Safety stocknorms in divergent systems with non-identical final products

BMA

Safety stocknorms in divergent systems with non-identical final products

by K. van Donselaar

Report ARW-03 TUE/BDK/ORS/88/04
Technische Universiteit Eindhoven

Safety stocknorms in divergent systems
with non-identical final products.

by K. van Donselaar and J. Wijngaard.

1. Introduction

The goal of this paper is to determine safety stocknorms for divergent systems with non-identical final products. The final products are non-identical with respect to their lot-sizes and demand characteristics. The stocknorms should yield a pre-specified service level.

Stocknorms for divergent systems with identical products have been investigated already in [1]. The major conclusions for those systems were: 1) Divergent systems may face imbalance. A divergent system is out of balance if the inventory positions of the final products are not equivalent, e.g. due to large lot-sizes. 2) If divergent systems are controlled integrally, the integral stocknorm has to take into account the presence of imbalance if this imbalance is relatively large. Corresponding formulas for integral stocknorms are determined for systems with identical products.

If these results are to be extended to the non-identical products case several difficulties arise:

- the definition of imbalance is more complex. Simple defining imbalance as the deviation of the final products' inventories from the average inventory no longer makes sense. In fact it will become apparent that different definitions are needed for systems with and without depot.
- the allocation is more complex. For identical products allocation was simple: allocate to the product with the lowest inventory. If mean and standard deviation of demand as well as the lot-sizes and consequently the reorderpoints of all products are different, it is no wonder that the allocation has to be more complex.
- the system service level is more complex. Simply defining the system service level as the average of all final products' service level would be unfair to the products with a large average demand; Raising

their service level with 5% may require a larger amount of inventory than an equivalent raise of the service level of products with a smaller average demand. Therefore it is suggested to use as a system service level the weighted average of the final products' service level. The weight factors are equal to the ratio's of the final product's average demand over total average demand.

These aspects will be discussed in Section 2, where imbalance for a divergent system with non-identical products will be defined.
After finding an estimator for the variance of imbalance in Section 3, the quality of stocknorms based on this estimator is tested in Section 4.

It will appear that these stocknorms perform quite well. More important however is the observation that elements other than imbalance are more disruptive to the system. For a system with depot e.g. the main disruption is due to so-called 'dead stock'. Some remedies against these disruptive elements are suggested in Section 5. Conclusions are drawn in Section 6.

2. Definition of imbalance

2.1 Implicit definition

In [1] imbalance was studied for divergent systems with identical final products. There imbalance for a final product j was defined as the difference between the actual economic inventory of that final product (I_j) and the average inventory level of all final products:

$$\text{imbalance}_j = I_j - \Sigma I_i/N \qquad (1),$$

When the final products have different demand distributions and/or different lot-sizes, this definition no longer holds. Suppose for example that two final products each have 100 products on hand. According to (1) the system would be balanced. This makes sense if both products have equal average demand. However if the average demand of one of these products is ten times as high as the average demand of the other product, it is crystal clear, that the system is out of balance.

A sensible way of defining imbalance seems to be

$$\text{imbalance}_j = I_j - I_j^* \qquad (2),$$

where I_j^* is the amount of inventory for final product j after an optimal (imaginary) re-allocation of all economic inventory (ΣI_j) took place without lot-sizing-restrictions.
Note that $\Sigma E[\text{imbalance}_j] = 0$ since $\Sigma I_j = \Sigma I_j^*$. The variance of imbalance for final product j will be denoted by $\sigma^2_{imb,j}$. So $\sigma^2_{imb,j} = var(I_j - I_j^*)$.

2.2 Re-allocation

Having defined imbalance as in formula (2), the remaining problem to be solved here is finding the optimal re-allocation rule in case of a system with non-identical products and without lot-sizes. Once that re-allocation rule is defined, imbalance is known according to the above definition. The system with and without depot will be dealt with separately.

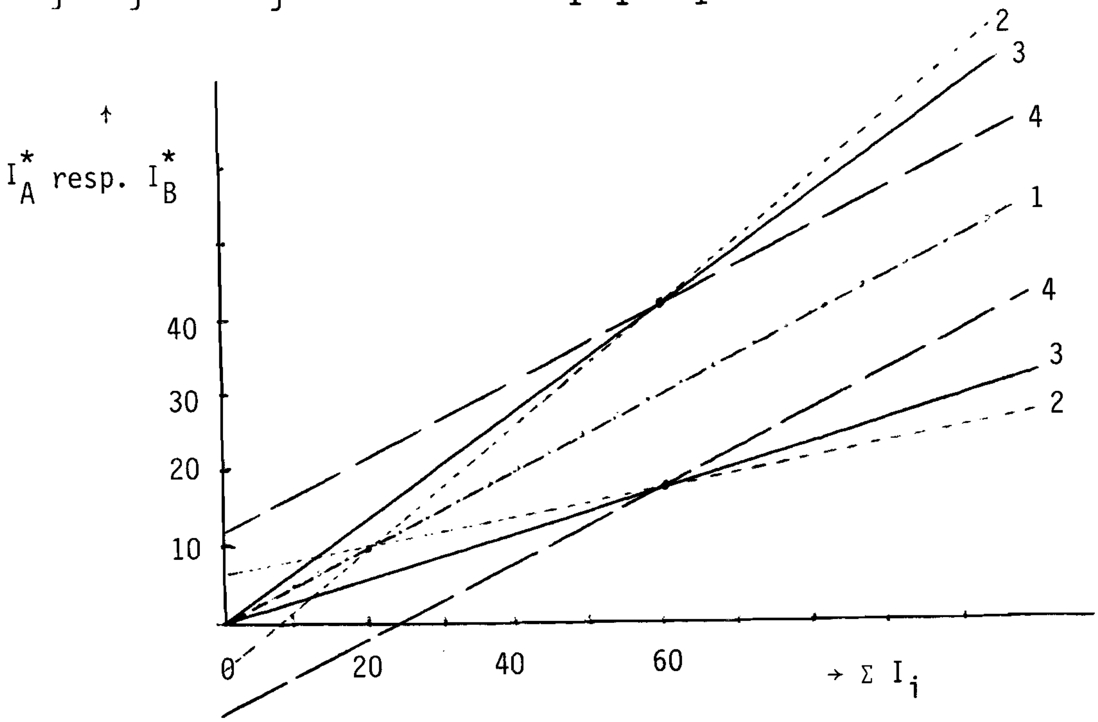
2.2.1 Re-allocation in a system with depot

Examples of re-allocation rules for the system with depot, which can be used to define imbalance, are:

1) Equalize the run-out-time (that is: the ratio of inventory over average demand) for all final products. This yields:
   $I_j^* = c \cdot \mu_j$, where $c = \Sigma I_i / \Sigma \mu_i$ (since $\Sigma I_j^* = \Sigma I_j$).
   In this way all final products will tend to run out of stock simultaneously if their leadtimes are equal.
2) Equalize the expected service-level for all final products.
   In case of normal distributed demand this yields:
   $I_j^* = (\ell_j+1)\mu_j + k^* \cdot \sqrt{(\ell_j+1)}\sigma_j$,
   where $k^* = (\Sigma I_i - \Sigma(\ell_i+1)\mu_i) / \Sigma\sqrt{(\ell_i+1)}\sigma_i$.
3) Equalize the stocknorm-ratio for all final products.
   This results in:
   $I_j^* = c \cdot r_j$, where $c = \Sigma I_i / \Sigma r_i$.
4) Fill the inventory of the final products up to their stocknorm and allocate the difference between total inventory available and the sum of the final products' stocknorms according to the average demand of the final products.

In this way all final products' economic inventories will tend to drop below their stocknorm simultaneously. The corresponding formula is:

$$I^*_j = r_j + c \cdot \mu_j, \quad \text{where } c = \Sigma(I_i - r_i)/\Sigma\mu_i \qquad (3)$$



The proposed allocations using different allocation rules for a system with two final products A and B having the following characteristics: $\mu(A)=\mu(B)=10$, $\sigma(A)=20$, $\sigma(B)=5$, $\ell(A)=\ell(B)=0$, $r(j)=\mu(j)+1.6\sigma(j)$ so $r(A)=42$, $r(B)=18$. The numbers in this figure correspond with the number of the allocation rule described above.

Figure 1.

It appears that every rule has its weak and strong points. Note for example that all rules take account of the variance for each final product, except for the first rule. So in case $\Sigma I=\Sigma r$, the first rule will allocate $0.5\Sigma r_j$ in stead of r_j to each product (see Figure 1). That implies that the expected service level of A and B will differ. In case $\Sigma I=\Sigma r$ and allocation rule 1 is used, the expected service levels will be $\Phi(1)\approx 84\%$ and $\Phi(4)\approx 100\%$ for A resp. B. With the other allocation rules the expected service levels of A and B will be the same: they will both be equal to $\Phi(1.6)\approx 95\%$.

The second and third rule will tend to allocate too much inventory to the product with the largest uncertainty if total inventory far exceeds the sum of the stocknorms (e.g. due to large lot-sizes).

Suppose for example that ΣI_j equals 220. According to allocation rule 2 the 'optimal' allocation is:
$I_A^* = 10 + [(220-20)/25] \cdot 20 = 170$, $I_B^* = 10 + [(220-20)/25] \cdot 5 = 50$.
Both products have equal service levels for the next period: $\Phi(8)$. The run-out-times however for products A and B are resp. 17 and 5 periods. Suppose that the total inventory of 220 products was meant to last for 6 periods. The expected service level for products A and B for 6 periods from now will be $\Phi((170-6 \cdot 10)/20\sqrt{6}) \approx 99\%$ resp. $\Phi((50-6.10)/5\sqrt{6}) \approx 21\%$. The reason for this imbalanced result is the greediness of allocation rule 2: It does not take into account the fact that total inventory may be needed for more than $(\ell+1)$ periods. Analogous results can be derived for allocation rule 3.

The fourth rule will tend to allocate too much inventory to the product with the largest uncertainty if total inventory is far below the sum of the stocknorms. Let e.g. $\Sigma I = \Sigma\mu = 20$. Then according to allocation rule 4: $I_A^* = 42 - 2 \cdot 10 = 22$ and $I_B^* = 18 - 2.10 = -2$. The expected service level for the next period equals $\Phi((22-10)/20) \approx 73\%$ for product A and $\Phi((-2-10)/5) \approx 1\%$ for product B. It seems more appropriate in this case to select $I_A^* = I_B^* = 10$.

Since in this paper the attention is focussed on getting an indication of the impact of imbalance rather than on finding the overall optimal allocation rule, a simple and robust allocation rule is selected: allocation rule number 4.

Note that in the special case of identical products all these policies are the same: the optimal allocation then with each of the above policies is to allocate to each product the average amount of inventory.

2.2.2 Re-allocation in a system without depot

In a system without depot a different re-allocation rule has to be chosen since there are no longer stocknorms for the final products. Federgruen and Zipkin [3] investigated the allocation of an amount of inventory among several final products with the restriction that all inventory had to be allocated. Their research revealed a.o. that a good, simple and robust allocation rule is the rule proposed by Eppen and Schrage [2]:

$$I'_j = (\ell_j+v+1)\mu_j + k' \cdot \sqrt{(\ell_j+v+1)} \cdot \sigma_j \qquad (4)$$

where $k' = [\Sigma(I_i-(\ell_i+v+1)\mu_i) + d] / \Sigma[\sqrt{(\ell_i+v+1)}\cdot\sigma_i]$

with I'_j, I_j : the inventory of final product j after, resp. before allocation.

$v+1$: the average time between two orders = $Q_{comm}/\Sigma\mu$.

d : amount available to be allocated.

This is a good rule, except for the cases where the system lot-size and variances are large. For those cases Federgruen and Zipkin propose a refinement of this formula:

$$v = \min([\Sigma I_i+d-\Sigma(\ell_i+1)\mu_i]^+/\Sigma\mu_i,\ Q_{comm}/\Sigma\mu-1).$$

This refinement states that if the total inventory which is available for the final products in the depot or downstream is small, that is: less than the lot-size of the common part plus the average demand for the final products times their leadtime, then the expected time up to the arrival of the next lot for the common part should be decreased accordingly.

The allocation rule (4) is based on the fact, that allocation takes place just after an order has arrived in the depot. Since that order contains v+1 periods demand on average, the allocation is chosen such, that the expected service level after ℓ+v+1 periods is equal for all products. However, imbalance is measured every period and not only after an order is allocated. The allocation rule mentioned above has to be adapted to this to come up with a good re-allocation rule to measure the imbalance. A sensible adaptation seems to be:

$$I^*_j = (\ell_j+v+1)\mu_j + k \cdot \sqrt{(\ell_j+v+1)}\cdot\sigma_j \qquad (5)$$

where $k = \Sigma(I_i-(\ell_i+v+1)\mu_i) / \Sigma[\sqrt{(\ell_i+v+1)}\cdot\sigma_i]$

and $v = \min ([\Sigma I_i-\Sigma(\ell_i+1)\mu_i]^+/\Sigma\mu_i,\ [Q_{comm}/\Sigma\mu-1-t]^+).$

with t : the number of periods after the last order arrived in the depot.

3. An estimator for the variance of imbalance

The goal of this paper is to gain insight in the relationship between stocknorms and service level. For a divergent system with identical products this relationship appeared to depend on the variance of imbalance. Generalisation of that relationship to the non-identical products case seems to be rather straightforward:

$$r_{comm} = \Sigma(\ell_{comm}+\ell_j+1)\mu_j - Q_{comm}/2 +$$

$$+ k\surd \left[\frac{Q^2_{comm}}{12} + \ell_{comm}\Sigma\sigma^2_j + \{\Sigma\surd\,[(\ell_j+1)\sigma^2_j + \sigma^2_{imb,j}]\,\}^2\,\right] \quad (6)$$

where r_{comm} is the integral stocknorm for the common part,
ℓ_{comm}, ℓ_j are the leadtimes for the common part resp. final product j,
Q_{comm} is the lot-size for the common part and
μ_j, σ_j are the average and standard deviation of demand.
For the relationship between the safety factor k in this formula and the expected service level, the reader is referred to [1], Section 5. According to formula (6) an estimator for the variance of imbalance is needed in order to be able to determine the stocknorm.

To derive an estimator on the variance of imbalance for the general systems considered here seems to be very complex. The only estimator at hand for the moment is the estimator for the system with identical products. By assuming that the variance of imbalance for each final product in the system with non-identical products is the same as in a system with all products being identical to that final product, the following estimator will result:

$$\hat{\sigma}^2_{imb,j} = \begin{cases} 0 & \text{if } N=1. \\ Q^2_j/12 & \text{if a depot is present and } N\neq 1. \\ \frac{Q^2_j}{12}\cdot\frac{(N-1)}{N}\cdot\frac{(N+2)}{N} + 0.5\left(\frac{Q_{comm}}{\Sigma\mu_j} - 1\right)\cdot\frac{N-1}{N}\cdot\sigma^2_j & \text{elsewhere.} \end{cases} \quad (7)$$

This certainly will not be a good estimator for the variance of imbalance for final product j. This can be easily seen from the fact that for N=2 the variance of imbalance for the two final products should be equal (since $\text{imbalance}_1 = -\text{imbalance}_2$). If the two products have different lot-sizes, formula (7) however proposes unequal variances of imbalance. So formula (7) will not be correct always. Nevertheless it is possible, although the spread over all final products is lost, that formula (7) still is a good representative of the total 'weight' of imbalance in the formula for the stocknorm.

By defining imbalance as in (2), (3) and (5) it is possible by means of simulation to measure imbalance and to investigate its influence on the system's performance. This will be described in more detail in the next Section, where stocknorms based on formula (6) and (7) are tested.

4. Testing the quality of the estimators.

4.1 Description of the simulation experiments.

In order to check the quality of stocknorms based on formulas (6) and (7), a number of simulations are performed. The number of simulations is small, since the accent here is not on 'proving' formulas (6) and (7), but rather on getting an impression of the quality of the formulas and more importantly: getting an impression of the behaviour of divergent systems with non-identical products.
These systems differ a great deal from systems with identical products. To name a few differences:

- The definition of imbalance is far more complex. See formulas (2), (3) and (5).
- The allocation rule is far more complex:
  * For identical products all allocation rules mentioned in Section 2.2.1 are equivalent. This is not true for non-identical products. To define imbalance an allocation rule for systems without lot-sizes was used. In the simulation rule this same rule will be applied to actually allocate products, even though the simulated systems have order policies with lot-sizing.
  * For non-identical products it becomes questionable what to do if the content of the depot is too small (due to lot-sizing) for the product which needs it most and if another product, which has little need for it, has a lot-size smaller than the inventory in the depot.

  In the simulations this question was answered differently depending on whether the system has a depot or not. In general it was considered beneficial to allocate as much inventory as possible to the final products. For the system with depot therefore the content of the depot was allocated as long as one or more final products were below their stocknorm and their lot-size was smaller than the content of the depot. If this same rule were to be applied to the system without depot, the slowmovers would get allocated more inventory on average due to their smaller lot-size. Since this would yield very unbalanced inventories, it was decided to stop the allocation in a system without depot as soon as the content of the depot was smaller than the lot-size of the final product which needed it most.

An alternative solution here is to allocate the amount of inventory in the depot regardless of the fact that it is smaller than the lot-size. This solution is not used in the simulations however. Although it yields a substantial better service level, it is questionable whether this solution is economical and whether the production system is flexible enough to meet these order adaptations.

In total 10 systems were simulated. Each system consisted of 2 product groups: so called fastmovers and slowmovers. The fastmovers make up 80% of the total demand and have a relatively small coefficient of variation (0.5 as opposed to 2.0 for the slowmovers). The sum of the leadtimes for the common part and the final products was always equal to 6 periods. The demand per period for a slowmover was equal to 10. The parameters which were varied are:

N: Number of final products: N=2, 4 or 8.
The number of slowmovers was equal to 1, 3 resp. 6

ℓ: The leadtime for the final product: ℓ = 1 or 3.

f1: The ratio Q_j/μ_j: f1 = 1 or 3.

f0: The ratio $Q_{comm}/\Sigma Q_j$: f0 = 1 or 3.

dep: Depot available or not.

As a performance criterium the system service level α was used. This criterium was derived from the final products' service levels α_j: $\alpha = \Sigma\alpha_j\mu_j \ / \ \Sigma\mu_j$.

For each system three simulations were performed: In these simulations the estimator for the variance of imbalance was set equal to resp.

- the estimator in formula (7)
- the average variance of imbalance measured during the simulation which was based on formula (7).
- zero (to investigate the consequences of neglecting imbalance).

The corresponding service levels are denoted by α(est), α(sim) resp. α(0).

4.2 Simulation results.

Table 1 shows the service levels for each of the three simulations and the corresponding parameter setting:

						α		α(sim)	
dep	N	ℓ	f1	f0	est	sim	0	slowm	fastm
no	2	1	1	3	92.5	93.0	91.2	91.9	93.2
no	2	1	3	3	92.9	92.8	90.5	87.4	94.1
no	2	1	3	1	89.7	90.3	86.6	90.3	90.3
no	2	1	1	1	88.7	90.2	88.4	94.6	89.1
no	2	3	3	1	90.9	91.1	88.6	89.1	91.7
no	8	3	1	1	92.0	92.5	91.6	93.0	91.7
no	8	1	3	3	94.3	94.0	91.2	87.1	95.7
yes	2	1	3	1	88.4	86.0	84.2	92.7	84.3
yes	2	1	1	1	89.5	89.3	89.0	93.5	88.2
yes	8	3	1	1	92.7	92.7	92.3	94.1	92.4

Table 1. Simulation results for divergent systems with non-identical final products.

The last columns in Table 1 represent the simulated service level for the slowmovers resp. fastmovers.

From Table 1 the following observations can be made:

-1. The fact that α(est) and α(sim) are rather close suggests either that the estimator for the variance of imbalance performs quite well or that imbalance has hardly any impact on the service level. The latter is contradicted by columns α(sim) and α(0), at least in case no depot is present.

-2. The target service level of 95% is not achieved at all. From column α(sim), where full knowledge of the variance of imbalance is used, it is clear that this is not due to misestimating the variance of imbalance. Apparently there are other disturbing factors, which have more impact on the service level than the mis-estimation of the variance of imbalance.

A closer examination of the simulations revealed the following two extra disturbing factors:

a. Dead stock is inevitable.

b. The allocation rule is too simple.

ad a.: Dead stock is inevitable.

In systems with identical final products, the supply and retrieval of products at the depot are always multiples of the lot-size Q. Therefore if the simulation starts with a depot having a multiple of the lot-size Q on hand, the depot will remain to have exactly multiples of the lot-size. This changes as soon as either the supply or retrieval of products in the depot deviates slightly from the lot-size Q (or a multiple of it). If Q-1 products are supplied (e.g. due to random scrap), these products can not be shipped out of the depot, that is: if the lot-size strategy of the final products is interpreted strictly. So these Q-1 products have turned into 'dead stock': They are in the depot but can not be used. If the supply is random, dead stock will be Q/2 on average.

This phenomena is automatically present in a divergent system with non-identical products. Since demand is uncertain and the final products' lot-sizes are different, the inventory in the depot is no longer guaranteed to be a multiple of the lot-size of the final product which needs it most. To give an indication on the size of this dead stock: if supply is deterministic, the dead stock for the fastmover will approximate $(Q_{fastm}-Q_{slowm})/2$ if N=2 and $Q_{fastm}>Q_{slowm}$.

For systems with a depot the slowmover has a strong advantage because of the dead stock for the fastmover: That stock can be used by the slowmover, which results in a higher service level for the slowmover. This fact is reflected in Table 1 by the service levels for the fast- and slowmovers in systems with a depot.

This effect also occurs in systems without depot. This becomes visible if allocations are made regular (that is: if $Q_{comm}/N\mu$, which is equal to $f0 \cdot f1$, is small). From Table 1 it can be seen however that if in systems without depot these allocations are not made frequently, another phenomena has an opposite effect on the products' service level:

ad b.: The allocation rule is too simple.

For example in a system without depot, with f0=f1=3 and N=2, Table 1 shows that the service level of the fastmover (94.1%) is <u>higher</u> than the service level of the slowmover (87.4%). The cause for this is the fact that the allocation rule used was developed for systems without lot-sizes. The rule allocates an order to the product with the lowest 'inventory-equivalent' (e.g. run-out-time). This has a very different impact on the service levels of the fast- resp. slowmovers, since the lot-size of the fastmovers is larger and their coefficient of variation is smaller compared with the slowmovers.

Suppose e.g. that a fastmover (A) and a slowmover (B) have the following characteristics:

$\mu(A) = 40, \quad \sigma(A) = 80, \quad Q(A) = 120,$

$\mu(B) = 10, \quad \sigma(B) = 20, \quad Q(B) = 30.$

The leadtimes are zero for both products.
Furthermore suppose that before allocation takes place both products have equivalent inventories: inv(A) = 40, inv(B) = 10.
An allocation of one lot to A has far more impact on the service level than an allocation of one lot to B:
service level(A) = $\Phi[(40+120-40) / 20] = \Phi(6)$, whereas
service level(B) = $\Phi[(10+30-10) / 20] = \Phi(1.5)$ if demand is assumed to be normally distributed.

This shows the fact that the fastmover gets relatively more if it gets an order allocated compared with the slowmover, which results in unbalanced service levels.

5. Remedies for the extra disturbing factors.

Dead stock can be accepted or not.

If dead stock is <u>accepted</u>, a new allocation rule has to be developed which takes account of the dead stock effects on the service levels. This allocation rule should give a slight priority to the fastmovers.

Secondly the integral stocknorm for the common part has to be raised. As a rule of thumb the stocknorm may be raised by $(\max\{Q_1,\ldots,Q_N\} - \mathrm{HCF}\{Q_1,\ldots,Q_N\})/2$, where LCD stands for Highest Common Factor. Note that for N>1 combined with equal lot-sizes and for N=1 this raise equals zero.

In general this raise will be approximately $\max\{Q_1,...,Q_N\}/2$ since it is most likely that $HCF\{Q_1,...,Q_N\}$ is small if the lot-sizes are determined independently.

In case dead stock is <u>not accepted</u>, sophisticated 'allocate-or-not' decisions have to be made to decrease the amount of dead stock. An example of such a rule is: If the inventory in the depot is less than a minimum order quantity (e.g. $Q_j/2$), the inventory is given to product j, otherwise it is retained in the depot.

Such rules may have severe and complex impact on the capacity requirements (more orders usually imply more set-up times) and it is therefore questionable whether such rules should be formalized or just left over to the planner.

Note that although dead stock can be reduced, it is very dubious whether it can be avoided completely in an economical way.

The second disturbing factor (the fact that the allocation rule is not based on lot-sizes) also requires an adaptation of the allocation rule. As demonstrated in Section 4.2 the fastmovers get relatively more allocated compared with the slowmovers. This can be compensated by defining a new allocation rule which gives the slowmover a slight priority over the fastmover. Note that this adaptation of the allocation rule has an opposite effect compared with the adaptation necessary to deal with dead stock.

<u>5. Conclusions.</u>

a. In all simulations performed in this paper it appeared that the stocknorms for systems with non-identical products performed well, although they were based on the estimators for the variance of imbalance developed for systems with identical products.

b. Generally order policies with fixed order quantities for the final products lead to dead stock in divergent systems. This requires an increase of the integral stocknorm for the common part and an adaptation of the allocation rule.

c. When allocating inventory to the final products,
   - the products with a <u>large</u> lot-size should be given slight priority in case of divergent systems
      * with depot or

* without depot and short time between two allocations.

- the products with a small lot-size should be given slight priority in case of divergent systems
  * without depot and large time between two allocations.

Literature.

[1] Donselaar, K. van (1988). Integral stocknorms in divergent systems with lot-sizes. Working paper University of Technology Eindhoven nr. TUE/BDK/ORS/88/01.

[2] Federgruen, A. and P. Zipkin (1984). Allocation policies and cost approximations for multilocation inventory systems. Naval Research Logistics Quarterly, 31, page 97.

[3] Eppen, G. and L. Schrage (1981). Centralized ordering policies in a multi-warehouse system with lead times and random demand. Appeared in: Multi-level production/inventory control systems: theory and practice (ed: 1.B. Schwarz). North-Holland Publ. Co., Amsterdam.

Appendix: Minimal variance of imbalance.

Recall from [1] that the minimal variance of imbalance can be achieved if there is an infinite amount of inventory available in the depot. In that case it is known that the inventory levels of the final products will be uniformly distributed between r_j and r_j+Q_j. So $var(I_j)$ equals $Q_j^2/12$. This result together with formula (13) can be used to derive an estimator for the minimal variance of imbalance:

$$\sigma^2_{imb,j} = var(I_j - I_j^*) = var(I_j - r_j - \mu_j \cdot \Sigma(I_i - r_i)/\Sigma\mu_i)$$

$$= (1 - \mu_j/\Sigma\mu_i)^2 \cdot var(I_j) + (\mu_j/\Sigma\mu_i)^2 \cdot \sum_{k\neq j} var\ (I_k).$$

So $$\sigma^2_{imb,j} \geq (1 - \mu_j/\Sigma\mu_i)^2 \cdot Q_j^2/12 + (\mu_j/\Sigma\mu_i)^2 \cdot \sum_{k\neq j} Q_k^2/12. \qquad (5)$$

Note that if $\mu_j/\Sigma\mu_i \rightarrow 0$ this tends to $Q_j^2/12$ just like in the identical products case (see [1]).

All conclusions are drawn under the assumption of independent distributed demand for the final products and unlimited capacity.