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Nomenclature

b	*Width of cut*	m
b_g	*Limit value of width of cut*	m
F_{1i}	*Amplitude of the force component caused by the inner modulation of the chip*	N
F_{1o}	*Amplitude of the force component caused by the outer modulation of the chip*	N
ΔF_j	*Component of the resulting dynamic cutting force corresponding with the direction* j	N
ΔF_f	*Dynamic component of the feed force*	N
ΔF_v	*Dynamic component of the main cutting force*	N
h_o	*Nominal undeformed chip thickness*	m
Δh_o	*Amplitude of chip thickness modulation*	m
I_i, I_o	*Quadrature components of the dynamic stiffness per unit of width of cut caused by the inner chip modulation and the outer chip modulation respectively*	N/m^2
k	*Stiffness of the machine tool structure*	N/m
k_i	*Specific process stiffness*	N/m^2
k_{1f}	*Chip thickness coefficient of the feed force*	N/m

k_{1v}	*Chip thickness coefficient of the main cutting force*	N/m
N_{2j}	*Inner modulation coefficient of the component of the specific dynamic cutting force in the direction* j	Ns/m^2
R_i, R_o	*In-phase component of the dynamic stiffness per unit of width of cut caused by the inner chip modulation and the outer chip modulation respectively*	N/m^2
R_{ij}	*Coefficient of the component of the specific dynamic cutting force in the direction* j	N/m^2
s	*Feed*	mm/rev
v	*Cutting speed*	m/s
Y	*Amplitude of the inner chip thickness modulation*	m
Y^*	*Amplitude of the outer chip thickness modulation*	m
δ	*Phase shift (see eq. (6.1.))*	rad
ε	*Phase shift (see eq. (6.2.))*	rad
ϕ	*Phase shift between the inner and outer chip thickness modulations*	rad
Φ_m	*Mean shear angle*	o
ζ	*Damping ratio of the machine tool structure*	-
ω	*Angular frequency*	rad/s

1. Introduction

Over the last number of years there are investigators who advocate a more detailed approach of the dynamic cutting process. The introduction to this is found in the work of Das and Tobias (1). Starting from a shear plane model of the cutting process, these investigators present a pure geometrical analysis of the wave-on wave-cutting process that occurs when the tool vibrates. They consider separately the influence of the inner and the outer modulation of the undeformed chip and derive the relations on the dynamic cutting forces on the basis of static parameters only. From this it follows that a phase shift is introduced by both the dynamic force component of the inner modulation and the force component of the outer modulation.

In this way of thinking, Polacek (2) developed a method to measure the various dynamic components of the cutting force applying a dynamometer.

Van Brussel and Vanherck (3), (4) carried out experiments on the same subject. They propose a method yielding the dynamic stiffness of the cutting process. As a matter of fact, this method is basically analogous to the one described in a previous paper by the author (5). The observations of Van Brussel and Vanherck confirm the theoretical results of Das and Tobias concerning the inner and outer modulation forces behaving independently of each other.

2. Discussion of the results

With respect to the different edges of the modulated chip, Van Brussel applies the following force equations:

$$F_{1i}\, e^{i(\omega t + \delta)} = Y\, e^{i \omega t} (R_i + i\, I_i)\, b \qquad \text{I}$$

$$F_{1o}\, e^{i(\omega t - \phi + \varepsilon)} = Y\, e^{i(\omega t - \phi)} (R_o + i\, I_o)\, b \qquad \text{II}$$

where

- index i refers to the direct chip modulation
- index o refers to the delayed chip modulation
- R is the in-phase component of the dynamic cutting stiffness
- I is the quadrature component of the dynamic cutting stiffness
- ϕ is the phase shift between the two chip thickness modulations
- δ and ε are the phase shifts of the dynamic cutting force components with respect to the direct and the delayed chip thickness modulation respectively.

The results of the various parameters obtained by Van Brussel (3) are shown in Table 1. The data show different values of R_i and R_o. However, from a physical point of view, it may be expected that for any cutting condition the values of R_i and R_o are equal. The outer modulation will only change the resulting depth of cut. This has been proved by Van Brussel and Vanherck to have no influence on the overall dynamic cutting force (4). Thus, one may conclude that the values of k_i obtained by the present author can stand for R_i as well as for R_o.

In this way of thinking, differences between the inner modulation stiffness and the outer modulation stiffness can exist only on account of the respective quadrature components I_i and I_o.

According to the theory of Das, Van Brussel explains the existence of the leading quadrature component I_o with the aid of a shear plane model (3). Das derived the next equation of the phase shift ε between the outer modulation and the inner modulation force component:

$$\varepsilon \simeq \frac{\omega}{v} h_o \cot \Phi_m \qquad \text{III}$$

where Φ_m represents the average value of the shear angle.

Eq. III shows that for small values of the chip thickness and for high values of the cutting speed the influence of ε can be ignored.

The stability criterion as applied by Van Brussel and Vanherck yields the limit value b_g when the dynamic stiffness of the machine tool equals the negative value of the resulting stiffness of the cutting process.

Fig. 1 shows the graphical solution method based on the stability criterion mentioned. A straight line approximates the dynamic stiffness of a low-damped machine tool. The intersection of this line with the out-of-phase axis of the polar diagram represents the dynamic stiffness $2\zeta k$ at natural frequency.

The inclination at the point of intersection approximates 2ζ rad. The components R_i and I_i are plotted with reversed sign, whilst a circle, having 0' as centre and $\sqrt{(R_o^2 + I_o^2)}$ as the radius, gives all the loci of the stiffness of the cutting process per unit of width of cut. In order to obtain the limit value b_g, straight lines are drawn passing through the origin 0, intersecting the circle and the machine tool stiffness locus. The minimum of the ratios AO/BO, A'0/B'0, etc. brings in b_g. The different lines correspond to different relative phase shifts between the inner and the outer modulation as indicated by the angle ϕ.

With the aid of this method, the influence of the quadrature components I_o and I_i on the threshold of stability has been calculated. Fig. 2 shows the influence of the ratios I_o/R_o, I_i/R_i on the limit value for the generalized situation that $R_o = R_i$, $R_o/\zeta k$ being constant, and $\zeta = 0.025$. From the figure it can be concluded that the influence of I_i on b_g is considerably stronger than the influence of I_o on b_g. It should be noticed that in general Van Brussels' results lead to about the same values of I_o/R_o and I_i/R_i.

With respect to the parameters of the outer modulation, another important remark can be made. Contrary to what may be expected from the foregoing, the results of Table 1 show values of R_o which are considerably smaller than those of R_i. However, it is remarkable that for all the different cutting speeds applied, the values of R_i and $\sqrt{(R_o^2 + I_o^2)}$ are approximately the same. This is diagrammatically shown in Fig. 3. With the aid of the graphical solution method of Fig. 1 it can be seen that a substitution of R_o and I_o by the real component $\sqrt{(R_o^2 + I_o^2)}$ does not affect the limit value. The introduction of I_o will only increase the phase shift between R_o and R_i.

Van Brussel and Vanherck (4) computed stability charts for their special tool holder. Comparing the theoretical and the experimental values of ϕ, it follows that the discrepancy between the values of both series of results generally is of the same magnitude as the influence of I_o on ϕ (5%).

The facts mentioned do not support the relevancy of I_o, the more so as the present author's results, which have been obtained excluding the influence of a quadrature outer modulation component, show a very good agreement between various series of practical and theoretical results. With respect to this, it is mentioned that in some cases the ε-values can increase up to 0.7 rad.

Table 2 shows the results obtained by Polacek. The equation of the dynamic cutting force derived by the latter can be written as

$$\Delta F_j = -b \left[(R_{1j} + i\, R_{2j} + i\, \omega\, N_{2j})\, Y - (R_{3j} + i\, R_{4j})\, Y^{*} \right] \qquad \text{IV}$$

Polacek shows this equation to be similar to Van Brussel's equation with the only difference that the parameters in eq. IV relate to a particular direction j, whilst the parameters of the eqs. I and II stand for the resulting dynamic force. The analogy is

$$R_1 \rightarrow R_i \qquad\qquad R_3 \rightarrow R_o$$

$$R_2 + N_2\, \omega \rightarrow I_i \qquad\qquad R_4 \rightarrow I_o$$

A direct comparison of all the results mentioned with the author's findings is not possible, since Polacek used a different work material and Van Brussel did not mention any materials specification.

With respect to Polacek's results it draws attention that positive as well as negative values of R_4 are obtained. It is obvious that the negative results do not fit the theory of Das. Moreover, according to Das' theory, the values of R_4/R_3 should show an increase with respect to an increasing feed. This, however, is also not confirmed by the results of Table 2.

Resuming, one can conclude that, at least for feed values up to 0.2mm/rev, the physical meaning of I_o in relation to the shear plane theory is very doubtful. At this stage, one can make objections against the assumption made by Das that the orientation of the shear plane will remain unaffected by the vibration. Physical considerations suggest that the direction in which the shearing zone propagates will be controlled by the stress conditions close to the tip of the tool. Thus, the variation of the cutting force will be strongly affected by a dynamically changing shearing process.

In reference to the inner modulation damping, Polacek's results as well as the present author's results (6) show that in the directions of both the feed and the cutting speed, the damping can be positive and negative as well. Das' results only permit a negative damping with respect to the dynamic component of the force in the direction of the main cutting force, and a positive damping related to the component in the direction of the feed force, according to

$$\Delta F_v = k_{1v}\,\Delta h_o \sin \omega t - k_{1f}\,h_o \frac{\omega}{v}\,\Delta h_o \cos \omega t \qquad \text{V}$$

$$\Delta F_f = k_{1f}\,\Delta h_o \sin \omega t + k_{1v}\,h_o \frac{\omega}{v}\,\Delta h_o \cos \omega t \qquad \text{VI}$$

where k_{1v} is the chip thickness coefficient of the main cutting force

k_{1f} is the chip thickness coefficient of the feed force

Δh_o is the amplitude of chip thickness modulation

Finally, it should be mentioned that the assumption, that the component of a vibration in the direction of the main cutting force has no influence on the dynamic force (5) (6), is confirmed up to a great extent by experiments carried out by Polacek (2).

References

(1) Das, M.K., Tobias, S.A., Int. J. Mach. Tool Des. Res. 7 (1967) 63

(2) Polacek, M., Slavicek, J., Messen des Dynamischen Schnittkraftkoeffizienten und Berechnung der Stabilitätsgrenze.
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(3) Van Brussel, H., Vanherck, P., 11th Int. M.T.D.R. Conference, Manchester (1970)

(4) Van Brussel, H., Vanherck, P., Measurement of the dynamic cutting coefficient and prediction of stability.
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(5) Kals, H.J.J., C.I.R.P. Ann. 19 (1971) 297

(6) Kals, H.J.J., Fertigung 5 (1971) 165

SUBSCRIPTION OF THE FIGURES

Fig. 1	*The graphical solution method for the limit width of cut by Van Brussel and Vanherck*
Fig. 2	*The influence of the ratios* I_o/R_o *and* I_i/R_i *on the limit value* b_g *for a generalized situation*
Fig. 3	*The agreement between the resulting dynamic stiffness of the outer modulation* $\sqrt{(R_o^2 + I_o^2)}$ *and* R_i, *after Van Brussels' results*
Table 1	*Results after Van Brussel and Vanherck*
Table 2	*Results after Polacek*
Table 3	*The author's results*

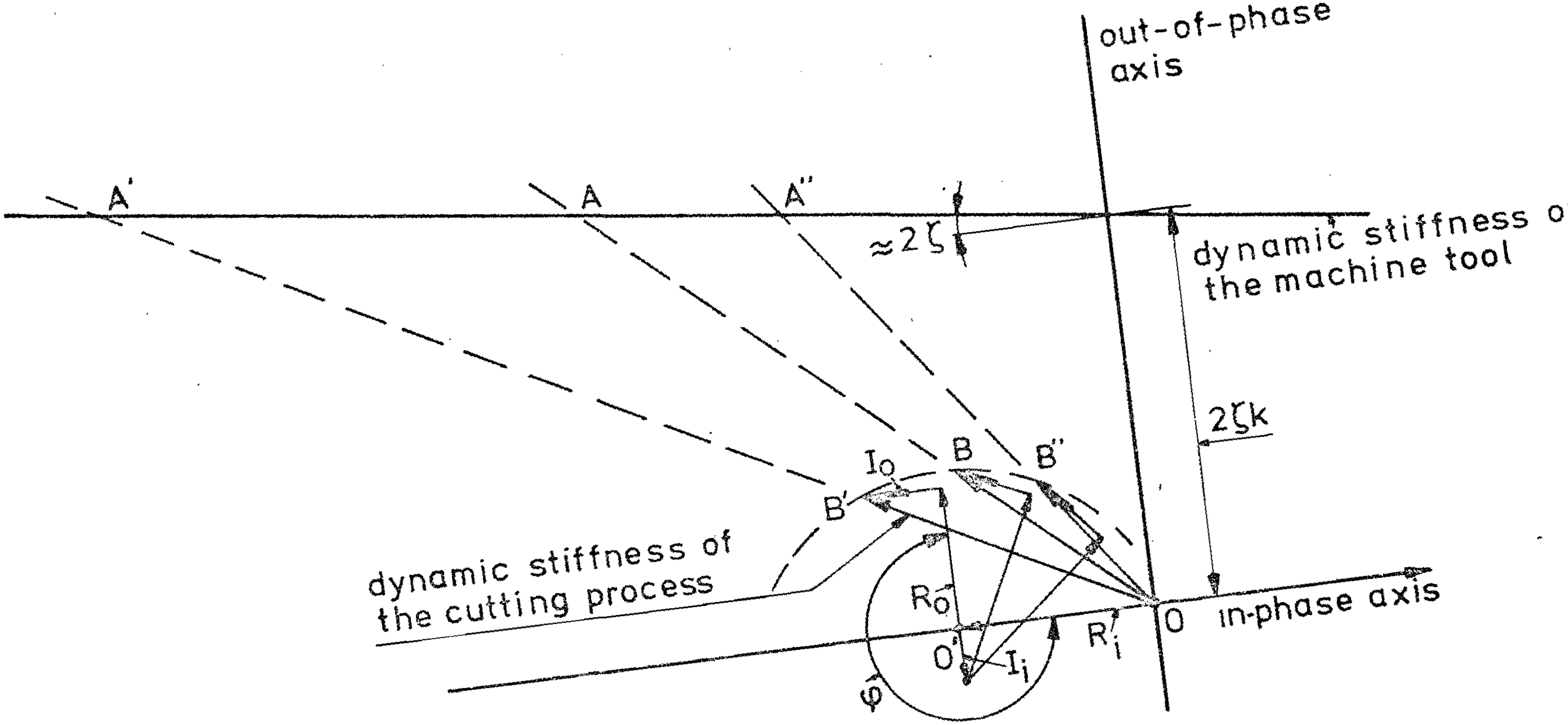
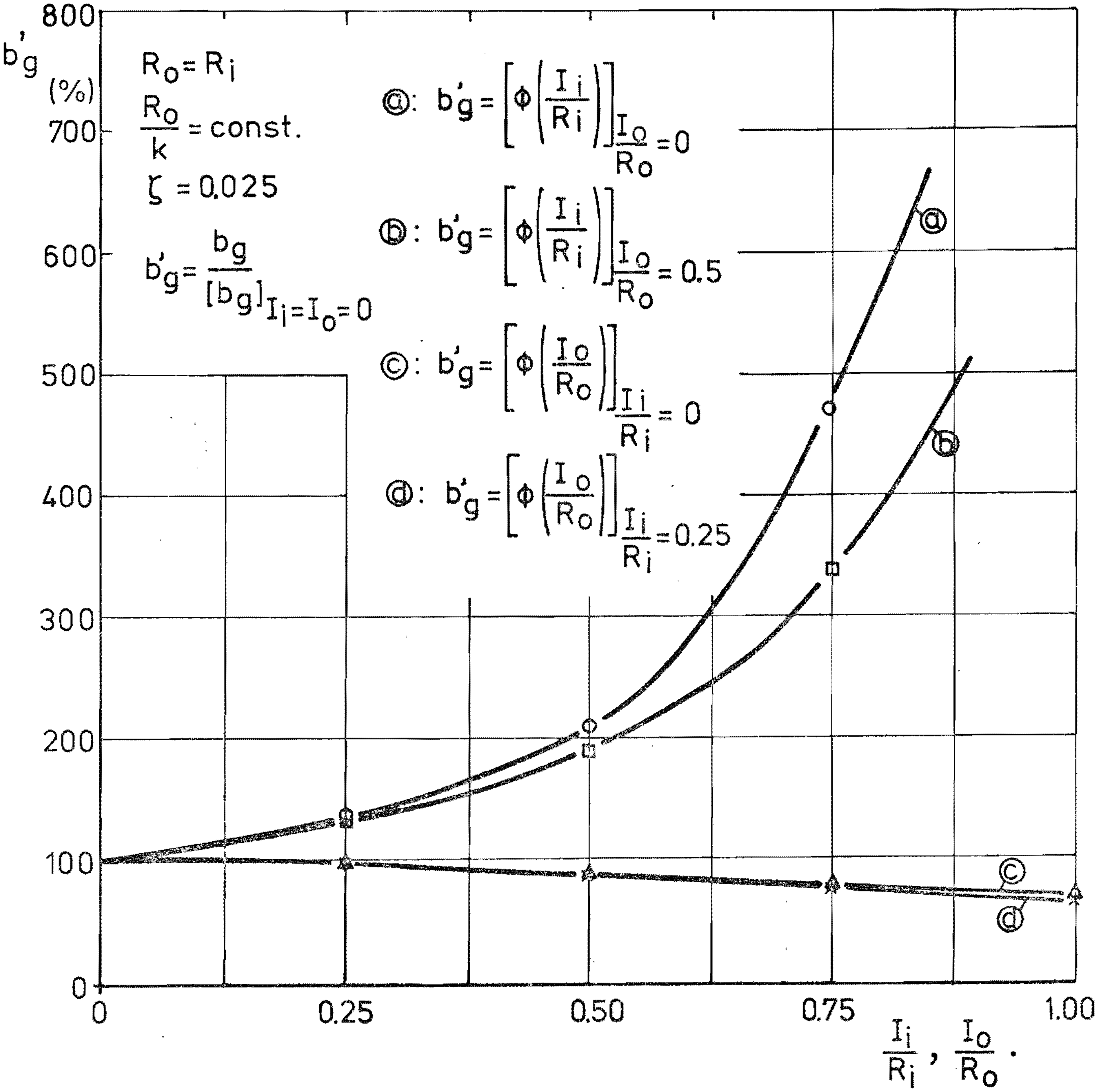

out-of-phase
axis
A'
A
A''
≈2ζ
dynamic stiffness of
the machine tool
2ζk
I₀
B
B''
B'
dynamic stiffness of
the cutting process
R₀
O
in-phase axis
R'ᵢ
O'
Iᵢ
φ

Fig. 1.



b'g (%)
800
700
600
500
400
300
200
100
0
0
0.25
0.50
0.75
1.00
$R_o = R_i$
$\frac{R_o}{k}$ = const.
$\zeta = 0.025$
$b'_g = \frac{b_g}{[b_g]_{I_i=I_o=0}}$
ⓐ: $b'_g = \left[\Phi\left(\frac{I_i}{R_i}\right)\right]_{\frac{I_o}{R_o}=0}$
ⓑ: $b'_g = \left[\Phi\left(\frac{I_i}{R_i}\right)\right]_{\frac{I_o}{R_o}=0.5}$
ⓒ: $b'_g = \left[\Phi\left(\frac{I_o}{R_o}\right)\right]_{\frac{I_i}{R_i}=0}$
ⓓ: $b'_g = \left[\Phi\left(\frac{I_o}{R_o}\right)\right]_{\frac{I_i}{R_i}=0.25}$
ⓐ
ⓑ
ⓒ
ⓓ
$\frac{I_i}{R_i}, \frac{I_o}{R_o}$.

Fig. 2.

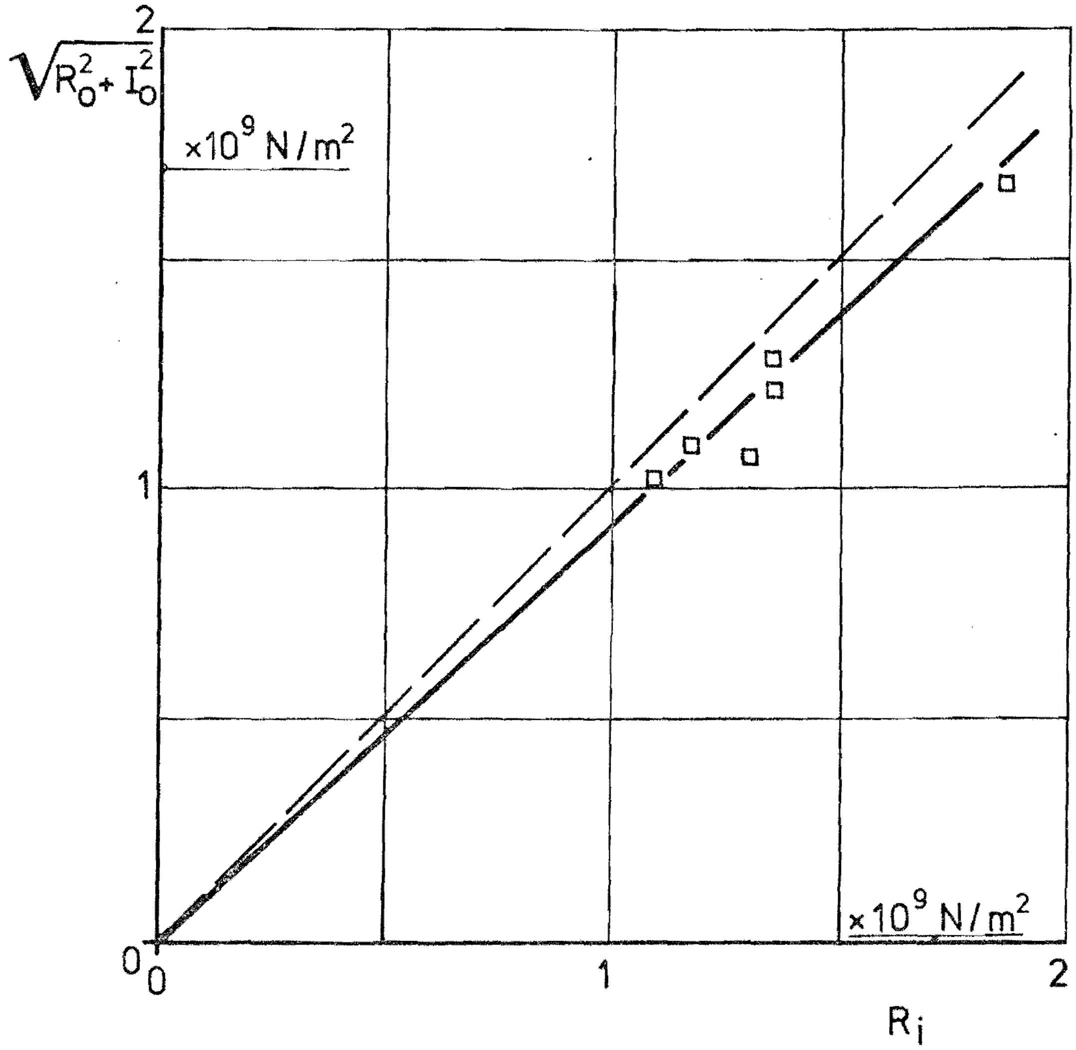


Fig. 3.

Table 1

s = 0.22 mm/rev.		frequency of excitation 150 Hz		
v (m/s)	R_i (10^9 N/m^2)	R_o (10^9 N/m^2)	I_i (10^9 N/m^2)	I_o (10^9 N/m^2)
0.37	1.83	0.48	1.58	0.73
0.47	1.33	0.73	1.05	0.70
0.58	1.08	0.78	0.65	0.35
0.75	1.28	0.93	0.55	0.45
0.93	1.15	0.95	0.55	0.35
1.50	1.35	1.15	0.60	0.40

Table 2

s (mm/rev)	0.05						0.1						0.2					
v (m/s)	0.47		0.83		1.67		0.47		0.83		1.67		0.47		0.82		1.47	
freq. of excitat. 50÷200 Hz	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v	ΔF_f	ΔF_v
R_1 (10^9 N/m^2)	0.09	0.67	0.27	1.05	0.59	1.51	0.29	0.85	0.36	1.45	0.54	1.34	0.08	0.88	0.32	1.24	0.15	1.09
R_2 (10^9 N/m^2)	0.17	0.23	-0.05	-0.11	0.06	0.08	0.19	0.26	-0.04	-0.24	-0.03	-0.3	0.11	0.24	-0.08	-0.08	0.05	-0.49
N_2 (10^5 Ns/m^2)	1.5	0.54	0.84	0.74	1.17	1.07	4.7	1.07	1.93	2.35	6.62	6.62	4.9	1.07	2.78	1.49	1.71	5.87
R_3 (10^9 N/m^2)	0.12	0.5	0.17	0.88	0.4	1.36	0.09	0.57	0.27	1.41	0.21	1.08	0.13	0.67	0.19	1.21	0.17	1.14
R_4 (10^9 N/m^2)	0	0.08	-0.07	-0.11	-0.13	-0.15	-0.05	-0.18	-0.11	-0.17	-0.09	-0.24	0.02	0.04	-0.11	-0.05	-0.08	-0.04

v [m/s]	s = 0.072 mm/rev.				s = 0.208 mm/rev.			
	$\|\vec{k}_i\|$ [N/m²]	β_o [°]	$\|\vec{c}_i\|$ [N.s/m²]	γ_o [°]	$\|\vec{k}_i\|$ [N/m²]	β_o [°]	$\|\vec{c}_i\|$ [N.s/m²]	γ_o [°]
0.500	–	–	–	–	1.56×10^9	73	1.03×10^6	112
0.667	1.08×10^9	41	0.45×10^6	334	1.80×10^9	74	1.43×10^6	118
0.833	1.59×10^9	81	1.77×10^6	100	1.41×10^9	26	0.19×10^6	274
1.000	1.45×10^9	82	0.66×10^6	138	2.26×10^9	66	1.35×10^6	91
1.167	–	–	–	–	1.98×10^9	72	1.26×10^6	92
1.250	1.01×10^9	79	0.69×10^6	211	–	–	–	–
1.333	2.36×10^9	85	1.19×10^6	121	–	–	–	–
1.667	2.16×10^9	69	0.75×10^6	78	–	–	–	–
2.000	2.57×10^9	67	1.25×10^6	71	–	–	–	–
2.333	1.69×10^9	59	0.67×10^6	50	–	–	–	–

Table 3 SKF 1550