Circuits containing periodically-operated switches in particular resonant-transfer circuits

CIRCUITS CONTAINING PERIODICALLY-OPERATED SWITCHES

in particular

RESONANT-TRANSFER CIRCUITS

by

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CONTENT

1. INTRODUCTION.

1.1. Circuits containing periodically-operated switches.

Circuits containing periodically-operated switches are in some respects the simplest circuits with time-varying elements. Nevertheless, a general theory of these circuits is not known although various systematic methods of analysis [13,30, 37, 66-69] and synthesis [37, 67, 68] have been published by Bennett, Desoer, Kurth and the author, following an earlier analysis of a particularly simple case by Belevitch [10]. A certain amount of additional work has been done by the author (including a treatment by means of an integral equation related to the Wiener-Hopf equation of the theory of optimum filters), but this has not been published.

For certain subclasses, however, theories have been developed which are valid within certain limitations. For switched modulators (such as the ringmodulator, etc.) a simplified theory exists which assumes the modulators to be connected to strongly idealized filters. This theory was first proposed by Caruthers [20] and Kruse [64] and has been extended by various authors. It is described in the books by Tucker [99, 100], Belevitch [12] and (to a lesser extent) Henkler [55]. Its main deficiency is that it assumes in fact the filters to be "overidealized" in the sense that they cannot be approximated arbitrarily closely by means of realizable networks.

The sampled data systems form another subclass which seems to have been described first by Oldenbourg and Sartorius [75] and which has been treated extensively in the literature (among the many books on this subject, see e.g. [58, 59, 97, 98]). In this case however the approach is closer to control system theory than to true network theory; different parts of the system under consideration are usually assumed to be interconnected by means of decoupling devices so that the analysis problem is appreciably simplified.

In the present series of lectures, we shall deal exclusively with a third subclass formed by the resonant transfer circuits. For these, a theory has now evolved which can be considered to be a true and general network approach.

In several respects, this theory bears a close relationship to the theory of sampled data control systems.

There exist, of course, many other types of circuits containing periodically-operated switches, which do not fall, or at least, do not necessarily fall into one of the subclasses mentioned above (see e.g.[2, 47, 52, 65, 71, 89, 95]). For the sake of brevity, however, we have to omit discussing them here any further.

1.2. Resonant-transfer circuits.

The resonant-transfer principle has been discovered independently by Haard and Svala [54] in 1952, by Cattermole [21, 22] in 1954, by French [48] in 1955 and by Lewis [72] in 1957, although the basic idea behind it is not really new (see e.g. [51]). The aim of all these inventors was to find an efficient and bidirectional method of PAM modulation/demodulation for use in TDM-PAM (time-division multiplex, pulse-amplitude modulation) electronic telephone exchanges *. For such exchanges, the modulation/demodulation process has to be almost lossless and of equal quality in either direction of transmission.

Later on, it has been realized that the resonant-transfer principle could also be used advantageously for various other applications, in particular in PCM (pulse-code modulation) encoders [23, 24, 28] for delta modulation and for conventional AM (amplitude modulation) with SSB (single sideband) or DSB (double sideband) transmission. One can, of course, also take advantage of the possibility that resonant-transfer circuits can serve simultaneously as PAM and as AM modulators. In an electronic TDM-PAM exchange one can e.g. enter the exchange at audio frequency at the subscriber side and leave it again at carrier frequency at the junction side. In this case the frequency transposition should not only be done towards frequencies of the form $F \pm f$ (F the sampling frequency, f the input frequency) as is commonly done in ordinary modulators, but also towards frequencies of the form $nF \pm f$ (n a positive integer).

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* For a general discussion of electronic TDM-PAM exchanges, see e.g. [7, 8, 9, 26, 34, 53, 63, 73, 76, 77, 81, 90, 91, 92, 96].

Among the first studies on resonant-transfer circuits, the most important is certainly the one by Cattermole [22, 23] . The paper by Kraus [61] takes up similar ideas but complements them in various respects. Some of the results first given by Cattermole have later been found independently by Thomas [94]. Stimulated by Cattermole's work, a general theory of resonant-transfer circuits has been developed by the author [38 - 46]. Other methods of analysis have been published by Desoer [29] , Svala [93] , French and Harding [49],Perkins [78] , May and Stumps [74] , Kaden [60] , Feder [36] , Darré [27] and Leberwurst [70] . Broux has examined certain crosstalk [16] and amplification [17, 18] problems. Various non-reciprocal resonant transfer devices have been described by Adelaar [3] and Edrich [32, 33] . Further contributions are due to Adelaar [4, 5, 6], Price [83] , Svala [92] , Rosenoer [private communication],Aagaard [1] and Kraus [62] . A very incomplete list of additional patents is [35, 85-88] . Many of the papers on electronic TDM-PAM switching systems mentioned in the footnote on page 1.2. also contain descriptions of the resonant-transfer principle.

A new possibility of application for the resonant-transfer principle has been discovered more recently. Poschenrieder [81] had noticed that a certain chain matrix derived in[43] is very similar to the chain matrix of various distributed-parameter two-ports. This has led him to propose the realization of filter networks built of capacitances and resonant-transfer switches only and having properties mathematically identical to those of certain transmission-line networks and mechanical filters. Further research into this subject by the author has shown that even more flexible designs than those proposed by Poschenrieder can be realized. These correspond in a certain sense to the most flexible of the transmission-line circuits originally proposed by Richards [84] , (i.e. those in which capacitances and inductances of a lumped parameter circuit are individually replaced by transmission-line elements), but without involving the technological difficulties which make these transmission-line circuits impractical. These results appear here for the first time.

It is clear that only a summary of the various results obtained so far in the theory of resonant-transfer circuits can be given in these lectures, and that proofs, if given at all, can only be sketched. For further results as well as for all details, the original literature should be consulted.

2. GENERAL PROPERTIES OF PERIODICALLY-VARYING CIRCUITS.

In order to simplify the subsequent discussions, we shall summarize in this section some general properties of periodically-varying circuits, to which also belong the circuits containing periodically-operated switches. We assume throughout that we are dealing with linear circuits, even if this is not explicitly stated.

2.1. Fundamental properties.

Since all differential equations describing the network are linear and have real coefficients, the method of analysis by means of complex exponentials remains valid, just as for linear constant networks.

Consider then a network whose parameters are varying with period T and assume that a complex excitation

$$x(t) = A\, e^{pt} \tag{2.1}$$

is applied to it, where A and p are complex constants. As Zadeh [102] and Belevitch [10] have pointed out the steady-state response to (2.1) can be written

$$y(t) = Y\, e^{pt}, \qquad Y = A\, H(p,t) \tag{2.2}$$

where the system function H(p,t) is independent of A and periodic in t with period T. If the actual excitation is equal to the real part of (2.1), the actual steady-state response will be equal to the real part of (2.2), and every other possible response differs only by an additive term corresponding to a free oscillation.

Unless a system function is in fact independent of t, its value depends also on the choice of the time origin. If the time origin is shifted from zero to t_o, H(p,t) is changed into $H(p,t+t_o)$. H(p,t) can also be developed into a Fourier series,

$$H(p,t) = \sum_{n=-\infty}^{\infty} H_n\, e^{jn\Omega t} \tag{2.3}$$

where

$$\Omega = 2\pi F, \qquad F = 1/T\ . \tag{2.4}$$

A quantity such as $H_n e^{jn\Omega t}$ will be called the component of order n of H. By shifting the time origin to t_o, H_n is changed into $H_n e^{jn\Omega t_o}$.

From these considerations, we conclude that if a periodically-varying circuit is driven by an excitation of the form (2.1), the factor e^{pt} is just as superfluous as in case of constant circuits. Thus, if i(t) and v(t) are e.g. a current and a voltage under exponential steady-state conditions, it will in general be sufficient to consider the periodic functions I=I(t) and V=V(t) defined by

$$i = I\, e^{pt}, \qquad v = V\, e^{pt} \tag{2.5}$$

Just as in case of constant networks, we may then speak of the current I and the voltage V instead of the current i and the voltage v. If clarity requires, these latter ones will also be called respectively the instantaneous current and the instantaneous voltage.

We shall henceforth assume that rms values are used throughout. This way, the physical excitation actually applied is $\sqrt{2}$ times the real part of the complex excitation, and the response actually obtained is then also $\sqrt{2}$ times the real part of the corresponding complex response.

2.2. Power transmitted.

Consider a port of a periodically-varying circuit and let V and I be respectively the voltage and the current at this port under exponential steady-state conditions. We have

$$V = \sum_n V_n\, e^{jn\Omega t}, \qquad I = \sum_n I_n\, e^{jn\Omega t}. \tag{2.6}$$

Assuming $p = j\omega$, with ω real, and

$$\omega \neq k\Omega/2 \qquad (k = \text{any integer}), \tag{2.7}$$

the average power P delivered through the port is given by

$$P = \overline{vi} = \sum_n P_n, \qquad P_n = \mathrm{Re}\, V_n^* I_n \tag{2.8}$$

Whenever we write $p = j\omega$, we shall always assume that ω is real. Furthermore, whenever power is considered we shall always assume $p = j\omega$, with ω satisfying (2.7).

2.3. Effective behaviour.

Consider a periodically-varying circuit N terminated at port 2 by a constant resistance R_2 and at port 1 by a source of voltage $e = E\,e^{pt}$ (E a constant) in series with a constant resistance R_1 (fig. 2.1). The following system functions are particularly important, the transfer function S_{21} defined by

$$S_{21}(p,t) = 2\sqrt{\frac{R_1}{R_2}} \cdot \frac{V_2}{E} = -2\sqrt{R_1R_2}\; I_2/E \tag{2.9}$$

and the reflection function S_{11} defined by

$$S_{11}(p,t) = (2V_1 - E)/E = (E - 2R_1I_1)/E. \tag{2.10}$$

These definitions are analogous to those of the transfer and reflection coefficients of constant networks.

The Fourier expansions of V_1, I_1, V_2 and I_2 can be written, with $i = 1$ or 2,

$$V_i(t) = \sum_n V_{in}\, e^{jn\Omega t}, \qquad I_i(t) = \sum_n I_{in}e^{jn\Omega t},$$

and those of S_{21} and S_{11}

$$S_{21}(p,t) = \sum_n S_{21n}\, e^{jn\Omega t}, \qquad S_{11}(p,t)= \sum_n S_{11n}e^{jn\Omega t}.$$

We have for the S_{21n}, the conversion coefficient of order n,

$$S_{21n} = 2\sqrt{\frac{R_1}{R_2}}\; \frac{V_{2n}}{E} = -2\sqrt{R_1R_2}\,I_{2n}/E \tag{2.11}$$

and for S_{11n}, the reflection coefficient of order n,

$$\begin{aligned} S_{110} &= (2V_{10} - E)/E = (E - 2R_1I_{10})/E \\ &= (Z_{10} - R_1)/(Z_{10} + R_1), \end{aligned} \tag{2.12}$$

$$S_{11n} = 2V_{1n}/E = -\,2I_{1n}R_1/E, \qquad n \neq 0 \tag{2.13}$$

where $Z_{10} = V_{10}/I_{10}$ is the effective input impedance at the complex frequency p.

Finally, let P_{max} be the maximum power which can be delivered by the source, P_{1n} the power delivered through port 1 by the component of order n, and P_{2n} the power delivered to the load by the same component, we have

$$P_{max} = |E|^2/4R_1, \qquad P_{in} = \mathrm{Re}\ V^*_{in} I_{in} \qquad i = 1 \text{ or } 2,$$

$$|S_{21n}|^2 = P_{2n}/P_{max},$$

$$|S_{110}|^2 + P_{10}/P_{max} = 1,$$

$$|S_{11n}|^2 + P_{1n}/P_{max} = 0, \qquad n \neq 0.$$

2.4. Reciprocity and quasi-reciprocity.

If a linear constant network is reciprocal, we have $S_{21}=S_{12}$, i.e. the transmission properties between ports 1 and 2 are the same in the direction $1 \to 2$ as in the direction $2 \to 1$. Extension of the concept of reciprocity to linear periodically-varying networks requires that the frequency transposition, which then usually takes place, and the dependence of the system functions on the choice of the time origin be taken into account. We shall therefore say that a periodically-varying network is reciprocal between ports 1 and 2 with respect to t_o if, after shifting the time origin to t_o, the transmission properties are the same in both directions under the following conditions: if for the direction $1 \to 2$ the (complex) frequency injected at port 1 is p_1 and the useful frequency received at port 2 is p_2, the useful frequency received at port 1 for the direction $2 \to 1$ is again p_1 if p_2 is injected at port 2. Taking into account the fact that p_2 is necessarily of the form $p_1 + jn\Omega$ as well as the rule given in par. 2.1. for the influence of a change of the time origin on a system function, the conditions for reciprocity between ports 1 and 2 can be written as follow: There exists a value t_o such that the conversion coefficients satisfy the two (equivalent) relations

$$S_{21n}(p) = S_{12,-n}(p + jn\Omega)e^{-j2n\Omega t_o} \qquad (2.14)$$

$$S_{12n}(p) = S_{21,-n}(p + jn\Omega)e^{-j2n\Omega t_o} \tag{2.15}$$

After shifting the time origin to t_o, the following relations are then satisfied

$$S_{21n}(p) = S_{12,-n}(p + jn\Omega),$$

$$S_{12n}(p) = S_{21,-n}(p + jn\Omega).$$

As Duinker [31] and Belevitch [12] have shown, a circuit is reciprocal with respect to a time t_o if its only time-varying elements are resistances varying symmetrically in time with respect to t_o. As a switch is by definition a limiting case of a varying resistance, circuits with periodically-operated switches will be reciprocal if all switches operate symmetrically in time with respect to a certain time t_o.

From a practical point of view, the reciprocity so defined is too severe and cannot be satisfied by certain networks which, however, are perfectly usable in bidirectional circuits. For this reason, we shall make use of the more general concept of quasi-reciprocity which we define as follows. We shall say that a periodically-varying circuit is quasi-reciprocal if the transmission in the direction 2 $\rightarrow$1 differs from the one in the direction 1$\rightarrow$2 only by a uniform delay Δ_1 at the stage of the frequency p_1 and a uniform delay Δ_2 at the stage of the frequency p_2. This implies that the following two (equivalent) conditions be satisfied

$$S_{21n}(p) = S_{12,-n}(p + jn\Omega)e^{2p\Delta + jn\Omega(\Delta - \Delta')}, \tag{2.16}$$

$$S_{12n}(p) = S_{21,-n}(p + jn\Omega)e^{-2p\Delta - jn\Omega(\Delta + \Delta')} \tag{2.17}$$

where $$2\Delta = \Delta_1 + \Delta_2, \qquad 2\Delta' = \Delta_1 - \Delta_2. \tag{2.18}$$

From the point of view of a signal traversing the circuit, the total delay difference 2Δ is the only one of importance*. Moreover, if we change the time origin to t_o, this difference 2Δ

* According to the choice of the signs of Δ_1 and Δ_2 adopted here, 2Δ expresses in fact the additional delay of direction 2 $\rightarrow$ 1 with respect to the delay of direction 1$\rightarrow$2.

is not altered while Δ' is replaced by $\Delta'-2t_o$.

The concept of quasi-reciprocity can, of course, also be applied to constant circuits. Such a circuit will be called quasi-reciprocal between ports 1 and 2 if we have

$$S_{21}(p) = S_{12}(p)\, e^{2p\,\Delta} \tag{2.19}$$

Note that for a gyrator we have $S_{21} = - S_{12}$ which cannot be satisfied for all frequencies by this expression.

3. BASIC PRINCIPLES OF RESONANT-TRANSFER CIRCUITS.

3.1. Basic resonant transfer circuits.

Consider first the circuit of Fig. 3.1. It contains two equal capacitances C, one transfer inductance 2L and a switch S, which together form the simplest type of resonant-transfer arrangement.

Suppose that the switch S is being closed at t=0 and that it is being opened again at $t = \tau$, where

$$\tau = \pi \sqrt{LC} \tag{3.1}$$

is equal to half the resonant period of the circuit. The charge initially present on the left-hand capacitance will then precisely have been transferred to the right-hand capacitance and vice-versa. With other words, by keeping the switch closed for a period equal to τ , we have obtained a means to produce a controlled interchange of the charges initially present on the two capacitances.

Another circuit which produces the same result is shown in Fig. 3.2. where we assume that the switches S_1 and S_2 are closed simultaneously for a period still given by (3.1). The advantage of this new circuit is that certain parasitic capacitances which are unavoidable in practical situations can be absorbed into the central capacitance 2C/3.

Consider next the simple resonant-transfer circuit shown in Fig. 3.3.It comprises two two-ports N_1 and N_2 which, for simplicity, we may assume to be identical. At port 1, it is fed by a source of voltage E, resistance R_1 and frequency $p = j\omega$, and at port 2 it is terminated by a resistance R_2. Ports 3 and 4 are interconnected by a small inductance 2L and a switch S. At high frequency, N_1 and N_2 reduce at ports 3 and 4 to simple capacitances C. The two capacitances C, the inductance 2L and the switch S together clearly form a resonant-transfer arrangement with τ given by (3.1). We assume that τ is very small compared to all other resonant periods and time constants of the complete circuit (i.e. the circuit including N_1, N_2 and the terminations).

Suppose now that S operates periodically at a rate $F = 1/T$; more precisely, suppose that S is closed during <u>transfer periods</u> defined by

$$t_m - \frac{\tau}{2} < t < t_m + \frac{\tau}{2}\,, \qquad (3.2)$$

$$t_m = m\,T, \qquad m = \ldots\ldots -2,\ -1,\ 0,\ 1,\ 2,\ldots \qquad (3.3)$$

and that it is open otherwise. If we assume furthermore that $\tau \ll T$ we can clearly distinguish between a <u>short-time</u> and a <u>long-time</u> behaviour. The short-time behaviour is determined by the resonant-transfer arrangement alone. For the long-time behaviour, however, the detailed phenomena in this resonant transfer arrangement are without importance. It simply acts as if at regular discrete instants t_m given by (3.3) the two capacitances C would instantaneously interchange their charges.

The current i flowing between terminals 3 and 4 must be of the general form (2.5). On the other hand, it is composed of short pulses so that we can write

$$i = Ie^{pt}, \qquad I = J\Delta(t) \qquad (3.4)$$

where

$$\Delta(t) = T \sum_{m=-\infty}^{\infty} \delta(t - mT), \qquad (3.5)$$

$\delta(t)$ being, in the usual notation, a unit impulse which we may assume to be ideal and J being a constant (in general complex) having the dimensions of a current. Moreover, if N_2 has suitable low-pass properties and if $\omega < \Omega/2 = \pi F$, the output signal appearing at port 2 will again be an almost sinusoidal signal of the same frequency ω as the input signal. This shows that the circuit of Fig. 3.3. can serve to transform a sinusoidal input signal first into a corresponding PAM-signal i(t) and then back into a continuous signal similar to the original one. Furthermore, as N_1 has been assumed to be identical to N_2, the low-pass properties of N_1 in turn will prevent the various frequency components (except ω) contained in (3.4) form reaching the source resistance R_1.

This is essential if high overall transmission efficiency should be obtained since these other frequency components would dissipate part of the available energy in R_1.

An important result of Cattermole's theory can now be stated: lossless transmission in either direction is obtained between the input and output ports 1 and 2 if N_1 and N_2 are ideal open-circuit low-pass filters with cut-off frequency f_c equal to half the sampling rate, i.e. if $f_c = F/2$. This result is quite remarkable since no similar result for conventional modulator circuits is known. As has been mentioned in the introduction, the ideal filters used in the theory of conventional modulators are in fact "overidealized", but this is clearly not the case here.

Cattermole, [22, 23] has shown that the same ideal open-circuit filter as the one mentioned above can also serve as ideal filter for various other types of resonant transfer circuits. One of these is the circuit with <u>intermediate storage</u> shown in Fig. 3.4 where S_1 and S_2 operate both at the same rate but not simultaneously. The resonant-transfer arrangement is formed in this case by the three capacitances C, the two inductances 2L and the two switches S_1 and S_2. The transfer in either direction therefore occurs in two steps, the middle capacitance C serving as temporary storage device.

Despite the importance of Cattermole's results, we shall omit discussing his theory any further, since all results obtained by him also follow from the more general theory te be presented here.

3.2. <u>Need for a more general theory</u>.

The discussion given in par. 3.1 leaves many questions open. Among these, the most important are perhaps the following:

1. How can we analyse the circuit if the <u>two networks</u> N_1 and N_2 <u>are distinct</u> ? In this case, we also want to tolerate that the two capacitances, which have been designated both by C in Fig.3.3 and which we shall henceforth designate respectively by C_1 and C_2, may be distinct. This problem is of practical importance. In a telephone exchange, a local line may e.g. be connected to N_1 and a trunk line to N_2.In this case,N_1 and N_2 may still both be

low-pass filters, but economy considerations require that the filter connected to the local line be less elaborate than the one connected to the trunk line. It may also be, however, that we want to select an AM-modulated signal for the trunk transmission, in which case N_2 will have to be a band-pass filter while N_1 may still be a low-pass filter.

2. In addition to the arrangements considered by Cattermole, various other resonant transfer arrangements are of interest. The question is, therefore, how the behaviour of the most general resonant-transfer arrangement can be described in a simple and general way.

3. What are the general expressions for the conversion and reflection coefficients ?

4. What is the influence of the unavoidable losses in the resonant-transfer arrangement as well as the influence of the timing errors (duration of the transfer period not equal to the ideal)?

5. What are the general conditions which must be satisfied in order to insure equal quality of transmission in both directions ? As true reciprocal behaviour is usually no longer possible for time-varying circuits, we want at least to obtain quasi-reciprocal behaviour, i.e. we want the transmission properties in both directions to differ at most by a constant (frequency independent) delay.

6. What are the general conditions which must be satisfied in order to insure absence of reflection at one or both terminal ports ?

7. What are the general conditions which must be satisfied by N_1 and N_2 to be ideal filters, i.e. filters such that lossless transmission of the overall circuit is obtained ? Such ideal filters are desired not only for audio-to-audio transmission but also if frequency translation is involved.

8. In a TDM-PAM exchange, the networks N_1 and N_2 must work under different operating conditions (in particular together with different types of resonant-transfer arrangements) depending on the way the connection is set up in the exchange. What are the conditions for N_1 and N_2 to be ideal universal filters, i.e. ideal filters providing equal perfomance in these different situations ?

9. In practical low-pass situations, one always wants to have a cut-off frequency lower than half the sampling rate, i.e. $f_c < F/2$. Under these circumstances, is it still possible to conceive ideal realizable networks having the properties of ideal filters, and more specifically of ideal universal filters ? More precisely, can an algorithm be given allowing to design networks N_1 and N_2 whose performance is arbitrarily close to the ideal one?

10. Similar questions arise for band-pass problems.

11. Although an algorithm such as the one mentioned under 9 . allows to design filters with arbitrary good perfomance, it does not furnish optimum filters, i.e. filters whose performance is the best possible under given conditions such as number of elements, structure etc. The question is thus how optimum filters can be designed for given perfomance criteria.

12. What simplifications arise for the filter design problem in case of narrow band transmission ?

13. Although lossless transmission can theoretically be obtained, some losses , which occur mainly in the decoupling transformers contained in N_1 and N_2, are unavoidable. If these losses are too large, they have to be compensated, and one may even wish to realize an overall amplification. This can e.g. be achieved by including bidirectional active elements (negative resistances), in the resonant-transfer arrangements. The question is then how to design the arrangement under these conditions in order to insure proper behaviour.

14. What are the consequences for the stability of the circuit if active elements are provided and if at the same time absence of reflections is required ? In particular, what general stability criterion can be given and how does such a criterion depend on the amount of loss compensation or amplification required ?

15. What are the consequences for the filter design problem resulting from the stability criterion mentioned under 14?

16. What is the crosstalk between various channels in case of time-division multiplexing ?

17. How can the theory be extended to various other domains of application of the resonant-transfer principle, such as resonant-transfer n-ports, PCM and delta modulation circuits, resonant-transfer N-path filters, etc. ?

In the subsequent sections, we shall briefly expose a very general theory by means of which the above questions can be answered, or at least partially be answered. In this theory, we shall always assume that from the point-of-view of the long-time behaviour, the pulse duration may be considered to be infinitely small. It is true that for certain applications it would be useful to dispose of first order expressions which take into account the deviation from this assumption. Such expressions, however, have not yet been computed, although the theories exposed in [29] and [37] could be useful for this problem.

In the last section, finally, we shall show how the general theory, which will be discussed hereafter, leads to a completely new type of application of the resonant-transfer principle, the realization of filters built of capacitances and resonant-transfer switches only. Such a resonant-transfer switch may be formed by a transfer inductance and an ordinary (electronic) switch, (as is also the case in the circuits of Fig. 3.3 and 3.4), or by such an ordinary switch together with additional transistors and capacitors [87] . This way, completely inductorless filters can even be obtained.

4. PULSE IMPEDANCES.

4.1. Definition of the concepts pulse impedance and step resistance.

Consider an impedance $Z(p)$ which reduces at high frequency to a capacitance C defined by

$$C = 1/\lim_{p \to \infty} p\, Z(p) \,. \tag{4.1}$$

Suppose that we apply to it a current of the form

$$i = I\, e^{pt}, \qquad I = I(t) = J\Delta(t-t_o) \tag{4.2}$$

where J is a constant having the dimensions of a current and where $\Delta(t)$ is the pulse-train given by (3.5). The current pulses thus occur at the instants

$$t_m = t_o + mT, \qquad m = \ldots\ldots -2,-1,0,1,2, \ldots\ldots \tag{4.3}$$

where we may e.g. assume without any restrictions that

$$0 \leqslant t_o < T. \tag{4.4}$$

Let $v = V\, e^{pt}$ be the voltage appearing across Z due to the current (4.2). Clearly, $V = V(t)$ is a periodic function of time of period T. At the instants t_m given by (4.3), V jumps suddenly from a value V_b to a value V_a, the subscripts "b" and "a" standing respectively for "before" and "after". V_a and V_b are both proportional to J and we can write

$$U = \tilde{Z}J \quad (4.5), \qquad (V_a - V_b)/2 = R_C J \tag{4.6}$$

where

$$U = (V_a + V_b)/2 \,. \tag{4.7}$$

The proportionality coefficients $\tilde{Z}$ and R_C have the dimensions of impedances and will be called respectively <u>pulse impedance</u> and <u>step resistance</u>. $\tilde{Z}$ is dependent on p and has a real and an imaginary part,

$$\tilde{Z} = \tilde{R} + j\,\tilde{X} \,. \tag{4.8}$$

R_C turns out to be a positive constant given by (see also par. 4.2)

$$R_C = T/2\,C. \tag{4.9}$$

If we interpret U as a voltage and J as a current, (4.5) can be considered to express Ohm's law between U, J and $\tilde{Z}$. Various other reasons for calling $\tilde{Z}$ an impedance will be seen later. We also have

$$V_a = U + R_C J \,, \qquad V_b = U - R_C J, \tag{4.10}$$

$$V_a = \tilde{Z}_a J \,, \qquad V_b = \tilde{Z}_b J \tag{4.11}$$

where
$$\tilde{Z}_a = \tilde{Z} + R_C \,, \qquad \tilde{Z}_b = \tilde{Z} - R_C. \tag{4.12}$$

Suppose now that Z is the input impedance of a network N to which a source of voltage $E\,e^{pt}$ (E a constant) is applied at terminals 1-1' and which is fed at 2-2' by a current (4.2) (Fig.4.1a, the superfluous factor e^{pt} having been dropped). The voltage $v = V\,e^{pt}$ appearing across 2-2' can be computed by applying the superposition principle. With V_a and V_b having the same meaning as before, and U being still defined by (4.7), we obtain

$$U = E_o + \tilde{Z}J \tag{4.13}$$

where E_o is the voltage which would appear across 2-2' if the voltage source were acting alone and where the pulse impedance $\tilde{Z}$ is the same as in (4.5). Expression (4.6) is still valid but a term E_o has to be added to the right-hand members of all four equations (4.10) and (4.11). Expression (4.13) clearly can be represented by the equivalent circuit of Fig. 4.1b.

The significance of this equivalent circuit can considerably be enhanced by calculating the average power transmitted to N via port 2. This can be done either by a frequency domain analysis, using (2.8), or by a time domain analysis calculating the power transmitted by each pulse. One obtains

$$P = \mathrm{Re}\; U^*J = \mathrm{Re}\; U\,J^*, \tag{4.14}$$

which is the expression one would obtain from Fig. 4.1b if U and J are considered to be ordinary steady-state quantities. For E = 0, i.e. for the situation under which (4.5) is valid, we obtain as for ordinary impedances, using (4.8),

$$P = \tilde{R}\,|J|^2 \,. \tag{4.15}$$

It should be clear that any other conclusions which can be drawn from expressions such as (4.5), (4.13), (4.14) and (4.15) in case of constant networks in the sinusoidal steady-state also remain valid. Thus, e.g., the maximum power P_{max} which can be derived from the circuit of Fig. 4.1a by means of a current of the form (4.2) is given by

$$P_{max} = |E_o|^2 / 4\,\tilde{R} \quad . \tag{4.16}$$

4.2. Explicit expressions for the pulse impedance.

By means of methods similar to those used in the theory of the z-transform, one obtains for the pulse impedance the following expressions

$$\tilde{Z} = \sum_{n=-\infty}^{\infty} Z(p + jn\Omega) \tag{4.17}$$

$$\tilde{Z} = R_C + T \sum_{m=1}^{\infty} A(mT)e^{-mpT} \tag{4.18}$$

where Ω is given by (2.4) and where $A(t)$ is the impulse response corresponding to $Z(p)$, i.e.

$$Z(p) = \mathcal{L}\,\{A(t)\} \;, \tag{4.19}$$

the symbol $\mathcal{L}$ standing, as usual, for the Laplace transform operator. (4.17) is easiest derived by means of a frequency domain analysis, noting that $(V_a + V_b)/2$ must be equal to the value of the Fourier development of $V(t)$ at the points of discontinuity $t = t_m$. (4.18) is easiest derived by means of a time domain analysis by summing over the effects of all previous pulses up to $t=-\infty$, The expression (4.9) for R_C can be obtained by either of these analyses.

An integral expression for $\tilde{Z}$ is

$$\tilde{Z} = \frac{T}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{Z(q)}{1 - e^{(q-p)T}}\, dq \tag{4.20}$$

where

$$\sigma_o < \sigma < \operatorname{Re} p \;, \tag{4.21}$$

σ_o being the largest of the real parts of the poles of $Z(p)$.

For $p = j\omega$, (4.20) is strictly valid only if Z is minimum-reactive, but it can easily be extended to the limiting case of non-minimum-reactive impedances by proper modification of the path of integration. (4.17) can be derived from (4.20) by closing the contour of integration by means of a large semi-circle in the right-half plane, while (4.18) can be obtained by series expansion of the integrand of (4.20), making use of (4.19).

As Z(p) is capacitive at high frequency, we can write

$$Z(p) = \sum_{i=1}^{N} \frac{B_i}{p-p_i}, \tag{4.22}$$

with

$$R_C = T/2\,C = \frac{T}{2} \sum_{i=1}^{N} B_i, \tag{4.23}$$

or, if multiple poles are present,

$$Z(p) = \sum_{i=1}^{N} \sum_{k=1}^{N_i} \frac{B_{ik}}{(p-p_i)^k} , \tag{4.24}$$

with

$$R_C = T/2C = \frac{T}{2} \sum_{i=1}^{N_i} B_{i1}, \tag{4.25}$$

where N is the number of distinct poles and N_i the degree of multiplicity of the i^{th} pole. $\tilde{Z}$ can now be computed by means of (4.17), (4.18) or (4.20); in the latter case, the contour of integration has to be closed by means of an infinite half-circle in the left half-plane. The result is best expressed in a new variable

$$\psi = \tanh\,(p\,T/2) \tag{4.26}$$

or, for $p = j\omega$,

$$\psi = j\varphi , \qquad \varphi = \tan\,(\omega T/2). \tag{4-27}$$

We obtain from (4.22),

$$\tilde{Z} = \tilde{Z}(\psi) = \frac{T}{2} \sum_{i=1}^{N} B_i \frac{1-\psi\psi_i}{\psi - \psi_i} \tag{4.28}$$

and from (4.24),

$$\tilde{Z} = \tilde{Z}(\psi) = \psi R_C + (1-\psi^2)\,\frac{T}{2} \sum_{i=1}^{N} \sum_{k=1}^{N_i} \frac{B_{ik}}{(k-1)!} \frac{\partial^{k-1}}{\partial p_i^{\,k-1}} \frac{1}{\psi - \psi_i} \tag{4.29}$$

where

$$\psi_i = \tanh(p_i\,T/2) \tag{4.30}$$

The derivation of (4.28) and (4.29) assumes in fact that Z(p) is minimum-reactive. These expressions remain valid, however, even in case of non-minimum-reactive impedances since impedances of this latter type have a physical meaning only as limiting cases of minimum-reactive impedances.

For $p = j\omega$, we also obtain from (4.8) and (4.17),

$$\tilde{Z}(j\varphi) = \tilde{R}(\varphi) + j\,\tilde{X}(\varphi) \tag{4.31}$$

where

$$\tilde{R}(\varphi) = \sum_{n=-\infty}^{\infty} R(\omega + n\Omega), \qquad \tilde{X}(\varphi) = \sum_{n=-\infty}^{\infty} X(\omega + n\Omega). \tag{4.32}$$

Henceforth, $\tilde{Z}(\psi)$ shall also be called the <u>ψ-transform</u> of Z(p). Although $\tilde{Z}(\psi)$ is unique for a given Z(p), there exist an infinite number of inverse transforms Z(p) for a given $\tilde{Z}(\psi)$. This follows from the fact that there exist an infinite number of different p_i which satisfy (4.30) for a given ψ_i.

It is often advantageous to make use of normalized quantities which we shall define as follows, R_o being a normalizing resistance to be specified in each individual case:

$$z = r + jx = Z/R_o, \qquad r = R/R_o, \qquad x = X/R_o \quad ; \tag{4.33}$$

$$\tilde{z} = \tilde{r} + j\tilde{x} = \tilde{Z}/R_o, \qquad \tilde{r} = \tilde{R}/R_o, \qquad \tilde{x} = \tilde{X}/R_o \quad ; \tag{4.34}$$

$$C_o = T/2R_o, \qquad c = C/C_o = R_o/R_C \quad ; \tag{4.35}$$

$$\lambda = pT/2 \quad , \qquad \lambda_i = p_i T/2 \quad ; \tag{4.36}$$

$$\nu = \omega T/2 = \pi f/F \quad ; \tag{4.37}$$

$$b_i = B_i C_o \ . \tag{4.38}$$

We then have e.g.

$$\psi = \tanh \lambda \quad (a), \qquad \varphi = \tan \nu \qquad (b) \tag{4.39}$$

$$z(\lambda) = \sum_{i=1}^{N} \frac{b_i}{\lambda - \lambda_i} \quad , \tag{4.40}$$

$$1/c = \lim_{\lambda \to \infty} \lambda z(\lambda) = \sum_{i=1}^{N} b_i \ , \tag{4.41}$$

$$\tilde{z}(\phi) = \sum_{i=1}^{N} b_i \frac{1-\phi\phi_i}{\phi-\phi_i} \ . \qquad (4.42)$$

The variable ϕ corresponds to the variable w introduced by Johnson, Lindorff and Nordling [57] and is related to the variable $z= e^{pt}$ used in the z-transform theory by

$$\phi=(z-1)/(z+1) \ .$$

It has the interesting property that

$$\text{Re}\,\phi \gtrless 0 \quad \text{if} \quad \text{Re}\,p \gtrless 0 \qquad (4.43)$$

where equally placed symbols correspond to one another.

From an expression such as (4.17), one might conclude that, except for a missing factor 1/T, $\tilde{Z}$ is identical to the usual z-transform [59] . This is not so since in the usual z-transform theory an expression such as (4.17) is only valid for transfer impedances having at least a double zero at infinity, while here this zero is essentially simple.

The pulse sequence impedances G and G_1 used by Cattermole [22,23] are related to $\tilde{Z}_a$ and $\tilde{Z}_b$ by

$$\tilde{Z}_a = G\,T \ , \qquad \tilde{Z}_b = G_1\,T \ .$$

Kraus [62] has proposed to replace the variables U and J by new variables U' and J' defined by

$$U' = U\,e^{pt_o} \qquad J' = J\,e^{pt_o} \ .$$

In this case, the definition of $\tilde{Z}$ is not modified and the power relations also remain valid. Although there are certain advantages to using these new variables, we have prefered not to do so in order to keep the formulas given here in agreement with those given in [43].

4.3. Properties of the pulse impedance and the step resistance.

From the results of the two previous paragraphs, many useful properties of the pulse impedance and the step resistance can be obtained. Among these, the following should be mentioned:

1. $\tilde{Z}(\phi)$ is a Brune function (rational positive real function) of the variable ϕ if Z(p) is a Brune function of the variable p (follows from (4.28),(4.29)and (4.43)together with(4.15)or(4.32)).

The converse, however, is not always true, i.e. none of the inverse transforms of a Brune function $\tilde{Z}(\psi)$ has to be a Brune function; example: a function $\tilde{Z}(\psi)$ for which $\tilde{R}(\varphi)$ is zero for a certain real value of φ without being identically zero for all φ cannot correspond to any Brune function $Z(p)$ (cf. property 3 mentioned below).

2. $\tilde{Z}(\psi)$ is minimum-reactive in ψ if and only if $Z(p)$ is minimum-reactive in p (follows from (4.30) and (4.43)).
3. If for $\psi = j\varphi$, $\tilde{R}(\varphi)$ is zero for any particular value of φ , it is identically zero for all φ , (follows from (4.32) by noting that $R(\omega)$ cannot be zero in an infinite number of points unless it is identically zero).
4. $\tilde{Z}(\psi)$ is a reactance function of the variable ψ if $Z(p)$ is a reactance function of the variable p, and vice versa. Furthermore, although $Z(p)$ has to be capacitive at high frequency, $\tilde{Z}(\psi)$ may have a pole at $\psi = \infty$. More precisely we have the following correspondences

$$z(\lambda) = 1/\lambda c, \qquad \tilde{z}(\psi) = 1/\psi c \tag{4.44}$$

$$z(\lambda) = \frac{\lambda}{c(\lambda^2+\nu_o^2)}, \qquad \tilde{z}(\psi) = \frac{\psi}{c'(\psi^2+\varphi_o^2)} \tag{4.45}$$

where

$$\varphi_o = |\tan \nu_o|, \qquad \nu_o = |\tan^{-1} \varphi_o| \tag{4.46}$$

$$c' = c/(1 + \varphi_o^2) = c \cos^2 \nu_o \tag{4.47}$$

Note that $\tilde{z}(\psi)$ in (4.45) reduces to $1/\psi c$ for $\nu_o = n\pi$, i.e. for $f_o = nF$ (n an integer), and that it reduces to ψ/c for $\nu_o = (2n + 1)\pi/2$, i.e. for $f_o = (2n + 1)F/2$.

5. $\tilde{Z}(\psi)$ is the input impedance of an ideal open-circuit filter in the ψ-domain if $Z(p)$ is the input impedance of an ideal open-circuit filter in the p-domain, and vice versa. More precisely, limiting ourselves to low-pass filters with cut-off frequency $f_c \leqslant F/2$ and $R = R_o$ in the pass-band, we have for $z(j\nu) = r(\nu)+jx(\nu)$,

$$r(\nu) = \frac{R}{R_o} = \begin{cases} 1 & \text{for } |\nu| < \nu_c = \pi f_c/F \leqslant \pi/2, \\ 0 & \text{for } |\nu| > \nu_c, \end{cases} \tag{4.48}$$

$$x(\nu) = \frac{X}{R_o} = -j\,\frac{1}{\pi}\,\ln\left|\frac{\nu + \nu_c}{\nu - \nu_c}\right| , \tag{4.49}$$

and for the corresponding $\tilde{z}(j\varphi)$

$$\tilde{r}(\varphi) = \frac{\tilde{R}}{R_o} = \begin{cases} 1 & \text{for } |\varphi| < \varphi_c = \tan \nu_c \\ 0 & \text{for } |\varphi| > \varphi_c , \end{cases} \tag{4.50}$$

$$\tilde{x}(\varphi) = \frac{\tilde{X}}{R_o} = -\,j\,\frac{1}{\pi}\,\ln\left|\frac{\varphi+\varphi_c}{\varphi-\varphi_c}\right|. \tag{4.51}$$

Similar relations have been obtained in case of band-pass filters. All these relations can best be proved by first computing $\tilde{r}(\varphi)$ by means of (4.32), which is relatively simple to do. The imaginary part $\tilde{x}(\varphi)$ can then be obtained directly by noting that $\tilde{z}(\psi)$ is minimum-reactive at the same time as $z(p)$ so that the Bayard-Bode relations [15] can be applied.

6. $\tilde{Z}$ and R_C are related by

$$\tilde{Z}(1) = R_C, \qquad \tilde{Z}(-1) = -R_C \tag{4.52}$$

$$\frac{\pi}{2c} \geqslant \int_{0}^{\pi/2} \tilde{r}\, d\nu = \int_{0}^{\infty} \frac{\tilde{r}(\varphi)}{1+\varphi^2}\, d\varphi \tag{4.53}$$

where in this last expression the equality or inequality sign holds depending on whether $z(\lambda)$, and therefore also $\tilde{z}(\psi)$ is minimum-reactive or not.

7. $\tilde{Z}(\psi)$ and R_C are permanent, i.e. independent of the choice of the time origin. This needs some explanation as we have nowhere assumed in section 4 that we are dealing with time-variable circuits. In fact, however, we may assume that the circuit under consideration belongs to a larger circuit containing e.g. periodically-operated switches. In this case, a change of the time origin will modify the parameter t_o in (4.2) as well as the system functions of the total circuit, but will leave unchanged the values of $\tilde{Z}$ and R_C.

5. GENERAL PROPERTIES OF RESONANT-TRANSFER CIRCUITS.

5.1. Mathematical definition of the resonant transfer.

A general resonant-transfer two-port is shown in Fig. 5.1 where the following assumptions are supposed to be fulfilled: S_1 and S_2 are switches* which operate periodically at a rate $F=1/T$. N_1 and N_2 are constant networks (usually filters) while N_o may contain further periodically-operated switches operating at the same rate as S_1 and S_2. Z_3 is the input impedance of N_1 at port 3 when terminated at port 1 by resistance R_1 (i.e. with $E=0$). Similarly, Z_4 is the input impedance of N_2 at port 4 when terminated at port 2 by R_2. C_1 and C_2 are related to Z_3 and Z_4 by

$$1/C_1 = \lim_{p\to\infty} p\, Z_3(p), \qquad 1/C_2 = \lim_{p\to\infty} p\, Z_4(p) \qquad (5.1)$$

The switches S_1 and S_2 are closed during very short transfer periods

$$t_{1m} - \frac{\tau_1}{2} < t < t_{1m} + \frac{\tau_1}{2} \quad \text{and} \quad t_{2m} - \frac{\tau_2}{2} < t < t_{2m} + \frac{\tau_2}{2} \qquad (5.2)$$

of duration τ_1 and τ_2 respectively, where

$$t_{1m} = t_1 + mT, \qquad t_{2m} = t_2 + mT, \qquad (5.3)$$

$$m = \ldots\ldots -2, -1, 0, 1, 2, \ldots\ldots$$

and are open otherwise. t_{1m} and t_{2m} do not have to correspond to the same instants, i.e. we may have $t_1 \neq t_2$; more precisely, we may write

$$t_2 = t_1 + T_1, \qquad T = T_1 + T_2 \qquad (5.4)$$

where we may e.g. assume without any restriction that

$$0 \leqslant T_1 < T, \qquad 0 < T_2 \leqslant T. \qquad (5.5)$$

Similarly, we may have $\tau_1 \neq \tau_2$, although this is usually less essential. The circuit is fed by a source of voltage E and complex frequency p; the currents traversing the ports 3 and 4 are I_3 and I_4 respectively, and the voltages across these ports are V_3 and V_4, the factor e^{pt} being again omitted everywhere.

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* In practical situations, one of the switches S_1 or S_2 may often be omitted without affecting the behaviour of the circuit.

τ_1 and τ_2 are of such short duration that we can again distinguish between short-time and long-time behaviour. The short-time behaviour is determined by the resonant-transfer arrangement alone, i.e. by C_1, C_2, S_1, S_2 and N_o. This resonant-transfer arrangement is such that V_3 and V_4 change appreciably during the corresponding transfer periods. The currents I_3 and I_4 are thus composed of short pulses which carry appreciable charges across S_1 and S_2 respectively. From the point of view of the long-time behaviour, we may thus write

$$I_3 = J_3\,\Delta(t-t_1), \qquad I_4 = J_4\,\Delta(t-t_2) \tag{5.6}$$

where the "currents" J_3 and J_4 are (complex) constants and where Δ (t) is defined by (3.5). Furthermore, we may assume that V_3 changes suddenly at each $t = t_{1m}$ from a value V_{3b} to V_{3a}, and V_4 at each $t = t_{2m}$ from V_{4b} to V_{4a}. We can thus define "voltages" U_3 and U_4 by

$$U_3 = (V_{3a}+V_{3b})/2, \qquad U_4 = (V_{4a}+V_{4b})/2 \tag{5.7}$$

and we can write, according to what we have seen in par. 4.1,

$$U_3 = E_o - \tilde{Z}_3 J_3, \qquad U_4 = \tilde{Z}_4 J_4, \tag{5.8}$$

$$(V_{3b}-V_{3a})/2 = R_{C1}J_3, \qquad (V_{4b}-V_{4a})/2 = R_{C2}J_4 \tag{5.9}$$

where

$$R_{C1} = T/2C_1\ , \qquad R_{C2} = T/2C_2 \tag{5.10}$$

are the step resistances corresponding to C_1 and C_2 respectively, $\tilde{Z}_3$ and $\tilde{Z}_4$ the pulse impedances corresponding to Z_3 and Z_4 respectively and where E_o is the open-circuit voltage measured at port 3 of network N_1 when port 1 is fed by the source E in series with R_1.

Without going into any physical detail, we can now formally define a resonant-transfer arrangement as a means for establishing two independent linear* relations between the quantities V_{3a}, V_{3b}, V_{4a} and V_{4b}. More precisely, we postulate at present that

* As we are dealing with linear circuits, we assume that the resonant-transfer arrangement is linear too.

the arrangement composed of N_o, C_1, C_2, S_1 and S_2(Fig.5.1) imposes the existence of two such relations whose coefficients depend exclusively on this arrangement but not on the rest of the circuit (they may depend, of course, on p and T). Solving these equations with respect to V_{3a} and V_{4a}, we can write

$$V_a = B\, V_b \tag{5.11}$$

where V_a and V_b are the vectors

$$V_a = \begin{Bmatrix} V_{3a} \\ V_{4a} \end{Bmatrix}, \qquad V_b = \begin{Bmatrix} V_{3b} \\ V_{4b} \end{Bmatrix} \tag{5.12}$$

and where B is the <u>voltage transfer matrix</u>

$$B = \begin{pmatrix} B_{33} & B_{34} \\ B_{43} & B_{44} \end{pmatrix} \tag{5.13}$$

whose elements only depend on the resonant-transfer arrangement itself.

Taking into account (5.7) and (5.9), we can now transform (5.11) into two linear relations between U_3, U_4, J_3 and J_4. These two new relations thus define a certain equivalent two-port N_i, called <u>interconnecting two-port</u>, with port voltages U_3 and U_4 and port currents J_3 and J_4. Finally, taking into account (5.8) we obtain for the original circuit of Fig. 5.1 the <u>equivalent circuit</u> shown in Fig. 5.2.

The two-port N_i can, of course, be characterized by the various matrices used in conventional two-port theory. Hereafter, we shall make use of its chain matrix A defined by

$$U_3 = A_{33}U_4 - A_{34}J_4 \tag{5.14}$$

$$J_3 = A_{43}U_4 - A_{44}J_4 \tag{5.15}$$

and related to B by

$$A_{33} = [\,(1+B_{33})(1-B_{44}) + B_{34}B_{43}\,] \,/2\, B_{43} \tag{5.16}$$

$$A_{34} = [\,(1+B_{33})(1+B_{44}) - B_{34}\, B_{43}\,]\, R_{C2}/2\, B_{43}\ , \tag{5.17}$$

$$A_{43} = [\,(1-B_{33})(1-B_{44}) - B_{34}\, B_{43}\,] \,/2\, R_{C1}\, B_{43}\ , \qquad (5.18)$$

$$A_{44} = [\,(1-B_{33})(1+B_{44}) + B_{34}\, B_{43}\,]\, R_{C2}\, /2\, R_{C1}\, B_{43}\ , \qquad (5.19)$$

as well as (incidentally) of its impedance matrix

$$W = \begin{pmatrix} W_{33} & W_{34} \\ W_{43} & W_{44} \end{pmatrix} . \qquad (5.20)$$

The matrix B, however, presents also many advantages. It can be shown that it cannot cease to exist for Re $p \geqslant 0$ if the two-port N_o (Fig. 5.1) is composed of passive elements, including, possibly, periodically-operated switches. The scattering matrix S_i of N_i with respect to terminating resistances R_{C1} at port 3 and R_{C2} at port 4 is related to B by

$$S_i = \begin{pmatrix} B_{33} & B_{34}\sqrt{\gamma} \\ B_{43}/\sqrt{\gamma} & B_{44} \end{pmatrix} \qquad (5.21)$$

where

$$\gamma = R_{C2}/R_{C1} = C_1/C_2 . \qquad (5.22)$$

The interconnecting two-port clearly is reciprocal if

$$\det A = \frac{W_{34}}{W_{43}} = \frac{C_1 B_{34}}{C_2 B_{43}} = 1 \qquad (5.23)$$

and it is quasi-reciprocal if

$$\det A = \frac{W_{34}}{W_{43}} = \frac{C_1 B_{34}}{C_2 B_{43}} = e^{-p\Delta} \qquad (5.24)$$

where Δ is a certain constant.

Note that the various two-port parameters of N_i do not have to be constants but may depend on p.

5.2. Computation of the conversion and reflection coefficients.

With the aid of the equivalent circuit of Fig. 5.2, the conversion and reflection coefficients of the original circuit of Fig. 5.1 can now be computed.

Using the definitions given in section 2, we obtain* e.g. in terms of the chain matrix of N_i,

$$S_{21n}(p) = 2\sqrt{R_1R_2}\, M_1(p)\, M_2(p+jn\Omega)e^{-jn\Omega t_2}/D_A, \tag{5.25}$$

$$S_{110}(p) = \rho_1 - 2R_1N_{A4}\, M_1^2(p)/D_A, \tag{5.26}$$

$$S_{11n}(p) = -2R_1N_{A4}M_1(p)\, M_1(p+jn\Omega)e^{-jn\Omega t_1}/D_A \quad ; \tag{5.27}$$

$$S_{12n}(p) = 2\sqrt{R_1R_2}\, M_1(p+jn\Omega)M_2(p)e^{-jn\Omega t_1}\det \dot{A}/D_A, \tag{5.28}$$

$$S_{220}(p) = \rho_2 - 2R_2N_{A3}\, M_2^2(p)/D_A, \tag{5.29}$$

$$S_{22n}(p) = -2R_2N_{A3}M_2(p)\, M_2(p+jn\Omega)e^{-jn\Omega t_2}/D_A, \tag{5.30}$$

where

$$\rho_1 = \frac{Z_{11}(p)-R_1}{Z_{11}(p)+R_1}, \qquad \rho_2 = \frac{Z_{22}(p)-R_2}{Z_{22}(p)+R_2}, \tag{5.31}$$

$$N_{A3} = A_{43}\tilde{Z}_3 + A_{33}, \qquad N_{A4} = A_{43}\tilde{Z}_4 + A_{44}, \tag{5.32}$$

$$D_A = A_{43}\tilde{Z}_3\tilde{Z}_4 + A_{44}\tilde{Z}_3 + A_{33}\tilde{Z}_4 + A_{34}. \tag{5.33}$$

In these expressions, Z_{11}, Z_{22}, M_1 and M_2 are the following quantities: Z_{11} is the open-circuit input impedance of N_1 seen from port 1, Z_{22} the open-circuit input impedance of N_2 seen from port 2, M_1 the open-circuit voltage ratio of N_1 in the direction $1 \rightarrow 3$ if N_1 is fed at port 1 by a voltage source of resistance R_1, and M_2 is the open-circuit voltage ratio of N_2 in the direction $2 \rightarrow 4$ if N_2 is fed at port 2 by a voltage source of resistance R_2. As we assume N_1 and N_2 to be reciprocal, M_1 is also equal to the current transfer ratio of N_1 in the direction $3 \rightarrow 1$, when port 1 is terminated by R_1, and M_2 is also equal to the current transfer ratio of N_2 in the direction $4 \rightarrow 2$ when port 2 is terminated by R_2.

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* The expressions given in [43] follow from those given here if we chose $t_1 = 0$, $t_2 = T_1$.

5.3. General properties of the interconnecting two-port.

Some of the important properties of the interconnecting two-port N_i are the following:

1. The only parameters of N_1 and N_2 having any influence on the various matrices by means of which N_i can be described are the capacitances C_1 and C_2. Consequently, the properties of N_i depend exclusively on the resonant-transfer arrangement itself.
2. The interconnecting two-port is permanent, i.e. its two-port parameters are independent of the choice of the time origin.
3. The average power P_o delivered to the two-port N_o of Fig. 5.1 is given by

$$P_o = \mathrm{Re}(U_3 J_3^* + U_4 J_4^*). \tag{5.34}$$

This corresponds to the power dissipated in the two-port N_i of the circuit of Fig. 5.2 if this circuit is interpreted as in ordinary steady-state analysis.

4. N_i is passive at the same time as N_o.

5.4. Reciprocity and quasi-reciprocity.

As has been done above, we shall assume hereafter that the networks N_1 and N_2 are reciprocal. Under these conditions, the circuit of Fig. 5.1 will be reciprocal in the sense discussed in par. 2.4. if, and only if the following conditions are satisfied by the interconnecting two-port:

1. N_i itself is reciprocal, i.e. (5.23) is satisfied.
2. The coefficients A_{33}, A_{34}, A_{43} and A_{44} are either all single-valued (e.g. rational) functions of ϕ or they are all the product of $\sqrt{1-\phi^2}$ or $1/\sqrt{1-\phi^2}$ by single-valued functions of ϕ.

In terms of the matrices B or W, condition 2 just mentioned can also be replaced by the following two conditions to be satisfied simultaneously:

3. The coefficients B_{33} and B_{44} (the coefficients W_{33} and W_{44}) are single-valued functions of ψ.
4. The coefficients B_{34} and B_{43} (W_{34} and W_{43}) are either both single-valued functions of ψ or both the product of $\sqrt{1-\psi^2}$ or $1/\sqrt{1-\psi^2}$ by single-valued functions of ψ.

Similarly, the circuit of Fig. 5.1 will be quasi-reciprocal in the sense discussed in par. 2.4 if, and only if the following conditions are satisfied by the interconnecting two-port:

1. N_i itself is quasi-reciprocal, i.e. (5.24) is satisfied.
2. The coefficients A_{33}, A_{34}, A_{43} and A_{44} are either all the product of $e^{-p\Delta}$, where Δ is a certain constant, by single-valued functions of ψ, or they are all the product of $\sqrt{1-\psi^2}\, e^{-p\Delta}$ or $e^{-p\Delta}/\sqrt{1-\psi^2}$ by single-valued functions of ψ.

Here again, in terms of the matrices B and W, condition 2 just mentioned can also be replaced by the following two conditions to be satisfied simultaneously:

3. The coefficients B_{33} and B_{44} (W_{33} and W_{44}) are both single-valued functions of ψ.
4. The quantities $B_{34}\, e^{p\Delta}$ and $B_{43}\, e^{-p\Delta}$ ($W_{34}\, e^{p\Delta}$ and $W_{43}\, e^{-p\Delta}$) are both the product of $\sqrt{1-\psi^2}$ or $1/\sqrt{1-\psi^2}$ by single-valued functions of ψ.

5.5. Absence of reflection at the terminal ports.

In this paragraph, we shall assume throughout that $p=j\omega$. Furthermore, for reasons of simplicity we assume that only one useful frequency has to be considered at each terminal port; this excludes in particular DSB input or output signals. We shall say that no reflection occurs at a port at which the circuit is being fed by a source if no power is returned neither at the frequency of the source itself nor at any of the additional frequencies generated in the circuit.

Assume first that the circuit under consideration is fed at port 1 as shown in Fig. 5.1, with $p=p_1=j\omega_1$.

No reflection will occur if for $p=j\omega_1$ the reflection function $S_{11}(p,t)$ is identically zero for all t, or, what amounts to the same, if all the reflection coefficients $S_{11n}(p)$ (n= ..... -2, -1, 0, 1, 2, ...) are zero for $p=j\omega_1$. Using the results of par. 5.2, and assuming that N_1 is lossless, this leads to the following two conditions to be satisfied for $p=j\omega_1$,

$$M_1(p + jn\Omega)=0 \qquad \text{for } n \neq 0, \tag{5.35}$$

$$\tilde{Z}_3^* = (A_{33}\tilde{Z}_4 + A_{34}) / (A_{43}\tilde{Z}_4 + A_{44}), \tag{5.36}$$

the asterisk expressing, as usual, the complex conjugate of the corresponding quantity, Expression (5.36) can equivalently be expressed in terms of the matrix B by

$$B_{33} + B_{44}\rho_3{}^*\rho_4 = \rho_3^* + \rho_4 \det B \tag{5.37}$$

where $\rho_3=(\tilde{Z}_3 - R_{C1})/(\tilde{Z}_3 + R_{C1})$, $\rho_4=(\tilde{Z}_4-R_{C2})/(\tilde{Z}_4+R_{C2})$. (5.38)

These conditions can be interpreted as expressing that the power available from the source, i.e. $|E|^2/4\,R_1$, is completely transmitted to N_o via port 3.

Assume next that the circuit is fed at port 2 instead of 1, with $p=p_2=j\omega_2$, and that N_2 is lossless. No reflection will occur in this case if we have for $p=j\omega_2$,

$$M_2(p + jn\Omega)=0, \qquad \text{for } n \neq 0, \tag{5.38}$$

and

$$\tilde{Z}_4^* = (A_{44}\tilde{Z}_3 + A_{34})/(A_{43}\tilde{Z}_3 + A_{33}), \tag{5.39}$$

this last condition being equivalent with

$$B_{44} + B_{33}\rho_3\rho_4^* = \rho_4^* + \rho_3 \det B. \tag{5.40}$$

In practice absence of reflections will usually be required simultaneously in both respective pass-bands of the filters N_1 and N_2, i.e. for all useful frequencies ω_1 at port 1 and for all useful frequencies ω_2 at port 2.

In this case, we may always assume that a frequency ω_1 injected at port 1 is received as ω_2 at port 2 and vice versa, i.e.

$$\omega_2 - \omega_1 = m\Omega \tag{5.41}$$

where m is an integer. Assuming that N_1 and N_2 are lossless (at least at the useful frequencies ω_1 and ω_2 respectively), the conditions (5.35) to (5.40) must then be satisfied simultaneously, the first three at $p=j\omega_1$ and the last three at $p=j\omega_2=j\omega_1+jm\Omega$. For circuits which are at least quasi-reciprocal, however, (5.36), (5.37), (5.39) and (5.40) do not change if p is replaced by $p+jn\Omega$. In this case, these four conditions may thus be tested either at $p=j\omega_1$ or at $p=j\omega_2$.

As we shall see later, the simplest ideal filters for resonant-transfer circuits are those for which $\tilde{Z}$ is equal to R_C in the pass-band. If N_1 and N_2 are such filters, we thus have

$$\tilde{Z}_3 = R_{C1}, \qquad \tilde{Z}_4 = R_{C2}, \tag{5.42}$$

i.e. $\rho_3 = \rho_4 = 0$. (5.37) and (5.40) then reduce respectively to

$$B_{33} = 0 \qquad \text{and} \qquad B_{44} = 0\ , \tag{5.43}$$

which are remarkably simple. They express that V_{3a} does not depend on V_{3b}, and V_{4a} not on V_{4b}.

If the conditions (5.43) are fulfilled, they will usually be fulfilled at all values of p, in contrast to (5.35) to (5.40) which will usually be realized at most in the respective pass-bands of the filters N_1 and N_2.

6. DIRECT AND INDIRECT RESONANT-TRANSFER.

6.1. Direct resonant-transfer.

6.1.1. General properties of a direct resonant-transfer arrangement.

Suppose that the two switches S_1 and S_2 operate simultaneously, i.e. that $t_1=t_2$ and $\tau_1=\tau_2=\tau$. Suppose furthermore that the network N_o in Fig. 5.1 is without memory, i.e. that no energy is stored in it at the beginning of each transfer period, and that N_o is constant during each transfer period. If these various conditions are fulfilled, we say that the resonant-transfer is direct, and the network N_o is then called a resonant-transfer switch.

Consider under these conditions the network N_a corresponding to the resonant-transfer arrangement during a transfer period and let Z_{33}, Z_{34}, Z_{43} and Z_{44} be the elements of its impedance matrix (Fig. 6.1). We obtain

$$B_{33}=C_1\mathcal{L}_\tau^{-1}\{Z_{33}\}, \qquad B_{34}=C_2\mathcal{L}_\tau^{-1}\{Z_{34}\}, \tag{6.1}$$

$$B_{43}=C_1\mathcal{L}_\tau^{-1}\{Z_{43}\}, \qquad B_{44}=C_2\mathcal{L}_\tau^{-1}\{Z_{44}\} \tag{6.2}$$

where $\mathcal{L}_\tau^{-1}\{\ \}$ represents the inverse Laplace transform taken at $t=\tau$. In this case, the two-port N_i is thus constant. It is furthermore reciprocal if $Z_{34}=Z_{43}$, as can be seen from (5.23). All the reciprocity conditions given in par. 5.4. are then satisfied, the overall circuit of Fig. 5.1 being reciprocal with respect to $t_o=t_1$ or $t_o=t_1+T/2$. This is in agreement with the general reciprocity condition discussed in par. 2.4. The equivalent circuit of N_i for $Z_{34}=Z_{43}$ is a resistive two-port.

In most practical situations, N_a can be separated into two parts which differ at most by a different choice of the impedance level. Designating in this case by Z_o and Z_c respectively the open-circuit and the short-circuit impedance of the left one of these two parts, the relations (6.1) and (6.2) can be replaced by simpler relations leading to the equivalent circuits shown in Fig. 6.2 where

$$\varepsilon = (1+B_c)/(1-B_c), \qquad \eta = (1+B_o)/(1-B_o), \tag{6.3}$$

$$B_o = C_1 \mathcal{X}^{-1}_{\tau}\{Z_o\}, \qquad B_c = C_1 \mathcal{X}^{-1}_{\tau}\{Z_c\}, \tag{6.4}$$

$$R_{Cs} = R_{C1} + R_{C2}, \qquad R_{Cp} = R_{C1}R_{C2}/(R_{C1}+R_{C2}). \tag{6.5}$$

In practice, one usually has

$$0 \leqslant \varepsilon \ll 1, \qquad 1 \ll \eta \leqslant \infty. \tag{6.6}$$

In the ideal case $\varepsilon = 0$, $\eta = \infty$, N_i thus reduces to a simple through-connection.

6.1.2. Basic direct resonant-transfer arrangements.

Consider first the arrangement of Fig. 6.3 where R_s and R_p take into account the unavoidable losses in the coil. We use the following notation:

$$C = \frac{2C_1C_2}{C_1+C_2}, \qquad \omega'_c = \frac{1}{\sqrt{LC}}, \qquad \tau' = \pi/\omega'_c, \tag{6.7}$$

$$Q_s = \frac{\omega'_c L}{R_s}, \qquad Q_p = \frac{R_p}{\omega'_c L}, \qquad Q = \frac{Q_s Q_p}{Q_s + Q_p}. \tag{6.8}$$

In this case, we always have $\eta = \infty$ while ε can be computed by means of the results of par. 6.1.1. Limiting ourselves here to high quality inductors and small timing errors, i.e. assuming that

$$Q \gg 1 \quad \text{and} \quad |\tau-\tau'|/\tau' \ll 1 \tag{6.9}$$

we obtain

$$\varepsilon = \pi/4Q + \pi^2(\tau-\tau')^2/4\tau'^2. \tag{6.10}$$

Consider next the arrangement of Fig. 6.4 where we assume

$$\frac{L_1}{L_2} = \frac{C_2}{C_1} = \frac{R_{s1}}{R_{s2}} = \frac{R_{p1}}{R_{p2}} \quad \text{and} \quad C_o = \frac{C_1+C_2}{3} \tag{6.11}$$

We still use the notation (6.7) and (6.8) as well as

$$Q' = 2Q_sQ_p/(4Q_s+Q_p) \quad , \quad L = (L_1+L_2)/2 \ . \tag{6.12}$$

Assuming again that (6.9) holds as well as $Q' \gg 1$, we obtain for ε the same expression (5.53) as before while we obtain for η

$$1/\eta = \pi/8Q' + \pi^2(\tau-\tau')^2/4\ \tau'^2. \tag{6.13}$$

6.1.3. Inductorless resonant-transfer arrangement.

If the arrangements of Figs. 6.3 and 6.4 are lossless ($Q_s=Q_p=\infty$) and if the timing error is zero ($\tau=\tau'$), the total charge q_o transmitted in the direction from C_1 to C_2 is given by

$$q_o = \frac{2C_2}{C_1+C_2}\,q_{1b} - \frac{2C_1}{C_1+C_2}\,q_{2b}, \tag{6.14}$$

where q_{1b} is the charge initially present on C_1 and q_{2b} the charge initially present on C_2. This is precisely twice the charge which would be transmitted if we had simply interconnected C_1 and C_2 via an arbitrary resistance (which may simply be the lead resistance) until the voltages across C_1 and C_2 have become equal. In this case the precise duration of the transfer period is unimportant as long as it is long enough compared to the time constant of the transient behaviour. The great disadvantage is, however, that a considerable loss of energy now occurs, the equivalent circuit of N_1 being in this case a simple series resistance equal to $R_{Cs}=R_{C1}+R_{C2}$.

All this suggests to conceive a different type of arrangement in which C_1 and C_2 are first simply interconnected via an ordinary (electronic) switch and in which some electronic means is provided which subsequently doubles the charge transmitted via this switch. A device having these properties is is described in a recent Siemens patent [87] and is also mentioned in [82] . It is shown in Fig. 6.5 where the resonant-transfer switch N_o comprises two auxiliary transistors and two auxiliary capacitances, with

$$C_3=C_1 \ll C_5 , \qquad C_4 = C_2 \ll C_6 . \tag{6.15}$$

The time constant of the discharge across the switch S has to be much smaller than the duration τ of a transfer period while the time constant of C_5 (C_6) with the input resistance of transistor T_1 (T_2) has to be large compared to τ. The advantages of the circuit of Fig. 6.5 are thus that precise timing becomes much less important and that no coils are needed.

6.1.4. Direct resonant-transfer circuit.

For a direct resonant-transfer circuit with a resonant-transfer arrangement as described in par. 6.1.1., the complete equivalent circuit becomes as shown in Fig. 6.6. The conversion and reflection coefficients can be computed either directly from this circuit or from the general expressions derived in par. 5.2. Neglecting higher order terms in ε and $1/\eta$, we obtain e.g.

$$S_{21n}=\frac{2\sqrt{R_1R_2}\, M_1(p)M_2(p+jn\Omega)}{\tilde{Z}_3+\tilde{Z}_4+\varepsilon R_{Cs}+\tilde{Z}_3\tilde{Z}_4/\eta\, R_{Cp}} \tag{6.16}$$

In particular, for $\varepsilon= 1/\eta = 0$ and $p=j\omega$, we obtain, assuming the networks N_1 and N_2 to be lossless,

$$|S_{21n}(j\omega)|^2 = 4R_3(\omega)R_4(\omega+n\Omega)/|\tilde{Z}_3+\tilde{Z}_4|^2 \tag{6.17}$$

where $R_3(\omega)$ is the real part of $Z_3(j\omega)$ and $R_4(\omega)$ the real part of $Z_4(j\omega)$.

6.2. Resonant-transfer with intermediate storage.

The most important circuits with indirect resonant-transfer are those with (capacitive) intermediate storage. A circuit of this type is shown in Fig. 6.7 where the transfer occurs in two steps. S_1 and S_2 are never closed simultaneously, i.e., referring to our earlier notation defined in par. 5.1, we have

$$0 < T_1 < T \ , \qquad 0 < T_2 < T \ , \tag{6.18}$$

$$T_1 > (\tau_1+\tau_2)/2 \ , \qquad T_2 > (\tau_1+\tau_2)/2 \ . \tag{6.19}$$

On the other hand, S_3 opens and closes at the same time as S_1, and S_4 at the same time as S_2. N_o' and N_o'' are resonant-transfer switches, i.e. N_o' together with C_1 and C_o forms a direct resonant-transfer arrangement N_a', and N_o'' together with C_o and C_2 a direct resonant-transfer arrangement N_a''. R_o' may be the leakage resistance of C_o while S_o and R_o may or may not be present. If they are, R_o may, according to a proposal by Adelaar [6] , be a negative resistance connected periodically across C_o when S_3 and S_4 are open in order to produce an amplification of the sample stored on C_o. S_o and R_o may, however, also be used as a clamping device if unilateral transmission is required (par. 9.2).

From the point of view of the long-time behaviour, the currents I_5 and I_6 across the switches S_3 and S_4 respectively can be written

$$I_5=J_5\Delta\,(t-t_1) \ , \qquad I_6 = J_6\Delta(t-t_2) \tag{6.20}$$

where $\Delta(t)$ is again defined by (3.5). For the (instantaneous) voltage $v_o=V_oe^{pt}$ across C_o, we can write

$$v_o(t_{1m}-0) = e^{-\alpha_2}\, v_o(t_{1m}-T_1+0) \tag{6.21}$$

$$v_o(t_{2m}-0) = e^{-\alpha_1}\, v_o(t_{2m}-T_2+0) \tag{6.22}$$

where t_{1m} and t_{2m} are defined by (5.3) and where the (positive or negative) constants α_1 and α_2 characterize the influence of S_o, R_o and R'_o. At each instant t_{1m}, V_o changes suddenly from a value which we call V_{5b} to a value V_{5a}, and at each t_{2m}, it changes suddenly from V_{6b} to V_{6a}. This allows us to define "voltages" U_5 and U_6 by

$$U_5=(V_{5a}+V_{5b})/2\ , \qquad U_6=(V_{6a}+V_{6b})/2,$$

while we also have

$$(V_{5a}-V_{5b})/2 = J_5R_{Co}\ , \quad (V_{6a}-V_{6b})/2 = J_6R_{Co} \tag{6.23}$$

where

$$R_{Co} = T/2C_o\ . \tag{6.24}$$

The interconnecting two-port N_i of the equivalent circuit of Fig. 5.2 can now be considered to be composed of three two-ports N'_i, $N''_i{}'$ and N''_i in cascade (Fig. 6.8). N'_i is simply the interconnecting two-port corresponding to N'_a; it can be characterized e.g. by its chain matrix A' which can be computed as discussed in section 6.1. Similarly, N''_i corresponds to N''_a and can be characterized by its chain matrix A". The two-port $N''_i{}'$ has constant image impedances equal to R_{Co}. Its scattering matrix with respect to terminations R_{Co} is given by

$$\begin{pmatrix} 0 & e^{-pT_2-\alpha_2} \\ e^{-pT_1-\alpha_1} & 0 \end{pmatrix} \tag{6.25}$$

and its chain matrix A"' is given by

$$A''' = e^{-p\Delta-\alpha'}\begin{pmatrix} \cosh(\alpha+pT/2) & R_{Co}\sinh(\alpha+pT/2) \\ \sinh(\alpha+pT/2)/R_{Co} & \cosh(\alpha+pT/2) \end{pmatrix} \tag{6.26}$$

or equivalently by

$$A'''=A''_o\ \cosh\alpha\, e^{-p\Delta-\alpha'}/\sqrt{1-\psi^2}, \tag{6.27}$$

where

$$\Delta = \frac{T_2-T_1}{2}, \qquad \alpha = \frac{\alpha_1+\alpha_2}{2}, \qquad \alpha' = \frac{\alpha_2-\alpha_1}{2}, \tag{6.28}$$

$$A_o''' = \begin{pmatrix} 1 + \phi \,\mathrm{tgh}\,\alpha & R_{Co}(\phi + \mathrm{tgh}\alpha) \\ (\phi + \mathrm{tgh}\alpha)/R_{Co} & 1 + \phi\, \mathrm{tgh}\,\alpha \end{pmatrix} . \tag{6.29}$$

If N_i' and N_i'' are reciprocal, the original circuit will be quasi-reciprocal if $\alpha'=0$, and it will be reciprocal if in addition $\Delta = 0$. A possible equivalent circuit of the original circuit is shown in Fig. 6.9 which comprises two three-port gyrators , two delay lines (T_1 and T_2) and two attenuators ($\alpha_i > 0$, i=1 or 2) or amplifiers ($\alpha_i < 0$), all with characteristic impedance equal to R_{Co}.

All the transmission and reflection properties of the circuit of Fig. 6.7 can now be computed. In the simplest case N_i' and N_i'' are simple through connections while $\alpha_1=\alpha_2=0$. We then have e.g. *

$$S_{21n} = \frac{2\sqrt{R_1R_2}\, R_{Co}M_1(p)M_2(p+jn\Omega)\sqrt{1-\varphi^2}\, e^{p\Delta - jn\Omega t_2}}{\phi(\tilde{Z}_3\tilde{Z}_4+R_{Co}^2) + R_{Co}(\tilde{Z}_3+\tilde{Z}_4)}, \tag{6.30}$$

and for $p=j\omega$, assuming N_1 and N_2 to be lossless,

$$|S_{21n}(j\omega)|^2 = \frac{4(1+\varphi^2)R_{Co}^2\, R_3(\omega)\, R_4(\omega+n\Omega)}{|j\varphi(\tilde{Z}_3\tilde{Z}_4+R_{Co}^2)+R_{Co}(\tilde{Z}_3+\tilde{Z}_4)|^2} \tag{6.31}$$

while similar expressions hold for S_{12n}.

6.3. Ideal filters.

We shall say that the networks N_1 and N_2 are ideal filters for direct resonant-transfer circuits if they are such that in their respective pass-bands (6.17) becomes unity, i.e. equal to its highest possible value. Similarly, we shall say that N_1 and

* Here again, the formulas given in [43] follow from those given here if we choose $t_1=0$, $t_2=T_1$.

N_2 are ideal filters for resonant-transfer circuits with intermediate storage if they are such that in their respective pass-bands (6.31) becomes equal to unity. The general conditions for ideal filters are different in these two situations and shall not be reproduced here. We shall limit ourselves instead to what we call <u>ideal universal filters</u>.

The concept of ideal universal filter arises e.g. in the following way. In an electronic TDM exchange based on the resonant-transfer principle, a same filter will at certain times have to work together with an arrangement with direct transfer and at other times with an arrangement with intermediate storage, depending on the way the connection is established in the exchange. One can thus wonder if it is possible to specify N_1 and N_2 in such a way that (6.17) and (6.31) both become equal to unity. It turns out that in order to obtain this we must have for the value of n and all the values of ω under consideration

$$R_3(\omega) = R_4(n\Omega+\omega) = \tilde{R}_3(\varphi) = \tilde{R}_4(\varphi) = R_{Co}\,, \tag{6.32}$$

$$\tilde{X}_3(\varphi) = \tilde{X}_4(\varphi) = 0 \tag{6.33}$$

where R_3, R_4, $\tilde{R}_3$ and $\tilde{R}_4$ are the real parts of Z_3, Z_4, $\tilde{Z}_3$ and $\tilde{Z}_4$ respectively, while $\tilde{X}_3$ and $\tilde{X}_4$ are the imaginary parts of $\tilde{Z}_3$ and $\tilde{Z}_4$. Consequently, an ideal universal filter can be defined as follows.

Consider the network N of Fig. 6.10 which is terminated at port 1 by a resistance R_1. We assume that the input impedance Z(p) at port 2 reduces at high frequency to a capacitance C, i.e. we assume (4.1) to hold. Let M(p) be the open-circuit voltage ratio in the direction $1 \to 2$ when N is fed at port 1 by a voltage source in series with R_1. We shall say that N is an ideal universal filter if the following conditions are satisfied for all pass-band frequencies ω:

$$\tilde{Z}(j\varphi) = R(\omega) = R_o, \qquad |M(j\omega)|^2 = R_o/R_1 \tag{6.34}$$

where R_o is a constant, the condition for $M(j\omega)$ expressing in fact the losslessness of N at the frequencies ω.

All ideal filters as well as the ideal universal filters discussed so far are in fact filters for single-side-band (including low-frequency to low-frequency) transmission. These concepts can however easily be extended to situations where the signal is transmitted in double-sideband form at one or both terminal ports. This leads to the concept of ideal universal filter for double-sideband transmission for which (6.34) has to be replaced by

$$\tilde{Z}(j\varphi)=2R(\omega)=R_o, \qquad |M(j\omega)|^2 = R_o/R_1. \tag{6.35}$$

So far, we have assumed in (6.34) and (6.35) that R_o is any constant. With the aid of a theorem due to Kintchine and Ostrowski (see e.g. [14], vol. II, pp. 157-158), as well as (4.52) it can be shown, however, that this constant R_o is necessarily equal to the step resistance $R_C=T/2C$ corresponding to Z(p). With other words, the input capacitance of an ideal universal filter is necessarily equal to $C_o=T/2\ R_o$.

6.4. Influence of losses and timing errors on the overall performance.

In order to get an idea of the influence of the losses and timing errors on the overall performance, it is useful to compute S_{21n} under the assumption that (6.32) and (6.33) are fulfilled. Retaining only first order terms in ε and $1/\eta$, we obtain for direct resonant-transfer circuits

$$1/|S_{21n}| = 1 + \varepsilon + 1/\eta , \tag{6.36}$$

and for circuits with intermediate storage

$$1/|S_{21n}| = 1 + \varepsilon' + \varepsilon'' + 1/\eta' + 1/\eta'' \tag{6.37}$$

where ε' and η' refer to N_i', and ε'' and η'' to N_i''. In first approximation the influence of the losses and timing errors is thus to add a frequency independent loss.

7. THE FILTER DESIGN PROBLEM

7.1. Realization of ideal universal filters.

7.1.1 General principles of the reactance compensation procedure.

We shall assume in this and the following paragraphs that we are dealing with low-pass filters. Some particularities of the band-pass problem will be discussed later.

Normalized quantities will be used throughout, the constant resistance R_o in the pass-band being, of course, chosen as normalizing resistance. The conditions (6.34) can then be written

$$\tilde{z}(j\varphi)=1, \qquad r(\nu)=1 \ , \qquad |M(j\omega)|^2=1/r_1 \qquad (7.1)$$

where $r_1=R_1/R_o$ is the normalized terminating resistance.

For the ideal open-circuit low-pass filter studied in par. 4.3, the two last conditions (7.1) are clearly satisfied. If in addition, $f_c=F/2$, i.e. $\varphi_c=\infty$, the first condition (7.1) is also satisfied. This corresponds to the ideal universal filter already studied by Cattermole [22,23] .

Consider next the two-port N shown in Fig. 7.1 where we assume N' to be an ideal open-circuit low-pass filter of cut-off frequency $f_c<F/2$ and $z_n(\lambda)$ to be a reactance which is capacitive at high frequency. The input impedance $z'(\lambda)$ of N' is then given by (4.48) and (4.49), with z replaced by z', and the corresponding pulse impedance is given by (4.50) and (4.51), with $\tilde{z}$ replaced by $\tilde{z}'$. The two-port N of Fig. 7.1 can thus be made arbitrarily close to an ideal universal filter with an arbitrary cut-off frequency $f_c < F/2$ if we can indicate a succession of reactance functions

$$z_1(\lambda) \ , \ z_2(\lambda), \ \ldots\ldots\ , \ z_n(\lambda), \ \ldots\ldots \qquad (7.2)$$

such that in the pass-band of N' their ϕ-transforms

$$\tilde{z}_1(\phi), \ \tilde{z}_2(\phi) \ , \ \ldots\ldots \ , \ \tilde{z}_n(\phi), \ \ldots\ldots \qquad (7.3)$$

tend more and more towards the value $\tilde{z}_o$ defined by

$$\tilde{z}_o(\phi) = \frac{j}{\pi} \ln \frac{j\varphi_c+\phi}{j\varphi_c-\phi} \qquad (7.4)$$

Since every reactance function in the ψ-domain can be transformed back into a reactance function in the λ domain having a zero at $\lambda = \infty$, the problem will be completely solved if we can indicate a succession of reactance functions (7.3) having the required property. Such a succession of reactance functions is indeed given by

$$\tilde{z}_n(\psi) = j\,\frac{2}{\pi}\,\frac{W_{n-1}(j\varphi_c/\psi)}{P_n(j\varphi_c/\psi)} \tag{7.5}$$

where P_n is the Legendre polynomial of degree n and where W_{n-1} is, in the notation of Jahnke and Emde, the auxiliary polynomial of degree n-1 occurring in the theory of the spherical functions of the second kind [56,101]. That (7.5) is indeed a reactance function can be shown by continued fraction expansion of (7.4) [79]. It can also be shown by direct calculation that the input capacitance of N has the value mentioned at the end of par. 6.3.

The procedure of obtaining an ideal universal filter by the method just described will be called reactance compensation. It can equally well be applied to the realization of ideal universal band-pass filters, starting from an ideal open-circuit band-pass filter.

7.1.2. Practical considerations.

In a practical situation, the open-circuit filter N' to be compensated will, of course, be non-ideal. Similarly the compensating reactance $z_n(\lambda)$ should be the simplest one which is still compatible with the given performance criterion. This does not imply, however, that the subscript n should necessarily be as small as possible. For low-pass filters, e.g., the number of inductances as well as the number of capacitances required for the realization of a $z_n(\lambda)$ with odd subscript n is the same as for the next highest even value of n. Consequently, only even values of n are of interest in this case.

In particular, $z_2(\lambda)$ corresponds to a parallel resonant circuit, $z_4(\lambda)$ can be realized by means of two parallel resonant circuits in series, $z_6(\lambda)$ by three such circuits, etc. As $z_n(\lambda)$ only has to produce a correcting effect, a single parallel resonant circuit will be sufficient in most practical situations.

The restriction to even values of n does not apply to band-pass filters. If compensation is required in the neighbourhood of both cut-off frequencies, the simplest possible compensating reactance is formed by a parallel resonant circuit in series with a simple capacitance.

The method of reactance compensation is in some respects similar to the well-known method of m-derivation. As can be concluded from a theory given by Belevitch [11] , filters equivalent to m-derived filters can be obtained by adding a suitable reactance either in series with the open-circuit port of an open-circuit filter or in shunt with the short-circuit port of a short-circuit filter.

7.1.3. Compensation procedure using auxiliary filters.

The compensating impedance used in the circuit of Fig.7.1 does not necessarily have to be a reactance. Consider e.g. the function $\tilde{z}''(\psi)$ which for imaginary values of ψ is given by

$$\tilde{z}''(j\varphi)=\tilde{r}''(\varphi) + j\tilde{x}''(\varphi) \qquad (7.7)$$

where

$$\tilde{r}''(\varphi) = \begin{cases} 0 & \text{for } |\varphi|< \varphi_c \text{ ,} \\ 1 & \text{for } |\varphi|> \varphi_c \text{ ,} \end{cases} \qquad 0 <\varphi_c < \infty \text{ ,} \qquad (7.8)$$

$$\tilde{x}''(\varphi) = j\,\frac{1}{\pi}\,\ln\left|\frac{\varphi+\varphi_c}{\varphi-\varphi_c}\right| \text{ .} \qquad (7.9)$$

Clearly, $\tilde{z}''(\psi)$ is the input impedance of an ideal open-circuit high-pass filter in the ψ-domain. In the λ-domain, however, the corresponding $z''(\lambda)$ is the input impedance of an ideal open-circuit band-pass filter of anyone of the following three types, f_c' and f_c'' being respectively the lower and the upper cut-off frequency and f_c being the smallest positive solution of $\varphi_c = \tan(\pi f_c/F)$,

1. $f'_{-c} = nF + f_c$, $f'_c = (n + 1/2)F$,

2. $f'_{-c} = (n + 1/2)F$, $f'_c = (n + 1)F - f_c$,

3. $f'_{-c} = nF + f_c$, $f'_c = (n + 1)F - f_c$,

where n is a non-negative integer. Furthermore, in the stop-band the real part $r''(\nu)$ of $z''(j\nu)$ is $r''(\nu)=0$ in all three cases, while in the pass-band $r''(\nu) = 1$ for type 1 and 2, and $r''(\nu) = 1/2$ for type 3.

If $z_n(\lambda)$ in Fig. 7.1 is replaced by anyone of the impedances $z''(\lambda)$ thus defined, the resulting two-port N is again an ideal universal low-pass filter. It is no longer purely reactive although it is still lossless in the pass-band of N'. A purely reactive compensating impedance can be derived from $z''(\lambda)$ by simply short-circuiting or open-circuiting the terminating resistance of the band-pass filter. This does not affect its input impedance in the stop-band (i.e. in the pass-band of N' as well as at all other frequencies corresponding to $|\varphi| < \varphi_c$.

The simplest practical realization of the impedances $z''(\lambda)$, clearly, is a parallel resonant circuit shunted by a resistance.

7.2. Effective (insertion) loss design.

7.2.1. Realizability conditions.

Although in principle, the methods described above allow to design arbitrarily good filters, they do not furnish filters which are optimum for a given performance criterion (e.g. smallest number of elements or smallest number of inductors for given attenuation requirements). The design of such optimum filters requires that we first establish a set of necessary and sufficient conditions for the realizability of the mathematical expressions to be considered.

We shall limit ourselves here to the case of a symmetrical lossless ($\varepsilon = 1/\eta = 0$, N_1 and N_2 purely reactive) direct resonant-transfer circuit. We shall assume in addition that the open circuit voltage ratios $M_1=M_2=M$ are normal, i.e. that they have no multiple poles. The conversion coefficient $S_{21n}(\lambda)$ of a direct resonant-transfer circuit having these various properties satisfies the following necessary and sufficient conditions (written in terms of normalized quantities):

1. It is of the form

$$S_{21n}(\lambda) = \frac{m(\lambda)m(\lambda + j2n\pi)}{\sum_{i=1}^{N} b_i \frac{1-\psi\psi_i}{\psi - \psi_i}} \tag{7.10}$$

where N is a positive integer and where the parameters ψ_i are related to certain other parameters λ_i by $\psi_i = \tanh \lambda_i$.

2a. The parameters λ_i are real or complex numbers with non-positive real parts. Moreover, if k is the number of parameters λ_i which are purely imaginary, we have $0 \leqslant k < N$. We shall assume that these purely imaginary parameters are those labelled $i=N-k+1$ to $i=N$.

2b. All complex λ_i occur in conjugate pairs.

2c. All λ_i are distinct.

3. $m(\lambda)$ is a rational function of the form

$$m(\lambda) = f(\lambda)/d(\lambda) \tag{7.11}$$

where the polynomial $d(\lambda)$ is related to the λ_i by

$$d(\lambda) = \prod_{i=1}^{N-k} (\lambda - \lambda_i) \tag{7.12}$$

and where $f(\lambda)$ is an arbitrary real even or odd polynomial in λ of degree smaller than N-k. The coefficient of the term of highest degree in $f(\lambda)$ may without any restriction be chosen equal to unity.

4a. For $1 \leqslant i \leqslant N-k$, the parameters b_i are related to the λ_i and to the polynomial $f(\lambda)$ by

$$\begin{aligned} b_i &= \pm 2\, f^2(\lambda_i)/d(-\lambda_i)d'(\lambda_i) \\ &= \mp f^2(\lambda_i) \Big/ \prod_{\substack{l=1 \\ \neq i}}^{N-k} (\lambda_l^2 - \lambda_i^2) \end{aligned} \tag{7.13}$$

where the upper sign corresponds to $f(\lambda)$ even and the lower sign to $f(\lambda)$ odd, and where $d'(\lambda)$ represents the derivative of $d(\lambda)$ with respect to λ.

4b. For $N-k+1 \leqslant i \leqslant N$, the parameters b_i are arbitrary real positive numbers, the only restriction being that the b_i's corresponding to two conjugate λ_i's have to be identical.

7.2.2. Some practical considerations.

With the aid of the above realizability conditions, the design of an optimum filter can now be reduced to a mathematical optimization problem which, however, is more difficult than in ordinary filter design.

Firstly we note that we have to consider simultaneously the transmission of the useful signal as well as the suppression of all unwanted signals generated in the circuit. Due to this, a certain number (in general at least two) of the functions $S_{21n}(\lambda)$, corresponding to different values of the subscript n, have to be considered simultaneously.

Let us examine next the pass-band attenuation A (i.e. the attenuation at the useful values of ω and n). We want to approximate A in such a way that

$$A_d \leqslant A_{dm}, \qquad A_s \leqslant A_{sm}, \tag{7.14}$$

where

$$A_d = (A_{max} - A_{min})/2, \qquad A_s = (A_{max} + A_{min})/2, \tag{7.15}$$

A_{max} and A_{min} being respectively the maximum and the minimum values taken by A, and A_{dm} and A_{sm} being preassigned quantities.

In contrast to conventional filters, the value of A_{min} cannot become zero since this would require the filters to block perfectly all unwanted frequency components, i.e. to have an infinite number of attenuation poles (the unwanted frequency components would otherwise dissipate part of the available energy in the terminating resistances). The value of A_{sm} can thus certainly not be smaller than a certain value $A_{s\ min}$, otherwise no solution can exist. Furthermore, for $A_s > A_{s\ min}$, the obtainable attenuation of the unwanted signals will be a function of the value of A_s and will be optimum for a certain value A_{so} of A_s, possibly for $A_s = A_{sm}$. On the other hand, it seems most likely that the optimum value of A_d corresponds to $A_d = A_{dm}$.

There may be different situations in which the optimum attenuation requirement is not simply given by (7.14), or in which e.g. also the shape of the phase characteristic is of importance. Whatever the requirements may be, it is certain that we cannot expect that an explicit solution of the optimization problem can ever be found. In practice, iterative methods of optimization will thus always have to be used.

Note also that in case of effective loss design, there is usually no advantage to choose for the parameter k mentioned in par. 7.2.1. any value other than k = 0.

7.3. Filters for narrow-band transmission.

We say that we are dealing with narrow-band transmission if the bandwidth of the filters is small compared to F. In this sense, low-pass filters may thus also be narrow-band filters.

It can be shown that in case of narrow-band direct resonant-transfer circuits, the design of the filters can often be reduced to an ordinary filter design problem.

In particular, if both networks N_1 and N_2 are low-pass filters, one may first design an ordinary filter with symmetrical structure which is subsequently split in two equal halves (see example Fig. 7.2). If a DSB signal is required at port 2, the filter N_2 may subsequently be transformed by a low-pass to band-pass transformation, after which its impedance level should be divided by 2 (Fig. 7.3).

8. AMPLIFICATION IN RESONANT-TRANSFER CIRCUITS.

8.1. Means of realizing amplification in resonant-transfer arrangements.

The simplest practical way of achieving loss compensation or amplification in resonant-transfer circuits for bidirectional transmission is to include active devices in the resonant-transfer arrangement. For TDM systems in particular, this often offers the possibility of using the same active device in common for a large number of channels.

The simplest active arrangement is perhaps the one proposed by Adelaar [6] and already mentioned in par. 6.2. In this case, no reflection occurs since the amplification takes place when the sample is stored in the intermediate capacitance C_o.

Many methods of amplification in direct resonant-transfer arrangements have been proposed [5,44,87,88,90,103,104] . In the arrangement of Fig. 6.5 , amplification can be obtained by choosing $C_3 > C_1$ and $C_4 > C_2$. An interesting method achieves amplification by parametrically varying one or several of the reactive elements contained in an arrangement such as the one of Fig. 3.1. This method has been proposed by Holzwarth, Sabban and Schlichte [88, 103,104] and is further discussed in [33] .

An active direct resonant-transfer arrangement which allows to satisfy a large number of requirements is shown in Fig. 8.1 [44] . In this case, an intermediate shunt capacitance C_o is provided which allows e.g. the parasitic capacitance of the highway in an electronic TDM exchange to be taken into account. Further requirements are that L_1, L_2 and C_o are discharged at the end of the transfer period (in order to avoid crosstalk as well as voltage surges when the switches S_1 and S_2 are being opened) and that the conditions (5.43), i.e. $B_{33}=B_{44}=0$, are fulfilled. The circuit has first been proposed in [43] and has been analyzed in the symmetrical case by Rosenoer [private communication] .

For the case of greatest practical importance, the element values found by him can be put into the following form (the capacitance C being assumed to be given):

$$LC = \frac{\tau^2}{\pi^2(1-\alpha^2)^2}, \qquad \frac{C_o}{C} = \frac{2(1+\alpha^2)^2}{3(1-\alpha^2)}, \tag{8.1}$$

$$RC = -\frac{\tau\alpha(3-\alpha^2)}{1-\alpha^4}, \qquad R'C = \frac{\tau}{\pi\alpha(1-\alpha^2)}, \tag{8.2}$$

$$R_oC = -\frac{3\tau(1-\alpha^2)}{2\pi\alpha(1+\alpha^2)(4+\alpha^2)} \tag{8.3}$$

where τ is the duration of the transfer period, α a gain coefficient defined by

$$B_{43} = B_{34} = e^{\alpha\pi} \tag{8.4}$$

and where $L = L_1 = L_2$, $C = C_1 = C_2$, $R = R_1 = R_2$ and $R' = R'_1 = R'_2$.

We conclude from the above expressions that for moderate gains ($0 < \alpha < 1$), only R and R_o become negative. The circuit then comprises three negative and two positive resistances. The highest possible gain (corresponding to $\alpha = 1$) which can be secured according to (8.1) to (8.4) is equal to π nepers or roughly 27 db .

Broux [17,18] has further analyzed the arrangement of Fig. 8.1 as well as various related ones. He has examined the possibility of realizing not only a gain but also a partial compensation of the unavoidable losses, taking into account the inherent imperfectionsof the negative resistances as well as the various operating positions of the switches S_1, S_2 and S_o (this last one not shown; it may have to be provided across C_o as a clamping device in order to avoid crosstalk). In any case, proper behaviour of the arrangement of Fig. 8.1 for $0 < \alpha < 1$ requires that R_1 and R_2 be open-circuit stable and R_o short-circuit stable.

8.2. Stability of active resonant-transfer circuits.

If a resonant-transfer circuit contains an arrangement with active elements, the overall circuit may easily become unstable. We shall discuss hereafter a stability criterion which has been found under the assumption that the circuit is at least quasi-reciprocal, that the conditions (5.43), i.e. $B_{33}=B_{44}=0$, are fulfilled at all frequencies and that the networks N_1 and N_2 are lossless in their respective pass-bands. We also assume that $\tilde{Z}_3$ and $\tilde{Z}_4$ can be considered to be equal to R_{C1} and R_{C2} in the passbands of N_1 and N_2 respectively, but not outside of these pass-bands. If ω falls into the pass-band of N_1 and $|\omega+n\,\Omega|$ into the pass-band of N_2, we have under the various assumptions just mentioned

$$|S_{21n}(j\omega)|^2 = |S_{12,-n}(j\omega + jn\,\Omega)|^2 = |b| \tag{8.5}$$

where

$$b = B_{34}B_{43} \ . \tag{8.6}$$

Thus, there will be overall amplification if $|b|>1$ and overall attenuation if $|b|<1$.

Consider next the conversion and reflection coefficients derived in par. 5.2. They all contain a denominator term which, for $B_{33}=B_{44}=0$, can be written

$$D=1-b\ \rho_3\rho_4 \tag{8.7}$$

where ρ_3 and ρ_4 are given by (5.38) and b by (8.6). If B_{34} and B_{43} have no poles in the right half plane, as will usually be the case, instability can only occur if D has zeros in the right half plane, i.e. if the plot of $b\,\rho_3\rho_4$ for $p=j\omega$ encircles the point +1. This will never occur if

$$|b| < 1/\rho^2_{max} \tag{8.8}$$

where ρ_{max} is the maximum value of $\sqrt{|\rho_3\rho_4|}$ at real frequencies, i.e. because of the periodicity of ρ_3 and ρ_4, between $\omega=0$ and $\omega=\Omega/2$.

At first glance, the criterion (8.8) may seem too severe, but this is usually not the case. This is due to the fact that in an electronic TDM exchange many different combinations of filters and resonant-transfer arrengements may be established depending on the way the connection is established in the exchange.

The coefficients ρ_3 and ρ_4 behave like reflection coefficients of ordinary impedances with respect to positive resistances. Their moduli can thus not exceed unity. According to (8.5) and (8.8), no gain is possible for $\rho_{max}=1$. Even the losses ocurring in the terminating transformers of practical circuits cannot be compensated in this case since these losses can be considered to occur in the terminations rather than in the circuit itself.

We conclude from all this, that filters obtained by pure reactance compensation (pars. 7.1.1 and 7.1.2) cannot be used in combination with active resonant-transfer arrangements since in this case the input pulse impedance is reactive in the stop-band. ρ_{max} can however be made as small as we like by means of the compensation method described in par. 7.1.3. The original filter N' should then not be unnecessarily steep since otherwise a slight misadjustment of the compensating impedance may substantially increase the value of ρ_{max} and thus lead to instability. Although this has not been mentioned in par. 7.1.3, note that ideal universal filters can also be obtained by means of the same compensation method even if the transition range between pass-band and stop-band is not infinitely steep.

9. SOME SPECIAL APPLICATIONS OF THE RESONANT-TRANSFER PRINCIPLE.

9.1. Resonant-transfer circuits for pulse-code modulation.

A resonant-transfer circuit for PCM application is shown in Fig. 9.1. The load circuit consists in this case of a simple capacitance C_2. In addition to this, a clamping switch (not shown) may be provided across C_2, which discharges this capacitance each time before the switch S closes again. Usually, the resonant-transfer arrangement will be of the simplest possible type as indicated in the figure. In this case, the equivalent circuit is as shown in Fig. 9.2, although a more general interconnecting two-port could equally well be provided between ports 3 and 4. We shall use hereafter the same notation as in the previous paragraphs.

For a circuit with clamping, we have $V_{4b}=0$, whence from (5.7) to (5.9), $\tilde{Z}_4=R_{C2}$. For a circuit without clamping, $\tilde{Z}_4=R_{C2}/\phi$.

The main quantity of interest is now V_{4a}. We shall define a transfer coefficient S_a by

$$S_a=\sqrt{\frac{R_1}{R_{C2}}}\cdot\frac{V_{4a}}{E}. \qquad (9.1)$$

It has the property that

$$|S_a|^2=W_2/W_{max} \qquad (9.2)$$

where

$$W_2=|V_{4a}|^2\,C_2/2\,, \qquad W_{max}=|E|^2T/4R_1, \qquad (9.3)$$

W_2 being the average energy per sample stored on C_2 and W_{max} being the maximum average energy available from the source during the time interval T.

For the circuit with clamping, we obtain for S_a

$$S_a(p)=2\sqrt{R_1R_{C2}}\,M_1(p)/[\tilde{Z}_3+R_{C2}+\varepsilon R_{Cs}]\,, \qquad (9.4)$$

i.e., if N_1 is lossless,

$$|S_a(j\omega)|^2 = 4R_{C2}R_3(\omega)/|\tilde{Z}_3+R_{C2}+\varepsilon R_{Cs}|^2. \tag{9.5}$$

No amplitude distortion will occur if N_1 is an ideal universal filter ($\tilde{Z}_3=R_3=R_{C1}$ in the pass-band), and we then have

$$|S_a(j\omega)| = 2\sqrt{C_1C_2}/(C_1+C_2)(1+\varepsilon) . \tag{9.6}$$

This expression reaches its maximum value $1/(1+\varepsilon)$ for $C_1=C_2$.

For a circuit without clamping, we obtain for S_a

$$S_a(p) = \frac{\sqrt{R_1R_{C2}}(1+\psi)\ M_1(p)}{R_{C2} + \psi(\tilde{Z}_3+\varepsilon R_{Cs})} \tag{9.7}$$

i.e., if N_1 is lossless,

$$|S_a(j\omega)|^2 = (1+\varphi^2)R_{C2}R_3(\omega)/|R_{C2}+j\varphi(\tilde{Z}_3+\varepsilon R_{Cs})|^2 \tag{9.8}$$

No amplitude distortion will occur only if N_1 is an ideal universal filter with

$$C_2/C_1 = R_{C1}/R_{C2}=(1-\varepsilon)/(1+\varepsilon), \tag{9.9}$$

and we then have

$$|S_a(j\omega)|^2 = (1-\varepsilon)/(1+\varepsilon) . \tag{9.10}$$

It is clear that general realizability criteria similar to those mentioned in par. 7.2.1. can also be given for the circuits described here.

9.2. Non-reciprocal resonant-transfer circuits.

The resonant-transfer principle can also be applied to the realization of essentially non-reciprocal circuits. A few examples of such circuits will be discussed. As can be concluded from the general discussion in par. 2.4., all these circuits must contain at least two switches which do not both operate symmetrically in time with respect to a same instant t_o.

A first example is offered by the circuit of Fig. 6.7., discussed in par. 6.2, where we assume that the samples transmitted via C_o in the direction $5 \rightarrow 6$ do not undergo any attenuation (α_1=0) while perfect clamping is applied by means of S_o to the samples transmitted in the opposite direction ($\alpha_2=\infty$). We also assume $C_1=C_2=C$, i.e. $R_{C1}=R_{C2}=R_C$. The two-port N_i''' (Fig. 6.8) is then determined by

$$B_{55}''' = B_{66}''' = B_{56}''' = 0, \qquad B_{65}'' = e^{-pT_1},$$

It can be represented as an isolator of impedance matrix

$$\begin{pmatrix} R_C & 0 \\ 2R_C & R_C \end{pmatrix} \tag{9.11}$$

in cascade with a delay line of delay T_1 and characteristic impedance R_C (Fig. 9.3). The isolator itself can be represented by a gyrator of gyration resistance R_C in series with a two-port consisting of a single shunt resistance R_C.

An even simpler resonant-transfer arrangement for the realization of an isolating device is shown in Fig. 9.4 where we assume that the two capacitances C, the inductance 2L and the switch S form an ideal resonant-transfer arrangement as discussed in par. 3.1 (Fig. 3.1). To this is added a clamping device consisting of an auxiliary switch S_a and a very small resistance R_a. Immediately after each transfer period, S_a closes for a very short but long enough period to produce a practically complete discharge of the left-hand capacitance. We then have $B_{33}=B_{44}=B_{34}=0$ and $B_{43}=1$ so that the corresponding two-port N_i is a simple isolator of impedance matrix (9.11). This is the same two-port as the one obtained from Fig. 9.3 in the limiting case $T_1 \to 0$.

A gyrator resonant-transfer arrangement has been proposed by Edrich [32,33] (Fig. 9.5a, with $C_1=C_2=C$). The switch S is closed during transfer periods of duration

$$\tau = \pi\sqrt{LC} \tag{9.12}$$

which are immediately followed by periods of duration

$$\tau_a = \pi\sqrt{2LC} \tag{9.13}$$

during which the switch S is open and S_a closed. The corresponding two-port N_i is determined by

$$B_{33}=B_{44}=0, \qquad B_{34}=-B_{43}=1 \tag{9.14}$$

Its impedance matrix

$$\begin{pmatrix} 0 & R_C \\ -R_C & 0 \end{pmatrix} \tag{9.15}$$

corresponds to a simple gyrator of gyration resistance $R_C=T/2C$.

In view of the discussion to be given in par. 10.2, it is useful to generalize the Edrich gyrator to the case $C_1 \neq C_2$. The capacitance C must then be replaced in (9.12) by $C=2C_1C_2/(C_1+C_2)$ and in (9.13) by C_2, while (9.14) must be replaced by

$$B_{33}=B_{44}=\rho, \qquad B_{34}=1-\rho, \qquad B_{43}=-1-\rho \tag{9.16}$$

where

$$\rho=(C_1-C_2)/(C_1+C_2). \tag{9.17}$$

This leads again to an impedance matrix of the simple form (9.15), with R_C replaced by $R_{C2}=T/2C_2$.

A resonant-transfer circulator [19] device first proposed by Adelaar [3] is shown in Fig. 9.6. We assume that the three switches operate periodically but not simultaneously in the order S_1, S_2, S_3. Transmission thus always takes place via the intermediate storage capacitance. From Fig. 9.6, we can still derive an equivalent circuit as has been done in par. 5.1, but this equivalent circuit is now a three-port. In particular, N_i is now a threeport circulator with a delay line in cascade with each one of its ports, all characteristic impedances being equal to R_C. According to the order in which the switches are assumed to operate, the direction of circulation is clockwise.

Another resonant-transfer circulator device has been proposed by Edrich [32,33]. The corresponding resonant-transfer arrangement is shown in Fig. 9.7. The switch S_1 is closed during transfer periods of duration $\tau = \pi\sqrt{LC'}$. These are immediately followed by transfer periods of the same duration during which S_2 is closed. The interconnecting three-port N_i is a simple circulator of characteristic impedance R_c and clockwise direction of circulation.

The Edrich circulator is simpler than the one proposed by Adelaar.This latter one,however, offers greater flexibility when used e.g. as a device for realizing 2-wire/ 4-wire transitions in an electronic TDM-exchange. It is also more advantageous if more than three ports have to be provided.

Various other non-reciprocal resonant-transfer circuits, including devices with non-reciprocal parametric amplification, have also been proposed by Edrich [32,33] . He has made no use, however, of the representation of these circuits by means of the equivalent circuit first derived in [43] . That this representation is particularly simple also for non-reciprocal circuits is clear from the few examples which we have just discussed.

In all these examples, we have assumed that the resonant-transfer switches are composed of ordinary switches and inductances. One could, of course, equally well use other types of resonant-transfer switches. In this case, some of the above circuits may have to be modified slightly. A circuit which e.g. produces the same gyrator effect as the one of Fig. 9.5a but which in a certain sense is more general, is shown in Fig. 9.5b, the resonant-transfer switches RS_1 and RS_2 being represented by ordinary switches surrounded by circles. From a circuit such as the one of Fig. 9.5b, one may thus not conclude that all parts of the resonant-transfer switches involved do necessarily have to be duplicated. This is certainly not the case for the circuit of Fig. 9.5a, and is also not the case if resonant-transfer switches of the type discussed in par. 6.1.3. are used.

10. FILTERS COMPOSED OF CAPACITANCES AND RESONANT-TRANSFER SWITCHES.

10.1 Filters composed of capacitances and resonant-transfer switches in cascade.

Consider first the circuit with intermediate capacitive storage shown in Fig. 6.7. We assume that neither S_o nor R_o are present, and we also neglect R_o', i.e. $\alpha_1=\alpha_2=0$. Under these circumstances, the expression (6.26) for the chain matrix of the two-port N_i''' becomes

$$A= e^{-p\Delta}\begin{pmatrix}\cosh (pT/2) & R_{Co} \sinh (pT/2)\\ \sinh (pT/2)/R_{Co} & \cosh (pT/2)\end{pmatrix}. \qquad (10.1)$$

Except for the factor $e^{-p\Delta}$, this is precisely the chain matrix of a lossless transmission line or a lossless element of a mechanical filter, the characteristic impedance being in both cases R_{Co} and the length $l=v_pT/2$ where v_p is the speed of propagation. This has led Poschenrieder [82] to propose the realization of filters composed of resonant-transfer switches and capacitances having properties similar to those of filters built by connecting transmission line elements or mechanical elements in cascade, with the advantage, of course, that T is now determined very precisely by the clock pulses and not by the mechanical length l of the element*.

The circuit proposed by Poschenrieder is shown in Fig. 10.1. In addition to the networks N_1 and N_2 and their terminations, it comprises a certain number of capacitances C_3, C_4 ....., C_n, all separated from one another as well as from N_1 and N_2 by resonant-transfer switches (represented again by the ordinary symbol of a

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* For a summary of the state of the art in the theory of transmission line filters, see e.g. the session devoted to this subject at the recent PIB symposium [80] . There exists also an extensive literature on mechanical filters. A book devoting a chapter to this subject is [50] .

switch surrounded by a circle). Two resonant-transfer switches connected to a same capacitance may not be closed at the same time. The networks N_1 and N_2 may simply consist of the capacitances C_1 and C_2 respectively, or they may be more elaborate RC-networks. From the point of view of the general theory to be exposed hereafter, there is no reason not to accept inductances too. These should however be excluded since one of the purposes of the circuit of Fig. 10.1 is precisely the realization of inductorless filters. If all inductances are to be proscribed, the resonant-transfer switches may e.g. be as described in par. 6.1.3. There may however also be cases in which at least the small (and cheap) inductors required by the circuit of Fig. 3.1 will be acceptable.

If we assume that all resonant-transfer switches are ideal, the equivalent circuit of Fig. 10.1 is as shown in Fig. 10.2 where the network N_{ik} ($k=3,4, \ldots\ldots n$) has a chain matrix as given by (10.1), with R_{Co} replaced by $R_{Ck} = T/2C_k$ and Δ replaced by Δ_k. The factors $e^{-p\Delta_k}$ simply produce a factor $e^{-p\Delta}$, with $\Delta = \Delta_3 + \Delta_4 + \ldots\ldots + \Delta_n$, in the resulting chain matrix of the total interconnecting two-port N_i. At worst, i.e. for $\Delta \neq 0$, this factor simply causes the overall circuit to be quasi-reciprocal. If even this is not acceptable, one can still adjust the timing of the various switches in such a way that $\Delta = 0$. In any case, we may omit this factor from our further considerations.

If we do this, the circuit of Fig. 10.2 is equivalent to a cascade of transmission line elements or mechanical filter elements of characteristic impedances R_{C3}, R_{C4}, ... R_{Cn}, connected between terminal impedances $\tilde{Z}_3$ and $\tilde{Z}_4$. The problem of designing the circuit of Fig. 10.1 thus becomes similar to a classical filter design problem. The networks N_1 and N_2 should be simple RC-filters required to eliminate the additional sidebands which are located further away.

As is also true for simple transmission line and mechanical filters, the circuit of Fig. 10.1 has the disadvantage that attenuation poles at real frequencies cannot be realized. Poschenrieder [82] has indicated a method to overcome this objection, but this method is less flexible than the one to be described in par. 10.2. We shall therefore not discuss it here.

10.2. General LC-networks in the ψ-domain, using inductorless networks in the p-domain.

10.2.1. Generalization of the resonant-transfer principle.

Consider a n-port N built of capacitances only (Fig.10.3). Suppose that we connect to each terminal port k (k=1, 2, ... n) an inductance L_k (corresponding to the situation of Fig.10.3 with all the switches S_k closed). The resulting LC-circuit will have n different natural frequencies ω_1, ω_2, ... , ω_n. We shall choose the L_k in such a way that $\omega_k=(2k-1)\omega_1$. All frequencies ω_k are thus odd harmonics of ω_1.

Suppose next that we add a switch S_k in series with each L_k, as indicated in Fig. 10.3. If we close all the switches S_k at t=0, assuming that some charges were present in N for t< 0, the resulting currents i_k in the inductances will all go through zero again at $t=\tau$, where

$$\tau = \pi/\omega_1 \quad . \tag{10.2}$$

We may thus open all the switches at $t=\tau$, after which all the voltages $v_k=v_{ka}$ will be precisely equal to the negative of their values v_{kb} before the switches had been closed, i.e.

$$v_{ka} + v_{kb} = 0 \; . \tag{10.3}$$

The charges q_k which are transmitted across the switches S_k to produce this effect are precisely double those which would be transmitted if the L_k were replaced by small resistances and the switches were kept closed until the circuit has come to rest. This remark makes clear that the inductances could in fact be replaced by electronic charge doubler devices similar to the one described in par. 6.1.3. The property of such devices to require less precise timing is particularly important for the present application since the higher harmonics involved in the circuit with inductances increase considerably the influence of timing errors.

A circuit such as the one of Fig. 10.3 may still be called a resonant-transfer arrangement. Similarly, each S_k with its associated L_k or electronic charge doubler device may still be called a resonant-transfer switch. As these resonant-transfer switches have to work together as a group, it may be better, however, to call them collectively coupled resonant-transfer switches.

10.2.2. Realization of inductances in the ψ-domain by means of capacitances in the p-domain.

We have seen in par. 9.2. that a gyrator can easily be realized in the equivalent circuit. It must thus be possible without difficulty to realize the dual of a given pulse impedance.

Assume e.g. that the gyrator device of Fig. 9.5 is used as resonant-transfer arrangement in the circuit of Fig. 5.1. The driving-point impedance seen in Fig. 5.2 from port 3 to the right is then equal to $R_{C2}^2/\tilde{Z}_4$, i.e. the dual of $\tilde{Z}_4$. In particular, if the network N_2 in Fig. 5.1 reduces simply to the capacitance C_2 and if $R_2 = \infty$, i.e. if $\tilde{Z}_4 = R_{C2}/\psi$, this expression becomes equal to ψR_{C2}, or in normalized quantities ψ/c_2 where $c_2=C_2/C_o$, with $C_o=T/2R_o$, R_o being the normalizing resistance. The new pulse impedance thus corresponds to a normalized inductance $l_2=1/c_2$ whereas the original pulse impedance corresponds to a normalized capacitance c_2. If the purpose of the gyrator device of Fig.9.5 is simply to realize an inductance in the ψ-domain, as has just been described, there is no need, of course, to have the switch S_a operate right after the switch S: The inversion of the charge on C_2 may now take place any time between two consecutive transfer periods.

It is interesting to derive the same results from the circuit of Fig. 6.7 where we omit again S_o, R_o and R'_o. If we assume that the two-port $N''_i{}'$ of Fig. 6.8 is short-circuited at port 6 ($U_6=0$), we obtain from the equations of $N''_i{}'$, independently of T_1,

$$U_5/J_5 = \phi R_{Co} .$$

$U_6=0$, however, corresponds precisely to $V_{6a} = -V_{6b}$, i.e. to a device which inverts the charge on C_2 at the instants $t=t_{2m}$.

10.2.3. Realization of equivalent LC-networks.

As an example, consider the circuit of Fig. 10.4. Its resonant-transfer arrangement contains main resonant-transfer switches (represented by the symbol of an ordinary switch surrounded by a double circle) and auxiliary resonant-transfer switches (represented by the symbol of an ordinary switch surrounded by a single circle). All switches operate periodically at a rate F=1/T. The main switches are closed during main transfer periods and the auxiliary switches during auxiliary transfer periods interlaced with the main ones. All capacitances of the resonant-transfer arrangement as well as the terminating resistances have been indicated in Fig. 10.4 by their normalized values. The notation used hereafter is the same as in the previous sections.

Let us assume first that the auxiliary switches are not present. We can number the branches by attributing to them the values of the indices of the corresponding capacitances c_m. For m=1 or 2, branch m is formed by the input of network N_m (together with the termination) seen from 3-3' or 4-4' respectively. From the point of view of the long-time behaviour, the current in branch m can be written, omitting as usual the factor e^{pt},

$$I_m = J_m \Delta(t-t_o) \qquad (10.4)$$

where t_o is independent of m. Furthermore, we can define voltages V_{ma}, V_{mb} and

$$U_m = (V_{ma}+V_{mb})/2 \tag{10.5}$$

corresponding to the voltage V_m across c_m.

For each node, an equation such as

$$\Sigma J_m = 0 \tag{10.6}$$

can now be written. For a loop such as the one formed by c_5, c_6 and c_7 we have similarly $\Sigma V_{ma} = \Sigma V_{mb} = 0$, i.e.

$$\Sigma U_m = 0 . \tag{10.7}$$

This last equation also holds for a loop containing one or several resonant-transfer switches. Let us indeed designate by V'_k the voltage across resonant-transfer switch k, with corresponding voltages V'_{ka} and V'_{kb}. For a loop containing resonant-transfer switches, equations of the form

$$\Sigma V_{ma} + \Sigma V'_{ka} = \Sigma V_{mb} + \Sigma V'_{kb} = 0$$

can then certainly be written. According to (10.3) (with accents added to conform with our present notation) we have, however, $V'_{ka} + V'_{kb} = 0$, whence (10.7) follows immediately.

Additional relations between the J_m and U_m are determined by the elements present in Fig. 10.4. Thus, (5.8) can still be written for the branches 1 and 2 respectively, while the (normalized) pulse impedances of the branches containing a simple capacitance are given by $1/\psi c_m$.

It is easy now to reestablish the presence of the auxiliary resonant-transfer switches. According to par. 10.2.2, the normalized pulse impedances $1/\psi c_m$ of the branches in question will then simply have to be replaced by ψl_m, with $l_m = 1/c_m$. Hence, the equivalent circuit of Fig.10.4 becomes as shown in Fig. 10.5.

It should be clear now that arbitrary LC-networks can be realized in the ψ-domain by means of the method exposed here.

The number of auxiliary switches required is equal to the number of inductances to be realized. The number of main switches is in principle equal to the number of inductances plus two, although it may be smaller. One must simply make sure that for every loop of the equivalent circuit containing either $\tilde{Z}_3$, $\tilde{Z}_4$ or an inductance, the corresponding loop of the original circuit contains a main switch. This way one insures that the charges of all those capacitances of the resonant-transfer arrangement which either belong to a terminating network or are connected to an auxiliary switch can vary freely between two consecutive main transfer periods without affecting the charges on the other capacitances. In the example of Fig. 10.4, none of the main switches indicated can be eliminated without violating this rule.

The fact that all the main switches in a circuit such as the one of Fig. 10.4 have to operate simultaneously may cause difficulties if more complicated structures are involved. In this case, one can use resonant-transfer arrangements composed of several parts in cascade, seperated by intermediate storage capacitances. The filters obtained this way are thus in a certain sense a combination of those described in pars. 10.1 and 10.2 respectively.

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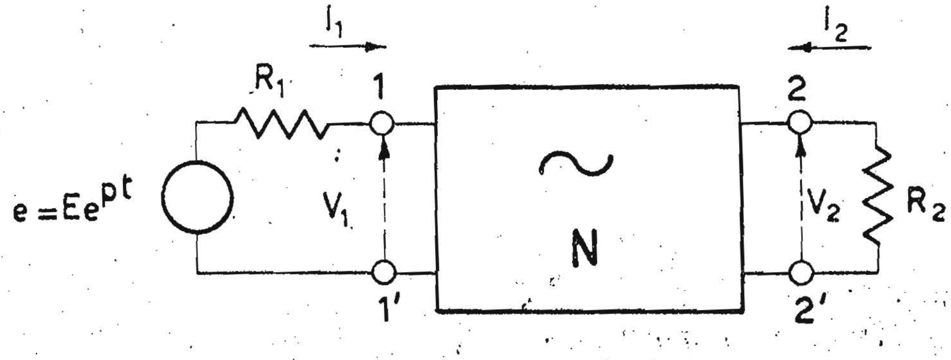


Fig. 2.1

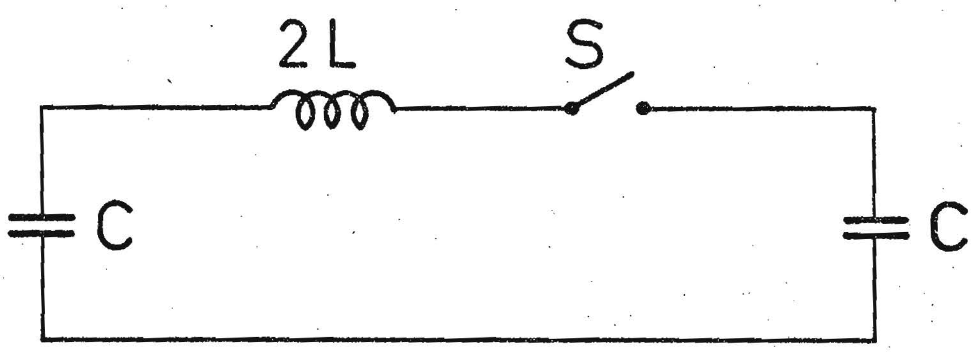


Fig. 3.1

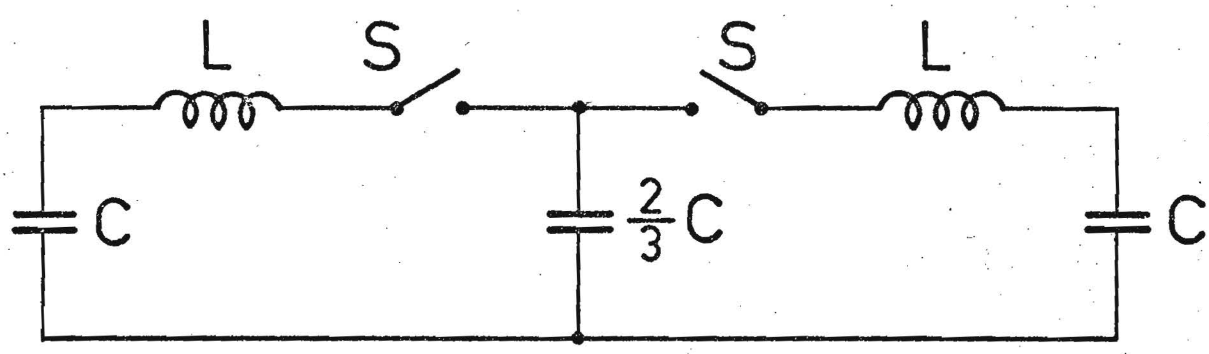


Fig. 3.2

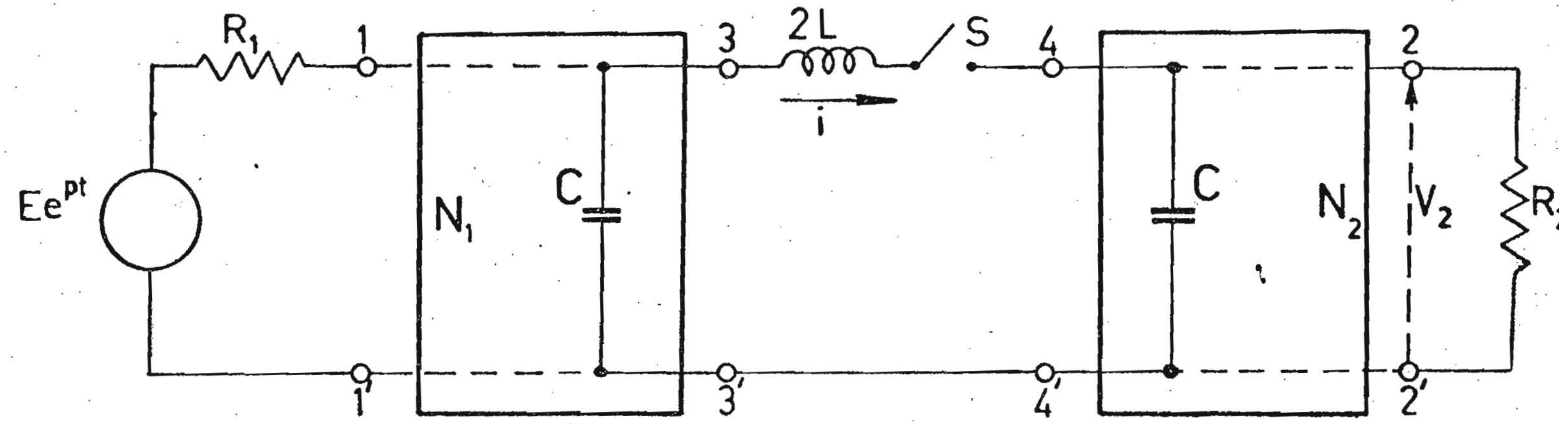


Fig.3.3

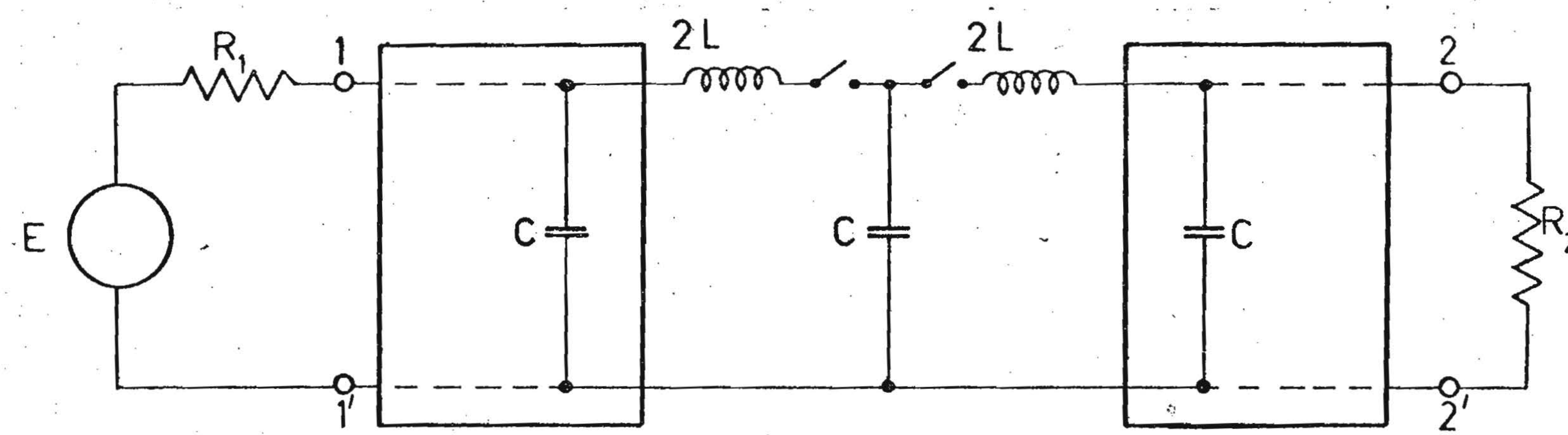


Fig.3.4

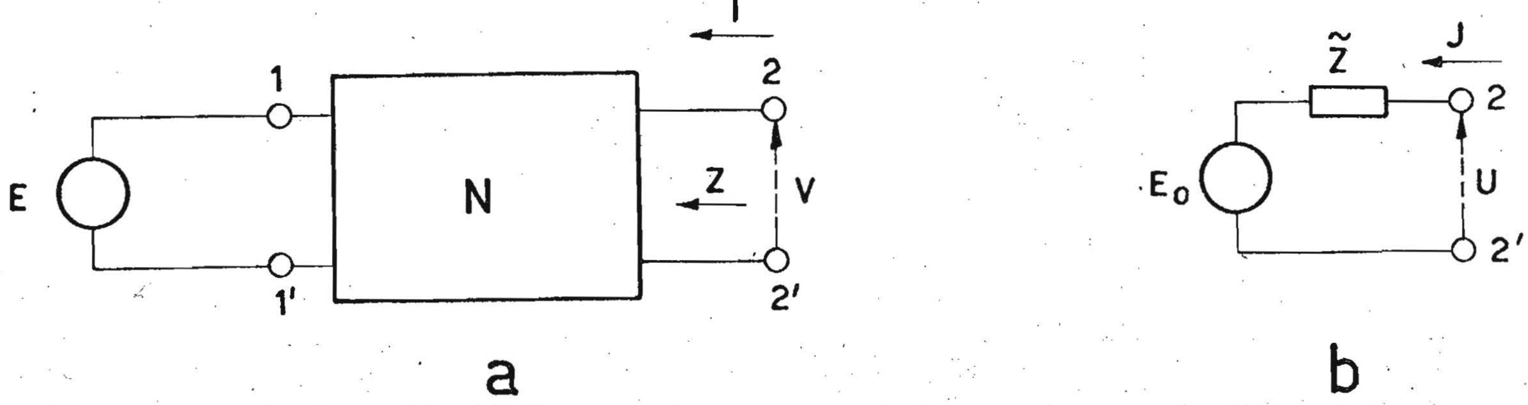


Fig.4.1

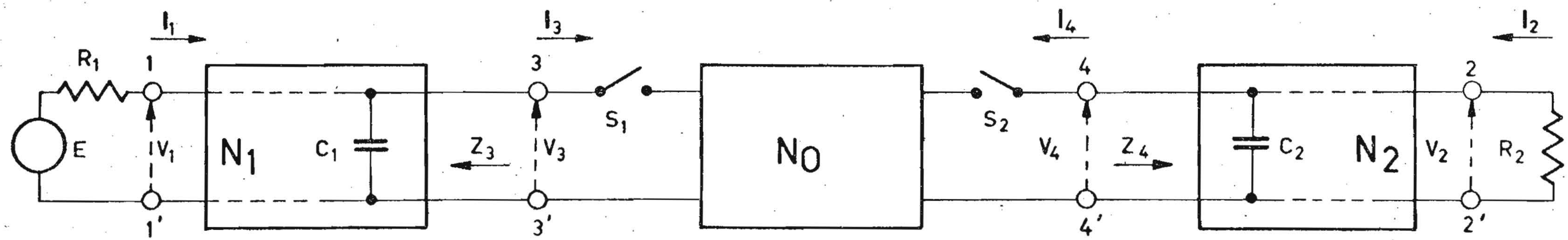


Fig.5.1

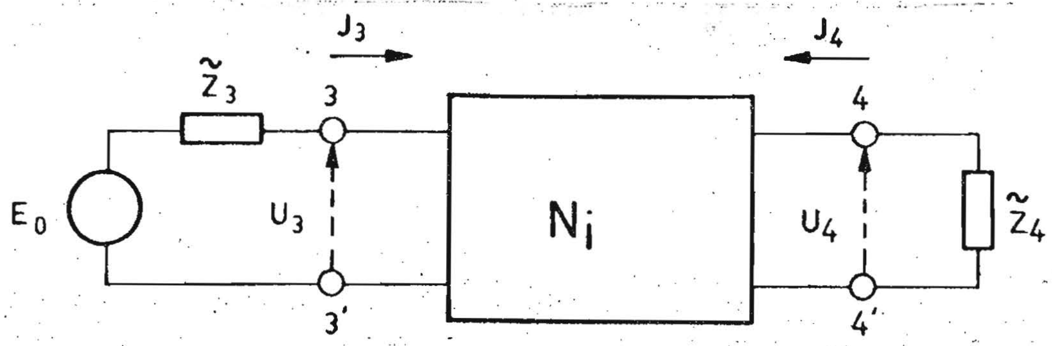


Fig.5.2

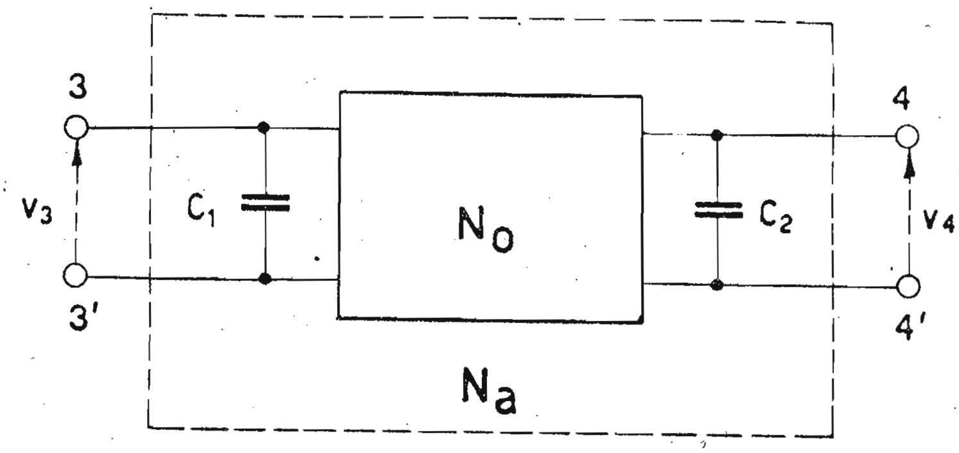


Fig.6.1

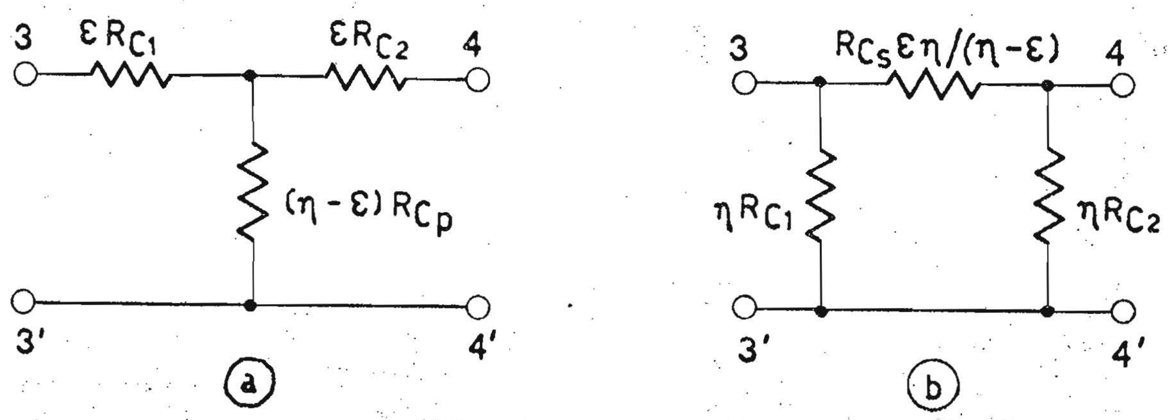


Fig.6.2

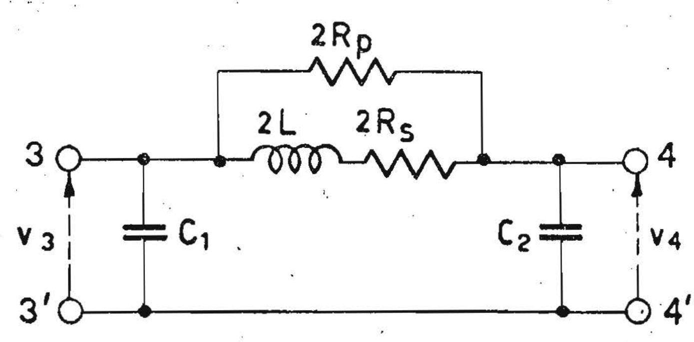


Fig.6.3

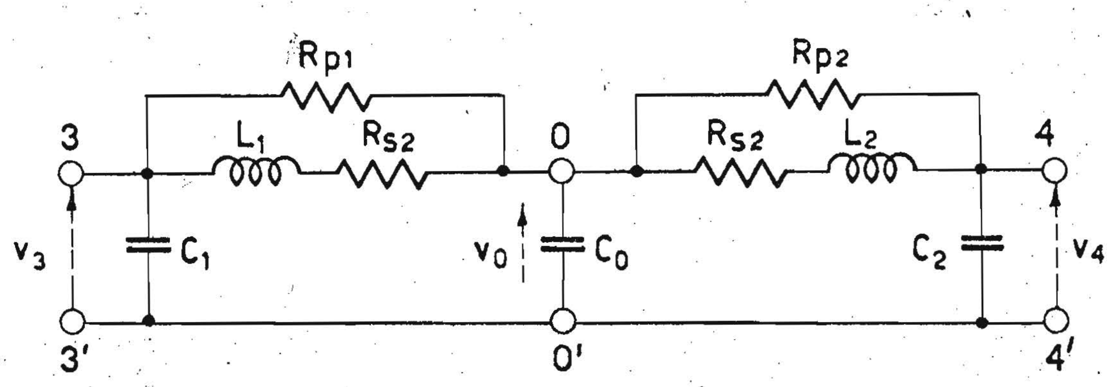
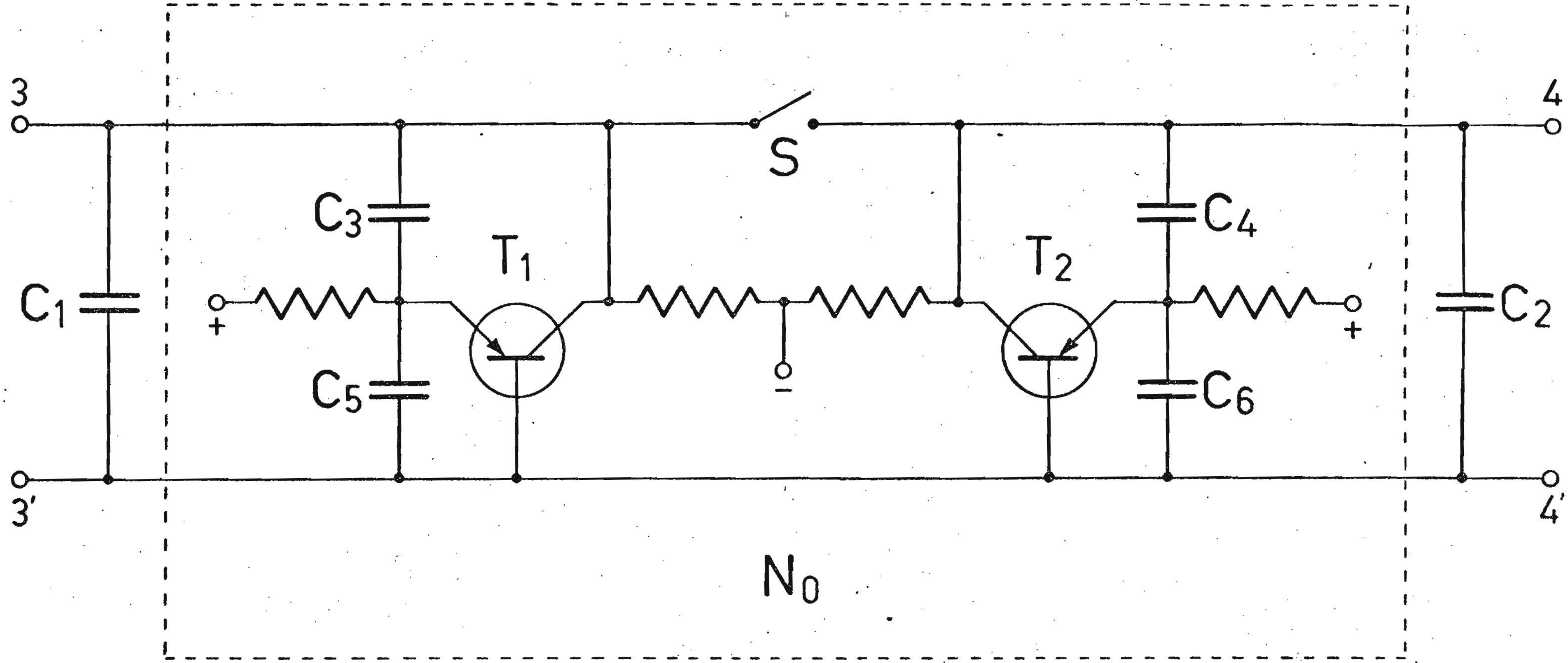
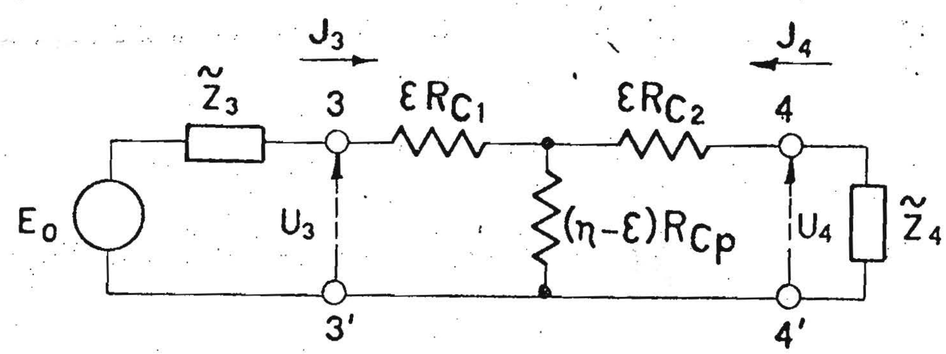


Fig.6.4



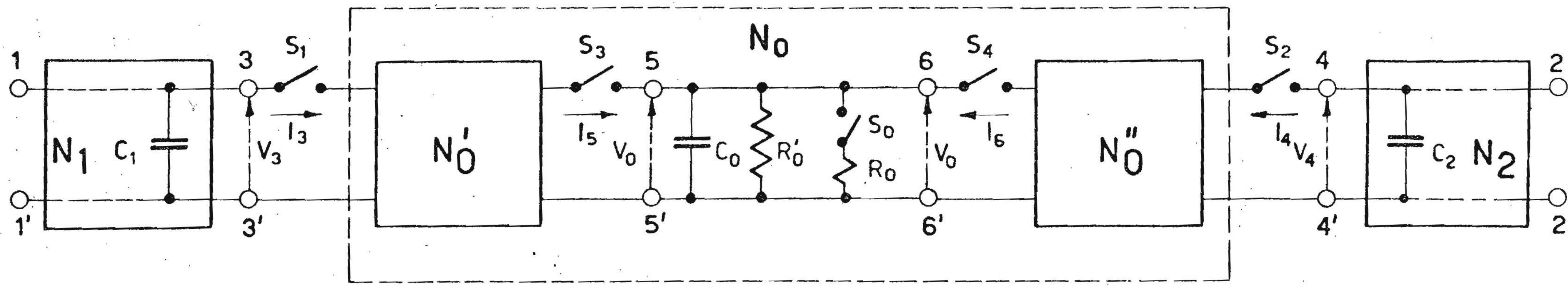
3
4
S
C3
C4
T1
T2
C1
C2
+
+
C5
C6
3'
4'
N0

Fig.6.5



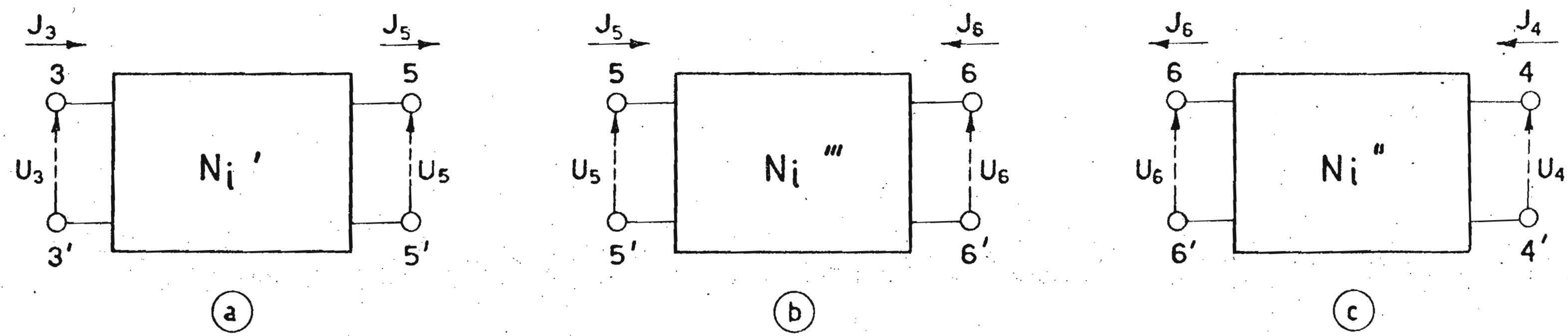
J3
J4
Z3
3
εRC1
εRC2
4
E0
U3
(η-ε)RCp
U4
Z4
3'
4'

Fig.6.6



1
1'
N_1
C_1
3
3'
V_3
S_1
I_3
N_0
N_0'
S_3
I_5
5
5'
V_0
C_0
R_0'
S_0
R_0
6
6'
V_0
S_4
I_6
N_0''
S_2
I_4
V_4
4
4'
C_2
N_2
2
2

Fig.6.7



J_3
J_5
3
5
U_3
N_i'
U_5
3'
5'
a
J_5
J_6
5
6
U_5
N_i'''
U_6
5'
6'
b
J_6
J_4
6
4
U_6
N_i''
U_4
6'
4'
c

Fig.6.8

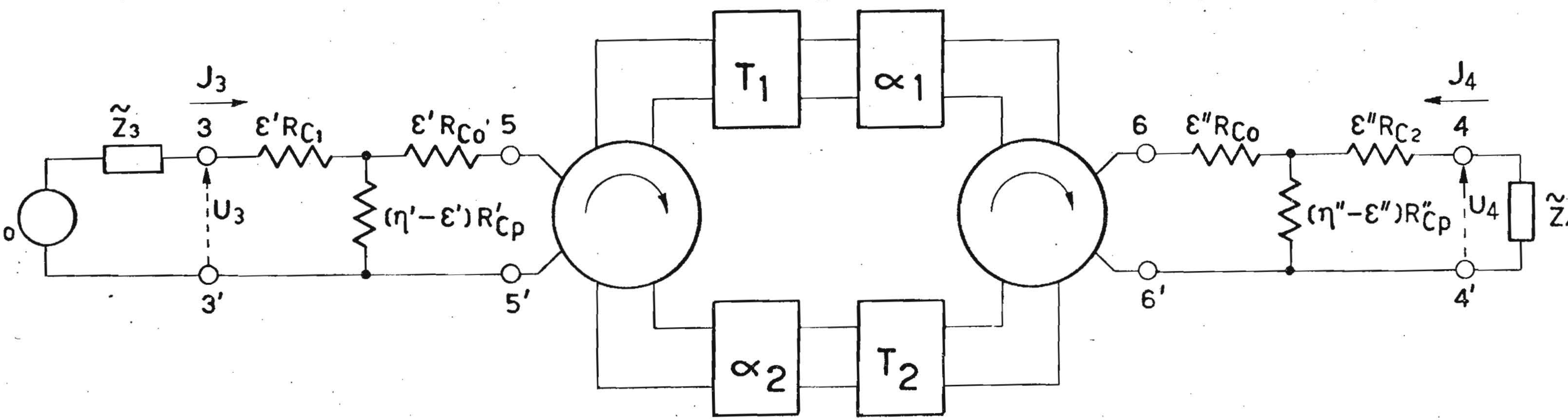


Fig.6.9

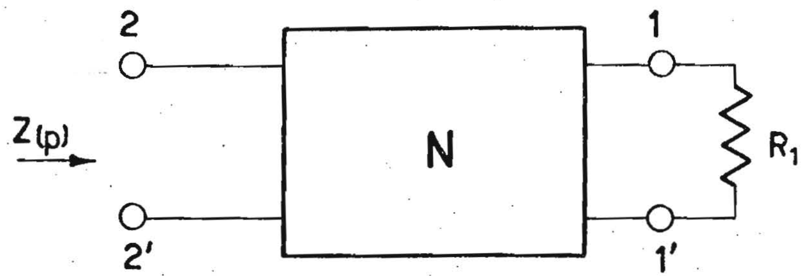


Fig.6.10

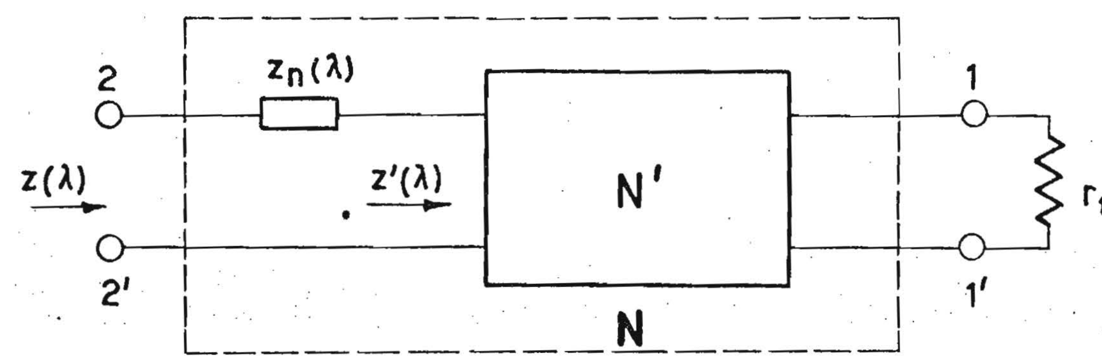


Fig.7.1

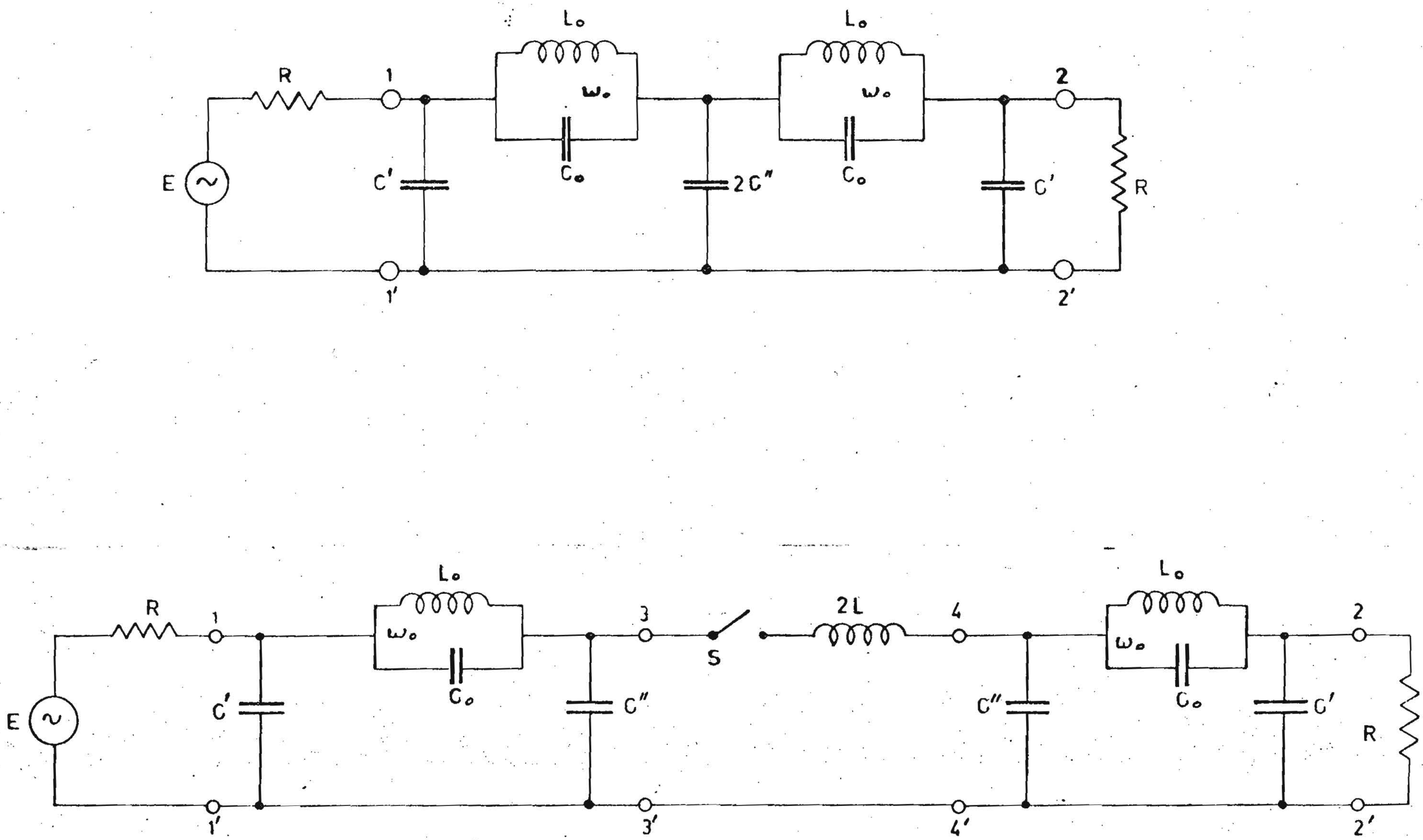
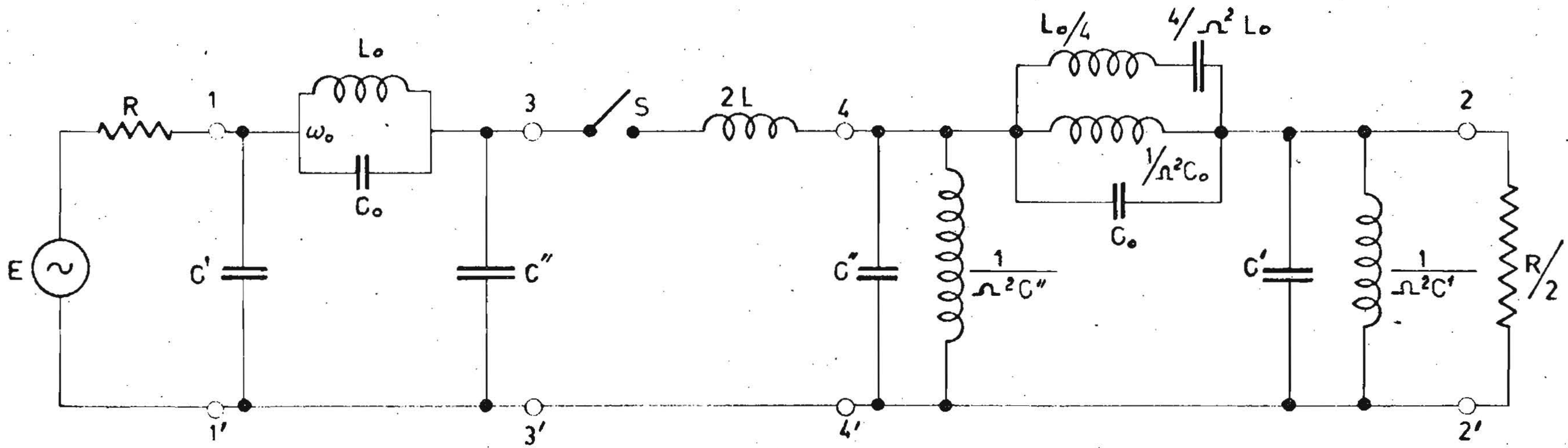
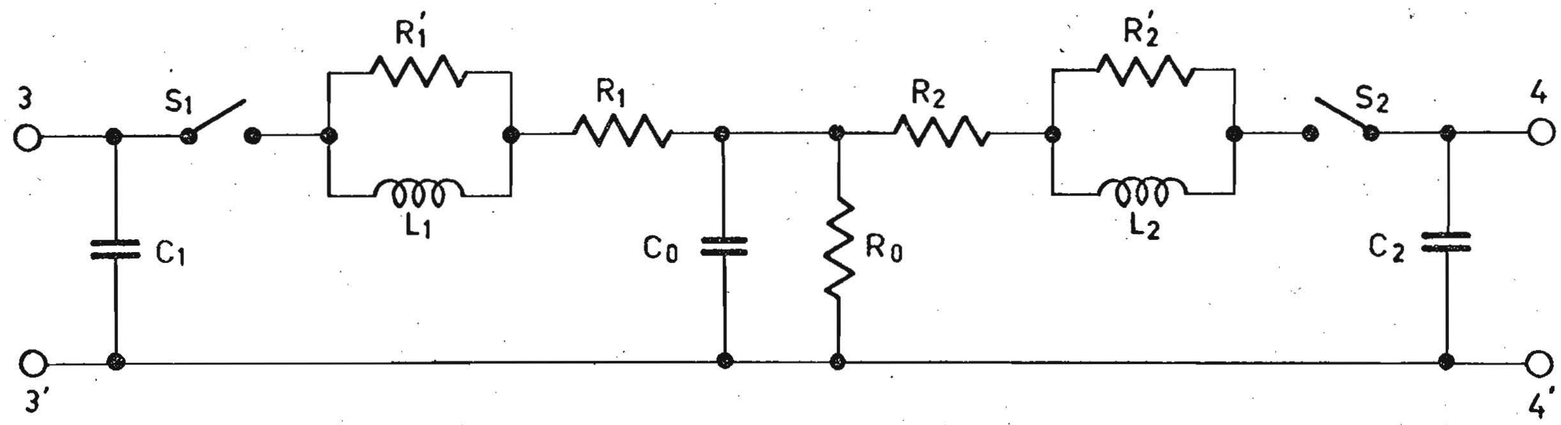


Fig. 7.2



R
1
1'
E
C'
L_o
ω_o
C_o
3
3'
C"
S
2L
4
4'
C"
$L_o/4$
$4/\Omega^2 L_o$
$1/\Omega^2 C_o$
C_o
$\frac{1}{\Omega^2 C''}$
C'
$\frac{1}{\Omega^2 C'}$
R/2
2
2'

Fig. 7.3



3
3'
S_1
C_1
R_1'
L_1
R_1
C_0
R_0
R_2
R_2'
L_2
S_2
C_2
4
4'

Fig. 8.1

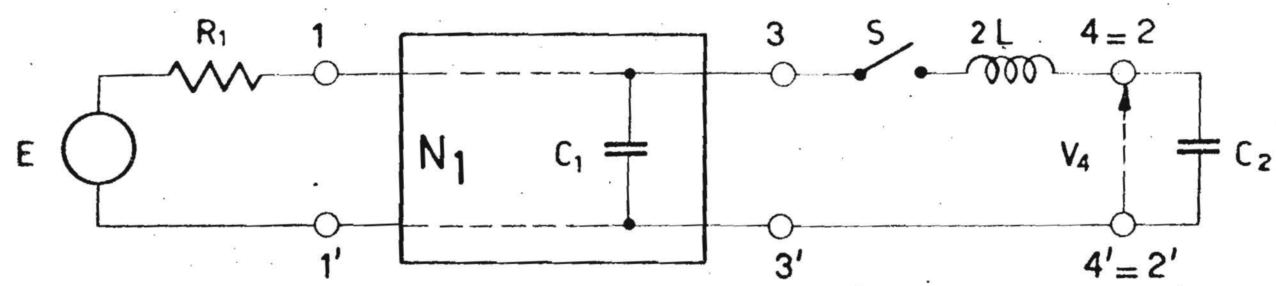


Fig.9.1

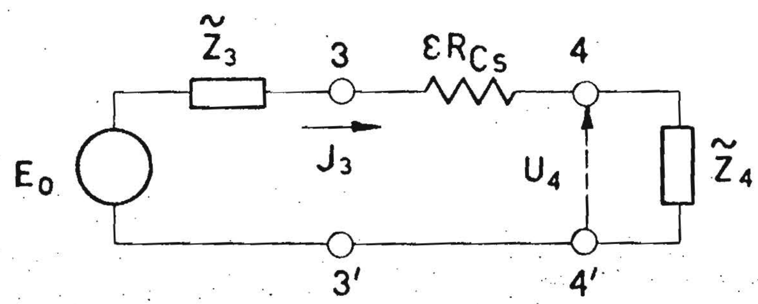


Fig.9.2

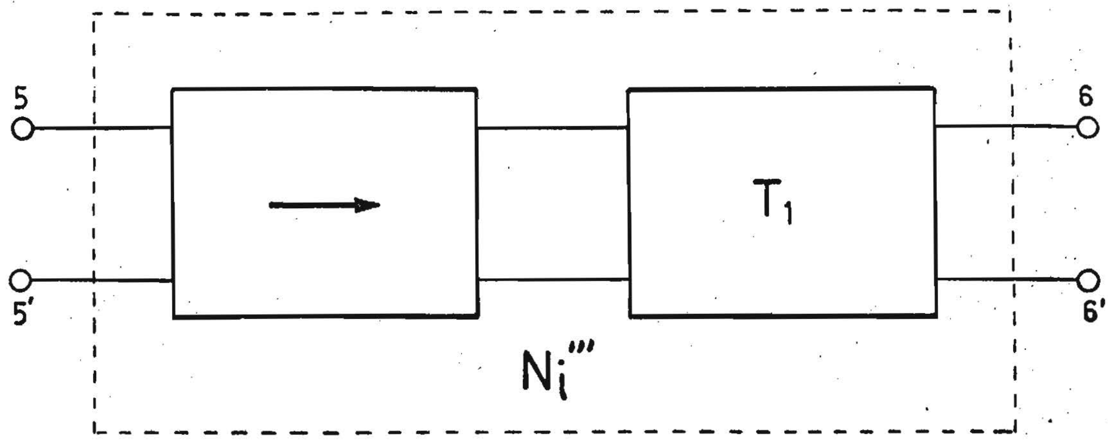


Fig. 9.3

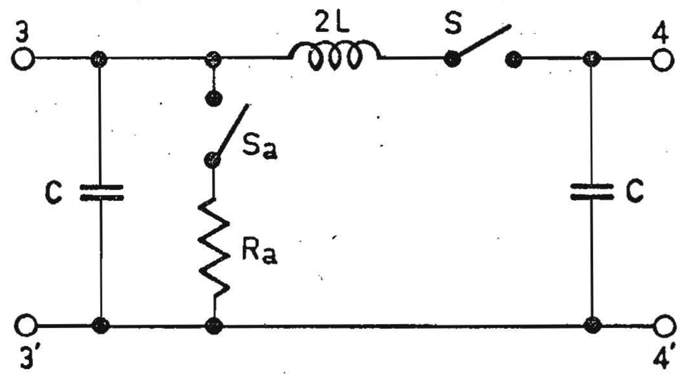


Fig. 9.4

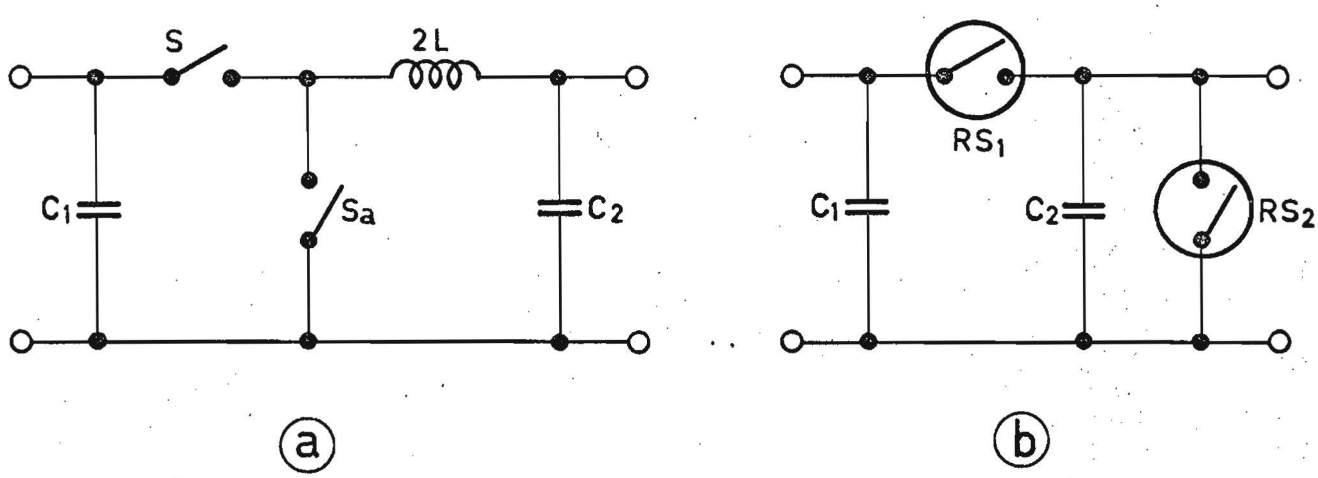


Fig. 9.5

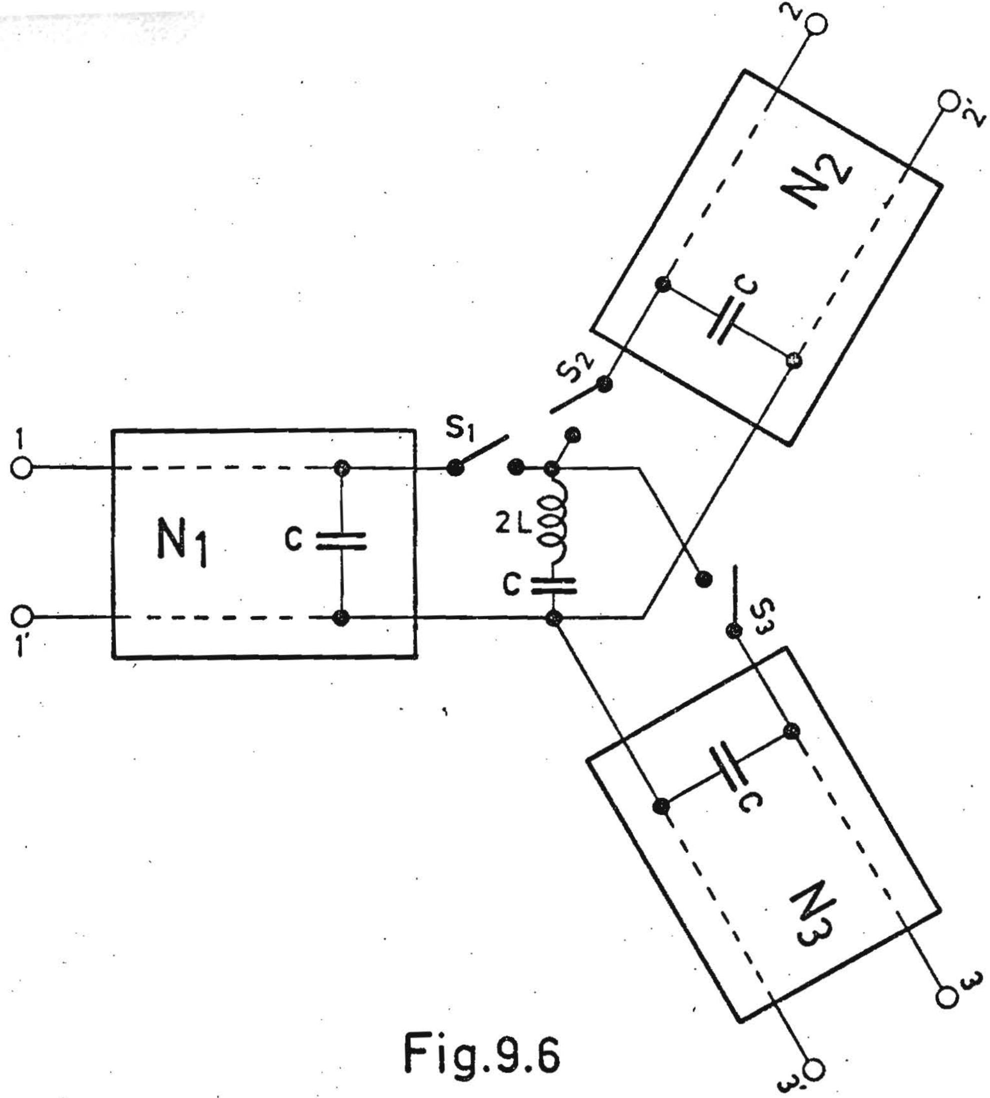


Fig.9.6

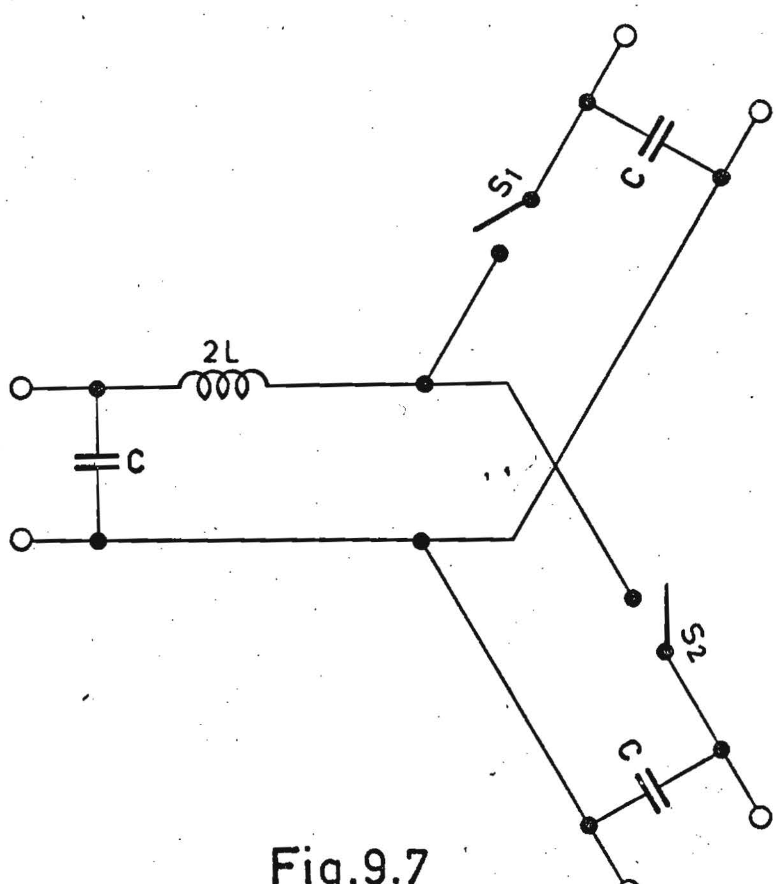


Fig.9.7

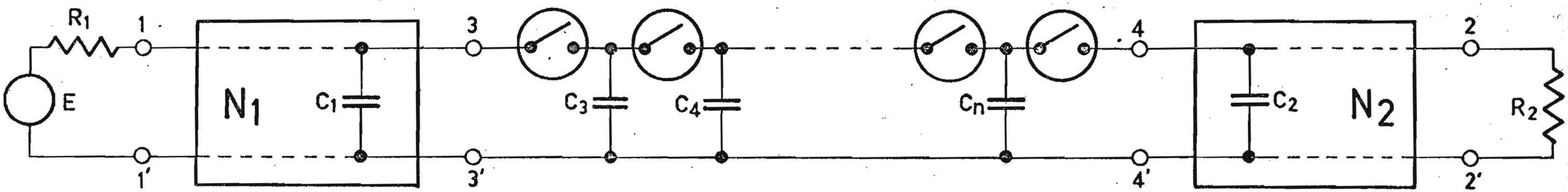


Fig.10.1

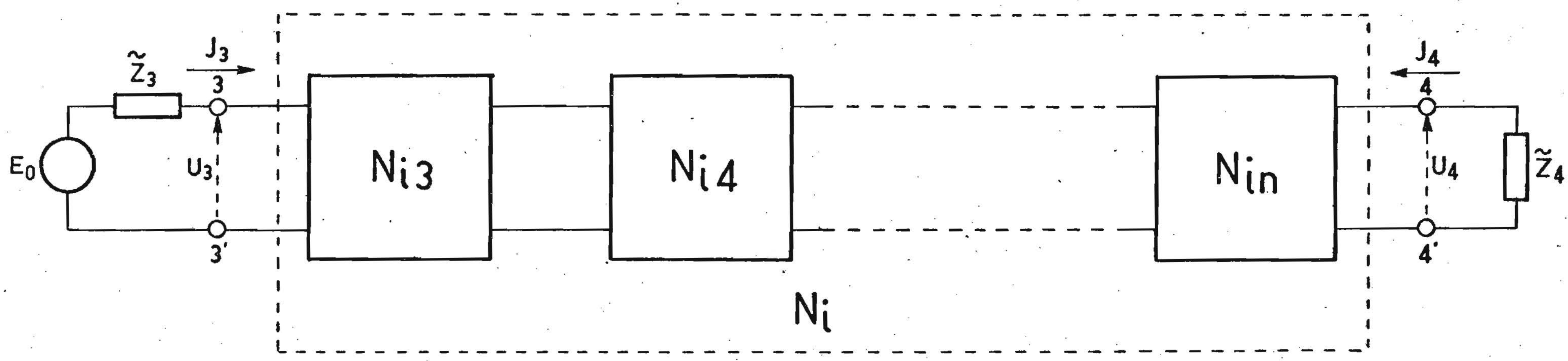


Fig.10.2

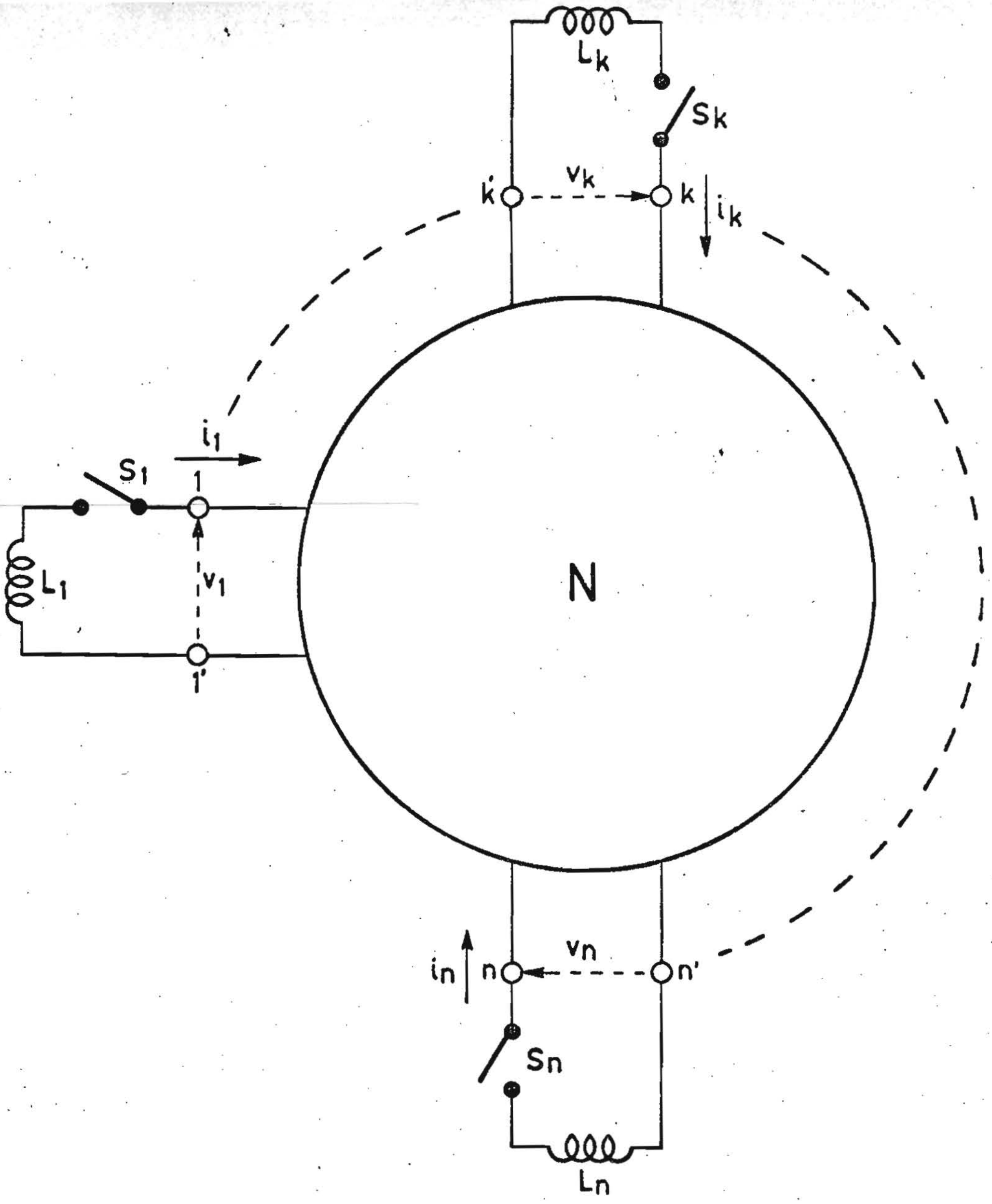


Fig.10.3

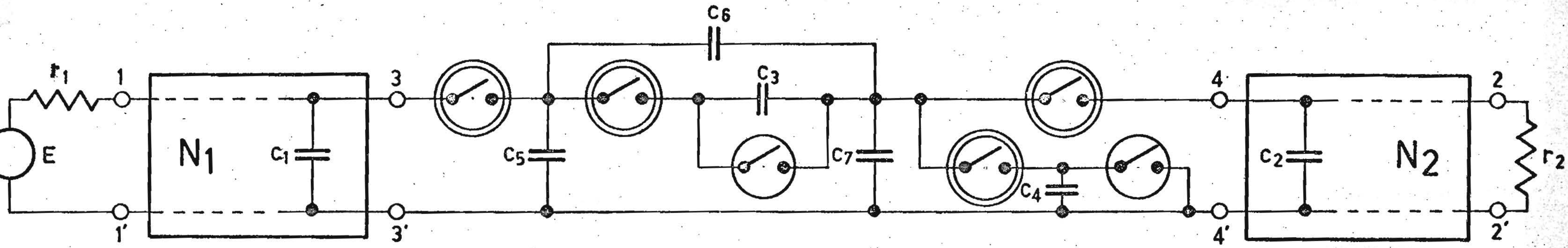


Fig.10.4

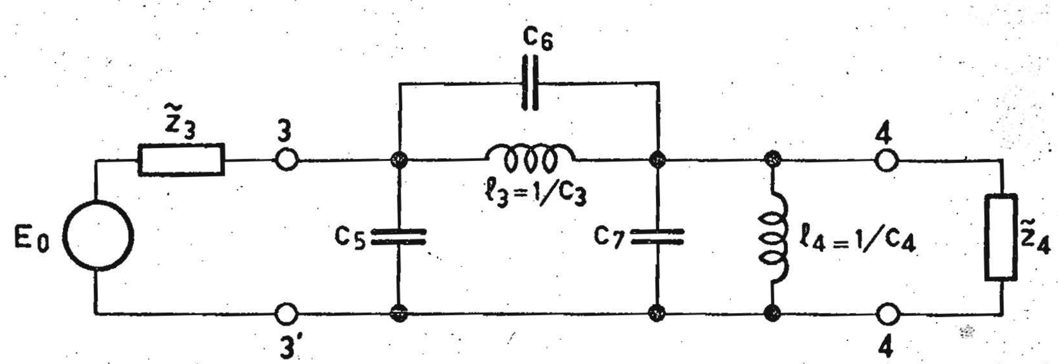


Fig.10.5