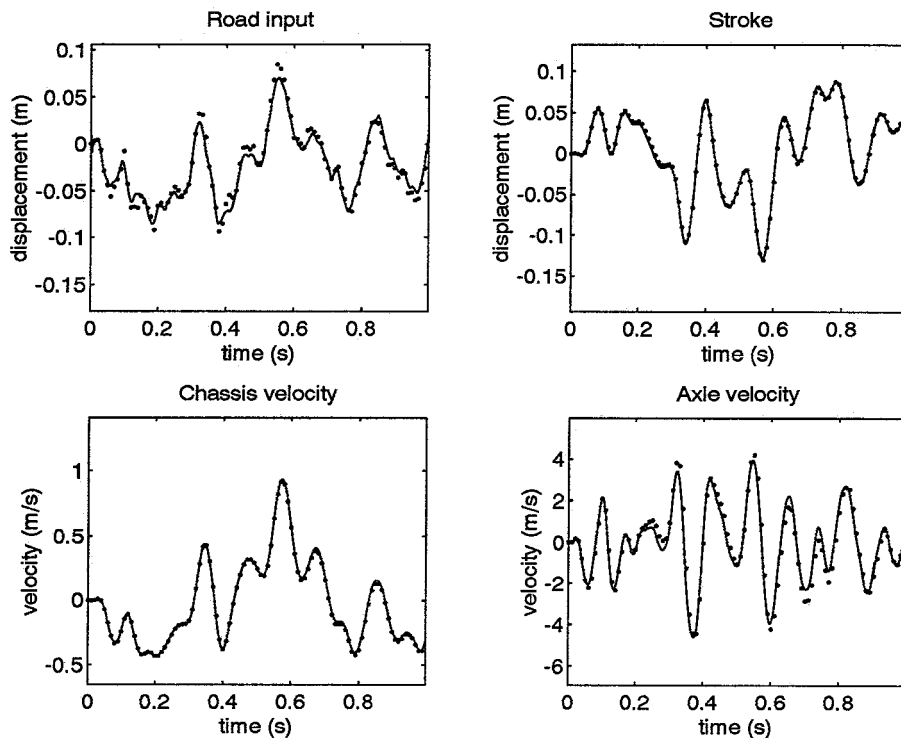


Figure 4.2: *Reconstruction of the road input and the suspension state for a stochastic sample function.* — actual values;  $\cdots$  reconstructed values



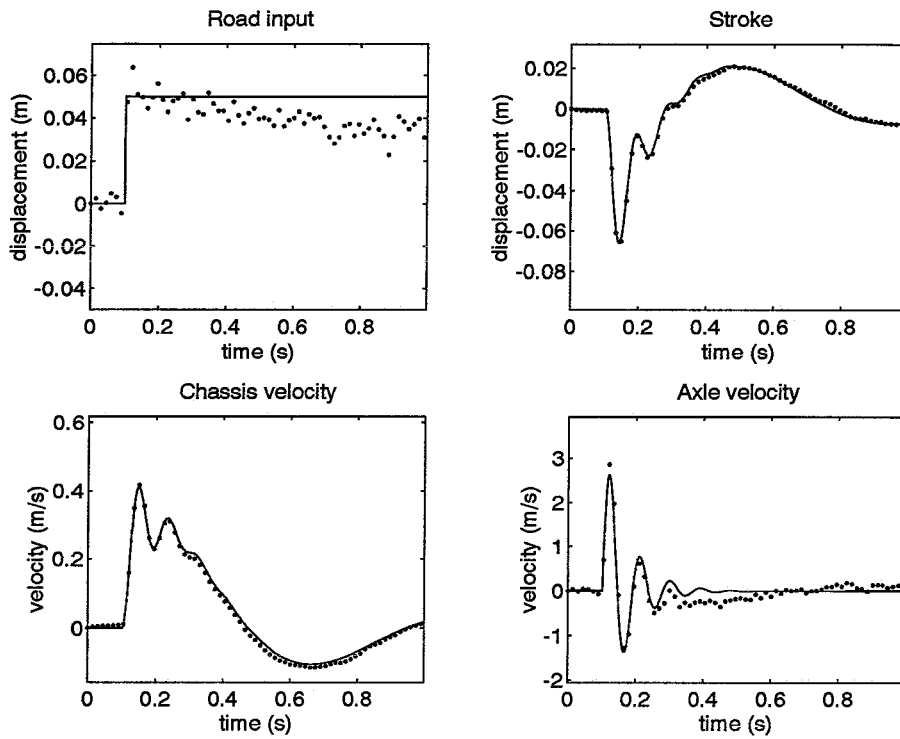


Figure 4.3: *Reconstruction of the road input and the suspension state for a step sample function.* — actual values; ··· reconstructed values

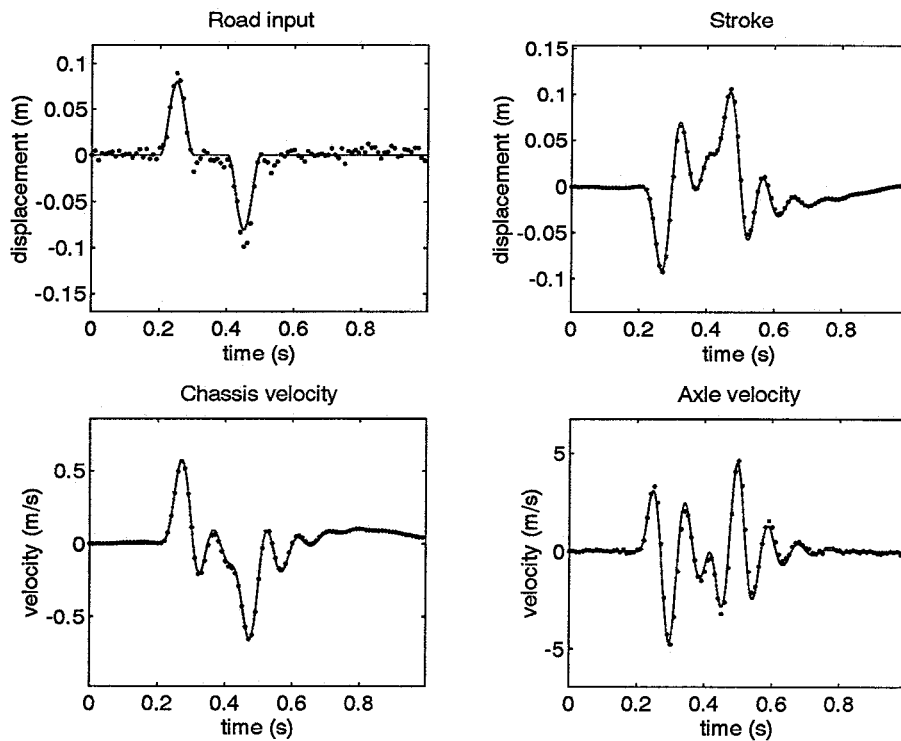


Figure 4.4: *Reconstruction of the road input and the suspension state for a bump/hole sample function.* — actual values; ··· reconstructed values

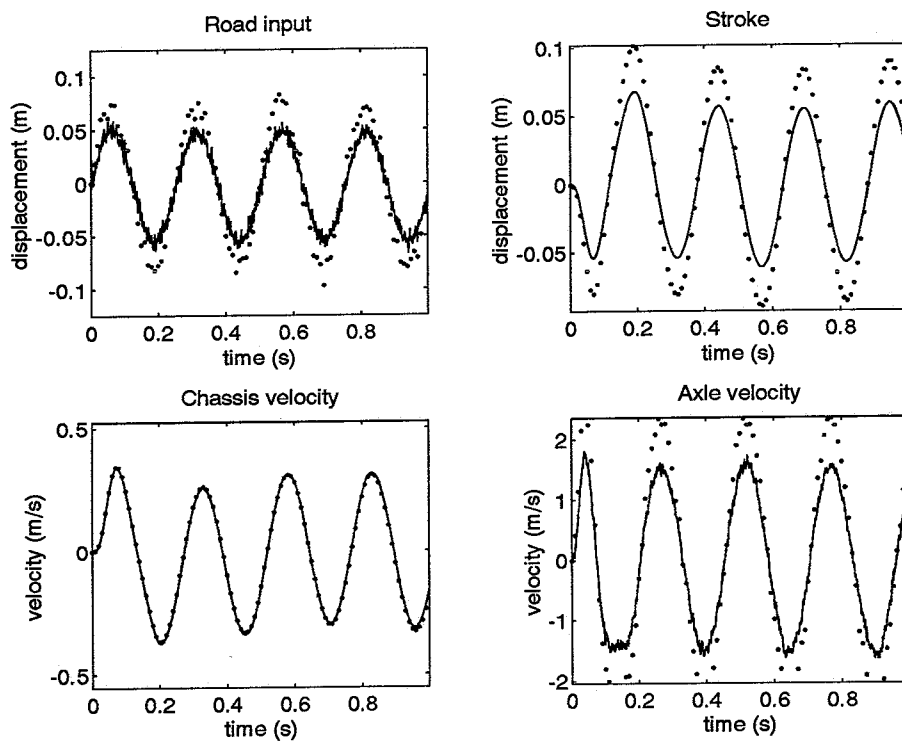


Figure 4.5: Influence of a parameter error in:  $\cdots m_c$ ; — actual value

### 4.3 Observing the state and road with a CSIO

As mentioned earlier in this chapter the Closed-Loop, State and Input Observer as it can be found in [23] is considered in this section. Appendix B gives a short description of the CSIO. The model of a system that can be observed by this CSIO is represented by the following equations

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y_1 &= Cx \\ y_2 &= Du\end{aligned}\tag{4.10}$$

In which  $x \in R^n$  is the state,  $u \in R^m$  is the complete set of known and unknown inputs,  $y_1 \in R^{k_1}$  is the output vector of state quantities and  $y_2 \in R^{k_2}$  is the output vector of input quantities. In this description one can see that the general description of the output  $y$ , i.e.  $y = Cx + Du$  is divided in two outputs  $y_1$  and  $y_2$ . This separation is to be explained further on.

Just like in the UIO design a reduced order pseudo-state  $w$  is defined. (Appendix B) A reduced-order unknown-input-free system is then defined as

$$\dot{w} = \bar{A}w + \bar{B}_1y_1 + \bar{B}_2y_2\tag{4.11}$$

For this system straightforward observer design methods as Kalman filter theory, Luenberger design or pole-placement can be used to reconstruct pseudo state  $w$  from measurements  $m_1$  and  $m_2$  of the outputs  $y_1$  and  $y_2$  respectively. A reduced-order observer for  $w$  is described as

$$\dot{\hat{w}} = \bar{A}\hat{w} + \bar{B}_1m_1 + \bar{B}_2m_2 + KV_{2M}^T(m_1 - C\hat{x})\tag{4.12}$$

Hence, an estimate of state  $x$  and the input vector  $u$  can be derived, i.e.

$$\hat{x} = V_{2N}\hat{w} + LMm_1\tag{4.13}$$

$$\hat{u} = -V_{2D}NAV_{2N}\hat{w} - V_{2D}NALMm_1 + (I - V_{2D}NB)D^+m_2 + V_{2D}Mm_1\tag{4.14}$$

The observer (4.12),(4.13) has a closed loop character. This can best be shown by the block diagram in figure 4.6. In this block diagram the bold lines represent the closed loop part of the observer. Substitution of the estimate  $\hat{x}$  in the differential equation of the pseudo state estimate  $\hat{w}$  easily shows that the closed loop dynamics can be manipulated by the eigenvalues of the matrix  $\bar{A} - KV_{2M}^T CV_{2N}$ . Pole placement methods, Kalman filter theory or Luenberger observer design can be used to determine an appropriate  $K$  and hence a suitable setting of the eigenvalues. Note that the structure of the CSIO observer (4.12)–(4.14) differs not so much with the UIO observer (4.3)–(4.5). The main distinction is the closed loop correction on the estimated state  $\hat{x}$ .

There are two disadvantages of the method. The first one is that no assumptions have been made about the measurement noise. The second one is that a linear combination of the state  $x$  and the input  $u$  is not considered in the system output  $y_i$ . Following the authors

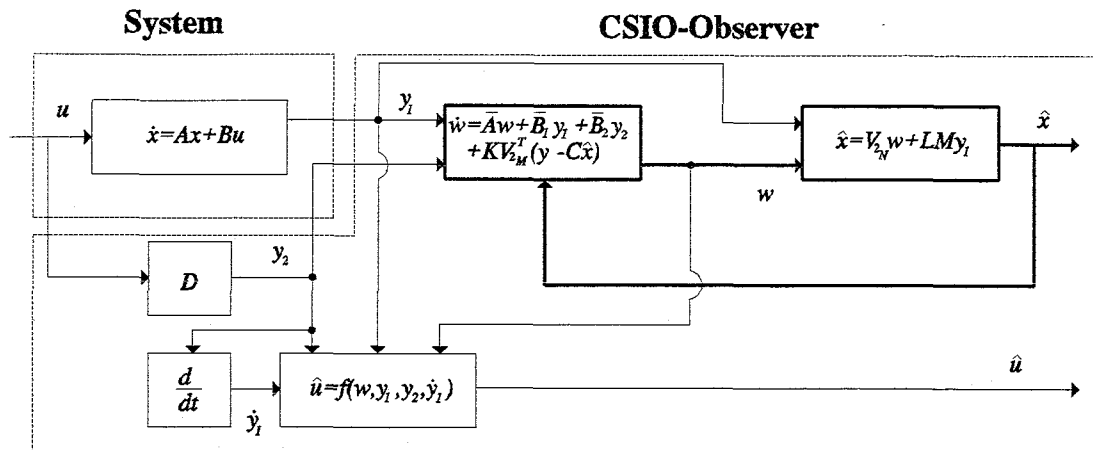


Figure 4.6: Block diagram of the system combined with the Closed Loop Observer

this case is not considered due to the fact that higher derivatives of the measurements would be necessary, which makes the method far less suitable in practice. Also the first derivative of  $y_1$  is a disadvantage, but in the foregoing section explicit formulas for  $\dot{y}_1$  have been derived to meet in this disadvantage.

Finally, the method will be tested in the next chapter. In this chapter the four-DOF model of Chapter 2 will be considered. The controller technique of Huisman will be used to evaluate the combination of this observer and preview controller.

## 4.4 Conclusions

In this chapter two *Unknown Input Observers* have been derived. From this chapter the following conclusions can be drawn.

- An unknown input observer is in fact a method which separates the state equations with unknown inputs from those without unknown inputs. The resulting reduced-order unknown-input-free system can be reconstructed with straightforward observer techniques. Afterwards, the estimate of the original state and unknown input can be derived from the reconstruction of the unknown-input-free system.
- The advantage of this method is that no assumptions on the shape of road input have to be made.
- Generally known methods like pole placement, Kalman filter theory and even conventional Luenberger observer techniques can be used to design the observers.
- Further investigation should be made to take measurement noise into account.
- The use of linear models makes it possible to derive explicit formulas for the derivatives of the two measurements (suspension deflection and chassis acceleration), which are needed in both methods to reconstruct the (unknown) input.

- Simulations show that the UIO performs in a stable manner, without bias and without drift within the considered time ranges. Nevertheless, it is rather difficult for this observer to find a good setting of observer gains, especially when *incidental* roads are used. This is probably the result of the low persistency of excitation in these kinds of road inputs. The excitation of an incidental road input is concentrated in just the small time range of the incident.

# Chapter 5

## Observing in a controller structure

In this chapter the closed loop observer of Chapter 5 is tested with an extended vehicle model. This four-DOF vehicle model is controlled with Huisman's preview controller. The performance of the CSIO in this structure is emphasized more than the controller performance. Section 5.1 describes the CSIO design for the controlled system. In Section 5.2 numerical simulations show the performance of the controlled vehicle with reconstructed information in comparison with perfect knowledge of the required information. Finally, in Section 5.3 conclusions are drawn.

### 5.1 CSIO in four-DOF controlled model

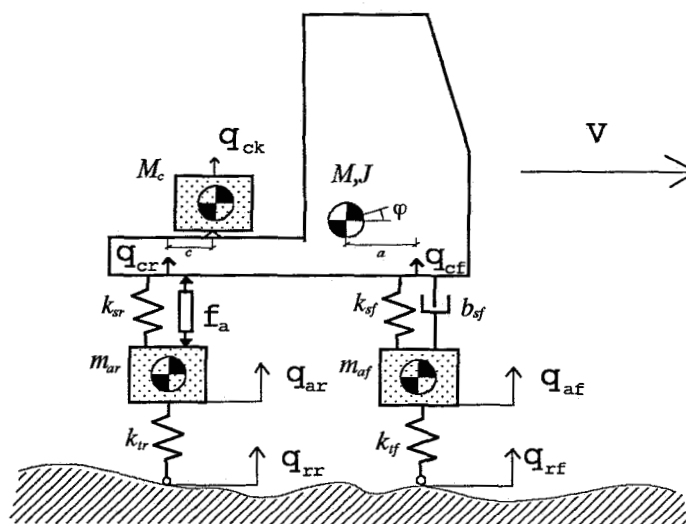


Figure 5.1: *Four-DOF controlled model with its parameters and states*

Because it is out of the scope of this research to investigate all the considered observers in a controlled structure one of them, the CSIO is chosen quite arbitrarily. This method is quite straightforward and easy to interpret. Moreover, the implementation is not complicated and not memory consuming, which makes it feasible for use in real time observing.

The four-DOF controlled vehicle model in figure 5.1 will be used as observer, controller and simulation model. The performance of the observer will be tested with the use of two road input sample functions.

The equations of motion of the model in figure 5.1 are

$$\ddot{q}_c = \gamma_1 f_f + \gamma_2 f_r \quad (5.1)$$

$$\ddot{\varphi} = \gamma_3 f_f + \gamma_4 f_r \quad (5.2)$$

$$m_{af} \ddot{q}_{af} = f_f + k_{if}(q_{rf} - q_{af}) \quad (5.3)$$

$$m_{ar} \ddot{q}_{ar} = f_r + k_{ir}(q_{rr} - q_{ar}) \quad (5.4)$$

in wich

$$f_f = k_s f(q_c + a\varphi - q_{af}) + b_s f(\dot{q}_c + a\dot{\varphi} - \dot{q}_{af})$$

$$f_r = k_s r(q_c - b\varphi - q_{ar}) - f_a$$

$$\gamma_1 = \frac{-(a+b-c)(b-c)M_c - J}{MJ + M_c(b-c)^2 + J}$$

$$\gamma_2 = \frac{c(b-c)M_c - J}{MJ + M_c(b-c)^2 + J}$$

$$\gamma_3 = \frac{(a+b-c)M_c + aM}{MJ + M_c(b-c)^2 + J}$$

$$\gamma_4 = \frac{-cM_c - bM}{MJ + M_c(b-c)^2 + J}$$

In these equations  $q_c$  is the displacement of the centre point of mass  $M$  and  $b$  is the distance between this point and the rear axle. The other quantities are defined in figure 5.1.

The state is defined by  $x = [q_{cf} - q_{af}, \dot{q}_{cf}, q_{af}, \dot{q}_{af}, q_{cr} - q_{ar}, \dot{q}_{cr}, q_{ar}, \dot{q}_{ar}]^T$ . The applied controller strategy, which synthesizes the value of  $f_a$  can be found in [14].

To fit the CSIO into the four-DOF model a few adjustments have to be made. The linear model on which the CSIO design is based is as follows

$$\dot{x} = Ax + Bu$$

$$y_1 = Cx$$

$$y_2 = Du$$

in which

$$u = [q_{rf}, q_{rf} - q_{af}, q_{rr}, q_{rr} - q_{ar}, f_a]^T$$

$$y_1 = [q_{cf} - q_{af}, \ddot{q}_{cf}, q_{cr} - q_{ar}, \ddot{q}_{cr}]^T$$

$$y_2 = [q_{rf} - q_{af}, q_{rr}, q_{rr} - q_{ar}, f_a]^T$$

and  $A, B, C, D$  matrices of appropriate order. It can easily be seen by equations (5.1) and (5.2) that the controller input  $f_a$  will be present in the front and rear chassis acceleration measurements  $\ddot{q}_{cf} = \ddot{q}_c + a\ddot{\varphi}$  and  $\ddot{q}_{cr} = \ddot{q}_c - b\ddot{\varphi}$ . The output vector  $y$ , which models the available measurements should then be written as  $y = Cx + Du$ . This is not the case for reasons given in the foregoing chapter. To overcome this problem the reconstruction is divided into two parts. The first part is the use of the CSIO strategy for a *passively suspended* four-DOF vehicle model<sup>1</sup>. This part is merely concentrated on the reconstruction at the front suspension. The second part is the use of a straightforward filter based reconstructor at the *actively suspended* rear side of the tractor. The idea behind this is that the *unknown road input* has to be reconstructed from the *passive* front suspension. This can best be done by the CSIO. At the rear side it is assumed that the road input is known from the reconstruction at the front. This implies that at the rear side we have a controlled system without any unknown inputs. Such a system can easily be observed by straightforward methods like Linear Quadratic Estimation, Luenberger Observer Design, Kalman Filter Theory, etc. To observe the rear side LQE is used here.

The definition of the output  $y_2$  suggests that the tire deflection at the front and the rear wheels, i.e.  $q_{rf} - q_{af}$  and  $q_{rr} - q_{ar}$  are available as measured quantities. In practice, this is impossible. However, if the axle acceleration at the front and at the rear wheels are measured too, it is possible to reconstruct the measured values of the tire deflections, i.e.

$$q_{rf} - q_{af} = -\frac{k_{sf}}{k_{tf}}(q_{cf} - q_{af}) - \frac{b_{sf}}{k_{tf}}(\dot{q}_{cf} - \dot{q}_{af}) + \frac{m_{af}}{k_{tf}}\ddot{q}_{af} \quad (5.5)$$

$$q_{rr} - q_{ar} = -\frac{k_{sr}}{k_{tr}}(q_{cr} - q_{ar}) + \frac{m_{ar}}{k_{tr}}\ddot{q}_{ar} + \frac{1}{k_{tr}}f_a \quad (5.6)$$

The derivative of the front suspension deflection  $\dot{q}_{cf} - \dot{q}_{af}$  can be reconstructed in an analogous manner as was done in Section 4.2. For the design of the LQE the following linear system is assumed

$$\dot{x} = Ax + B_L u_L + Gw \quad (5.7)$$

$$z_L = C_L x + D_L u_L + v \quad (5.8)$$

with process noise and measurement noise covariances  $E[w] = E[v] = 0$ ,  $E[ww^T] = Q$ ,  $E[vv^T] = R$ ,  $E[wv^T] = 0$ . A Linear Quadratic Gaussian optimal estimate of  $x$  can then be found by the stationary Kalman filter

$$\dot{\hat{x}} = A\hat{x} + B_L + L(z - C_L\hat{x} - D_L u_L) \quad (5.9)$$

via the filter gain matrix  $L$ . In this design the input and output quantities are chosen respectively as

$$u_L = [f_a, q_{rf}, q_{rr}]$$

$$z_L = [q_{cr} - q_{ar}, \ddot{q}_{cr}, \ddot{q}_{ar}, q_{rr} - q_{ar}]^T$$

<sup>1</sup>In the passive case the actuator force  $f_a$  is not a part of the input vector and so the definition  $y_1 = Cx$  can be used.



and  $A, B_L, C_L, D_L$  matrices of appropriate order.

This completes the design of the observer for the controlled four-DOF vehicle model. In the next section the strategy will be tested with a few sample functions.

## 5.2 Numerical Simulations

In this section the performance of the CSIO/LQE observer in a controlled four-DOF model is briefly tested by simulations. The results of the controlled vehicle are compared with a passive vehicle model for two realistic sample functions, i.e. the rounded pulse and the sinusoidal. These two sample functions are chosen to investigate the performance for incidental road inputs and continuously varying road inputs.

The measurements  $m_1$  and  $m_2$  of the corresponding outputs  $y_1$  and  $y_2$  are corrupted with noise. In both simulations the preview time  $t_p$  is 0.13 seconds (90 km/h is a maximum cargo vehicle speed in realistic situations)

The computation of  $\dot{m}_1$ , needed to reconstruct the unknown road input, can not be made completely explicit, in this four-DOF model.<sup>2</sup> Therefore, a combination of explicit formulas and numerical filter algorithms is made to derive the derivatives.

The observer gain  $K$  in equation (4.10) is derived by the use of Kalman filter theory for the system (4.9). The gain is optimized with trial and error adjustments in the Kalman filter design procedure. The rounded pulse is used for these adjustments.

In figure 5.2 the rounded pulse is reconstructed. It can be seen that the unknown input observer is not able to reconstruct the road very accurately. This is merely caused by the fact that the CSIO observer model is a passive four-DOF model, contrary to the active four-DOF simulation model. Nevertheless the reconstruction of the road occurs without drift and moreover, the main information (the top of the pulse) can be copied with by the CSIO.

The influence of the reconstructed road and suspension state on the control performance can be seen in figure 5.3. In this figure the four control objectives discussed by Huisman are shown in comparison with the corresponding passive quantities. The control objectives—or performance quantities—are simulated for *perfect* knowledge of the suspension state and road input together with *reconstructed* knowledge of the road and state. It is conspicuous that the control objectives do far less oscillate when the reconstructed values are used instead of the perfect values. This is one of the reasons that reconstructed information does not necessarily have to be perfect. The performance of a controlled system, which uses reconstructed information in practice, must always be tested in combination with the present observer.

The CSIO is also tested for a continuously varying road input, i.e. a sinusoidal road input. The reconstruction of the road can be seen in figure 5.4. This road is far better reconstructable than the rounded pulse. It is quite remarkable that continuously varying (high persistency of excitation) inputs can be better reconstructed than incidental inputs

<sup>2</sup>The derivatives  $\dot{q}_{er} - \dot{q}_{ar}$  and  $\dot{f}_a$  are not a part of the equations (5.1)–(5.4). This implies that they can never be written explicitly, which is required in all components of  $y_1$ , except for  $q_{ef} - q_{af}$ .

(persistency of excitation concentrated in a small time span). Probably, numerical fluctuations will be absorbed by the continuously varying quantities then. The control objectives for the sinusoidal input can be found in figure 5.5. Contrary to the previous sample function the simulated performance quantities for reconstructed information does not differ significantly from the corresponding quantities for perfect information. This is due to the fact that the road and the state can be reconstructed rather accurately.

Further investigation to the CSIO in a controlled structure, such as the influence of parameter errors, is not made in this limited report. Nevertheless, the method of unknown input observing is promising and deserves further investigation.

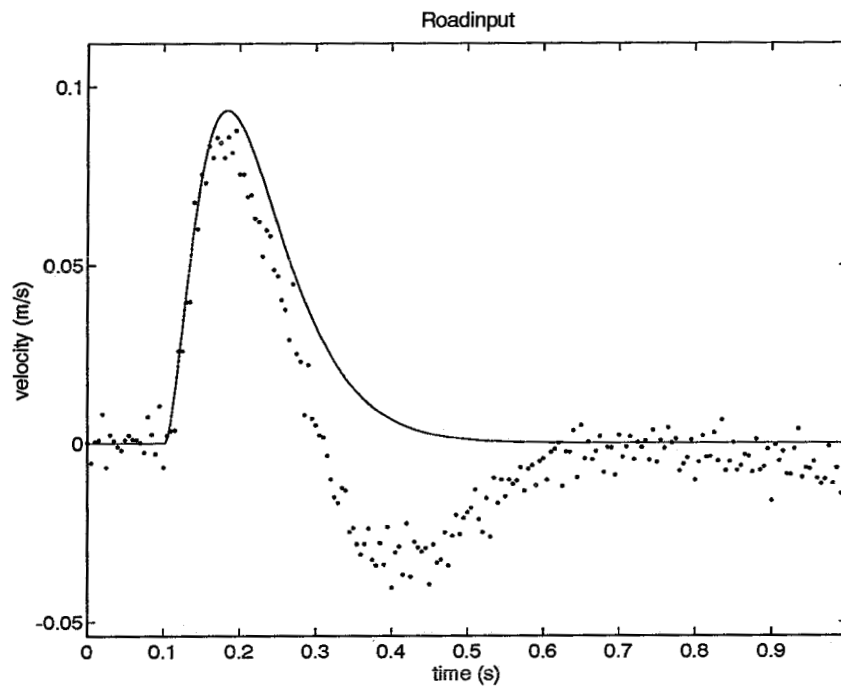


Figure 5.2: *Reconstruction of the road input for a rounded pulse sample function.* — actual values;  $\cdots$  reconstructed values

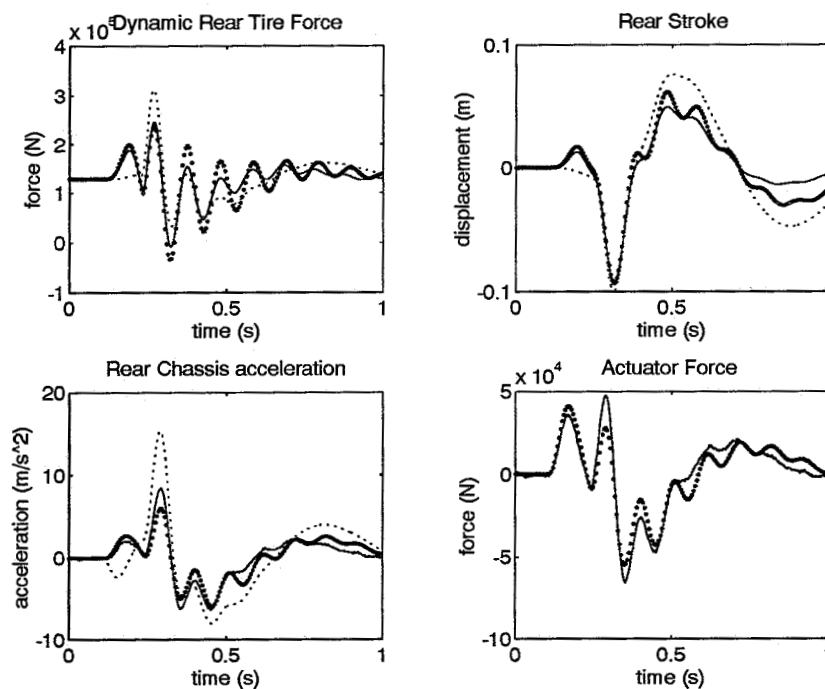


Figure 5.3: *Performance quantities of controlled vehicle model compared with passive vehicle model for a rounded pulse* - - - passive suspension;  $\cdots$  controlled with perfect state/road information; — controlled with reconstructed state/road information

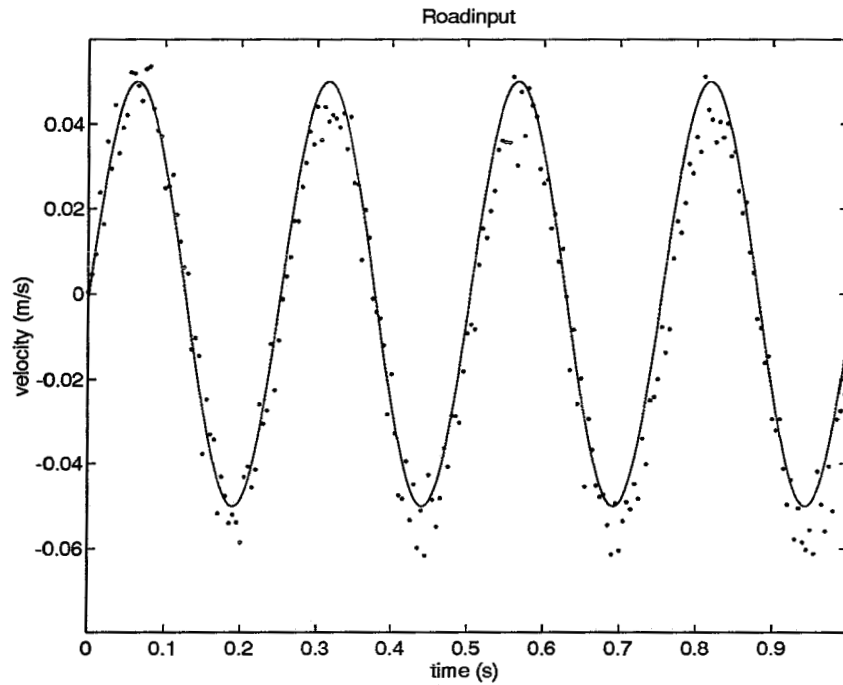


Figure 5.4: *Reconstruction of the road input for a sinusoidal sample function.* — actual values;  $\cdots$  reconstructed values

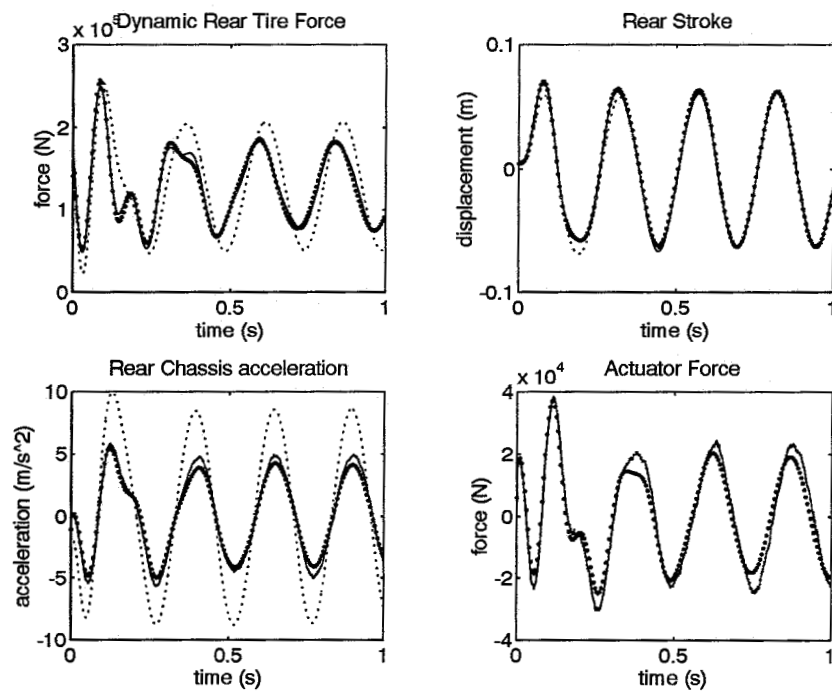


Figure 5.5: *Performance quantities of controlled vehicle model compared with passive vehicle model for a sinusoidal* - - passive suspension;  $\cdots$  controlled with perfect state/road information; — controlled with reconstructed state/road information

### 5.3 Conclusions

This chapter described the implementation of a CSIO-observer in the four-DOF controlled model. Some adjustments had to be made to make the observer suitable for this application. The use of a conventional LQE-observer is one of these adjustments. Numerical examples showed the performance of this observer for some characteristic road functions. From this chapter the following conclusions can be drawn:

- With the use of the axle acceleration as an extra measurement it is possible to design an unknown input observer of the considered type. To obtain the unknown input, derivatives of the first measurement vector  $m_1$  are required. These derivatives can not be computed completely explicitly, unlike in the passive two-DOF model. Derivatives of measurements which cannot be derived explicitly are computed by numerical filter algorithms.
- The description of the CSIO observer model is not general in the sense that the output vector is a linear combination of the state and input. Still, in the definition of the output vector this linear combination is required. Therefore a second observer is introduced. The reconstruction of the front suspension state and road input is merely taken into account by the CSIO, while the rear suspension state is reconstructed by a LQE-observer design.
- In the first reconstruction, the observer design is based on a passive four-DOF model.
- In the latter reconstruction the assumption is made that the reconstructed road input at the front is a good representation of the road input at the rear wheels after time delay  $t_p$ . This makes the rear system a linear system with known inputs and straightforward Linear Quadratic Estimation design can be used to estimate this linear system with process and measurement noise.
- Numerical simulations show that the performance of the CSIO is promising. The control objectives remain the same or are even better when the control force is derived from the reconstructed values, instead of the use of perfect information.
- The method is rather promising and deserves further investigation to the robustness and performance in more extended simulation models or real time implementations.

# Chapter 6

## Conclusions and Recommendations

In this chapter conclusions are drawn with respect to the research objectives as described in Section 1.2. Moreover, some recommendations for future investigations are given.

In Chapter 5 a continuous smoother is derived to determine the preview information and suspension state from simple measurements, i.e. chassis acceleration, axle acceleration and suspension deflection. The smoother is designed from the minimization of the fixed interval integral of an error index together with an initial value error. The following conclusions can be drawn:

- Drift due to integration of noisy measurements does not occur, even so for a bias.
- To design the smoother an extra measurement, i.e. the axle acceleration is required.
- The smoother is able to derive the estimate of the unknown road input separately from the state quantities. Since this was also done in the *unknown input observers* of Chapter 4 and 5 one can look at the derived smoother as an *unknown input smoother*.
- Longer preview times result in a better performance of the smoother.
- The method is very memory and CPU-time consuming.

The subject of Chapter 4 and Chapter 5 was the application of *unknown input observers* to the system under consideration. The unknown road input can be derived without assumptions on the shape of the road. Two methods—UIO and CSIO—have been derived and researched for their properties and performances. From these chapters the following conclusions can be drawn:

- Unknown input observers transform the original system into a reduced order unknown-input-free system. Straightforward reconstruction of the reduced system gives the basis for the reconstruction of the original state and unknown input.
- In the method derivatives of the measurements are required to derive an estimate of the unknown input. If linear passive models are used these derivatives can be written explicitly as a linear combination of the actual measurements.

- Numerical simulations show that the UIO is able to reconstruct the state and road in a stable manner without drift or significant bias. Difficulties arise when roads with low persistency of excitation are used.
- The combination of a CSIO and LQE-observer makes it possible to reconstruct an active four-DOF model. The results of using the reconstructed information for control purpose were promising.

The unknown input methods discussed in this report are specifically appropriate for the problem under consideration. With some further assumptions it was possible to reconstruct the preview information and suspension state without drift and bias. Variations due to measurement noise were not significant.

The implementation of the smoother in practice requires some further investigation. The use of advanced interpolation techniques can make it possible to reconstruct the state and road with far less computational efforts. This is desirable in real time application. Future research can tell if it is possible to smooth with interpolation algorithms.

The unknown input observers are based on models without measurement and process noise. It is interesting to investigate if the addition of these quantities in the observer model makes it possible to reconstruct the road and state even better.

In past studies it was shown that drift occurs when straightforward filter algorithms are used. An idea for future research is to investigate if it is possible to transform the suspension state and road in such a way that the measured suspension deflection can directly be compared with a transformed state of the chassis displacement or axle displacement as separate values. These separate values don't have to be reconstructed by integrating the noisy measurements of the chassis acceleration or axle acceleration then (which caused the drift). The introduction of an extra dummy state might be useful.

An other method of reducing drift in straightforward filtering is trying to detect the trend in the drifting signals. The comparison of this trend with polynomial functions, which can be subtracted from the drifting signals, can result in a satisfying performance. In this method the case of constant road slopes will also be seen as flat surface roads. However, this does not affect the performance of the controlled system, because the control force is especially to be synthesized for incidental and continuously varying road types.

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# Appendix A

## General solution of the continuous smoother problem

Consider the following linear time-invariant system which models the reality.

$$\begin{aligned}
 s(\tau) &= Ax(\tau) + Bu(\tau) + Er(\tau) + \xi_2(\tau) \\
 \dot{x}(\tau) &= s(\tau) + \xi_1(\tau) ; \quad x(t) = x_t \\
 y(\tau) &= Cx(\tau) + Du(\tau) + Fr(\tau) + \zeta(\tau)
 \end{aligned} \tag{A.1}$$

$x(\tau) \in \mathcal{R}^n$  is the state,  $u(\tau) \in \mathcal{R}^m$  is the known input,  $r(\tau) \in \mathcal{R}^p$  is the unknown input,  $y(\tau) \in \mathcal{R}^k$  the output,  $\xi_1(\tau) \in \mathcal{R}^n$  unmodelled nonlinearities etc.,  $\xi_2(\tau) \in \mathcal{R}^n$  modelling errors and disturbances and  $\zeta$  denotes the measurement noise and errors.

If  $u(\tau)$  and measurements  $m(\tau)$  of  $y(\tau)$  are supposed to be known over the interval  $\tau \in [t, t+t_p]$  and  $x_t$  is also known then the following quadratic error index can be minimized in order to find the best estimation of state  $x(\tau)$  and unknown input  $r(\tau)$ :

$$J(t) = \frac{1}{2} \int_t^{t+t_p} [\zeta^T V \zeta + \xi_1^T W_1 \xi_1 + \xi_2^T W_2 \xi_2] d\tau + \frac{1}{2} [x(t) - x_t]^T S [x(t) - x_t] \tag{A.2}$$

A new error value  $\lambda \equiv W_1 \xi_1$  is defined. After some lengthy but straightforward calculations the solution which minimizes (A.2) is

$$\begin{aligned}
 \dot{\hat{x}}(\tau) &= A_r \hat{x}(\tau) + B_r \lambda(\tau) + B_u u(\tau) + M_r m(\tau) \\
 \dot{\lambda}(\tau) &= -A_r^T \lambda(\tau) - C_r C \hat{x}(\tau) + C_r (m(\tau) - Du(\tau)) \\
 \hat{r}(\tau) &= E_r \lambda(\tau) + K_r (m(\tau) - C \hat{x}(\tau) - Du(\tau)),
 \end{aligned} \tag{A.3}$$

with the boundary conditions  $\lambda(t) = S_1 (\hat{x}(t) - x_t)$  and  $\lambda(t+t_p) = 0_n$ . In the following the first boundary condition is reformulated.

The matrices in (A.3) are defined as

$$\begin{aligned}
 A_r &= A - R^{-1} M C ; \quad B_r = R^{-1} + W_1^{-1} ; \quad B_u = B - R^{-1} M D ; \quad M_r = R^{-1} M ; \\
 C_r &= C^T (M^T R^{-1} M - Q) ; \quad E_r = H^{-1} E^T W_2 R^{-1} ; \quad K_r = H^{-1} (E^T W_2 R^{-1} W_2 E + H) H^{-1} F^T V,
 \end{aligned}$$

in which

$$\begin{aligned} H &= E^T W_2 E + F^T V F ; \quad M = W_2 E H^{-1} F^T V ; \\ Q &= V - V F H^{-1} F^T V ; \quad R = W_2 - W_2 E H^{-1} E^T W_2. \end{aligned}$$

We define an augmented state equation as follows

$$\dot{x}^* = A^* x^* + B^* u^* \quad (\text{A.4})$$

$$\text{where } x^* = \begin{bmatrix} x \\ \lambda \end{bmatrix}, u^* = \begin{bmatrix} u \\ m \end{bmatrix}, A^* = \begin{bmatrix} A_r & B_r \\ C_r & -A_r^T \end{bmatrix} \text{ and } B^* = \begin{bmatrix} B_u & M_r \\ -C_r D & C_r \end{bmatrix}.$$

As can be seen  $A^*$  is Hamiltonian which means that if  $\gamma$  is an eigenvalue of  $A^*$  than also  $-\gamma$  will be an eigenvalue. So, the system (A.4) will always give numerical instability.

Therefor define

$$\hat{x}(\tau) = P\lambda(\tau) + \mathcal{X}(\tau) \quad (\text{A.5})$$

with  $P$  an arbitrary constant symmetric matrix. From this, a set of new equations arise as

$$\begin{aligned} \dot{\mathcal{X}}(\tau) &= \{A_r - PC_r C\} \mathcal{X}(\tau) + \{A_r P + P A_r^T + B_r - PC_r C P - \dot{P}\} \lambda(\tau) \\ &\quad + B_u u(\tau) + M_r m(\tau) - PC_r (m(\tau) - Du(\tau)) \\ \dot{\lambda} &= \{PC_r C - A_r\}^T \lambda(\tau) + C_r C \mathcal{X}(\tau) + C_r (m(\tau) - Du(\tau)) \end{aligned} \quad (\text{A.6})$$

We use the freedom of the choice of  $P$  and define it as the solution of the Algebraic Ricatti equation

$$A_r P + P A_r^T + B_r - PC_r C P = 0 \quad (\text{A.7})$$

This will decouple  $\mathcal{X}(\tau)$  of  $\lambda(\tau)$ . A stable solution can be found if  $A_s = A_r - PC_r$  is Hurwitz.  $A_s$  is Hurwitz if the Lyapunov equation

$$A_s P + P A_s^T = -(B_r + PC_r C P) \quad (\text{A.8})$$

has an unique positive definite solution  $P$ . This unique solution can be found by equations (A.7) if the system  $(A, C)$  is observable.

Finally we have a stable, optimal and unique solution of the smoothing problem with unknown input as follows

$$\hat{x}(\tau) = P\lambda(\tau) + \mathcal{X}(\tau) \quad (\text{A.9})$$

$$\hat{r}(\tau) = E_r \lambda(\tau) + K_r (m(\tau) - C\hat{x}(\tau) - Du(\tau)) \quad (\text{A.10})$$

with  $\lambda(\tau)$  and  $\mathcal{X}(\tau)$  the solutions of:

$$\dot{\mathcal{X}}(\tau) = A_s \mathcal{X}(\tau) + B_u u(\tau) + M_r m(\tau) - PC_r (m(\tau) - Du(\tau)) \quad (\text{A.11})$$

$$\dot{\lambda}(\tau) = -A_s^T \lambda(\tau) + C_r C \mathcal{X}(\tau) + C_r (m(\tau) - Du(\tau)) \quad (\text{A.12})$$

If we have knowledge of the initial state  $x(t) = x_t$  then the first boundary condition can be written as  $\lambda(t) = S_1(\hat{x}(t) - x_t) = P^{-1}(\hat{x}(t) - \mathcal{X}(t))$ . If we choose  $S_1 = P^{-1}$  then this boundary condition will be  $\mathcal{X}(t) = x_t - P\lambda(t)$ . This b.c. can initialize the integration of (A.11) when a solution of  $\mathcal{X}(\tau)$  has to be computed. The second b.c., i.e.  $\lambda = 0_n$  can initialize the backwards integration of (A.12) when a solution of  $\lambda(\tau)$  has to be computed.

# Appendix B

## Derivation of UIO and CSIO

### UIO Design

Consider the constant time-invariant linear system

$$\begin{aligned}\dot{x} &= Ax + Bu + Er \\ y &= Cx\end{aligned}\tag{B.1}$$

where  $x \in R^n$ ,  $u \in R^p$ ,  $r \in R^q$  and  $y \in R^m$  are the state vector, the known input vector, the unknown input vector and output vector respectively. A,B,C,E are constant matrices of appropriate dimensions. Theory assumes that  $m \geq q$  and  $\text{rank}(E) = q$  and  $\text{rank}(C) = m$  without loss of generality. Under these assumptions one can choose a nonsingular matrix as

$$T = [N \ E], \quad N \in R^{n \times (n-q)}\tag{B.2}$$

so that

$$\bar{E} = T^{-1}E = \begin{bmatrix} O \\ I_q \end{bmatrix}\tag{B.3}$$

This makes the system (B.1) equivalent to<sup>1</sup>

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \bar{E}r \\ y &= \bar{C}\bar{x}\end{aligned}\tag{B.4}$$

where

$$\begin{aligned}x &= T\bar{x} = T \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \quad \bar{A} = T^{-1}AT = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \\ \bar{B} &= T^{-1}B = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}, \quad \bar{C} = CT = [CN \ CE]\end{aligned}$$

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<sup>1</sup>see:[24] in which is proved that any restricted system  $\{A, B, C\}$  is equivalent to a transformed system  $\{\bar{A}, \bar{B}, \bar{C}\}$  when T is nonsingular

with  $\bar{x}_1 \in R^{n-q}$  and  $\bar{x}_2 \in R^q$ . From here the reduced order observer is designed, which is in fact the heart of the method. The transformed state vector is partitioned to make it possible to isolate the part in which the unknown input is present and a part in which this unknown input is not present. It can easily be seen that an unknown-input-free system can be described by

$$\begin{aligned}\dot{\bar{x}}_1 &= [\bar{A}_{11} \ \bar{A}_{12}] \bar{x} + \bar{B}_1 u \\ y &= [CN \ CD] \bar{x}.\end{aligned}\quad (\text{B.5})$$

If  $\text{rank}(CE) = q$  then define a matrix  $Q$  so that the following nonsingular matrix exists

$$\begin{aligned}U &= [CE \ Q], \quad Q \in R^{m \times (m-q)} \\ U^{-1} &= \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, \quad U_1 \in R^{q \times m}, \quad U_2 \in R^{(m-q) \times m}\end{aligned}\quad (\text{B.6})$$

When the output equation in (B.5) is premultiplied by  $U^{-1}$  the following reduced order system can be obtained

$$\begin{aligned}\dot{\bar{x}}_1 &= \tilde{A}_1 \bar{x}_1 + \bar{B}_1 u + G_1 y \\ \bar{y} &= \tilde{C}_1 \bar{x}_1\end{aligned}\quad (\text{B.7})$$

where

$$\tilde{A}_1 = \bar{A}_{11} - \bar{A}_{12} U_1 C N, \quad G_1 = \bar{A}_{12} U_1, \quad \tilde{C}_1 = U_2 C N, \quad \bar{y} = U_2 y$$

For the reduced order unknown-input-free system (B.7) a conventional Luenberger observer can be designed as

$$\dot{w} = (\tilde{A}_1 - L\tilde{C}_1)w + \bar{B}_1 u + L^* m_x \quad (\text{B.8})$$

where  $L \in R^{(n-q) \times (m-q)}$  and  $L^* = LU_2 + G_1$ ,  $w \equiv \hat{\bar{x}}_1$  and  $m_x$  is the measured value of  $y$ .

From the solution of  $w$  one can obtain the final estimation for  $x$  and  $r$  as

$$\begin{aligned}\hat{x} &= T\hat{\bar{x}} = T \begin{bmatrix} w \\ U_1 m_x - U_1 C N w \end{bmatrix} \\ \hat{r} &= U_1 \dot{m}_x + G_2 w + G_3 m_x + G_4 u\end{aligned}\quad (\text{B.9})$$

$$\begin{aligned}G_2 &= U_1 C N L U_2 C N + U_1 C N \bar{A}_{12} U_1 C N - U_1 C N \bar{A}_{11} - \bar{A}_{21} + \bar{A}_{22} U_1 C N \\ G_3 &= -U_1 C N L U_2 - U_1 C N \bar{A}_{12} U_1 - \bar{A}_{22} U_1 \\ G_4 &= -U_1 C N \bar{B}_1 - \bar{B}_2\end{aligned}$$

The existency conditions for UIO system described by (C.7) and (C.8) are

a)  $\text{rank } CD = \text{rank } D = q$

b)  $\text{rank} \begin{bmatrix} sI_{n-q} - \tilde{A}_1 \\ \tilde{C}_1 \end{bmatrix} = n - q \quad \forall s \in \mathcal{C}, \text{Re}(s) \geq 0.$

*Proof:* see[13].

## CSIO Design

Consider the following constant time-invariant linear system.

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y_1 &= Cx \\ y_2 &= Du\end{aligned}\tag{B.10}$$

where input vector  $u$  is composed by known and unknown inputs. If measurements  $m_1$  and  $m_2$  of the outputs  $y_1$  and  $y_2$  are available then a reduced order CSIO can be presented by the following equations

$$\begin{aligned}\dot{w} &= \bar{A}w + \bar{B}_1 m_1 + \bar{B}_2 m_2 + KV_{2M}^T(m_1 - C\hat{x}) \\ \hat{x} &= V_{2N}w + LMm_1 \\ \hat{u} &= -V_{2D}NAV_{2N}w - V_{2D}NALMm_1 + (I - V_{2D}NB)D^+m_2 + V_{2D}M\dot{m}_1\end{aligned}\tag{B.11}$$

in which  $K$  is an arbitrary observer gain matrix and

$$\begin{aligned}L &= BV_{2D}, \quad M = (CL)^+, \quad N = MC \\ \bar{A} &= V_{2N}^T(I - LN)AV_{2N} \\ \bar{B}_1 &= V_{2N}^T(I - LN)ALM \\ \bar{B}_2 &= V_{2N}^T(I - LN)BD^+\end{aligned}\tag{B.12}$$

The notation  $V_{2(\cdot)}$  stands for the kernel of  $(\cdot)$  which results from a singular value decomposition of  $(\cdot)$ . The notation  $(\cdot)^+$  stands for the generalized or pseudo inverse (Moore-Penrose inverse) of the singular matrix  $(\cdot)$ . This CSIO (B.11) can asymptotically observe the state and input of the system (B.10) if the following conditions are satisfied.

a)  $(CL)^+CL = NL = I_{m-q}$

b)  $(\bar{A}, \bar{C})$  is detectable where  $\bar{C} = V_{2M}^T CV_{2N}$

*Proof:* see [23]. With this method it is also possible to identify some unknown or time varying parameters. The algorithm for this identification can be found in [23].