Some basic ideas about a single-point path-mid range approximation concept

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Abstract

A suitable approximation concept can be effectively used to interface structural analysis software and mathematical programming algorithm. In a certain part of the design space, approximations of objective function and constraints are built. Then, the approximate optimization problem is solved by the mathematical programming algorithm. Often, a local approximation concept is introduced. A local approximation of objective function or constraint is based on function value and derivatives calculated by the structural analysis and design sensitivity analysis at a single design point of the design space. Since such an approximation is only locally valid, a sequence of approximate optimization cycles, called design cycles, has to be performed to reach an optimum design.

Local approximation concepts do not use the data of previous design cycles. Only few multi-point or mid-range approximation concepts have been published which try to improve the approximations with analysis data of more than one design point. Mid-range concepts are called single-point-path if during every design cycle only one design point is analyzed, otherwise the term multiple-point-path is used.

In this report a single-point-path mid-range concept is studied that takes into account all design sites at which both the analysis and the design sensitivity analysis has been performed. An approximation of objective function or constraint is composed of a basic model function and a second model part. The second model part exactly fits the residual function values and derivatives of the basic model. In this second model, a function has been inserted to charge the correlation between different design points. A design point is less correlated with a remote design point than with a point lying much closer.

Two different basic model function are considered. Firstly, the basic model has been taken equal to the linear approximation in the design site of the current optimization cycle. With this type of basic model function, the aim is to develop an approximation concept with an improved convergence behaviour compared with the linear concept. However, the second model part is not able to introduce a correct curvature into the approximation for a number of design cycles smaller than the number of design variables. Therefore, the mid-range concept with linear basic model does not generally improve the sequential linear approximate optimization process.

Secondly, a constant basic model is considered. Suppose a constraint is to be approximated and the constant value of the basic model is taken more positive than the critical constraint value. Then, an approximation with a variable conservativeness can be generated by the mid-range concept. The conservativeness depends on the degree of correlation. In the case of the optimization of a cantilever beam, a better convergence has been achieved compared with sequential linear programming. Therefore, it is recommended to further investigate the approximation concept with constant basic model and to compare it with the method of moving asymptotes.

Contents

Chapter 1

Introduction

1.1 Background and scope of the report

To interface a non-linear programming algorithm with a structural analysis program a suitable approximation concept should be introduced. The basic principle is to generate approximations of the objective function and constraints in a certain part of the design space, and to solve the optimum point for this approximate optimization problem. Since approximate objective function and constraints are explicitly known, the approximate problem formulation can be easily solved by a (non-) linear programming algorithm. Barthelemy and Haftka (1993) review the basic and more recently developed approximation concepts in structural optimization. They distinguish local, global and mid-range approximations.

Most often a local single point approximation concept is introduced. A local approximation of objective function or constraint is based on the function value and the derivative values with respect to the design variables in a single point of the design space. Using this type of approximation a sequential approximate optimization process results.

Vanderplaats (1993) describes the basic program structure for sequential approximate optimization. It starts with the structural analysis of the initially proposed design. Then, all constraint functions are evaluated and only the critical and potentially critical are retained for further consideration. Gradients of the objective function and the retained constraints are computed by a sensitivity analysis. These derivatives together with the function values are used to generate an approximate optimization problem. Since the approximations of objective function and constraints are only locally valid, movelimits are imposed on the design variables. A newly proposed design results from the solution of the approximate problem formulation, at which a new design cycle of creating and solving an approximate optimization problem can be started. The objective function and constraint values computed by the structural analysis of the new design can be compared with the approximated values, thus giving an indication of the quality and reliability of the approximations. This process is repeated until an acceptable optimum is reached.

The most simplest local approximation concept is the linear approximation based on the Taylor series. In many cases an approximation of a higher quality can be obtained by the introduction of suitable intermediate variables and responses. Insight in the mathematical behaviour is used to get the best approximation to a particular response.

However, local approximation concepts do not use structural analysis results at design sites of former optimization cycles. Only few extended local or mid-range concepts have been developed to improve the local approximation by using more than one design point. Haftka, Nachlas, Watson, Rizzo and Desai (1987), Fadel and Cimtalay (1993), and Belegundu, Rajan and Rajgopal (1993) worked on two or three point approximation concepts. Rasmussen (1990) tried to improve the approximation with function values of former design cycles by Lagrangian interpolation. Free, Parkinson, Bryce and Balling (1987), and Toropov, Filatov and Polynkin (1993) build response-surface approximation models in a certain part of the design space for every design cycle based on the structural analysis results of several design points according to some experimental design.

In this report some basic ideas about a single-point-path (SPP) mid-range concept are considered to construct an approximation near the present design point, based on both structural and sensitivity analysis results of all evaluated designs. For every design cycle an optimum solution of the approximate optimization problem is computed at which a new structural and sensitivity analysis is done. These results are added to the total set of design information gathered during the optimization. So, every cycle adds one point to the optimization path, and therefore the mid-range concept is called single-point-path. This in contrary to mid-range concepts that analyze several design sites during one optimization cycle. They are called multiple-point-path.

1.2 Sequential approximate optimization

The general optimization problem is formulated as: find the set of n design variables $\mathbf{x}$, that will minimize the objective function:

$$F_{obj}(\mathbf{x}) \tag{1.1}$$

subject to the constraints:

$$g_h(\mathbf{x}) \leq c_h \qquad h = 1, ..., m \tag{1.2}$$

and side-constraints:

$$x_k^l \leq x_k \leq x_k^u \qquad k = 1, ..., n \tag{1.3}$$

The scalar x_k is the k th element of the design vector $\mathbf{x}$. The side-constraints define the design space, i.e. the region in which is searched for an optimum.

Generally, objective function value and constraint values at a certain design point have to be computed from an expensive structural analysis, e.g. a finite element analysis. Therefore, in every design cycle approximation models of objective function and critical or potentially critical constraints are introduced to generate an explicitly known optimization problem. For the p th design cycle the problem is now to find the vector $\mathbf{x}_*^{(p)}$ that minimizes the approximate objective function:

$$\tilde{F}_{obj}^{(p)}(\mathbf{x}) \tag{1.4}$$

bounded by the approximate constraints:

$$\tilde{g}_h^{(p)}(\mathbf{x}) \leq c_h \qquad h = 1, ..., m \tag{1.5}$$

in a limited part of the design space:

$$\alpha_k^l \leq x_k \leq \alpha_k^u \qquad k = 1, ..., n \tag{1.6}$$

The movelimits α_k^l and α_k^u determine the region around the present design $\mathbf{x}^{(p)}$ in which the approximations are supposed to be valid. Measures of the quality of the generated approximations of objective function and constraints are the differences between the approximated values and the corresponding values following from the structural analysis. Therefore we define:

$$e_o^{(p)} = \left| \frac{\tilde{F}_{obj}^{(p)}(\mathbf{x}_*^{(p)}) - F_{obj}(\mathbf{x}_*^{(p)})}{F_{obj}(\mathbf{x}_*^{(p)})} \right| \tag{1.7}$$

$$e_j^{(p)} = \max_h \left| \frac{\tilde{g}_h^{(p)}(\mathbf{x}_*^{(p)}) - g_h(\mathbf{x}_*^{(p)})}{g_h(\mathbf{x}_*^{(p)})} \right| \tag{1.8}$$

These measures should give an indication whether the movelimits should be enlarged or tightened in the next design cycle. If one is satisfied with the quality of the proposed design $\mathbf{x}_*^{(p)}$, it can be chosen as starting design of the new cycle, otherwise for example $\mathbf{x}^{(p)}$ can be selected again.

1.3 Availability of design sensitivity analysis results

At a certain design cycle, approximation models of objective function and all critical or potentially critical constraints not explicitly known have to be introduced. In the case of a multi-point approximation concept, some remarks have to be made about the availability of design sensitivity analysis data of objective function and all considered constraints. The simplest case is that for every constraint and objective function to be approximated, function values and derivatives are available in all evaluated design sites stored in some database. This will usually be the case if the (semi-analytical) direct method or the finite difference method is used to calculate the gradient information. Consider for example the static finite element case;

$$K\mathbf{u} = \mathbf{f} \tag{1.9}$$

where K is the stiffness matrix, $\mathbf{u}$ the nodal displacement vector and $\mathbf{f}$ a load vector. To compute derivatives of stress or displacement constraints with a forward finite difference method, equation (1.9) has to be solved n extra times for the perturbed designs, which results in gradient information for all constraints. When using a(n) (semi-)analytical method the constraint derivative with respect to a design variable x is written as:

$$\frac{dg}{dx} = \frac{\partial g}{\partial x} + \mathbf{q}^T \frac{d\mathbf{u}}{dx} \tag{1.10}$$

where $\mathbf{q}$ is a dummy load vector with components:

$$q_i = \frac{\partial g}{\partial u_i} \tag{1.11}$$

The direct method solves $\frac{d\mathbf{u}}{dx}$ from

$$K\frac{d\mathbf{u}}{dx} = \frac{d\mathbf{f}}{dx} - \frac{dK}{dx}\mathbf{u} \tag{1.12}$$

for every design variable x, so n times. As a result for all constraints derivative information is available.

However, if the adjoint method is applied, equation (1.10) is written as:

$$\frac{dg}{dx} = \frac{\partial g}{\partial x} + \left(\mathbf{q}^T - \boldsymbol{\lambda}^T K\right)\frac{d\mathbf{u}}{dx} + \boldsymbol{\lambda}^T\left(\frac{d\mathbf{f}}{dx} - \frac{dK}{dx}\mathbf{u}\right) \tag{1.13}$$

Then, for every constraint the adjoint vector $\boldsymbol{\lambda}$ has to be solved from:

$$K\boldsymbol{\lambda} = \mathbf{q} \tag{1.14}$$

So it is possible to calculate only the gradient information of critical and potentially critical constraints. Often, this approach is computationally more attractive, especially in the multi-load case. However, this means that during the approximate optimization process derivative constraint information at a certain evaluated design site is not available for all constraints. Constraint derivatives are only available at those design sites for which during the corresponding design cycle the constraint was found to be critical or potentially critical.

To apply a multi-point concept, somewhat more administration in the database is required. To construct an approximation of a certain constraint only those N design points of the total number of design sites are considered for which sensitivity data of that constraint are available. If the adjoint method is used to compute derivatives, these points may be different for every constraint.

Finally, it is remarked that design sensitivities calculated by the semi-analytical method can be somewhat inaccurate. In this report the sensitivities are exactly predicted which may give rise to convergence problems near the optimum for certain values of inaccuracy. It has not yet been investigated how inaccuracy in design sensitivity data can be taken into account.

Chapter 2

Basic principles of the mid-range concept

2.1 Introduction

The basic principles of the proposed mid-range SPP approximation concept are considered. Starting point is that all available data is used to build the approximations. Every approximation of objective function and constraint consists of a basic model part and a part exactly fitting the residual function values and derivatives. Given the basic model and the non-linear parameters in the residual model, the remaining linear parameters in the second model can be computed. The basic model and the non-linear parameter values influence the approximation. In chapter 3 the basic model is chosen equal to the linear approximation in the design site of the current optimization cycle, around which an approximation is desired. Chapter 4 deals with a constant basic model.

2.2 Approximation model

Suppose at the p th design cycle for a certain objective function or constraint to be approximated, structural and design sensitivity results are available at N design points. The evaluated design sites are gathered in the design set S:

$$S = \{\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_N\} \tag{2.1}$$

The computed function values and derivatives belonging to this set S are written in a form:

$$\mathbf{y} = \left[\begin{array}{cccc} y(\mathbf{s}_1) & y(\mathbf{s}_2) & ... & y(\mathbf{s}_N) \end{array}\right]^T \tag{2.2}$$

$$\frac{\partial \mathbf{y}}{\partial x_k} = \left[\begin{array}{cccc} \frac{\partial y}{\partial x_k}(\mathbf{s}_1) & \frac{\partial y}{\partial x_k}(\mathbf{s}_2) & ... & \frac{\partial y}{\partial x_k}(\mathbf{s}_N) \end{array}\right]^T \qquad k = 1, ..., n \tag{2.3}$$

The j th element of a column $\mathbf{s}_i$ is denoted as $(s_j)_i$.

For the objective function or constraint the following approximation is introduced:

$$\tilde{f}(\mathbf{x}) = f_b(\mathbf{x}) + \sum_{i=1}^{N} \mathbf{a}_i^T \mathbf{z} R(\mathbf{s}_i, \mathbf{x}) \tag{2.4}$$

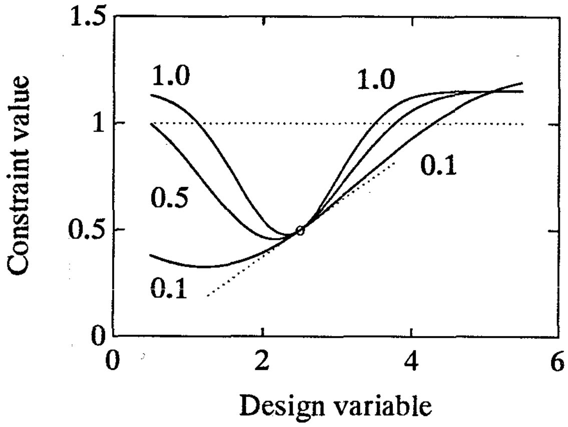

Figure 2.1: One point approximation for θ values of 0.1, 0.5 and 1.

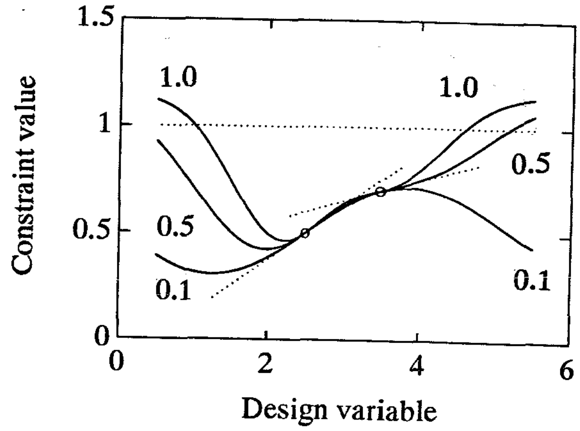

Figure 2.2: Two point approximation for θ values of 0.1, 0.5 and 1.

with an extended design variable column $\mathbf{z}$ and a parameter column $\mathbf{a}_i$ defined by:

$$\mathbf{z} = \begin{bmatrix} 1 & \mathbf{x}^T \end{bmatrix}^T \tag{2.5}$$

$$\mathbf{a} = \begin{bmatrix} a_0 & a_1 & \dots & a_n \end{bmatrix}^T \tag{2.6}$$

and a correlation function:

$$R(\mathbf{w}, \mathbf{x}) = \prod_{k=1}^{n} e^{-\theta_k (w_k - x_k)^2} \tag{2.7}$$

Approximation model (2.4) consists of a basic model and a summation of linear models belonging to every design site $\mathbf{s}_i$ multiplied by corresponding correlation functions. The basic model and the parameter θ_k in (2.7) are chosen beforehand, while the remaining linear parameters $\mathbf{a}_i$ ($i = 1,..,N$) for every design s_i are computed such that function value and derivatives in $\mathbf{s}_i$ are exactly predicted. The correlation function provides that function values and derivatives at a design point far away from $\mathbf{x}$ are less influential on the prediction than computational results more near by.

The behaviour of the model severely depends on parameters θ_k ($k = 1,..,n$). Consider for example a one dimensional optimization problem ($n = 1$) with one constraint $g(x) \leq 1$ to be approximated. Suppose at the initial design $x = 2.5$ structural and design sensitivity analysis have been performed, resulting in a constraint function value 0.5 and derivative value 0.25. The basic model is taken constant 1.15. In figure 2.1 it is clearly visible that the parameter θ value is directly connected with the region in which the approximation is supposed to be valid. Far away from the design site, the real constraint value is unknown and therefore the approximation tends to the basic model.

Now a new design is added at $x = 3.5$, so we have at two distinct design points two constraint values and two constraint derivative values. For a certain θ value and a basic line of 1.25 the unknown parameters $(a_0)_1$, $(a_1)_1$, $(a_0)_2$ and $(a_1)_2$ can be directly computed. Since the approximate model 2.4 has to predict the function and derivative values exactly, a set of four linear equations is found which can be easily solved. In figure 2.2 for some θ values the approximation model of the constraint is plotted. Again the influence of θ is clearly visible.

2.3 Estimation of the linear parameters

Suppose that for a certain constraint or objective function an approximation near the present design point based on model (2.4) has to be constructed and that structural and design sensitivity analysis results are available at N design points, collected in a form like (2.2) and (2.3). Starting from a given basic model and certain parameter θ_k values, the unknown parameters a_i ($i = 1, ..., N$) in model (2.4) should be determined such that the function values and derivatives will be exactly predicted. From equation (2.4) it can be derived that for every design site s_j ($j = 1, ..., N$) we have:

$$\tilde{f}(\mathbf{s}_j) - f_b(\mathbf{s}_j) = \sum_{i=1}^{N} \mathbf{a}_i^T \mathbf{t}_j R(\mathbf{s}_i, \mathbf{s}_j) \qquad j = 1, ..., N \tag{2.8}$$

$$\frac{\partial \tilde{f}}{\partial x_k}(\mathbf{s}_j) - \frac{\partial f_b}{\partial x_k}(\mathbf{s}_j) = \sum_{i=1}^{N} \Big((a_k)_i + 2\theta_k \mathbf{a}_i^T \mathbf{t}_j \left((s_k)_i - (s_k)_j \right) \Big) R(\mathbf{s}_i, \mathbf{s}_j) \qquad \begin{array}{l} j = 1, ..., N \\ k = 1, ..., n \end{array} \tag{2.9}$$

with the extended design vector $\mathbf{t}_j$ given by:

$$\mathbf{t}_j = \left[\begin{array}{cc} 1 & \mathbf{s}_j^T \end{array} \right]^T \tag{2.10}$$

Exact prediction of the data means:

$$\tilde{f}(\mathbf{s}_j) = y(\mathbf{s}_j) \qquad j = 1, ..., N \tag{2.11}$$

$$\frac{\partial \tilde{f}}{\partial x_k}(\mathbf{s}_j) = \frac{\partial y}{\partial x_k}(\mathbf{s}_j) \qquad j = 1, ..., N \tag{2.12}$$

The transformed function values and derivatives are defined by:

$$w(\mathbf{s}_j) = y(\mathbf{s}_j) - f_b(\mathbf{s}_j) \qquad j = 1, ..., N \tag{2.13}$$

$$\frac{\partial w}{\partial x_k}(\mathbf{s}_j) = \frac{\partial y}{\partial x_k}(\mathbf{s}_j) - \frac{\partial f_b}{\partial x_k}(\mathbf{s}_j) \qquad j = 1, ..., N \tag{2.14}$$

and put in a design data column $\mathbf{u}$:

$$\mathbf{u} = \left[\begin{array}{ccccccccc} w(\mathbf{s}_1) & \frac{\partial w}{\partial x_1}(\mathbf{s}_1) & ... & \frac{\partial w}{\partial x_n}(\mathbf{s}_1) & ... & w(\mathbf{s}_N) & \frac{\partial w}{\partial x_1}(\mathbf{s}_N) & ... & \frac{\partial w}{\partial x_n}(\mathbf{s}_N) \end{array} \right]^T \tag{2.15}$$

which has length $N(n+1)$. When also a column of unknown parameters b is formulated:

$$\mathbf{b} = \left[\begin{array}{cccc} \mathbf{a}_1^T & \mathbf{a}_2^T & ... & \mathbf{a}_N^T \end{array} \right]^T \tag{2.16}$$

then the following set of linear equations results:

$$T\mathbf{b} = \mathbf{u} \tag{2.17}$$

Matrix T directly follows from equation 2.8 and 2.9 and can be written as:

$$T = \left[\begin{array}{ccc} T_{11} & \dots & T_{1N} \\ \vdots & \ddots & \vdots \\ T_{N1} & \dots & T_{NN} \end{array} \right] \tag{2.18}$$

where the $n+1$ by $n+1$ submatrix T_{ji} given by:

$$T_{ji} = \begin{bmatrix} \mathbf{t}_j^T \\ 2\Theta(\mathbf{s}_i - \mathbf{s}_j)\mathbf{t}_j^T \end{bmatrix} R(\mathbf{s}_i, \mathbf{s}_j) + \begin{bmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & I \end{bmatrix} R(\mathbf{s}_i, \mathbf{s}_j) \qquad \begin{matrix} i = 1, ..., N \\ j = 1, ..., N \end{matrix} \tag{2.19}$$

with the n by n diagonal matrix Θ defined by:

$$\Theta = \begin{bmatrix} \theta_1 & & \emptyset \\ & \ddots & \\ \emptyset & & \theta_n \end{bmatrix} \tag{2.20}$$

Matrix I is an n by n identity matrix.

Chapter 3

Linear approximation as basic model

3.1 Introduction

In the preceding chapter an approximation concept has been introduced that consists of a basic model and a sum of linear models multiplied by correlation functions. Now, the basic model is set equal to the linear approximation in the design site of the present design cycle, as proposed in (Etman, 1992):

$$f_b(\mathbf{x}) = \mathbf{a}_0\mathbf{z} \tag{3.1}$$

Then, the multi-point concept should improve the convergence behaviour of the sequential linear approximate optimization process.

Vector $\mathbf{a}_0$ can be found from:

$$(a_0)_0 = y(\mathbf{s}_c) - \sum_{k=1}^{n}(a_k)_0(s_k)_c \tag{3.2}$$

$$(a_k)_0 = \frac{\partial y}{\partial x_k}(\mathbf{s}_c) \qquad k = 1, ..., n \tag{3.3}$$

with $\mathbf{s}_c$ the current cycle design. Remains a correct choice for parameters θ_k to be made. For the sake of simplicity, suppose that all design variables have been scaled such that the behaviour of functions to be approximated is almost the same in every design variable direction. This means that θ_k can be chosen equal for all k and that an absolute movelimit α can be introduced for all design variables.

3.2 Influence of θ

As already has been remarked, parameter θ is of direct influence on the size of the region in which the linear model is adapted by the second part of the approximation (2.4). This region size is related with the range of the region in which the approximation is supposed to be valid. Therefore, it is proposed to relate parameter θ with movelimit α.

The movelimits of all design variables build together an n-dimensional hypercube search subregion. The largest distance within this cube is from center $\mathbf{m}$ to corner $\mathbf{c}$, being $\sqrt{n\alpha^2}$.

If we define d to be the correlation function value between center and corner:

$$d = R(\mathbf{m}, \mathbf{c}) \tag{3.4}$$

and if we relate θ with this correlation value, then θ can be computed from:

$$\theta = -\frac{ln(d)}{n\alpha^2} \tag{3.5}$$

Remark that these considerations suggest to use a hyperglobe instead of a cube as search subregion, thus making the largest possible step independent of the dimension of the design space. This suggestion has not been investigated yet.

Now the problem is converted into the choice of a correct movelimit and a reasonable value of parameter d. The movelimit is determined by the smoothness of the response to be approximated. For a strong nonlinear behaviour smaller steps have to be made to keep the approximation valuable. The parameter d value - lying between zero and one - is less trivial. In any case d should be large enough that function value and derivatives of designs nearby do have influence and (hopefully) improve the linear approximation given in the basic model. Here it is assumed that steps made from cycle to cycle are not too large, i.e. the error $e_j^{(p)}$ is not too large.

For growing value of d a more stiff behaviour of the approximation between different design sites will result and the basic model will be adapted in a larger region in the design space. This also causes the extrapolative behaviour to become less predictable, like polynomials exactly predicting function values and derivatives.

Take for example the one dimensional situation of figure 3.1 in which function values and derivatives are known at two distinct design sites. Around the second point a new approximation is generated with the movelimit being the distance between the two points. The function value of the first point is slightly enlarged and reduced. For these three situations approximation models are generated with $d = 0.3$ (figure 3.1) and $d = 0.9$ (figure 3.2). At the same time, in figure 3.3 the corresponding polynomial approximation is given.

The two point approximation for d values near one (and therefor θ near zero) becomes a third order polynomial in a Taylor series expansion. This means that for d values near one the approximation around the present design site can show a rather wild extrapolative behaviour if function values and derivatives of the different designs are difficult to predict for the polynomial like model. This behaviour may be kept under control by not making too large steps from cycle to cycle. For d values near one the basic model has much less influence than for smaller values.

3.3 Test example

Consider the cantilever beam problem described by Svanberg (1987). The cantilever beam is plotted in figure 3.4. The design variables are the heights x_j of the different beam elements while the thicknesses are held fixed. Svanberg (1987) formulated the problem analytically as: minimize the weight:

$$F_{obj} = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5) \tag{3.6}$$

subject to the displacement constraint of the tip:

$$g = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \leq c \tag{3.7}$$

with $c = 1$, in the design space:

$$x_j > 0 \qquad j = 1, ..., 5 \tag{3.8}$$

The optimal solution is given by $x_1 = 6.016$, $x_2 = 5.309$, $x_3 = 4.494$, $x_4 = 3.502$, and $x_5 = 2.153$ with corresponding weight of 1.340.

Starting from initial design $x_j = 5.0$ ($j = 1, ..., 5$) it is tried to solve the problem with sequential linear approximate optimization keeping the movelimit constant 0.5. Although the errors made during the optimization remain within 10 % the process will not converge without the reduction of the movelimit (see table 3.1), because there is only one constraint. This means that the approximate optimum is always found on the boundary of the search subregion. Starting from cycle 8, with consequently reducing the movelimit, about five or six extra cycles are needed to get the infeasibility within 0.001 and the weight within 0.1 % of the known true optimal value.

Now, the cantilever beam problem is solved by the mid-range concept with the linear approximation of the last design cycle as basic model. The movelimit is set to 0.5. From the sequential linear approximate optimization process it can be concluded that for this value the steps made are not too large, since the errors stay within 10 %. The movelimit is kept constant and is not reduced such that the mid-range concept has to take care for convergence itself. The simulations are performed for d values of $0.001, 0.01, 0.1, 0.3, 0.5, 0.7$ and 0.9. The results are summarized in table 3.1.

Bad convergence for small d values is the first conclusion that can be drawn from this table. Around the optimum, relatively many computations are made before the true optimum is found. This is caused by the fact that the linear approximation is insufficiently adapted in the search subregion. In the major part of this region, the approximation is still almost linear such that, like the linear concept, the approximate solution is found on the boundary of the search subregion. For larger d values, better convergence is achieved, because the behaviour of the approximation is influenced in the complete subregion. A better reconstruction of the constraint results, making that the optimum is catched faster within the movelimits. However, remark that the convergence improves for values of d getting more near one. For these values the condition of matrix T becomes rather bad for growing number of designs, while the basic model has less influence, thus making our prior considerations about an improved linear approximation out of context. This is illustrated by taking the basic model constant c and repeating the optimization with $d = 0.9$. About the same convergence is found (see table 3.2).

Secondly, one can question the usefulness of taking into account the structural analysis and design sensitivity analysis results of all design sites, if one compares the first six cycle results of the linear concept with the results of the mid-range concept given in table 3.1. During the first design cycles the multi-point concept has hardly improved the optimization process compared with the linear concept. Only in the convergence stage a significant improvement has been achieved. The influence of the first cycle designs on the performance of the mid-range concept is investigated by taking the design of the 4, 5 and 6 th cycle of the linear concept as initial design of the mid-range concept with $d = 0.9$. Table 3.3 shows that for these three cases convergence occurs in six extra mid-range cycles. So in spite of the near optimal start design still six additional cycles are required. Therefore, a direct conclusion is that the automatic convergence for larger d-values seems to be dependent on the number of design variables. This, because of the six extra cycles for the five dimensional cantilever beam problem.

3.4 Discussion

A cantilever beam optimization problem of five design variables and one non-linear constraint has been used to illustrate the performance of the SPP mid-range concept. This cantilever beam problem is an underconstrained optimization problem, which means that the number of constraints that determine the optimum is smaller than the number of design variables. Therefore, the optimum is determined by the curvature of the displacement constraint. That is why sequential linear programming does not automatically converge without shrinkage of the search subregion.

For an underconstrained optimum, curvature in the approximate objective function and constraints is essential. In the case of a constrained optimum, curvature in the approximations may not be necessary. A linear approximation concept can perform quite well since the optimum lies in the vertex of the constraints. However, non-linearity of the constraints may give rise to a lot of linear approximate optimization cycles. Addition of curvature to the linear approximations can improve the convergence rate.

In this chapter, the basic model of the SPP mid-range approximation (2.4) has been chosen equal to the linear approximation in the design site of the present design cycle, to develop a concept that improves the linear concept. This improvement should be established by the second part of model (2.4) which adds curvature to the linear basic model. For the cantilever beam example, the convergence of the sequential approximate optimization process has been improved, compared with the linear concept. It has been noticed that the best improvements occur for highly correlated design points, i.e. for d values near one, and that a near optimum start design still requires six additional design cycles to reach the optimum solution.

High correlations give rise to unpredictable polynomial like approximations. This is caused by the exact prediction of function values and derivatives. To try to avoid this behaviour, the errors $e_j^{(p)}$ have been kept within ten to fifteen percent for every p th design cycle. However, this sensitive extrapolative behaviour remains undesirable.

The second observation of the six additional design cycles suggests a dependence between convergence and number of design variables. Starting from a near optimum design, six cycles were necessary to generate an approximation of the constraint with a correct curvature. A linear approximation of equation (2.4) for θ near zero (d near one) is given by:

$$\tilde{f}_L(\mathbf{x}) = \mathbf{a}_0^T\mathbf{z} + \sum_{i=1}^{N} \mathbf{a}_i^T\mathbf{z} - \theta \sum_{i=1}^{N} \mathbf{a}_i^T\mathbf{z} \sum_{k=1}^{n} \{(s_k)_i - x_k\}^2 \tag{3.9}$$

This polynomial model consists of the terms x_p, x_p^2, x_p^3, $x_p x_q$ and $x_p x_q^2$ with $p, q = 1, ..., n$ and a total of terms of $n(n+1)$. These terms are not necessarily independent, depending on N. For every design cycle one function value and n derivatives are calculated. Therefore, to independently estimate the parameters of all terms at least n cycles are necessary. Then, the number of cycles required for convergence is linearly dependent on the number of design variables. In the case of the cantilever beam at least 5 cycles are required.

Summarizing, the general conclusion is that the introduction of curvature in the approximations by a mid-range SPP concept with basic linear model as proposed in this chapter can not generally improve the sequential linear approximate optimization process. Only in lower dimensional optimization problems a better convergence may occur.

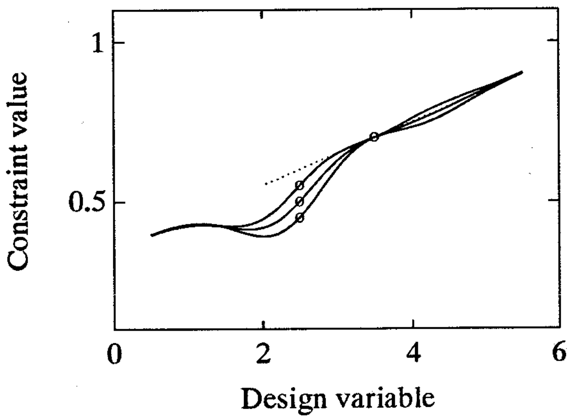

Figure 3.1: Two point approximation with $d = 0.3$.

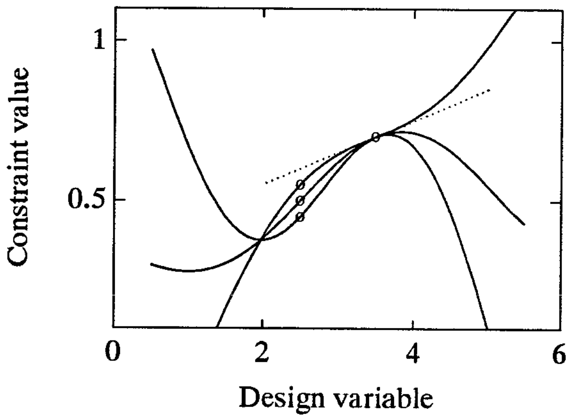

Figure 3.2: Two point approximation with $d = 0.9$.

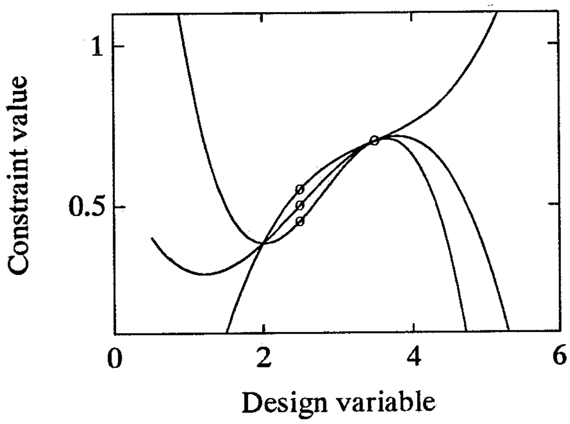

Figure 3.3: Polynomial approximation.

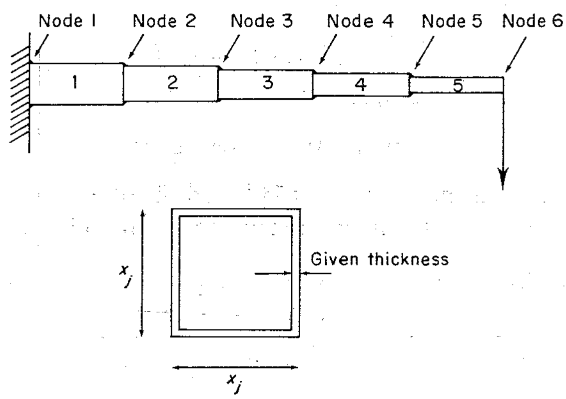

Figure 3.4: Cantilever beam.

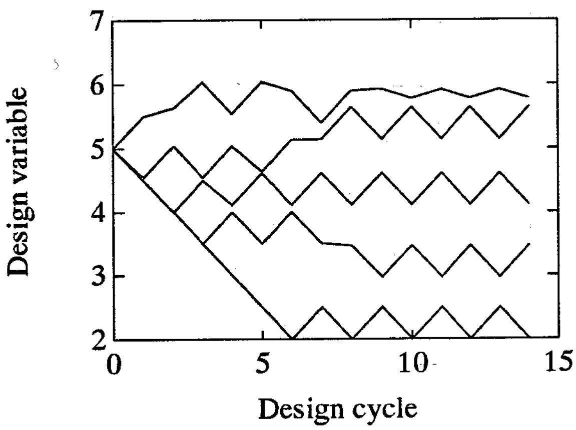

Figure 3.5: Design variable behaviour of the linear concept.

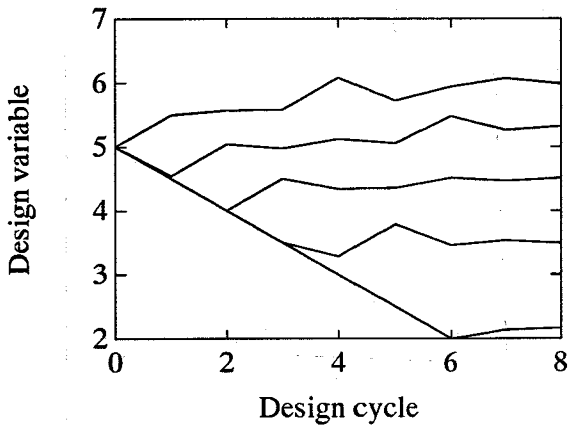

Figure 3.6: Design variable behaviour of the mid-range concept with linear basic model and $d = 0.9$.

cycle	linear concept	midrange concept $d = 0.001$	$d = 0.01$	$d = 0.1$	$d = 0.3$	$d = 0.5$	$d = 0.7$	$d = 0.9$
0	1.56	1.56	1.56	1.56	1.56	1.56	1.56	1.56
	0	0	0	0	0	0	0	0
1	1.469	1.469	1.469	1.469	1.469	1.469	1.469	1.469
	0.0582*	0.0582*	0.0582*	0.0582*	0.0582*	0.0582*	0.0582*	0.0582*
2	1.415	1.415	1.416	1.414	1.413	1.412	1.411	1.411
	0.0523*	0.0523*	0.0478*	0.0535*	0.0572*	0.0605*	0.0627*	0.0636*
3	1.378	1.378	1.378	1.377	1.377	1.376	1.376	1.376
	0.0675*	0.0675*	0.0663*	0.0680*	0.0700*	0.0723*	0.0521*	0.0460*
4	1.353	1.353	1.352	1.353	1.352	1.353	1.357	1.362
	0.0682*	0.0679*	0.0735*	0.0516*	0.0584*	0.0669*	0.0539*	0.0135*
5	1.328	1.328	1.324	1.32	1.321	1.327	1.33	1.337
	0.0695*	0.0693*	0.0833*	0.0837*	0.0840*	0.0527*	0.0400*	0.0340*
6	1.32	1.32	1.319	1.313	1.314	1.324	1.331	1.335
	0.0781*	0.0781*	0.0791*	0.0706*	0.0670*	0.0603*	0.0411*	0.0160*
7	1.32	1.321	1.318	1.333	1.334	1.331	1.337	1.34
	0.0806*	0.0793*	0.0741*	0.0423*	0.0278*	0.0374*	0.0209*	0.0010
8	1.318	1.319	1.32	1.328	1.333	1.335	1.338	1.34
	0.0698*	0.0657*	0.0774*	0.0676*	0.0478*	0.0936*	0.0091*	0.0003
9	1.319	1.322	1.314	1.319	1.335	1.329	1.34	
	0.0917*	0.0902*	0.0910*	0.0948*	0.0576*	0.111*	0.0009	
10	1.31	1.313	1.323	1.33	1.336	1.342		
	0.0892*	0.0859*	0.0653*	0.0263*	0.0106*	0.0147*		
11	1.319	1.326	1.32	1.331	1.339	1.333		
	0.0927*	0.0356*	0.0831*	0.0581*	0.0171*	0.0436*		
12	1.31	1.333	1.325	1.334	1.34	1.338		
	0.0891*	0.0335*	0.0360	0.0222*	0.0008	0.0053*		
13	1.319	1.326	1.331	1.338		1.34		
	0.0927*	0.0670*	0.0890*	0.0067		0.0005		
14	1.31	1.325	1.324	1.34				
	0.0891*	0.0904*	0.0744*	0.0015				

Table 3.1: Optimization history of the cantilever beam problem using a linear and a SPP mid-range concept with a linear basic model and a constant movelimit 0.5. The upper entry of each pair is the weight and the second is the infeasibility. Index * means: movelimit is active.

cycle	0	1	2	3	4	5	6	7	8
	1.56	1.469	1.413	1.378	1.365	1.343	1.334	1.34	1.34
	0	0.0582*	0.0561*	0.0442*	0.0018*	0.0079*	0.0275*	0.0012	$3\,10^{-5}$

Table 3.2: Optimization history of the cantilever beam problem using a mid-range SPP concept with a constant basic model c, $d = 0.9$ and a constant movelimit 0.5. The upper entry of each pair is the weight and the second is the infeasibility. Index * means: movelimit is active.

extra	start cycle linear concept				
cycle	4	5	6	7	8
0	1.353	1.328	1.32	1.32	1.318
	0.0682	0.0695	0.0781	0.0806	0.0698
1	1.328	1.32	1.32	1.318	1.319
	0.0695*	0.0781*	0.0806*	0.0698*	0.0917*
2	1.319	1.325	1.324	1.327	1.324
	0.0749*	0.0603*	0.0727*	0.0675*	0.0884*
3	1.329	1.334	1.334	1.332	1.334
	0.0782*	0.0681*	0.0463*	0.0958*	0.0419*
4	1.324	1.333	1.336	1.33	1.338
	0.1229*	0.0292*	0.0283*	0.0777*	0.0044*
5	1.336	1.339	1.338	1.340	1.337
	0.0108*	0.0029	0.0040*	0.0082*	0.0679*
6	1.34	1.34	1.34	1.336	1.34
	$1\,10^{-4}$	$2\,10^{-5}$	$6\,10^{-5}$	0.0326*	$5\,10^{-5}$
7				1.339	
				0.0189*	
8				1.34	
				$-2\,10^{-5}$	

Table 3.3: Optimization history of the cantilever beam problem using a mid-range SPP concept with a linear basic model, $d = 0.9$ and a constant movelimit 0.5. The start design is equal to the 4,5 or 6 th cycle design of the linear concept. The upper entry of each pair is the weight and the second is the infeasibility. Index * means: movelimit is active.

Chapter 4

A constant basic model

4.1 Introduction

Instead of a linear approximation, now a constant basic model is introduced:

$$f_b(\mathbf{x}) = \beta \tag{4.1}$$

It is supposed that only constraints have to be approximated and that the optimum design is determined by the constraints. Then, if we take β larger than c_h in equation (1.2) for all constraints $(h = 1, .., m)$, a more or less conservative approximation concept results depending on the values of $\theta_k (k = 1, ..., n)$. In accordance with the preceding chapter, it is again assumed that θ_k can be chosen equal for all k, i.e. the behaviour of the constraints is about the same in every design variable direction.

4.2 Influence of θ

In chapter two it has already been noted that for design sites with correlation function values near zero, an approximation results which tends to the basic model. Near design sites at which function values and derivatives are present, the approximation is adapted to predict this data exactly. A linear approximation as basic model proved to be not useful. However, in the case of a constant basic model, for certain values of β and θ a complete different behaviour is found.

Consider figure 2.1. Here, a constant basic model, lying above the critical constraint value, has been chosen. Far away from the computed function value and derivative, the correlation function value is near zero, and therefore the approximation almost equals the basic line. Since the basic line lies above the critical constraint value, a conservative approximation results. The conservativeness can be adjusted by the width of the correlation function (i.e. the value of θ) and indirectly also by the position of the basic line (i.e. the value of β). In this report β is kept constant, while θ can be used to change the curvature of the approximation.

As a consequence of the conservativeness, the optimum can be approached from the feasible region, when starting from a feasible initial design. During the optimization, it is tried to choose a parameter θ value such that the approximation is more conservative than the true functional behaviour. For large design variable changes the conservative approximations of the constraints may be inaccurate, but they always try to avoid you from making too large steps. This means that movelimits are not necessary anymore. The approximate constraints create their own search subregion.

Near the optimum, the design variable steps automatically become smaller without changing θ. If the approximations are conservative enough, no oscillations will occur. This step refinement leads to a more accurate approximation of the constraints. For smaller steps, neighbouring analysed design sites are situated more closely to each other. Then, the approximation is locally adapted by the mid-range SPP algorithm, which tries to exactly predict all available function values and derivatives of former design cycles. For near optimum designs lying closely to each other, the function values and derivatives do not change very much from one to another, and therefore it is expected that the approximation model will predict this data without difficulty. Although this data of former designs can not provide all second order information, it can influence the conservativeness of the approximation, and therefore improve the convergence behaviour. At the start of the approximate optimization process larger design variable steps will be made, and the influence of former design cycle results will be less than during the convergence stage.

4.3 Test example

Consider the cantilever beam example of section 3.3. The basic model is fixed at $1.15c$. In the present configuration, the initial design should be feasible or nearly feasible, but in any case be smaller than $1.15c$ to get an appropriate conservative approximation. For the cantilever beam this is satisfied.

Movelimits are not necessary, because of the conservative nature of the constraint approximation. However, to have an opportunity to compare the results with the preceding chapter, we do introduce a movelimit, and set it to a constant value of 0.5. Also the same relation between θ and d as given in 3.7 is assumed. So a d value near zero will result in a very conservative approximation.

Calculations are performed for d values of 0.01, 0.1, 0.3, 0.5, 0.7 and 0.9. Again the optimization is defined to be converged for an infeasibility within 0.001 and the weight within 0.1 % of the known true optimal value. The results are summarized in table 4.1.

For $d = 0.01$ the optimization process slowly approaches the optimum solution from the feasible design space. Too high a conservativeness has been introduced. The maximum absolute design variable step made is about 0.27, and never a movelimit has become active. Increasing d results in a less conservative behaviour of the approximate constraint, and larger design variable changes can be made. For a parameter d value that is equal to or larger than 0.3, movelimits have (unnecessarily) bounded the design variable changes during some design cycles. It is clearly visible that the best convergence occurs for $d = 0.3$. All calculated designs are feasible, and convergence is reached within 7 cycles. Higher d values give rise to a too less conservative approximation, and therefore infeasible designs occur. Again, $d = 0.9$ leads to an optimization process which is comparable with the results of the last column of table 3.1.

Illustrative is the comparison of the design variable optimization history of: the single point linear approximation concept, the mid-range SPP concept with linear approximation as basic model and $d = 0.9$, and the mid-range SPP concept with constant basic model $\beta = 1.15c$ and $d = 0.3$ (see figures 3.5, 3.6 and 4.1). It is clearly visible that the latter concept smoothly converges to the optimum design variable values, in contrary to the other concepts.

Starting from a near optimum design, the optimum design will be found within one or two extra design cycles for a correct conservativeness of the approximate constraint, since the optimum is determined by the curvature of the constraint. No oscillations will occur.

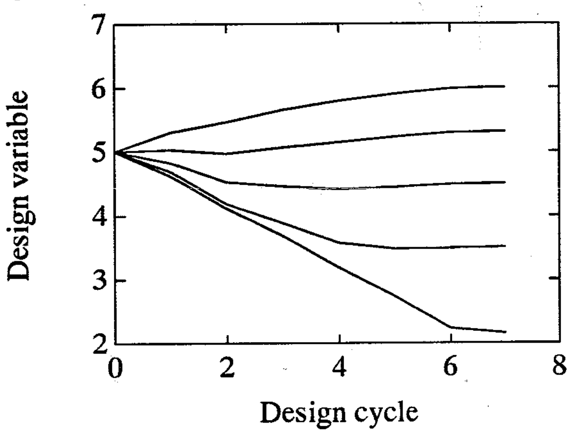

Figure 4.1: Design variable behaviour of the mid-range concept with constant basic model 1.15c and $d = 0.3$.

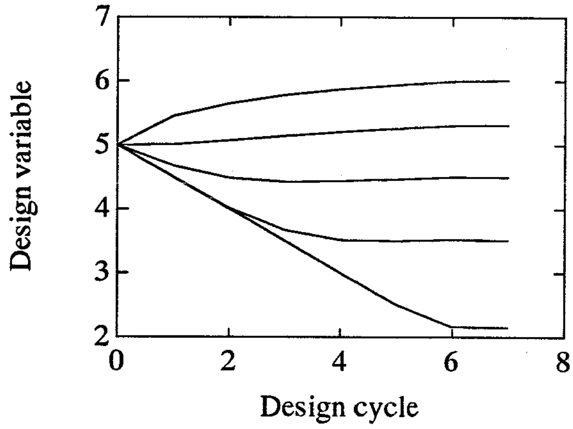

Figure 4.2: Design variable behaviour of the single point version of the mid-range concept with constant basic model 1.15c and $d = 0.5$.

4.4 Discussion

A better convergence behaviour of the mid-range concept is found, if a constant basic model is used instead of a linear approximation. Then, the correlations between different design sites determine the conservativeness of the approximation. However, just because of these lower correlation values, one can question the effect of previous design cycle data on the quality of the approximation.

To investigate the additional value of the data of previous design cycles, the cantilever beam problem has been solved by the same 'mid-range' concept, except that computational results are only known at the present design site. So the special single-point case of the mid-range concept is considered. The results are summarized in table 4.2.

For small d value the approximation of the constraint is too conservative, and a very slow convergence is noticed. The approximation is almost linear for a d value near one and therefore the same convergence behaviour with oscillations occurs. Near d is 0.5 the best convergence is found. All analyzed designs have been feasible. By comparison of table 4.1 and 4.2 can be concluded that the conservativeness of the approximation is influenced by analysis data of former design cycles. The approximation is made less conservative for $d = 0.3$ such that convergence is reached within 7 instead of 9 design cycles. However, if parameter d can be correctly valued during the optimization, the effect of the multi-point concept as proposed in this report may appear to be marginal.

cycle	midrange concept					
	$d = 0.01$	$d = 0.1$	$d = 0.3$	$d = 0.5$	$d = 0.7$	$d = 0.9$
0	1.56 0	1.56 0	1.56 0	1.56 0	1.56 0	1.56 0
1	1.55 -0.0217	1.541 -0.0359	1.526 -0.0526	1.506 −0.0553*	1.487 −0.0085*	1.474 0.0368*
2	1.517 -0.0215	1.489 -0.0226	1.451 −0.0091*	1.426 0.0162*	1.43 0.0043*	1.418 0.0442*
3	1.489 -0.0156	1.453 -0.0188	1.419 -0.0241	1.403 −0.0232*	1.391 0.0043*	1.38 0.0350*
4	1.462 -0.0145	1.418 -0.0175	1.38 −0.0091*	1.364 0.0061*	1.367 −0.0034*	1.366 −0.0003*
5	1.436 -0.0124	1.388 -0.0118	1.36 -0.0127	1.348 −0.0059*	1.345 0.0027*	1.344 0.0041*
6	1.414 -0.0103	1.365 -0.0095	1.341 −0.0011*	1.339 0.0026	1.339 0.0038	1.336 0.0257*
7	1.394 -0.0092	1.348 -0.0049	1.34 $-4\,10^{-5}$	1.34 $5\,10^{-6}$	1.34 $7\,10^{-6}$	1.339 0.0018
8	1.376 -0.0075	1.341 -0.0012				1.34 $1\,10^{-5}$
9	1.361 -0.0058	1.34 $-3\,10^{-5}$				
10	1.349 -0.0040					
11	1.342 -0.0018					
12	1.34 -0.0003					

Table 4.1: Optimization history of the cantilever beam problem using the mid-range concept with constant basic model 1.15c and a constant movelimit 0.5. The upper entry of each pair is the weight and the second is the infeasibility. Index * means: movelimit is active.

cycle	single point 'midrange' concept					
	$d = 0.01$	$d = 0.1$	$d = 0.3$	$d = 0.5$	$d = 0.7$	$d = 0.9$
0	1.56	1.56	1.56	1.56	1.56	1.56
	0	0	0	0	0	0
1	1.55	1.541	1.526	1.506	1.487	1.474
	-0.0217	-0.0359	-0.0526	−0.0553*	−0.0085*	0.0368*
2	1.533	1.511	1.48	1.449	1.435	1.421
	-0.0247	-0.0374	-0.0492	−0.0428*	−0.010*	0.0369*
3	1.515	1.483	1.439	1.405	1.394	1.381
	-0.0237	-0.0333	-0.0397	−0.0277*	−0.0058*	0.0430*
4	1.498	1.457	1.406	1.374	1.369	1.357
	-0.0218	-0.0286	-0.0306	−0.0197*	−0.0065*	0.0241*
5	1.483	1.435	1.38	1.352	1.347	1.338
	-0.0198	-0.0244	-0.0227	−0.0142*	−0.0019*	0.0493*
6	1.468	1.415	1.36	1.34	1.336	1.315
	-0.0180	-0.0207	-0.0151	-0.0010	0.0118*	0.064*
7	1.455	1.398	1.348	1.34	1.336	1.327
	-0.0163	-0.0174	-0.0078	$-2\ 10^{-6}$	0.0186	0.0672*
8	1.443	1.384	1.342		1.336	1.318
	-0.0148	-0.0145	-0.0024		0.0162*	0.0589*
9	1.431	1.372	1.34		1.332	1.326
	-0.0135	-0.0118	-0.0004		0.0285*	0.0690*
10	1.421	1.362			1.336	1.318
	-0.0122	-0.0092			0.0175*	0.0587*
11	1.411	1.354			1.332	1.326
	-0.0111	-0.0067			0.0290*	0.0690*
12	1.402	1.348			1.336	1.318
	-0.101	-0.0046			0.0179*	0.0587*
13	1.394	1.344			1.332	1.326
	-0.0091	-0.0028			0.0291*	0.0690*
14	1.386	1.342			1.336	1.318
	-0.0082	-0.0015			0.0180*	0.0587*

Table 4.2: Optimization history of the cantilever beam problem using the single point version of the mid-range concept with constant basic model 1.15c and a constant movelimit 0.5. The upper entry of each pair is the weight and the second is the infeasibility. Index * means: movelimit is active.

Chapter 5

Conclusions and recommendations

A single-point-path mid-range approximation concept has been studied. The concept generates an approximation of objective function or constraint that consists of a basic model part and a part exactly fitting the residual function values and derivatives. Two different basic models have been investigated: a constant model and a model which is equal to the linear approximation in the design site of the current optimization cycle. The mid-range model with linear basic model does not generally improve the sequential linear approximate optimization process. In the case of a constant basic model a better convergence of the optimization of a cantilever beam has been achieved. However, it is not exactly clear what the additional value is of the analyses and design sensitivity analyses of the previous design cycles.

It is recommended to further investigate the approximation concept with constant basic model starting from the single point version. A strategy has to be developed which determines the conservativeness of the approximation in every design variable direction at the current design cycle. Further, it has to be studied if analysis results of more than one design point can be effectively used to improve the approximation, and how intermediate design variables and responses can be implemented. Instead of using equation (2.4) one can think for example of

$$\tilde{f}(\mathbf{x}) = \beta + f_a(\mathbf{x}, \mathbf{x}_0) R(\mathbf{x}, \mathbf{x}_0) \tag{5.1}$$

Model $f_a(\mathbf{x}, \mathbf{x}_0)$ is a local approximation around $\mathbf{x}_0$ that can be based on intermediate (response) variables and analysis results of previous design cycles. This data might also be used to determine the conservativeness (i.e. the unknown parameters of the correlation function R) of the approximation. In any case, a comparison should be made with the method of moving asymptotes which is an alternative concept to generate conservative approximations.

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