# Elastic deformation of punch and die during the disk forging process

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# ELASTIC DEFORMATION OF PUNCH AND DIE DURING THE DISK FORGING PROCESS

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WPA Report Nr.0858, March, 1990

# CONTENT

## I. Introduction

Compression of a solid disk leads to elastic deformtions of the punch and die, especially in case of small height–diameter ratios. It will consquently affects the accuracy of the geometry of the products, and sometimes, it makes it impossible for the workpiece to expand because the surface of the die under compression becomes concave and the centripetal component force increases rapidly. If the deflection $w$ in the $z$–direction ( See Fig.1 ) is known before die–making, the surface of the die can beforehand be proccessed by making a compensation $\overline{w}$ which equalizes the deflection $w$ on the surface under the compression. In that way, the production can be carried out easilier and more accurately. However, first of all is to determine the deformation on the surface.

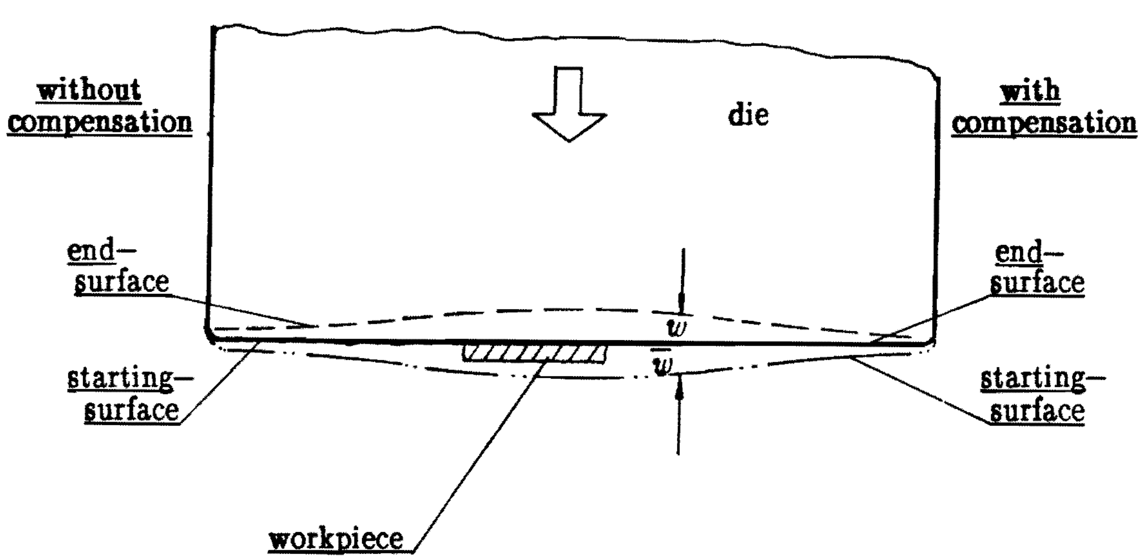


Fig.1 Comparing the dies with and without compensation

In general, there are compressive and friction forces on the surface. Using the "constant friction" model $\tau_{fr} = \frac{m}{\sqrt{3}}\sigma_f$ and applying "slabmethod" on the disk gives

$$\sigma_z = [\,1 - \frac{2m}{\sqrt{3}}(\frac{a}{h})(\frac{r}{a} - 1)\,]\,\sigma_f \tag{1}$$

where $\sigma_z$ is the stress in the $z$–direction, $\sigma_f$ the yield stress of material of the workpiece, $m$ the friction coefficient, and $h$ the thickness of workpiece.

It is assumed that the contribution of the friction $\tau_{fr}$ to $w$ ( See Fig.2 ), the deflection in the $z$–direction, is very small, and can be omitted. The main effect is caused by vertical force $q = q\,(\,r\,)$, which, from Eq.(1), acts on the surface of the die in a cone–shaped distribution, $q_e$ being on the edge of the loaled circle and $q_c$ at the centre. In this case, the load acting on the die can be considered as the combination of two parts. One is the uniform load

$$q_1 = q_e = \sigma_f \tag{1a}$$

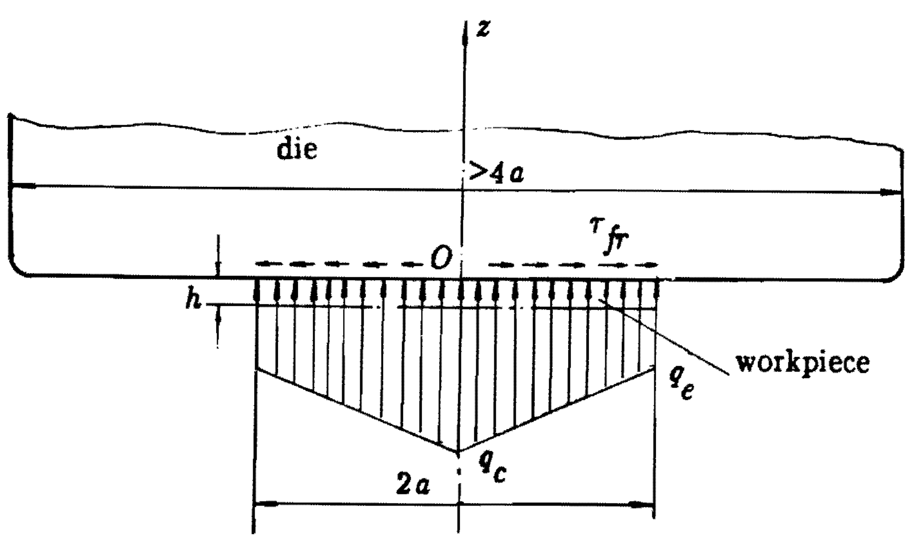


Fig.2 The distribution of forces on the die

the other, a cone–shaped load

$$q_2 = q_c - q_e = \frac{2m}{\sqrt{3}}\left(\frac{a}{h}\right)\sigma_f\left(1 - \frac{r}{a}\right) = q_0(1-k) \qquad (1b)$$

where $q_0 = \frac{2m}{\sqrt{3}}\left(\frac{a}{h}\right)\sigma_f$ and $k = r/a$.

This paper is going to determine the deformation $w$ of the surface produced by the compressive forces, by superposition of the contribution $w_1$ produced by uniform load $q_1$ and the contribution $w_2$ produced by $q_2$, that is,

$$w = w_1 + w_2 \qquad (2)$$

In the following sections we will review some general solutions of the linear–elastic problem and develop them to solve our project.

## II. A Review of Related Equations and Solutions

The project mentioned above is a space axisymmtric problem, in the cylindrical coordinate system, the deflections and the stresses are only depended on the $z$– and $r$–directions.

The basic equations of a space axisymmtric problem are

$$\frac{1}{1-2\nu}\cdot\frac{\partial e}{\partial r} + \nabla^2 u - \frac{u}{r^2} = 0 \qquad (3a)$$

$$\frac{1}{1-2\nu}\cdot\frac{\partial e}{\partial z} + \nabla^2 w = 0 \qquad (3b)$$

where $u$ and $w$ are the displacements in $r$– and in $z$–direction, and $e$ the atrain of volume

$$e = \epsilon_r + \epsilon_\theta + \epsilon_z = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}$$

and $\nabla^2$ is the Laplacian operator

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z}$$

There are two components of deformation in Eqs.( 3 ). Applying a suitable deformation function $\zeta = \zeta(r,z)$, which satifies $\nabla^4=0$, there are the relations between deformations and $\zeta$ below:

$$u = -\frac{1}{2G}\cdot\frac{\partial^2\zeta}{\partial r\partial z} \quad (4a)$$

$$w = \frac{1}{2G}[\,2(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\,]\,\zeta \quad (4b)$$

where $G = \frac{E}{2(1+\nu)}$, where $E$ and $\nu$ are the Young's modulus and the Posson's ratio of the die material.

Therfore, the expressions of stress components are

$$\sigma_r = \frac{\partial}{\partial z}(\,\nu\nabla^2 - \frac{\partial^2}{\partial r^2}\,)\,\zeta \quad (5a)$$

$$\sigma_\theta = \frac{\partial}{\partial z}(\,\nu\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r}\,)\,\zeta \quad (5b)$$

$$\sigma_z = \frac{\partial}{\partial z}[\,(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\,]\,\zeta \quad (5c)$$

$$\tau_{zr} = \frac{\partial}{\partial r}[\,(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\,]\,\zeta \quad (5d)$$

For the problem of a concentrated force $P$ on the boundary of a semi–infinite body ( See Fig.3a ), Love's deformation function $\zeta$ is suitable, which is

$$\zeta = A_1R + A_2\,[\,R - z\cdot\ln(R+z)\,] \quad (6)$$

where $R^2=z^2+r^2$, $A_1$ and $A_2$ are contants determined by boundary conditions.

Substitution of function ( 6 ) into Eqs.( 4 ) and ( 5 ) gives the deformation expressions

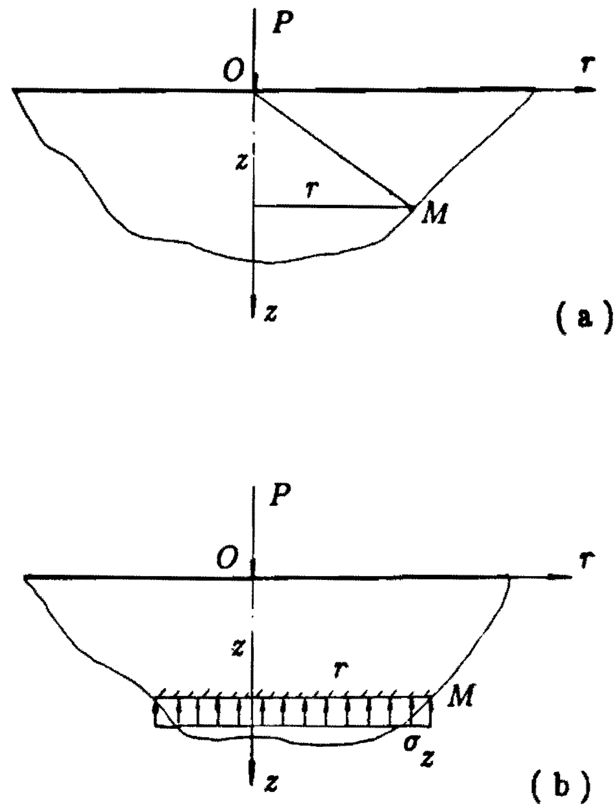


Fig.3 A concentrated force on the boundary of a semi–infinite body

$$u = \frac{1}{2G}\left[ A_1 \frac{rz}{R^3} + A_2 \frac{r}{R(R+z)} \right] \quad (7a)$$

$$w = \frac{1}{2G}\left[ A_1\left(\frac{r^2}{R^3} + \frac{3-4\nu}{R}\right) + A_2 \frac{1}{R} \right] \quad (7b)$$

and the stress expressions

$$\sigma_r = A_1\left[\frac{(1-2\nu)z}{R^3} - \frac{3r^2z}{R^5}\right] +$$

$$+ A_2\left[\frac{z}{R^3} - \frac{1}{R(R+z)}\right] \quad (8a)$$

$$\sigma_\theta = A_1 \frac{(1-2\nu)z}{R^3} + A_2 \frac{1}{R(R+z)} \quad (8b)$$

$$\sigma_z = -A_1\left[\frac{(1-2\nu)z}{R^3} + \frac{3z^3}{R^5}\right] - A_2 \frac{z}{R^3} \quad (8c)$$

$$\tau_{zr} = -A_1[\frac{(1-2\nu)r}{R^3} + \frac{3z^2r}{R^5}] - A_2\frac{r}{R^3} \qquad (8d)$$

The stress boundary conditions on the area out of the origin $O$, the point $P$ acts on, are

$$\sigma_z|_{z=0} = 0 \qquad (9)$$

$$\tau_{zr}|_{z=0} = 0 \qquad (10)$$

and the equilibrium in $z$-direction of the upper part of the semi-infinite body ( See Fig.3b ) gives

$$\int_0^\infty \sigma_z \cdot (2\pi r dr) + P = 0 \qquad (11)$$

The stress boundary condition Eq.( 9 ), from Eq.( 8c ), satisfies naturally. Subsitutions of Eq.( 8d ) and ( 8c ) into ( 10 ) and ( 11 ) give the contants

$$A_1 = \frac{1}{2\pi}P, \qquad A_2 = -\frac{1-2\nu}{2\pi}P \qquad (12)$$

Therfore, the solutions of deflection, from Eqs.(7a,b), are

$$u = \frac{(1+\nu)P}{2\pi ER}[\frac{rz}{R^2} - \frac{(1-2\nu)r}{(R+z)}] \qquad (13a)$$

$$w = \frac{(1+\nu)P}{2\pi ER}[2(1-\nu) + \frac{z^2}{R^2}] \qquad (13b)$$

The vertical displacement at the point far enough from $O$ on the surface, where $z=0$, from Eq.( 13b ), is

$$w|_{z=0} = \frac{(1-\nu^2)}{\pi Er} \cdot P \qquad (14)$$

This is the very important conclusion for us to solve the problem. It shows that the product $wr$ is a contant on the boundary. At the origin the displacement becomes infinite. We, according to Saint–Venant principle, imagine the concentrated force $P$ is replaced by statically equivalent forces over a hemispherical surface of small raduis around the origin.

## III. The Deflection of a Uniform Load Distributed over a Circle Aera

Having the solution for a concentrated force acting on the boundary of a semi–infinite body, we can, using superposition, find the displacement produced by a uniform load $q_1$ and cone–shaped load $q_2$ distributed over a circle area. First, we discuss the deflection produced by $q_1$.

1 ) The Deflection at the Point within the Loaded–Circle

Assuming a point $C$ at the distance $r$ away from the centre of the loaded circle ( See Fig.4a ), and taking a small element $\mathrm{d}F = s \cdot \mathrm{d}\varphi \cdot \mathrm{d}s$ within

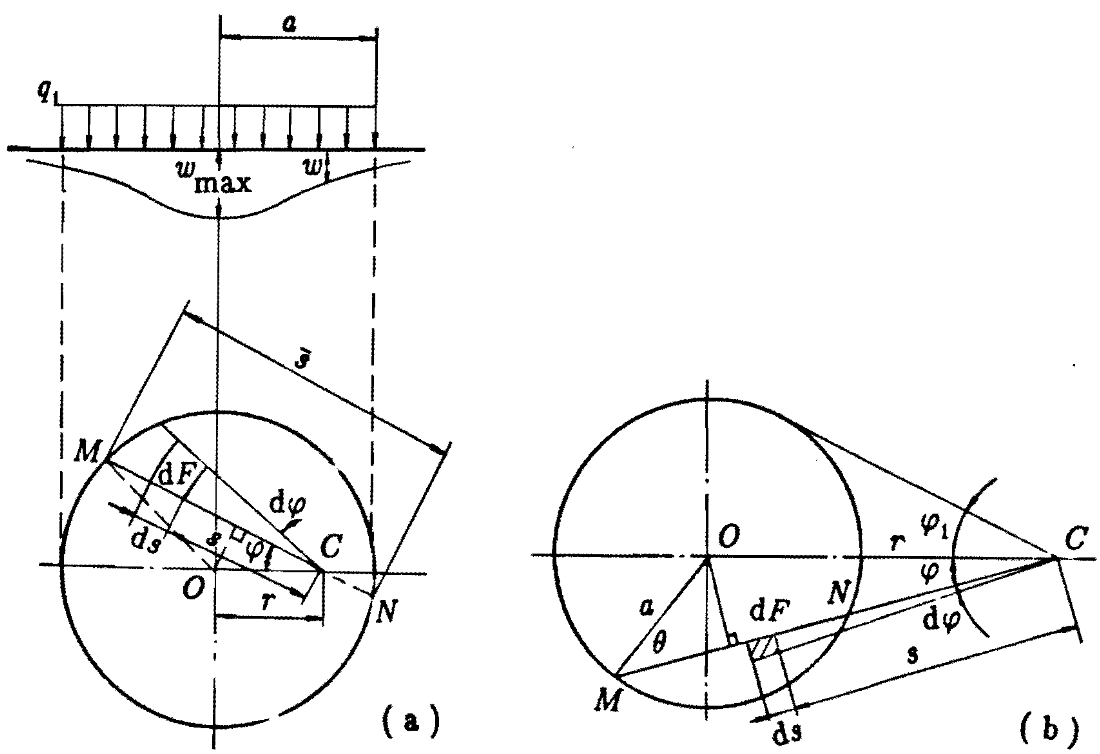


Fig.4 A uniform load distributed over a circle

the loaded area, then the load on this element is $\mathrm{d}P = q_1 \cdot \mathrm{d}F = q_1 \cdot s \cdot \mathrm{d}\varphi \cdot \mathrm{d}s$. The corresponding deflection at point $C$, from Eq.( 14 ), is

$$\mathrm{d}w = \frac{1-\nu^2}{\pi E s} \cdot \mathrm{d}P$$

$$= \frac{1-\nu^2}{\pi E} \cdot q_1 \mathrm{d}\varphi \mathrm{d}s \qquad (15)$$

The total deflection at point $C$ will be obtained by integration,

$$w = \int_A \mathrm{d}w$$

$$= \frac{1-\nu^2}{\pi E} q_1 \iint_A \mathrm{d}\varphi \mathrm{d}s \qquad (16)$$

where $A$ is the whole area loaded by $q_1$ which is a contant value. As the point $C$ is within the circle, or $r \leq a$, Eq.( 16 ) becomes

$$w = \frac{1-\nu^2}{\pi E} q_1 \left[ 2\int_0^{\frac{\pi}{2}} \mathrm{d}\varphi \int_{\bar{s}} \mathrm{d}s \right] \qquad (17)$$

wher $\bar{s}$ is the length of chord $\overline{MN}$, that is,

$$\bar{s} = 2a\sqrt{1-(r^2/a^2)\sin^2\varphi}$$

Finally, the deflection at the point $C$ in $z$–direction becomes

$$w = \frac{(1-\nu^2)}{E} q_1 a \lambda_1 \qquad (18)$$

where the parameter

$$\lambda_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\varphi}\, \mathrm{d}\varphi \qquad (19)$$

where $k = r/a$. Eq.( 19 ) can be rewritten in series form

$$\lambda_1 = 2[1-(\frac{1}{2})^2k^2-(\frac{1\cdot 3}{2\cdot 4})^2k^4-(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6})^2k^6-\cdots] \tag{19a}$$

The result of parameter $\lambda_1$ will be listed in Tab.1 and shown in Fig.6.

The maximum deflection, of course, occurs at the centre of the circle, so substitution of $r$=0 in Eq.( 18 ) gives

$$w_{\max} = \frac{2(1-\nu^2)}{E}q_1 a \tag{20}$$

At the point on the edge of the loaded–circle, substitution of $r$=$a$ in Eq.( 18 ) gives the deflection

$$w_a = \frac{4(1-\nu^2)}{\pi E}q_1 a \tag{21}$$

2 ) The Deflection at the Point out of the Loaded–Circle

If the point $C$ is out of the circle ( See Fig.4b ), that is, $r$>a, Eq.( 16 ) becomes

$$w = \frac{4(1-\nu^2)}{\pi E}q_1\int_0^{\varphi_1}\mathrm{d}\varphi\int_{\bar{s}}\mathrm{d}s$$

$$= \frac{4(1-\nu^2)}{\pi E}q_1 a\int_0^{\varphi_1}\sqrt{1-k^2\sin^2\varphi}\,\mathrm{d}\varphi \tag{22}$$

where $\varphi_1$ is the maximum value of $\varphi$ , that is , the angle between $r$ and the tangent to the circle. From the figure, there is the relation

$$a\cdot\sin\theta = r\cdot\sin\varphi \tag{23}$$

from which

$$\mathrm{d}\varphi = \frac{a\ \cos\theta}{r\ \cos\varphi}\mathrm{d}\theta$$
$$= \frac{a\cos\theta\,\mathrm{d}\theta}{r\sqrt{1-(a^2/r^2)\sin^2\theta}} \tag{24}$$

Substitution of ( 23 ) and ( 24 ) in ( 22 ), remembering that $\theta$ varies from 0 to $\frac{\pi}{2}$ when $\varphi$ changes from 0 to $\varphi_1$, gives

$$w = \frac{4(1-\nu^2)}{\pi E} q_1 \int_0^{\frac{\pi}{2}} \frac{a^2 \cos^2\theta d\theta}{r\sqrt{1-(a^2/r^2)\sin^2\theta}}$$

$$= \frac{1-\nu^2}{E} q_1 a \lambda_1 \tag{25}$$

where the parameter

$$\lambda_1 = \frac{4}{\pi k_1} \cdot \left[\int_0^{\frac{\pi}{2}} \sqrt{1-k_1^2 \sin^2\theta}\, d\theta - (1-k_1^2) \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k_1^2 \sin^2\theta}}\right] \tag{26}$$

where $k_1 = a/r$ $(r > a)$. The integrals in this expression are known as complete elliptic integrals, and their values for any value of $k_1$ can be taken from tables. For convenience of computation, Eq.( 26 ) can be written in series form,

$$\lambda_1 = 2\left\{\frac{1}{2} k_1 + \sum_{i=2}^{\infty}\left[\prod_{j=1}^{2i-3}\left(\frac{2j-1}{2j}\right)^2 - \left(1+\frac{1}{2i-1}\right)\prod_{j=1}^{2i-1}\left(\frac{2j-1}{2j}\right)^2\right] k_1^{2i-1}\right\} \tag{26a}$$

The result of $\lambda_1$ will be listed in Tab.1 and shown in Fig.6 later, where $k = 1/k_1 = r/a$.

## IV. The Deflection caused by a Cone–Shaped Load Distributed over a Circle Area

The deflection caused by a cone–shaped load distributed over a circle area can be obtained by a similar way as mentioned above ( See Fig.5 ). Since

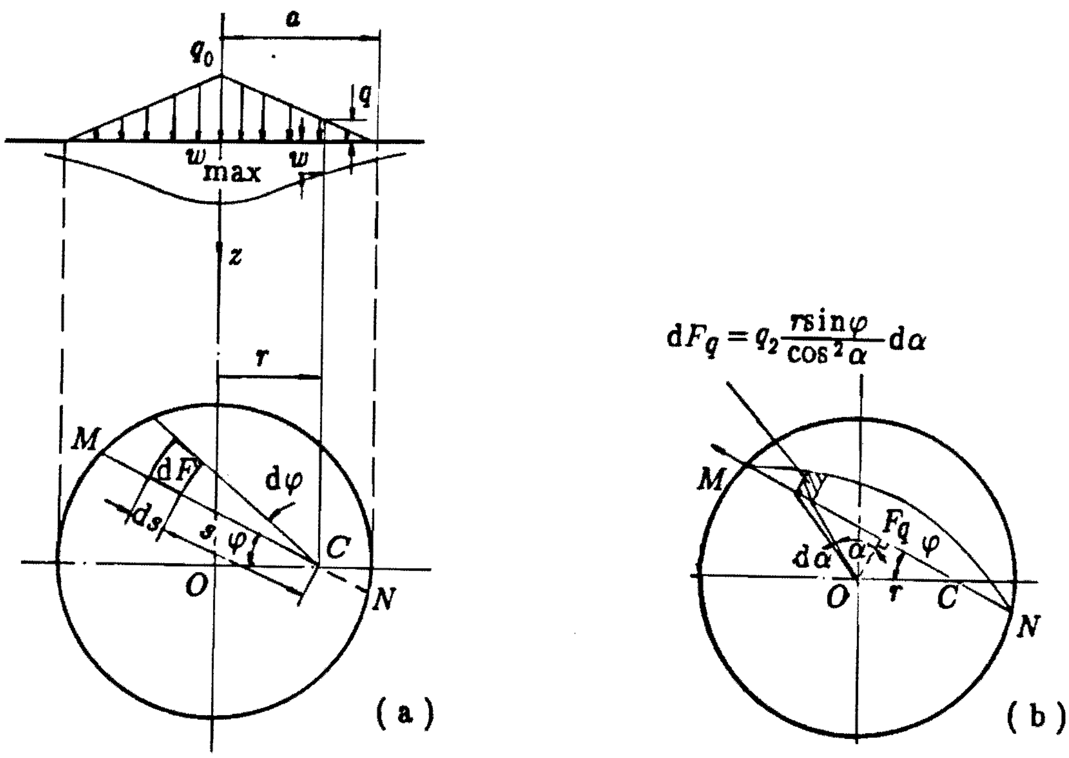


Fig.5 A cone–shaped load distributed over a circle

the load $q_2$ is not a contant value, it will be depended on the distance $r$ away from the centre, or $q = q(r) = q_0(1-k)$, where $k=r/a$ and $r \leq a$.

1 ) The Deflection at the Point within the Loaded–Circle

Assuming the point $C$, at the distance $r$ away from origin $O$, is within the circle, or $r \leq a$, taking a small element in the loaded circle, from Eq.( 16 ), the deflection

$$w = \frac{1-\nu^2}{\pi E}\left[2\int_0^{\frac{\pi}{2}} d\varphi \int_{\bar{s}} q_2 ds\right] \tag{27}$$

where $\bar{s}$ is the length of chord $\overline{MN}$, and $q_2$ the load distributed on the area $dF$,

$$q_2 = q_0\left(1 - k\frac{\sin\varphi}{\cos\alpha}\right) \tag{28}$$

In Eq.( 27 ), the second integral $\int_{\bar{s}} q_2 ds$ is the area of the segment of the circle, which is the load section cut by $\overline{MN}$ vertically. It is

$$F_q = \int_{\bar{s}} q_2 ds$$

$$= 2\int_0^{\alpha_1} \frac{q_2\rho\cdot \mathrm{d}\alpha}{\cos\alpha}$$

$$= 2\int_0^{\alpha_1} q_0\left( 1 - k\frac{\sin\varphi}{\cos\alpha} \right)\frac{r\sin\varphi}{\cos^2\alpha}\mathrm{d}\alpha$$

$$= 2q_0 r\sin\varphi \,\mathrm{tg}\,\alpha_1 - q_0 k r\sin^2\varphi\left[\frac{\sin\alpha_1}{\cos^2\alpha_1} + \ln\left(\frac{1}{\cos\alpha_1}+\mathrm{tg}\,\alpha_1\right)\right] \tag{29}$$

where $\alpha_1$ is the half of the angle between *OM* and *ON*, which is

$$\alpha_1 = \cos^{-1}(k\sin\varphi) \tag{30}$$

Substitution of it in ( 29 ) gives the area of segment section,

$$F_q = q_0 a\sqrt{1-k^2\sin^2\varphi} + q_0 k r\sin^2\varphi \ln\frac{k\sin\varphi}{1+\sqrt{1-k^2\sin^2\varphi}} \tag{31}$$

Therefore, substituting ( 31 ) in ( 27 ), we find

$$w = \frac{1-\nu^2}{E}q_0 a\lambda_0 \tag{32}$$

where the parameter

$$\lambda_0=\frac{2}{\pi}\left[\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2\varphi}\,\mathrm{d}\varphi + \int_0^{\frac{\pi}{2}} k^2\sin^2\varphi \ln\frac{k\sin\varphi}{1+\sqrt{1-k^2\sin^2\varphi}}\mathrm{d}\varphi\right] \tag{33}$$

The first item within the square brackets in Eq.( 33 ) is a complete elliptic integral, and its value can be obtained from table. The value of the second item can be obtained by numerical integral method. However, the maximum deflection occurs at the centre, or $r=0$,

$$w_{\max} = \frac{1-\nu^2}{E}q_0 a \tag{34}$$

and the deflection at the point on the edge, or $r=a$,

$$w\big|_{r=a} = \frac{1-\nu^2}{\pi E}\, q_0 a\left[ 2+ \int_0^{\frac{\pi}{2}}\sin^2\varphi \ln\frac{1-\cos\varphi}{1+\cos\varphi}\mathrm{d}\varphi \right] \tag{35}$$

or, by numerial integral, it approximattes

$$w\Big|_{r=a} \cong 0.37184\frac{1-\nu^2}{E}q_0 a \qquad (35a)$$

2 ) The Deflection at the Point out of the Loaded–Circle

If the point $C$ is out of the circle, or $r>a$, Eq.( 27 ) should be

$$w = \frac{1-\nu^2}{\pi E}\left[2\int_0^{\varphi_1} \mathrm{d}\varphi \int_{\bar{s}} q_2 \mathrm{d}s\right] \qquad (36)$$

and Eq.( 32 ) with ( 23 ) and ( 24 ) becomes

$$w = \frac{1-\nu^2}{E}q_0 a \lambda_0 \qquad (37)$$

where the parameter

$$\lambda_0 = \frac{2k_1}{\pi}\left[\int_0^{\frac{\pi}{2}} \frac{\cos^2\theta \mathrm{d}\theta}{\sqrt{1-k_1^2\sin^2\theta}} + \int_0^{\frac{\pi}{2}} \frac{\sin^2\theta\cos\theta}{\sqrt{1-k_1^2\sin^2\theta}} \ln\frac{\sin\theta}{1+\cos\theta}\mathrm{d}\theta\right] \qquad (38)$$

where $k_1=a/r$. The result of Eq.( 38 ) will be listed in Tab.1 and shown in Fig.6 below, where $k=1/k_1$.

Tab.1 The parameters $\lambda_1$ and $\lambda_0$ of $w$

| $k$ | $\lambda_1$ | $\lambda_0$ | $k$ | $\lambda_1$ | $\lambda_0$ | $k$ | $\lambda_1$ | $\lambda_0$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.000000 | 1.000000 | | | | | | |
| 0.1 | 1.994991 | 0.981560 | 1.1 | 1.060665 | 0.371808 | 2.1 | 0.490994 | 0.161611 |
| 0.2 | 1.979847 | 0.940161 | 1.2 | 0.937066 | 0.296149 | 2.2 | 0.467307 | 0.154005 |
| 0.3 | 1.954211 | 0.883825 | 1.3 | 0.845059 | 0.270224 | 2.3 | 0.445866 | 0.147095 |
| 0.4 | 1.917424 | 0.817036 | 1.4 | 0.771995 | 0.426357 | 2.4 | 0.426357 | 0.140788 |
| 0.5 | 1.868431 | 0.743168 | 1.5 | 0.711868 | 0.230718 | 2.5 | 0.408524 | 0.135007 |
| 0.6 | 1.805560 | 0.665122 | 1.6 | 0.661195 | 0.215202 | 2.6 | 0.392155 | 0.129687 |
| 0.7 | 1.726081 | 0.585647 | 1.7 | 0.617740 | 0.201719 | 2.7 | 0.377073 | 0.124776 |
| 0.8 | 1.625099 | 0.507625 | 1.8 | 0.579964 | 0.189879 | 2.8 | 0.363129 | 0.120227 |
| 0.9 | 1.491851 | 0.434498 | 1.9 | 0.546763 | 0.179389 | 2.9 | 0.350197 | 0.116002 |
| 1.0 | 1.273240 | 0.371808 | 2.0 | 0.517316 | 0.170025 | 3.0 | 0.338168 | 0.112066 |

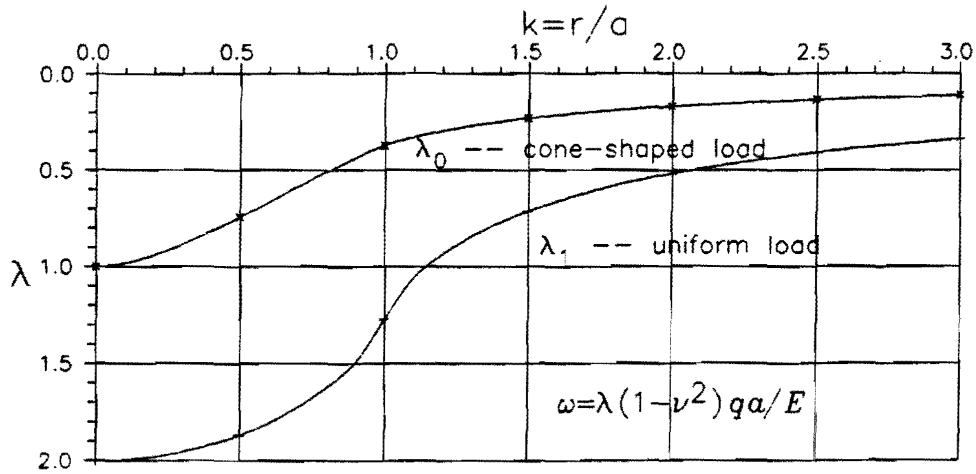


Fig.6 The parameter $\lambda$ of $w$

V. The Total Deflection

Having the deflection produced by a uniform load and a cone-shaped load, we can easily superpose them and get the total deflection on the surface of the die. That is,

$$w = w_1 + w_2 \qquad (2)$$

where $w_1$ is the deflection in the $z$-direction produced by the uniform load and $w_2$ produced by the cone-shaped load.

From Eq.( 1 ), the load function can be divided into two parts, one is the uniform load $q_1 = \sigma_f$, and the other $q_2 = \frac{2m}{\sqrt{3}}(\frac{a}{h})\sigma_f(1 - \frac{r}{a})$, which is a cone-shaped load and is written as $q_2 = q_0(1-k)$, where $q_0 = \frac{2m}{\sqrt{3}}(\frac{a}{h})\sigma_f$ and $k=r/a$ $(r \leq a)$. Substituting $q_1$ into Eq.( 18 ) and ( 25 ), they may have the same expression

$$w_1 = \frac{1-\nu^2}{E}\sigma_f a\lambda_1 \qquad (39)$$

Rewritten in dimensionless form, it becomes

$$\frac{w_1}{h} = (1-\nu^2)(\frac{\sigma_f}{E})(\frac{a}{h})\lambda_1 \qquad (40)$$

where the parameter $\lambda_1$ can be taken from Tab.1 and Fig.6 or calculated by Eq.( 19 ) or ( 26 ) according to the distance $r$ whether within or out of the loaded circle. Substitution of $q_0$ into Eq.( 32 ) and ( 37 ), they may also be written in the same expression

$$\begin{aligned} w_2 &= \frac{1-\nu^2}{E} q_0 a \lambda_0 \\ &= \frac{1-\nu^2}{E} \cdot \frac{2m}{\sqrt{3}} (\frac{a}{h}) \sigma_f a \lambda_0 \end{aligned} \qquad (41)$$

Rewritten in dimensionless form, it becomes

$$\frac{w_2}{h} = (1-\nu^2)(\frac{\sigma_f}{E}) \frac{2m}{\sqrt{3}} (\frac{a}{h})^2 \lambda_0 \qquad (42)$$

where the parameter $\lambda_0$ can be taken from Tab.1 and Fig.6 or calculated by Eq.( 33 ) or ( 38 ) according to the distance $r$ whether it is within or out of the loaded circle.

Substitutions of Eqs.( 39 ) and ( 41 ) into ( 2 ) give

$$w = \frac{1-\nu^2}{E} \sigma_f a [\lambda_1 + \frac{2m}{\sqrt{3}} (\frac{a}{h}) \lambda_0] \qquad (43)$$

also, combination of Eqs.( 40 ) and ( 42 ) gives

$$\frac{w}{h} = (1-\nu^2)(\frac{\sigma_f}{E})(\frac{a}{h})[\lambda_1 + \frac{2m}{\sqrt{3}} (\frac{a}{h}) \lambda_0] \qquad (44)$$

Asumming the material contants of die $E$=210000 $N/mm^2$, $\nu$=0.3, the yield stress of steel $\sigma_f$=600 $N/mm^2$, of aluminum $\sigma_f$=120 $N/mm^2$, the results are listed in tables from Tab.2 to Tab.6 and shown in figures from Fig.7 to Fig.9, where 'ss' means the materials of die and workpiece are both steel, and 'sa' the material of die is steel and workpiece is aluinum.

Tab.2 The relation between $w/h$ and $k$

ss : steel–steel  sa : steel–aluminum

( a ) $h/a$=0.001 $m$=0.00

| $k$ | $w/h$–ss | $w/h$–sa | $k$ | $w/h$–ss | $w/h$–sa | $k$ | $w/$h–ss | $w/h$–sa |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 5.200000 | 1.040000 | | | | | | |
| 0.1 | 5.186975 | 1.037395 | 1.1 | 2.757730 | 0.551546 | 2.1 | 1.276583 | 0.255317 |
| 0.2 | 5.147603 | 1.029521 | 1.2 | 2.436371 | 0.487274 | 2.2 | 1.214997 | 0.242999 |
| 0.3 | 5.080948 | 1.016190 | 1.3 | 2.197152 | 0.439430 | 2.3 | 1.159251 | 0.231850 |
| 0.4 | 4.985303 | 0.997061 | 1.4 | 2.007188 | 0.401438 | 2.4 | 1.108529 | 0.221706 |
| 0.5 | 4.857920 | 0.971584 | 1.5 | 2.197152 | 0.370171 | 2.5 | 1.062163 | 0.212433 |
| 0.6 | 4.694456 | 0.938891 | 1.6 | 1.719108 | 0.343822 | 2.6 | 1.019604 | 0.203921 |
| 0.7 | 4.487811 | 0.897562 | 1.7 | 1.606124 | 0.321225 | 2.7 | 0.980390 | 0.196078 |
| 0.8 | 4.225258 | 0.845052 | 1.8 | 1.507907 | 0.301581 | 2.8 | 0.944136 | 0.188827 |
| 0.9 | 3.878813 | 0.775763 | 1.9 | 1.421585 | 0.284317 | 2.9 | 0.910511 | 0.182102 |
| 1.0 | 3.310422 | 0.662085 | 2.0 | 1.345021 | 0.269004 | 3.0 | 0.879236 | 0.175847 |

( b ) $h/a$=0.001 $m$=0.20

| $k$ | $w/h$–ss | $w/h$–sa | $k$ | $w/h$–ss | $w/h$–sa | $k$ | $w/$h–ss | $w/h$–sa |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 305.4221 | 61.08443 | | | | | | |
| 0.1 | 299.8731 | 59.97462 | 1.1 | 101.3954 | 20.27909 | 2.1 | 49.79565 | 9.959130 |
| 0.2 | 287.4047 | 57.48095 | 1.2 | 91.3468 | 18.26936 | 2.2 | 47.45071 | 9.490142 |
| 0.3 | 270.4248 | 54.08500 | 1.3 | 83.3243 | 16.66485 | 2.3 | 45.32044 | 9.061850 |
| 0.4 | 250.2775 | 50.05550 | 1.4 | 76.7033 | 15.34065 | 2.4 | 1.108529 | 0.221706 |
| 0.5 | 227.9734 | 45.59467 | 1.5 | 71.1175 | 14.22350 | 2.5 | 1.062163 | 0.212433 |
| 0.6 | 204.3787 | 40.87573 | 1.6 | 66.3275 | 13.26551 | 2.6 | 1.019604 | 0.203921 |
| 0.7 | 180.3121 | 36.06242 | 1.7 | 62.1666 | 12.43332 | 2.7 | 0.980390 | 0.196078 |
| 0.8 | 156.6255 | 31.32510 | 1.8 | 58.5137 | 11.70275 | 2.8 | 0.944136 | 0.188827 |
| 0.9 | 134.3248 | 26.86496 | 1.9 | 55.2782 | 11.05564 | 2.9 | 0.910511 | 0.182102 |
| 1.0 | 114.9353 | 22.98706 | 2.0 | 52.3903 | 10.47807 | 3.0 | 0.879236 | 0.175847 |

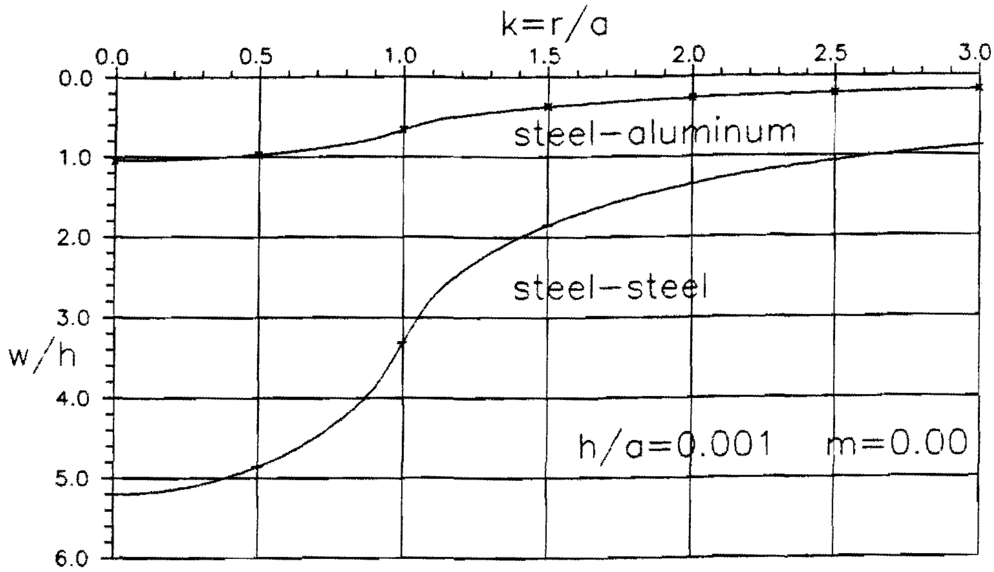


**Fig.7a The relation between $w/h$ and $k$**

**( $h$/a=0.001 $m$=0.00 )**

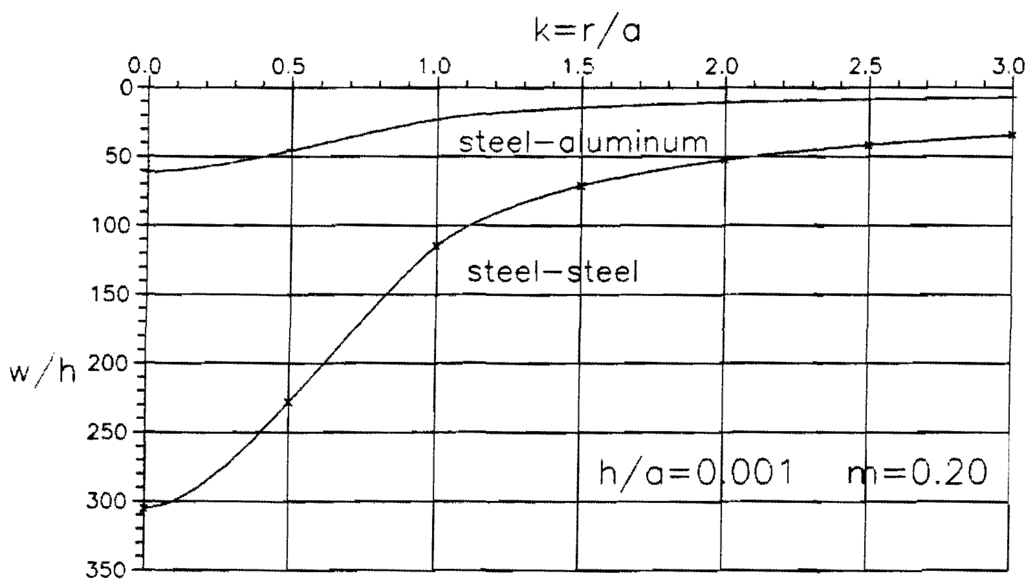


**Fig.7b The relation between $w/h$ and $k$**

**( $h$/a=0.001 $m$=0.20 )**

Tab.3 The relation between $w/h$ and $k$

( $m$=0.20 with $h$/a=0.025,0.050,0.075,0.100 )

ss:steel–steel sa:steel–aluminum

| $k$ | $h/a$=0.025 | | $h/a$=0.050 | | $h/a$=0.075 | | $h/a$=0.100 | |
|---|---|---|---|---|---|---|---|---|
| | $w/h$–s s | –sa | –s s | –sa | –s s | –sa | –s s | –sa |
| 0.0 | .688355 | .137671 | .224089 | .044818 | .122706 | .024541 | .082022 | .016404 |
| 0.1 | .678977 | .135795 | .221614 | .044323 | .121548 | .024310 | .081338 | .016268 |
| 0.2 | .657516 | .131503 | .215855 | .043171 | .118814 | .023763 | .079702 | .015940 |
| 0.3 | .627788 | .125557 | .207756 | .041551 | .114918 | .022984 | .077344 | .015469 |
| 0.4 | .591880 | .118376 | .197823 | .039565 | .110078 | .022016 | .074382 | .014876 |
| 0.5 | .551302 | .110260 | .186405 | .037281 | .104437 | .020392 | .070891 | .014178 |
| 0.6 | .507273 | .101455 | .173763 | .034753 | .098092 | .019618 | .066913 | .013383 |
| 0.7 | .460831 | .092166 | .160086 | .032017 | .091095 | .018219 | .062461 | .012492 |
| 0.8 | .412851 | .082570 | .145465 | .029093 | .083430 | .016686 | .057492 | .011499 |
| 0.9 | .363866 | .072773 | .129755 | .025951 | .074908 | .014982 | .051833 | .010367 |
| 1.0 | .311017 | .062203 | .110858 | .022172 | .063983 | .012797 | .044267 | .008853 |
| 1.1 | .268130 | .053626 | .094610 | .018922 | .054305 | .010861 | .037441 | .007488 |
| 1.2 | .239711 | .047942 | .084292 | .016958 | .048291 | .009658 | .033255 | .006651 |
| 1.3 | .217689 | .043538 | .076399 | .015279 | .043718 | .008744 | .030084 | .006017 |
| 1.4 | .199801 | .039960 | .070022 | .014004 | .040042 | .008008 | .027541 | .005508 |
| 1.5 | .184861 | .036972 | .064724 | .012945 | .036992 | .007398 | .025435 | .005087 |
| 1.6 | .172138 | .034428 | .060226 | .012045 | .034407 | .006881 | .023652 | .004730 |
| 1.7 | .161142 | .032228 | .056347 | .011269 | .032181 | .006436 | .022117 | .004423 |
| 1.8 | .151526 | .030305 | .052960 | .010592 | .030240 | .006048 | .020780 | .004156 |
| 1.9 | .143034 | .028607 | .049974 | .009995 | .028529 | .005706 | .019602 | .003920 |
| 2.0 | .135473 | .027095 | .047319 | .009464 | .027008 | .005402 | .018555 | .003711 |

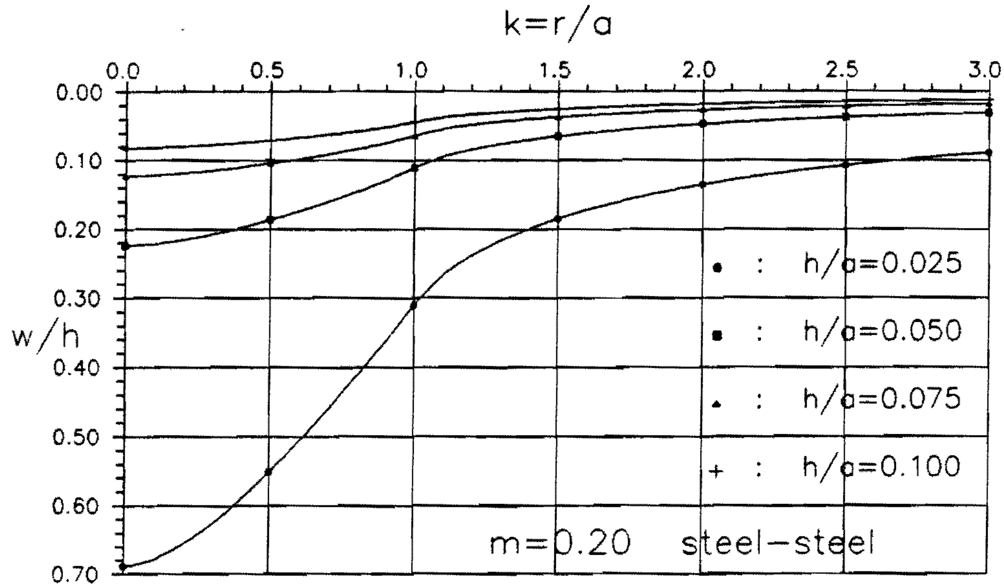


**Fig.8a The relation between $w/h$ and $k$ (steel–steel)**

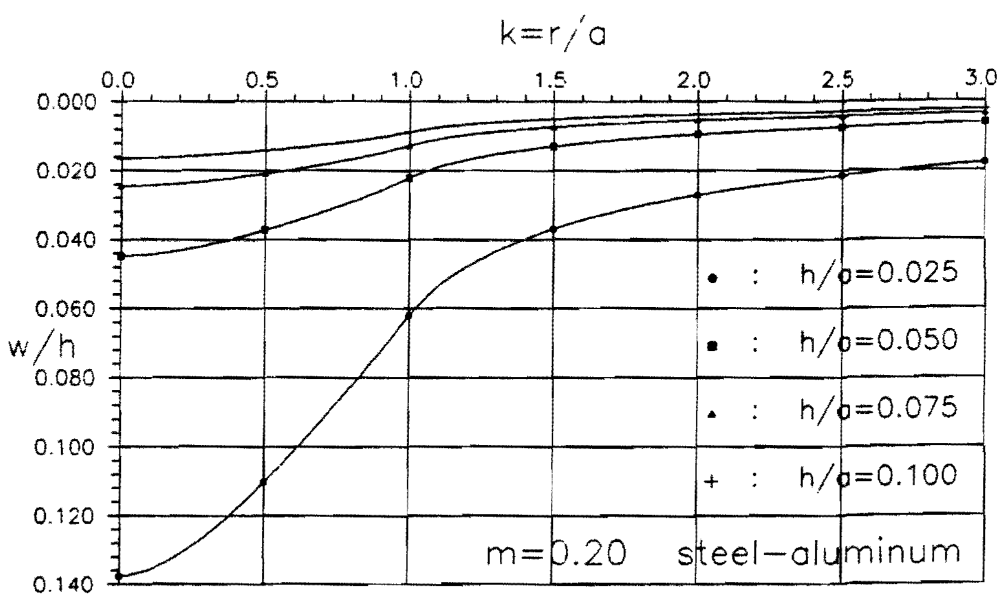


**Fig.8b The relation between $w/h$ and $k$ (steel–aluminum)**

Tab.4 The relation between $w_{max}/h$ and $h/a$ ($r/a$=0)

| $h/a$ | $m$ = 0.00 | | $m$ = 0.05 | | $m$ = 0.20 | | $m$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|
| | $w/h$—ss | —sa | —ss | —sa | —ss | —sa | —ss | —sa |
| 0.001 | 5.20000 | 1.04000 | 80.2553 | 16.0511 | 305.422 | 61.0844 | 755.755 | 151.151 |
| 0.005 | 1.04000 | 0.20800 | 4.04222 | 0.80844 | 13.0489 | 2.60978 | 31.0622 | 6.21244 |
| 0.010 | 0.52000 | 0.10400 | 1.27056 | 0.25411 | 3.52222 | 0.70444 | 8.02556 | 1.60511 |
| 0.015 | 0.34667 | 0.06933 | 0.68025 | 0.13605 | 1.68099 | 0.33620 | 3.68247 | 0.73649 |
| 0.020 | 0.26000 | 0.05200 | 0.44764 | 0.08953 | 1.01056 | 0.20211 | 2.13639 | 0.42728 |
| 0.025 | 0.20800 | 0.04160 | 0.32809 | 0.06562 | 0.68836 | 0.13767 | 1.40889 | 0.28178 |
| 0.030 | 0.17333 | 0.03467 | 0.25673 | 0.05135 | 0.50691 | 0.10138 | 1.00728 | 0.20146 |
| 0.035 | 0.14857 | 0.02971 | 0.20984 | 0.04197 | 0.39365 | 0.07873 | 0.76127 | 0.15225 |
| 0.040 | 0.13000 | 0.02600 | 0.17691 | 0.03538 | 0.31764 | 0.06353 | 0.59910 | 0.11982 |
| 0.045 | 0.11556 | 0.02311 | 0.15262 | 0.03052 | 0.26381 | 0.05276 | 0.48620 | 0.09724 |
| 0.050 | 0.10400 | 0.02080 | 0.13402 | 0.02680 | 0.22409 | 0.04482 | 0.40422 | 0.08084 |
| 0.055 | 0.09455 | 0.01891 | 0.11936 | 0.02387 | 0.19379 | 0.03876 | 0.34266 | 0.06853 |
| 0.060 | 0.08667 | 0.01733 | 0.10752 | 0.02150 | 0.17006 | 0.03401 | 0.29515 | 0.05903 |
| 0.065 | 0.08000 | 0.01600 | 0.09776 | 0.01955 | 0.15106 | 0.03021 | 0.25764 | 0.05153 |
| 0.070 | 0.07429 | 0.01486 | 0.08960 | 0.01792 | 0.13556 | 0.02711 | 0.22746 | 0.04549 |
| 0.075 | 0.06933 | 0.01387 | 0.08268 | 0.01654 | 0.12271 | 0.02454 | 0.20277 | 0.04055 |
| 0.080 | 0.06500 | 0.01300 | 0.07673 | 0.01536 | 0.11191 | 0.02238 | 0.18227 | 0.03645 |
| 0.085 | 0.06118 | 0.01224 | 0.07156 | 0.01431 | 0.10272 | 0.02055 | 0.16506 | 0.03301 |
| 0.090 | 0.05778 | 0.01156 | 0.06704 | 0.01341 | 0.09484 | 0.01897 | 0.15044 | 0.03009 |
| 0.095 | 0.05474 | 0.01095 | 0.06305 | 0.01261 | 0.08800 | 0.01760 | 0.13790 | 0.02758 |
| 0.100 | 0.05200 | 0.01040 | 0.05951 | 0.01190 | 0.08202 | 0.01640 | 0.12706 | 0.02541 |

Tab.5 The relation between $w/h$ and $h/a$ ($r/a$=0.5)

| $h/a$ | $m$ = 0.00 | | $m$ = 0.05 | | $m$ = 0.20 | | $m$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|
| | $w/h$—s s | −sa | −s s | −sa | −s s | −sa | −s s | −sa |
| 0.001 | 4.85792 | 0.97158 | 60.6368 | 12.1274 | 227.973 | 45.5947 | 562.646 | 112.529 |
| 0.005 | 0.97158 | 0.19432 | 3.20274 | 0.64055 | 9.89620 | 1.97924 | 23.2831 | 4.65663 |
| 0.010 | 0.48579 | 0.09716 | 1.04358 | 0.20872 | 2.71695 | 0.54339 | 6.06368 | 1.21274 |
| 0.015 | 0.32386 | 0.06477 | 0.57177 | 0.11435 | 1.31549 | 0.26310 | 2.80292 | 0.56058 |
| 0.020 | 0.24290 | 0.04858 | 0.38234 | 0.07647 | 0.80068 | 0.16014 | 1.63737 | 0.32747 |
| 0.025 | 0.19432 | 0.03886 | 0.28356 | 0.05671 | 0.55130 | 0.11026 | 1.08679 | 0.21736 |
| 0.030 | 0.16193 | 0.03239 | 0.22391 | 0.04478 | 0.40984 | 0.08197 | 0.78170 | 0.15634 |
| 0.035 | 0.13880 | 0.02776 | 0.18433 | 0.03687 | 0.32093 | 0.06419 | 0.59414 | 0.11883 |
| 0.040 | 0.12145 | 0.02429 | 0.15631 | 0.03126 | 0.26090 | 0.05218 | 0.47007 | 0.09401 |
| 0.045 | 0.10795 | 0.02159 | 0.13550 | 0.02710 | 0.21813 | 0.04363 | 0.38340 | 0.07668 |
| 0.050 | 0.09716 | 0.01943 | 0.11947 | 0.02389 | 0.18640 | 0.03728 | 0.32027 | 0.06405 |
| 0.055 | 0.08833 | 0.01767 | 0.10677 | 0.02135 | 0.16208 | 0.03242 | 0.27272 | 0.05454 |
| 0.060 | 0.08097 | 0.01619 | 0.09646 | 0.01929 | 0.14294 | 0.02859 | 0.23591 | 0.04718 |
| 0.065 | 0.07474 | 0.01495 | 0.08794 | 0.01759 | 0.12754 | 0.02551 | 0.20676 | 0.04135 |
| 0.070 | 0.06940 | 0.01388 | 0.08078 | 0.01616 | 0.11493 | 0.02299 | 0.18323 | 0.03665 |
| 0.075 | 0.06477 | 0.01295 | 0.07469 | 0.01494 | 0.10444 | 0.02089 | 0.16393 | 0.03279 |
| 0.080 | 0.06072 | 0.01214 | 0.06944 | 0.01389 | 0.09559 | 0.01912 | 0.14788 | 0.02958 |
| 0.085 | 0.05715 | 0.01143 | 0.06487 | 0.01297 | 0.08803 | 0.01761 | 0.13435 | 0.02687 |
| 0.090 | 0.05398 | 0.01080 | 0.06086 | 0.01217 | 0.08152 | 0.01630 | 0.12284 | 0.02457 |
| 0.095 | 0.05114 | 0.01023 | 0.05732 | 0.01146 | 0.07586 | 0.01517 | 0.11294 | 0.02259 |
| 0.100 | 0.04858 | 0.00972 | 0.05416 | 0.01083 | 0.07089 | 0.01418 | 0.10436 | 0.02087 |

Tab.6 The relation between $w/h$ and $h/a$ ($r/a$=1.0)

| $h/a$ | $m$ = 0.00 | | $m$ = 0.05 | | $m$ = 0.20 | | $m$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|
| | $w/h$—s s | −sa | −s s | −sa | −s s | −sa | −s s | −sa |
| 0.001 | 3.31042 | 0.66208 | 31.2166 | 6.24333 | 114.035 | 22.9871 | 282.373 | 56.4745 |
| 0.005 | 0.66208 | 0.13241 | 1.77833 | 0.35567 | 5.12708 | 1.02542 | 11.8246 | 2.36491 |
| 0.010 | 0.33104 | 0.06621 | 0.61010 | 0.12202 | 1.44729 | 0.28946 | 3.12166 | 0.62433 |
| 0.015 | 0.22069 | 0.04414 | 0.34472 | 0.06894 | 0.71681 | 0.14336 | 1.46097 | 0.29219 |
| 0.020 | 0.16552 | 0.03310 | 0.23529 | 0.04706 | 0.44458 | 0.08892 | 0.86318 | 0.17264 |
| 0.025 | 0.13242 | 0.02648 | 0.17707 | 0.03541 | 0.31102 | 0.06220 | 0.57892 | 0.11578 |
| 0.030 | 0.11035 | 0.02207 | 0.14135 | 0.02827 | 0.23437 | 0.04688 | 0.42042 | 0.08481 |
| 0.035 | 0.09458 | 0.01892 | 0.11736 | 0.02347 | 0.18571 | 0.03714 | 0.32239 | 0.06448 |
| 0.040 | 0.08276 | 0.01655 | 0.10020 | 0.02004 | 0.15253 | 0.03051 | 0.25717 | 0.05143 |
| 0.045 | 0.07356 | 0.01471 | 0.08735 | 0.01747 | 0.12869 | 0.02527 | 0.21137 | 0.04227 |
| 0.050 | 0.06621 | 0.01324 | 0.07737 | 0.01547 | 0.11086 | 0.02217 | 0.17783 | 0.03557 |
| 0.055 | 0.06019 | 0.01204 | 0.06941 | 0.01388 | 0.09709 | 0.01942 | 0.15244 | 0.03049 |
| 0.060 | 0.05517 | 0.01103 | 0.06293 | 0.01259 | 0.08618 | 0.01724 | 0.13244 | 0.02654 |
| 0.065 | 0.05093 | 0.01019 | 0.05753 | 0.01151 | 0.07735 | 0.01547 | 0.11698 | 0.02340 |
| 0.070 | 0.04729 | 0.00946 | 0.05299 | 0.01060 | 0.07007 | 0.01401 | 0.10424 | 0.02085 |
| 0.075 | 0.04414 | 0.00883 | 0.04910 | 0.00982 | 0.06398 | 0.01280 | 0.09375 | 0.01875 |
| 0.080 | 0.04138 | 0.00828 | 0.04574 | 0.00915 | 0.05882 | 0.01176 | 0.08498 | 0.01700 |
| 0.085 | 0.03895 | 0.00779 | 0.04281 | 0.00856 | 0.05440 | 0.01088 | 0.07757 | 0.01551 |
| 0.090 | 0.03678 | 0.00736 | 0.04023 | 0.00805 | 0.05056 | 0.01011 | 0.07123 | 0.01425 |
| 0.095 | 0.03485 | 0.00697 | 0.03794 | 0.00759 | 0.04721 | 0.00944 | | |
| 0.100 | 0.03310 | 0.00662 | 0.03589 | 0.00718 | 0.04427 | 0.00885 | | |

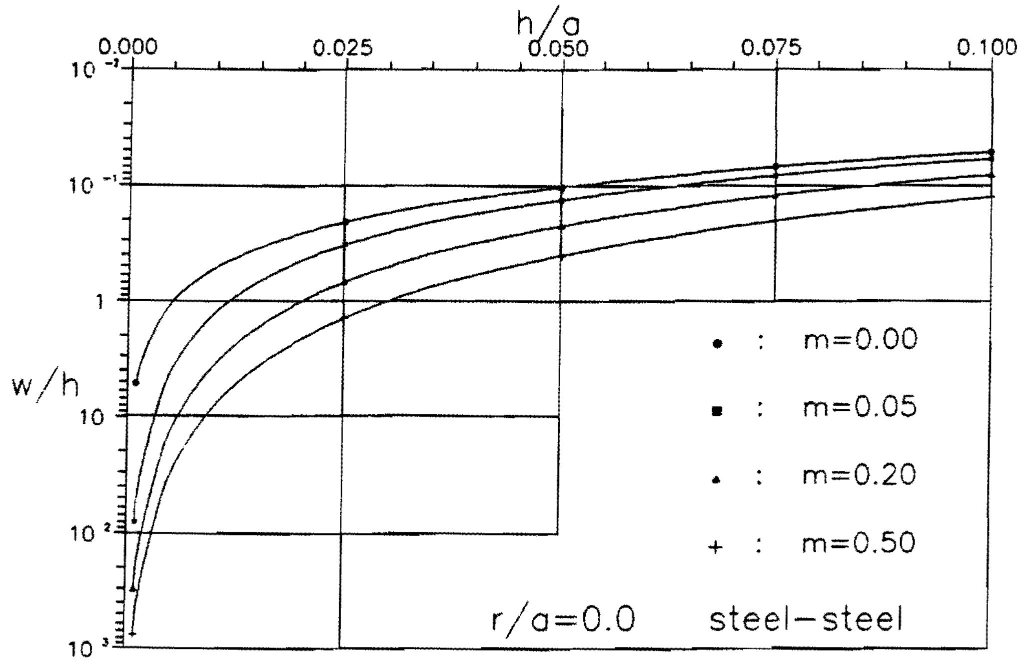


**Fig.9a The relation between $w_{max}/h$ and $h/a$ (ss, $k$=0.0)**

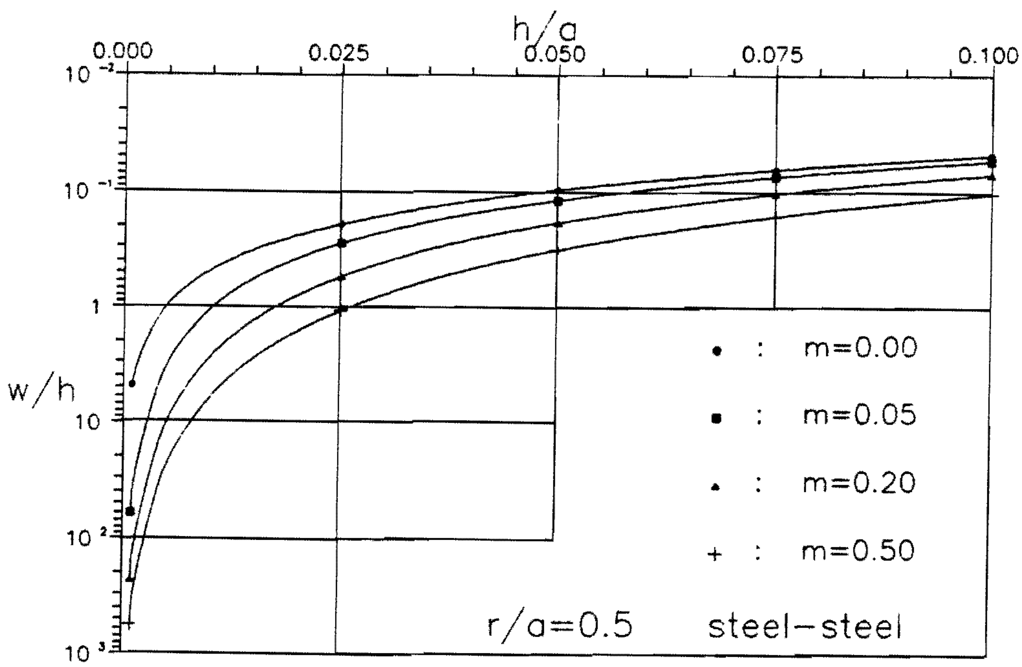


**Fig.9b The relation between $w/h$ and $h/a$ (ss, $k$=0.5)**

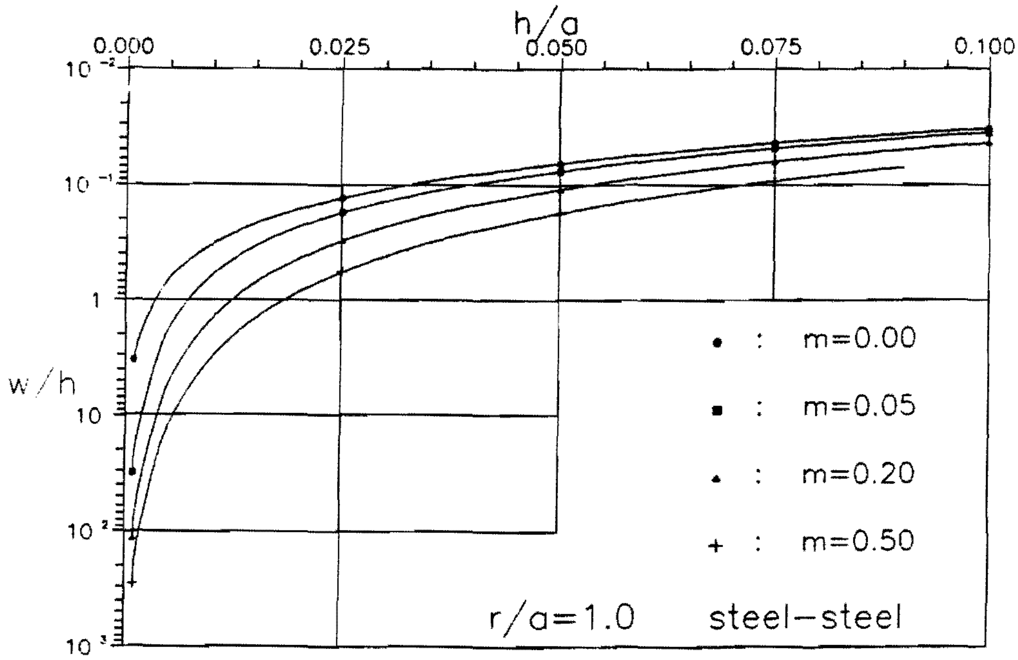


Fig.9c The relation between $w/h$ and $h/a$ (ss, $k$=1.0)

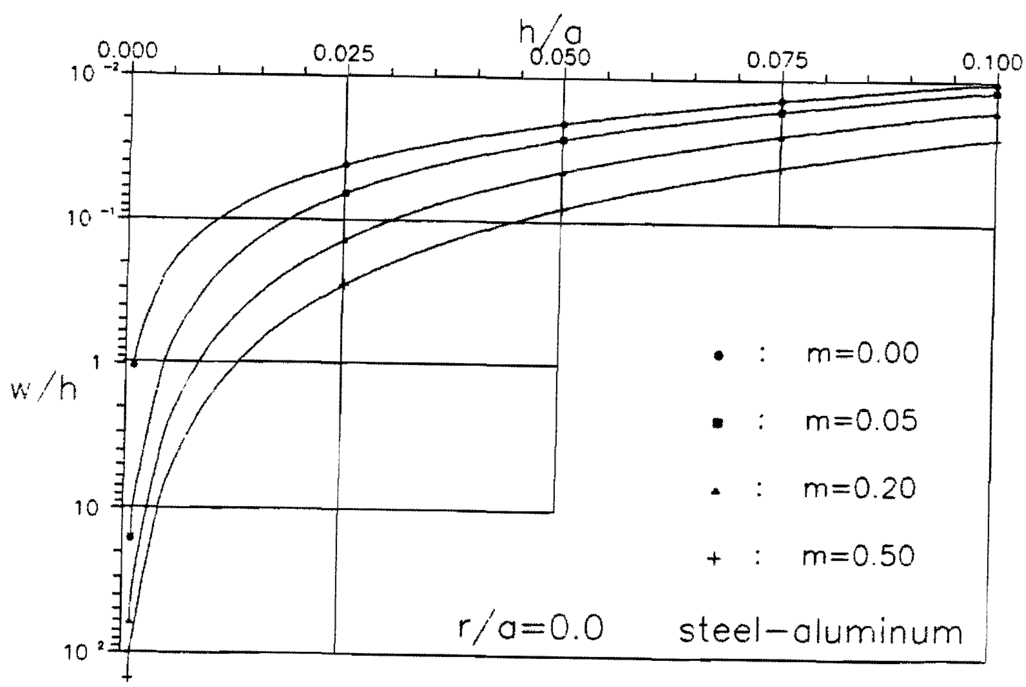


Fig.9d The relation between $w_{max}/h$ and $h/a$ (sa, $k$=0.0)

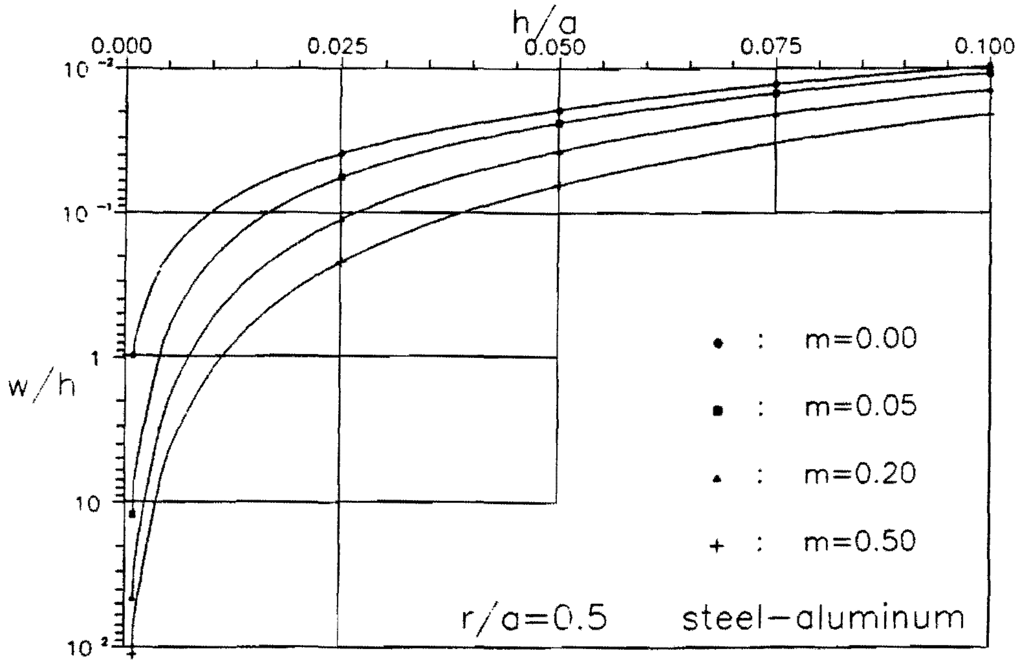


**Fig.9e** The relation between $w/h$ and $h/a$ (sa, $k$=0.5)

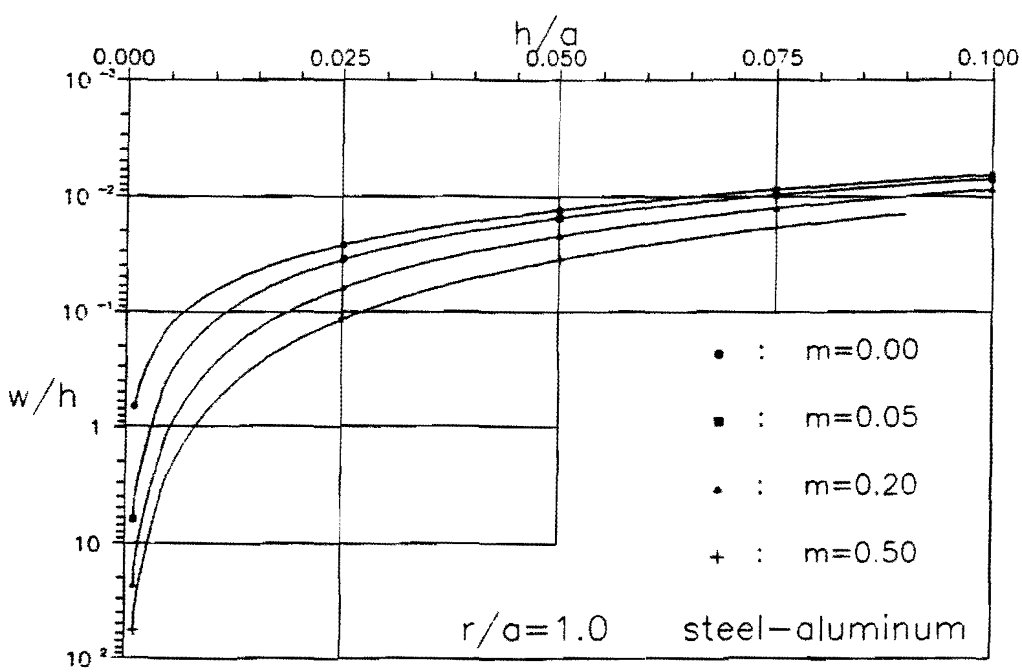


**Fig.9f** The relation between $w/h$ and $h/a$ (sa, $k$=1.0)

## VI. The Critical $h/a$ for the Die with Certain Size without Compensation

The surface of the die without compensation will deflect into concaved–shape, therefore, the upper and lower dies under compression will first contact each other at the point on the edge. This makes it impossible for the dies to move any more, and the ratio of $h/a$ has the minimum value called critical of $h/a$. So there is the necessity of determining the minimum value of $h/a$ for the die with a certain diameter $2R_d$ ( See Fig.10 ).

According to the analysis above, the maximum deformation occurs at the centre, and the combination ( 20 ) and ( 34 ) with $q_1=\sigma_f$ and $q_0=\frac{2m}{\sqrt{3}}(\frac{a}{h})\sigma_f$ gives

$$w_{max}=\frac{1-\nu^2}{E}\left[2+\frac{2m}{\sqrt{3}}(\frac{a}{h})\right]\sigma_f a \qquad (45)$$

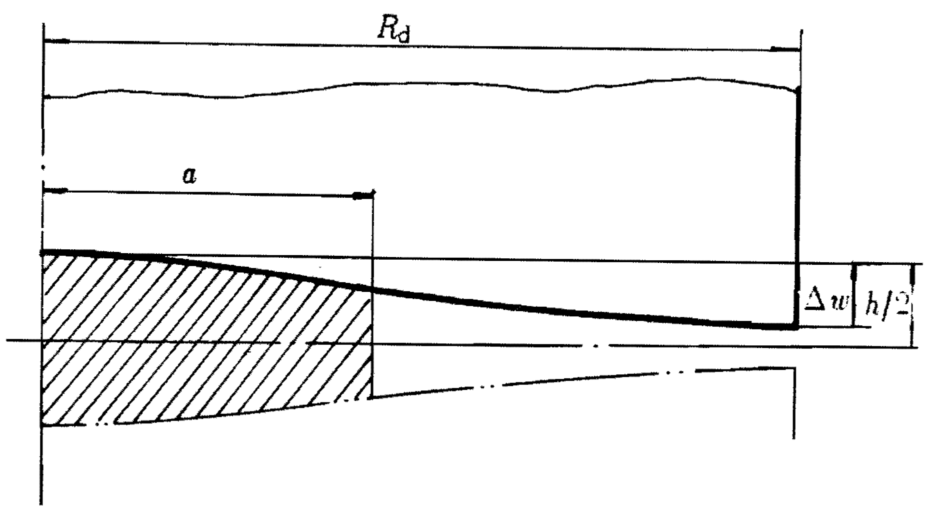


Fig.10 The difference of deflection between the centre and edge

The difference of deformation between the centre and a certain point is

$$\Delta w = w_{max} - w \qquad (46)$$

Substitution of ( 45 ) and ( 43 ) into ( 46 ) follows

$$\Delta w = \frac{1-\nu^2}{E}\left[ (2-\lambda_1) + \frac{2m}{\sqrt{3}}\left(\frac{a}{h}\right)(1-\lambda_0)\right]\sigma_f a \qquad (47)$$

From Fig.10, the difference of the deflection between the centre and the edge $\Delta w$ must be less than or equal to $h/2$, i.e., the critical point is

$$\frac{\Delta w}{h} = \frac{1}{2} \qquad (48)$$

so we get the condition equation

$$\frac{1-\nu^2}{E}\left[ (2-\lambda_1) + \frac{2m}{\sqrt{3}}\left(\frac{a}{h}\right)(1-\lambda_0)\right]\sigma_f\left(\frac{a}{h}\right) \leq \frac{1}{2}$$

that is,

$$(1-\nu^2)\left(\frac{\sigma_f}{E}\right)\frac{2m}{\sqrt{3}}(1-\lambda_0)\left(\frac{a}{h}\right)^2 + (1-\nu^2)\left(\frac{\sigma_f}{E}\right)(2-\lambda_1)\left(\frac{a}{h}\right) - \frac{1}{2} \leq 0 \qquad (49)$$

Solving this quadratic unequal equation and taking the real solution, we get the critical ratio of $h/a$, the reciprocal of the unkown $(\frac{a}{h})$ in Eq.( 49 )

$$\frac{h}{a} \geq \begin{cases} 2B & (m = 0) \\ \dfrac{2A}{-B+\sqrt{B^2+2A}} & (m \neq 0) \end{cases} \qquad (50)$$

where $A = (1-\nu^2)\left(\frac{\sigma_f}{E}\right)\frac{2m}{\sqrt{3}}(1-\lambda_0)$

$B = (1-\nu^2)\left(\frac{\sigma_f}{E}\right)(2-\lambda_1)$

For a die with a certain size, the different frictional coefficients gives different critical ratios of $h/a$, the results of the critical ratio of $h/a$ are listed in Tab.7 and shown in Fig.11. Avoiding contact between the edges of the upper and lower die means that the ratio of $h/a$ must be larger than the value given in the table or figure.

Tab.10 The Critical Ratio of $h/a$

| $m$ | $R_d = 2.0a$ | | $R_d = 2.5a$ | | $R_d = 3.0a$ | |
|---|---|---|---|---|---|---|
| | ss | sa | ss | sa | ss | sa |
| 0.00 | 0.0077 | 0.0015 | 0.0082 | 0.0016 | 0.0086 | 0.0017 |
| 0.05 | 0.0201 | 0.0079 | 0.0207 | 0.0081 | 0.0212 | 0.0082 |
| 0.10 | 0.0265 | 0.0108 | 0.0273 | 0.0111 | 0.0278 | 0.0112 |
| 0.15 | 0.0314 | 0.0130 | 0.0324 | 0.0133 | 0.0329 | 0.0135 |
| 0.20 | 0.0357 | 0.0149 | 0.0366 | 0.0153 | 0.0373 | 0.0155 |
| 0.25 | 0.0394 | 0.0166 | 0.0404 | 0.0170 | 0.0411 | 0.0172 |
| 0.30 | 0.0427 | 0.0181 | 0.0438 | 0.0185 | 0.0445 | 0.0188 |
| 0.40 | 0.0487 | 0.0208 | 0.0499 | 0.0212 | 0.0507 | 0.0215 |
| 0.50 | 0.0539 | 0.0231 | 0.0553 | 0.0236 | 0.0561 | 0.0240 |

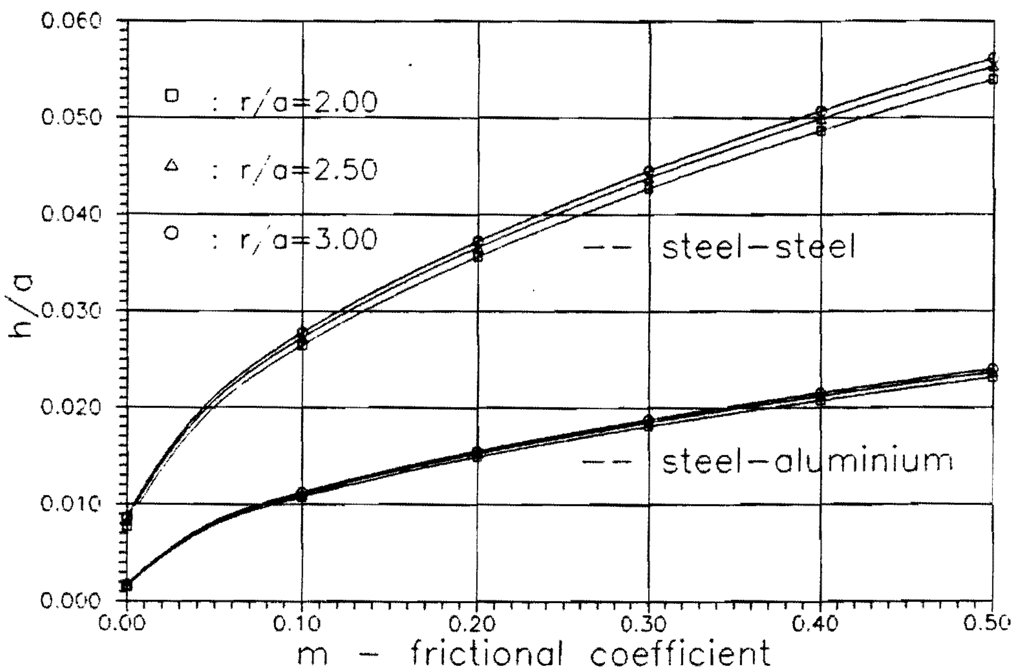


Fig.11 The critical value of $h/a$

## VII. Conclusions

The results mentioned above are based on the assumption that die and punch are semi–infinite boundary bodies. In fact, the die or punch has a limited size, therefore, there must exist difference from the reality. However, from figures shown above, the difference between the points where the ratio of $r/a$ is large enough, for example, $r/a > 2$, is comparatively small. Therefore, we may conclude that the results obtained above are very practical.

Now, we can draw out the following conclusions:

1. The surface of the die and punch under compression deforms according to

$$w = \frac{1-\nu^2}{E}\sigma_f a[\lambda_1 + \frac{2m}{\sqrt{3}}(\frac{a}{h})\lambda_0]$$

2. To increase the precision of the product geometry and make production going easier, deformation of the surface of the die and punch, especially those for the steel product, shoud be compensated by

$$\bar{w} = -w$$

3. Depending on the friction factor $m$ and the combination of the material of workpiece and die critical values of $h/a$ are found, near which upper and lower die contact each other at the edge, further compression is impossible.

4. The load function ( 1 ) is determined applying the slabmethod and assuming the upper and lower surface of the workpiece to be plane–parallel. Doing so the influene of the deformation of the die on the forming process of the disk is neglected. Taking this into account means that the radial

component of the contact pressure, which is caused by the slope of the $w(r)$ curve, should be incorporated into the equilibrium of the slab and therefore inflence the load function. The latter again gives another deformation of the die surface and so on. Among other things, this will be the subject of following studies.