

# Development and analysis of a vehicle model in SimMechanics for Formula Student

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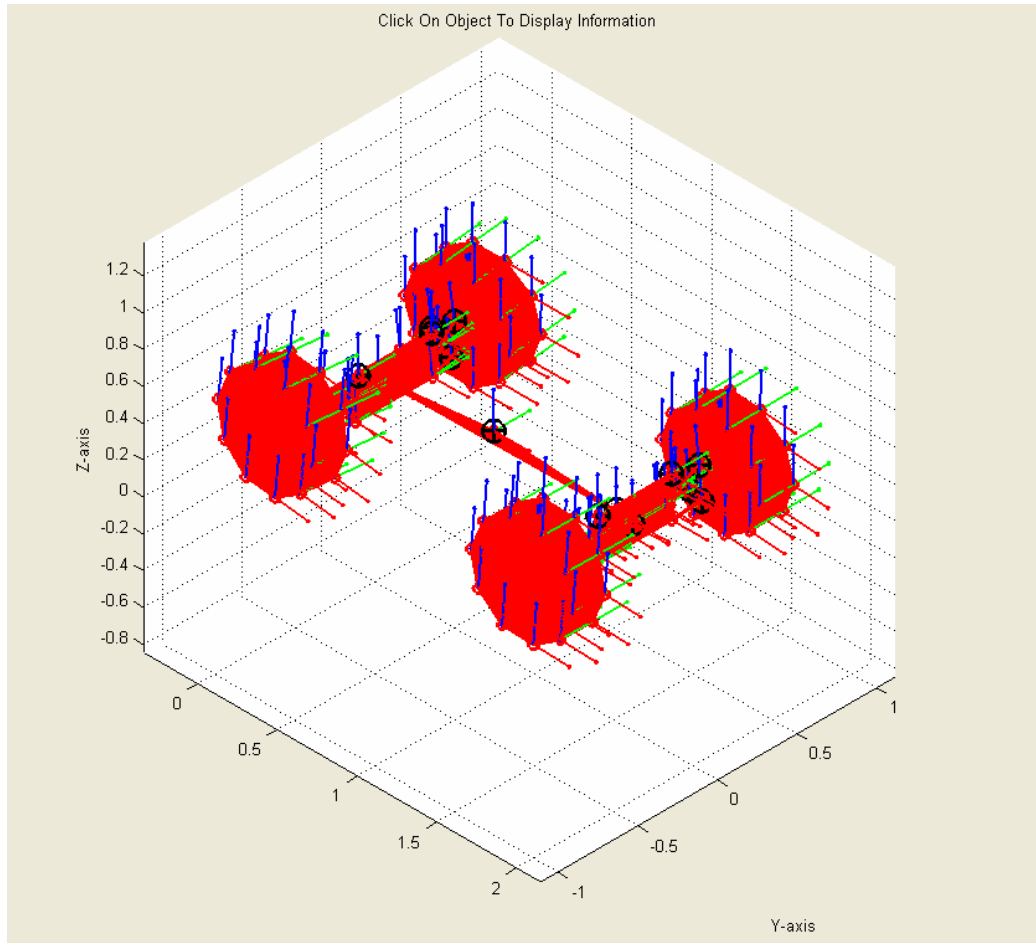
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# Development and analysis of a vehicle model in SimMechanics for Formula Student

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## Summary/ Samenvatting

### *Summary*

In this assignment, an already existing vehicle model has been validated, altered and improved. The model had to deal with some demands, concerning a faster calculation time and less complex modeling of the different parts, (like the A-arms) and no parametrisation. Therefore another approach has been chosen, which resulted in a totally new model. The model consists of 3 main parts, the front- and rear axle and the vehicle body. The axles themselves can be subdivided in a massless intermediate body to provide some hardpoints for the left and right suspension and the suspension system itself. When the model was build, the front tires had toe out and the rear tires had toe in. A thorough check of the suspension showed that this was a result of the coordinates used and the spring characteristics. Other (new) coordinates had to be used and prestress on the springs had to be added. There has been a modification, which has resulted in a single rod which replaces an A-arm which makes the car behave the same as with the old suspension. The model now also has a proper drivetrain on the rear wheels consisting of a differential and a simple engine model connected to a Continuously Variable Transmission (CVT). Thereby, a faster calculation time has been achieved with about a factor 10 and the structure of the model is clearer. Finally some basic handling tests have been done, to verify the model and predict the car's handling behavior. The car does safely understeer when doing a left corner at 30 degrees steering input at 20 m/s.

### *Samenvatting*

In deze opdracht is een al bestaand model gecontroleerd, aangepast en doorontwikkeld. Dit model moest aan een aantal eisen voldoen, zoals een snellere rekentijd en minder complexiteit m.b.t. de parametrisering van het model en de modellering van verschillende onderdelen zoals de A-armen. Daarom is voor een andere aanpak gekozen, wat resulteerde in een geheel nieuw model. Het model is opgebouwd uit drie hoofdonderdelen, te weten de voor- en achteras en het chassis. De assen zijn weer onder te verdelen in een massaloos tussenlichaam om de verbindingen te kunnen maken en een linker en rechter ophanging. De rest van de opbouw is hetzelfde gebleven als in het model van Bram de Jong. Nadat het model geheel in elkaar zat, bleken de voorwielen uitspoor en de achterwielen toespoor te hebben. Na een grondige controle van de ophanging, bleek dit te wijten aan de gebruikte coördinaten en veer karakteristiek. Hier is apart onderzoek naar gedaan, waaruit volgde dat er nieuwe (andere) coördinaten gebruikt moesten worden en dat de voorspanning op de veren nog niet gemodelleerd was in het huidige model. Er zijn een aantal zaken gemodificeerd, t.o.v. het voorgaande model, te weten een eenvoudiger ophanging die hetzelfde gedrag vertoont. Het model beschikt tevens over een aandrijving op de achterwielen bestaande uit een differentieel en een simpel model van de motor die weer gekoppeld is aan een Continu Variabele Transmissie (CVT). Verder is de rekensnelheid verhoogd met een factor 10 en is de structuur van het model overzichtelijker. Tenslotte zijn er tests uitgevoerd om het model te verifiëren en het gedrag van de auto te voorspellen. Het model vertoont veilig onderstuur wanneer er een bocht wordt gemaakt met een snelheid van 20 m/s en een stuurhoek van 30 graden.

# ***1. Introduction***

## ***1.1. Formula Student***

The challenge of Formula Student is to finance, market, design and build a small single seater racing car by students. Furthermore it is assumed that the students are engaged by a manufacturer that wants to build a thousand cars of 21.000 euro each for the non professional weekend autocross or sprint race. This is organised by the Institution of Mechanical Engineers (IMechE), in partnership with the Society of Automotive Engineers (SAE) and the Institution of Electrical Engineers (IEE). Of course, the highest goal is to win of the other teams. This can be done in several ways; it is a sum of different aspects. For example, the car is assessed on performance, cost, maintainability and marketability. All this is compared to determine the best overall car. The whole process provides the student with a real-life exercise in design and the business elements of automotive engineering [1].

## ***1.2. Background***

With the development of a new vehicle, simulations done by computers have become an essential part of the design proces. By using the computer, different vehicle alternatives can be analysed and optimised before the design is converted to a prototype. This also applies for the Formula Student racing car designed by students. For the analysis of the vehicle handling behaviour, a student who has worked on a comparable racing car model, Bram de Jong, started to create a model in SimMechanics, which is the multi-body toolbox of Matlab/SimMechanics. Using this tool, a complete model of a vehicle was developed, simulations are done and animations made. It was observed that the Formula Student vehicle in it's current set-up is possibly sensible for roll over by high lateral accelerations [2].

## ***1.3. Assignment***

The assignment can be subdivided in two parts.

First, the existing model by Bram de Jong has to be validated, improved and then further development has to be done. Secondly, some useful vehicle simulations have to be carried out to gain insight in the vehicle handling behaviour. One can think of under- and oversteer and roll-over-stability. The research should result in an improved vehicle model and recommendations for the design of the Formula Students racing car. First the new model will be presented and the general layout will be explained. Then adjustments from Bram's model will be presented and the different parts will be explained further in detail. Finally, some vehicle simulations will be done. After that, conclusions and recommendations are presented.

## 2. The model of the Formula Student car

### Introduction

In this assignment, an initial vehicle model is already available, which is described in the report written by Bram de Jong [2]. At first, the model of Bram had to be checked for faults. Because it was quite complex, the choice has been made to create a new model. Several reasons can be mentioned:

- 1) The calculation time of the model is quite long.  
Therefore an analysis is done to improve this calculation speed.  
It showed that the whole suspension (from tyre to spring/damper) makes the calculation slow. Exclusion of the tyre-model, suspension, rocker and spring/damper piece by piece, shows no difference.
- 2) Because all the coordinates and spring/damper properties are parameterised, access to these values is difficult and supervision is hard. To improve these drawbacks, a “back to basics” approach is chosen.

The basis of this new model is the one described in Bram’s report, but with some expansions and simplifications. A proper suspension, tyres, spring and dampers is already available, but one can think of different modelling of the A-arms or a drivetrain for expansion. Furthermore, all masses of the 2005 model are included.

### Matlab/SimMechanics

The initial and final model both are build with a toolbox of Matlab, namely SimMechanics. It has a library from which different bodies, constraints and drivers, joints, force elements, sensors and actuators can be selected and used to build a multi body model. The ground block defines “ground” and the world coordinates are relative to these ground coordinates. Furthermore, the length and Center of Gravity (cog) positions can be filled in at a body block. In this way bodies can be placed in 3-D space. Bodies are connected to each other with joints, force elements or constraints/drivers. It is thereby possible to read out different properties such as position and speeds and plot the results against time, or actuate bodies with for example a torque. Joints also can be actuated and their position can be read out. The model is described by a right-handed orthogonal set of axis and the coordinates and angles are defined as can be seen in figure 1. The model drives forward in positive x-direction and the left side of the car is described by the positive y-coordinates. Furthermore, the height is a positive z-coordinate and gravity is pointed downwards.

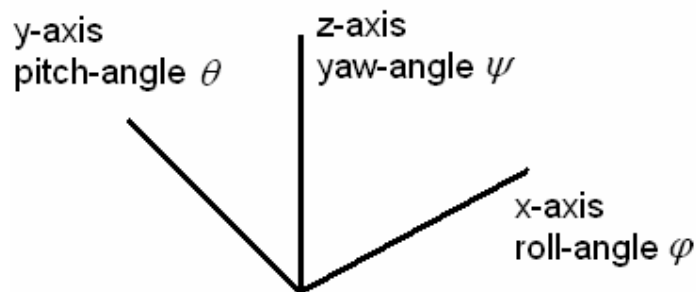


figure 1: orientation of the vehicle

## 2.1. General layout of the new model

### Introduction

The model consists of an independent front and rear axle connected to the vehicle body. The front tyres can be steered and the rear tyres are actuated by an engine torque, which delivers its power to the wheels via a differential. A body-spring-damper subsystem provides the suspension. The general layout can be seen in figure 2.

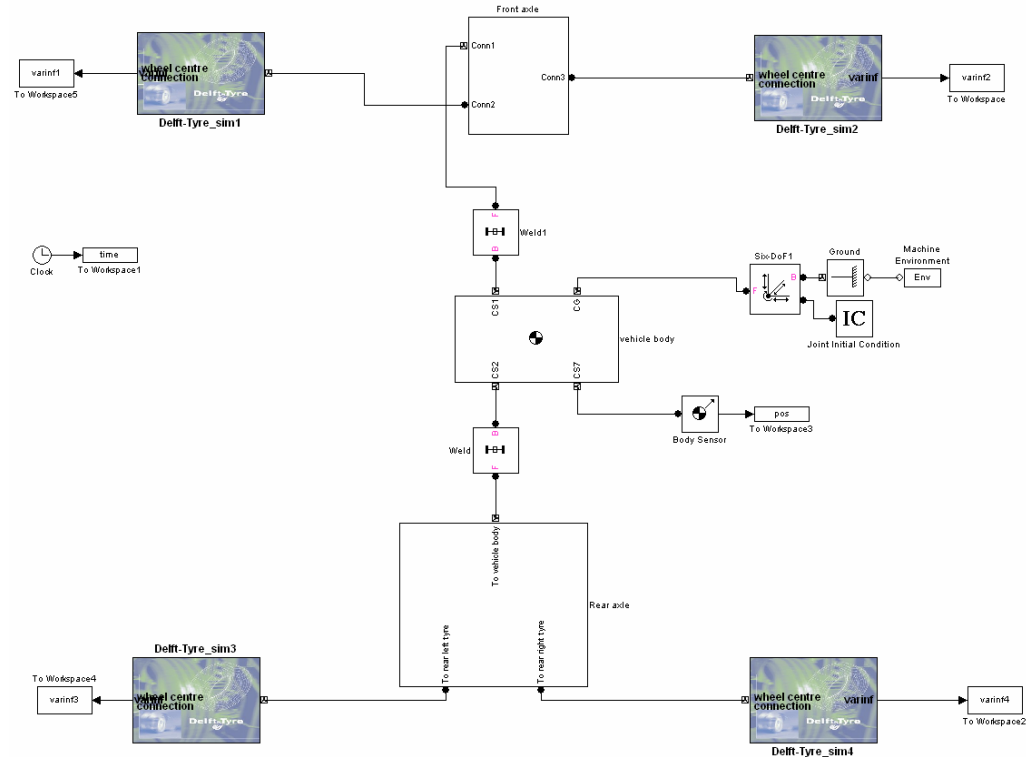


figure 2: general layout of the vehicle

### 2.1.1. The front axle

#### Introduction

The front axle consists of 4 parts. In the centre, an intermediate body can be seen, which only purpose is to connect both suspensions to the vehicle body. The right and left side of the car are symmetric about the x-axis. The suspension consists of an upper and lower A-arm, the upright and a tie rod and body-spring-damper subsystem. Via the tie rods, the uprights can be moved and in that way the car can be steered.

Figure 3 shows the layout of the front axle:

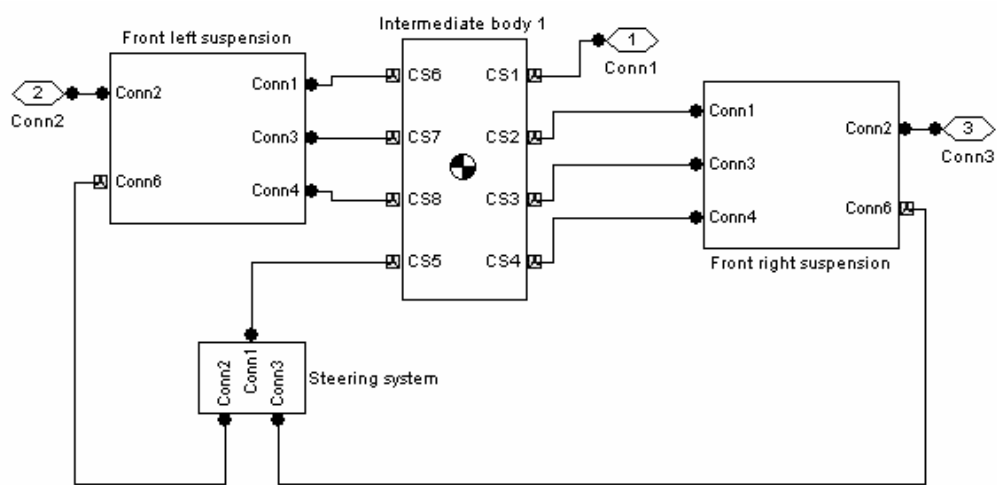


figure 3: the front axle

***The front suspension subsystem***

The front left - and front right suspensions are basically the same, only the y-coordinates differ, because of symmetry. In figure 4, the suspension is shown.

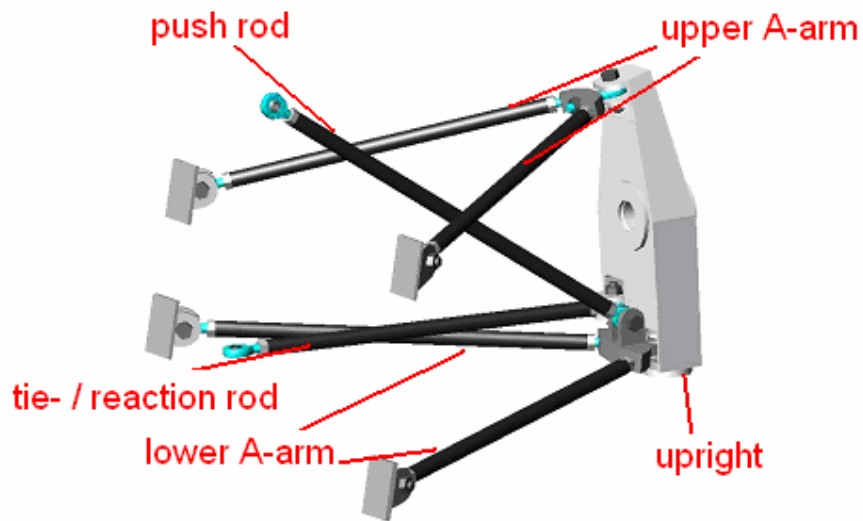


figure 4: example of the suspension



In figure 5, one can see how the suspension is built up in SimMechanics.

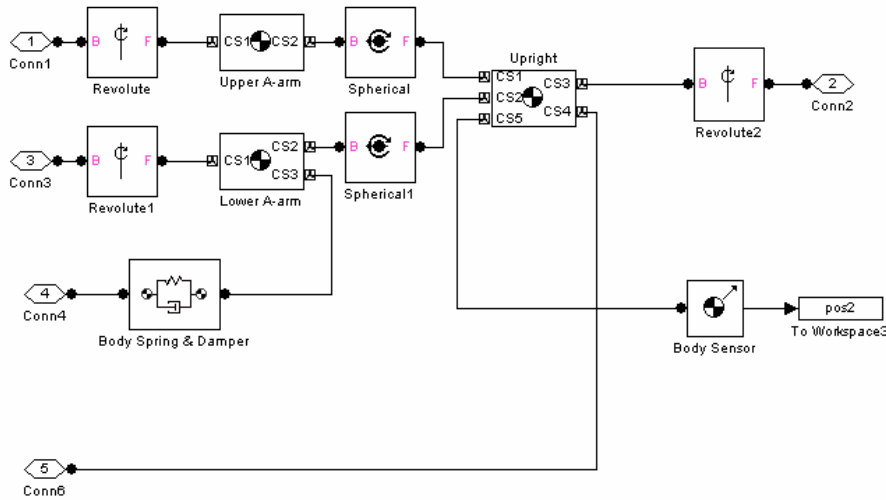


figure 5: the front right suspension

The suspension subsystem describes the double wishbone, upright, body-spring-damper subsystem and tie rod. These subsystems are positioned by coordinate systems relative to the world (the exact coordinates are given in appendix A).

Two important things have to be altered to the suspension in relation to the suspension in [2]. First of all, the suspension consists of an upper and lower A-arm, an upright in between them, a tie rod connected to the upright. In that way, the system is over constrained. The z-coordinate and y-coordinate, as well as the rotation around the y-axis  $\theta$ , are described twice per triangle. Therefore, the numerical solver can not compute forces and stresses inside the tubes.

To overcome this problem, two things can be done:

- 1) Extra degrees of freedom are introduced by adding a longitudinal degree of freedom at the front connection point of the A-arm to make sure the system is not over constrained. This is what Bram has done in his model by adding a longitudinal degree of freedom at the front connection point of the A-arm. In this way, the model represents the layout of the car as good as possible, but it does not resemble the physical reality.
- 2) The total suspension can be replaced by two revolute-body-sphericals. The revolute joint represents the connection with the vehicle body and describes the movement in a plane, where the spherical joint connects the upright with both rods.

The body-spring-damper subsystem connects the lower A-arm with the vehicle body. Steering can be done via a tie rod, which is a body connected to the upright by a spherical joint and connected to the vehicle body by a universal joint with the rotation around the y-axis blocked. In that way, the rotation of the tie rod itself is suppressed. Secondly, for the sake of convenience, the front- and rear suspension have to be translated at once, so adjustments concerning wheelbase can be made very easily.

To get this result, an intermediate body is added, where the suspension systems are fitted on. In that way one coordinate from the intermediate body has to be translated with respect to the vehicle body, and all the other coordinates are dependent of the previous ones. Due to time reasons, this could not yet be realised, but the structure is provided to do so. The positions of the hub centres can be read out in pos1 and pos2, for the front left – and front right tyre respectively.

### ***The steering subsystem***

In the model by Bram de Jong, not the steering angle, but a torque actuates the steering wheel. The steering angle  $\theta$  is given in and converted by a PD-controller to a torque. Because in reality, the driver controls the vehicle by means of a steering angle, the actuation by a torque is not correct. In the new configuration, with actuation of the steering wheel in a correct physical way, it needs an angle, steering-angular-velocity  $\dot{\omega}$  and the steering-angular-acceleration  $\ddot{\omega}$  as input. First, the steering angle input is led through a “smoother”, which is in fact a low pass filter. In that way, high frequencies are filtered out of the input signal. Furthermore, the angle is differentiated twice to get the steering-angular-velocity and the steering-angular-acceleration. The torque needed to hold the steering wheel in a certain position, which is in fact the reaction of the wheel, is read out. Due to stability reasons, a phase marge of at least 45 degrees at 10 Hz is provided. In figure 6 one can see the step response led through the low pass filter.

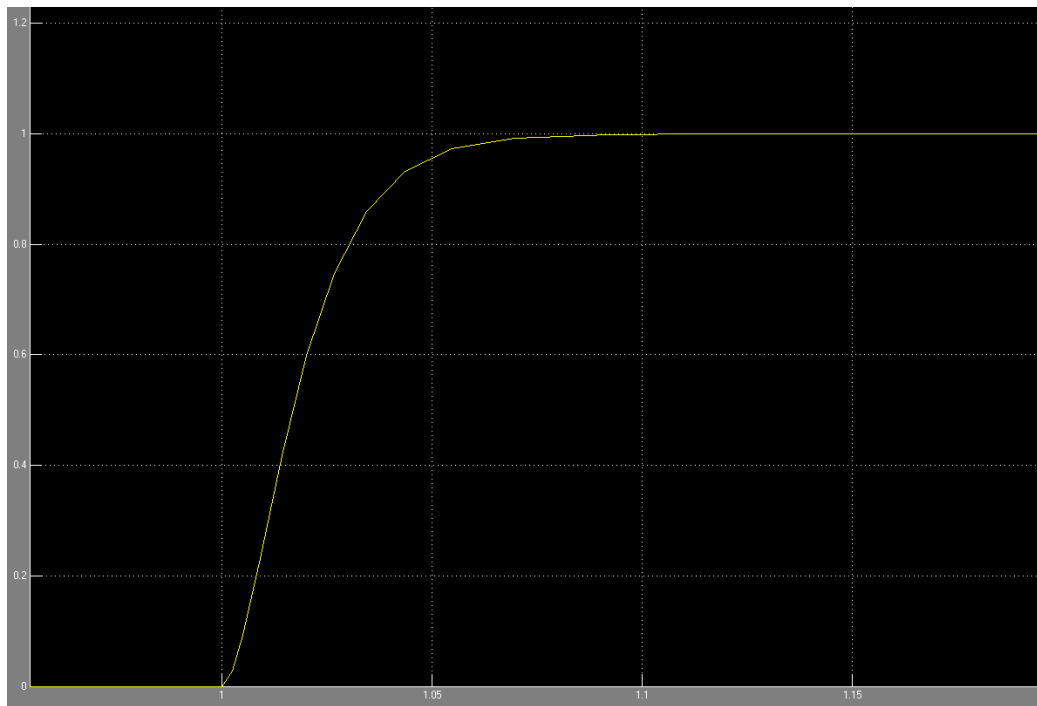


figure 6: angle input led through the low pass filter

As can be seen in figure 6, the step-response led through the low pass filter, is a lot smoothed and reaches its final value after about 75/1000 of a second, which is faster then 10 Hz. The response is quite fast.

In figure 7, one can see the steering system on the FS car.



figure 7: a) steering wheel b) steering system with tie rods

The steering-wheel-ratio has been checked and it showed that the gear constraint value was far too large. It has now been set to a displacement of the tie rods of 0.0426 m/turn of the steering wheel. The calculation of this value can be seen in Appendix B. The steering rack can be seen in figure 8.

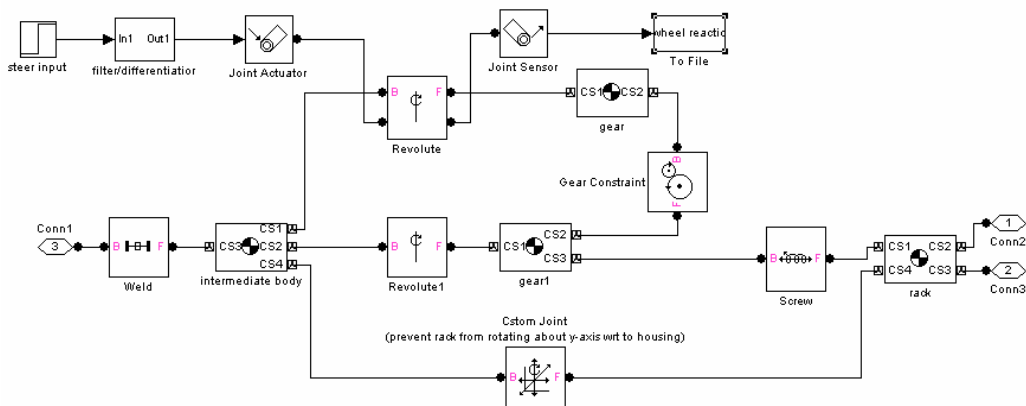


figure 8: the steering rack

The structure of the steering rack is as follows. The steering wheel normally directly drives the tie rods, which is called “rack and pinion”. In SimMechanics this is not possible so another way to accomplish this is to actuate the joint which connects a body (in fact the steering wheel) and put a gear constraint to another body (which is in fact a bolt). This bolt can rotate, but due to the pitch in the bolt and a screw joint, the body inside this bolt will translate. At this latter body, both left and right tie rods are connected. Due to translation of these tie rods, the wheels will rotate around the z-axis and the vehicle is able to steer. The reaction torque which is sensed at the steering wheel is read out by a joint sensor and put in Matlab/workspace “reaction\_torque\_steer”.

### 2.1.2. The vehicle body

Both front- and rear axle are connected with the vehicle body in which the driver is seated. The total weight of the car with driver is around 300 kg. The vehicle body is connected to the ground by a six Degrees of Freedom (6 DOF) block. Initial conditions can be given in at the joint initial condition block. The body's position can be read out by a body sensor and is put in the file "pos". The body can be seen in figure 9.

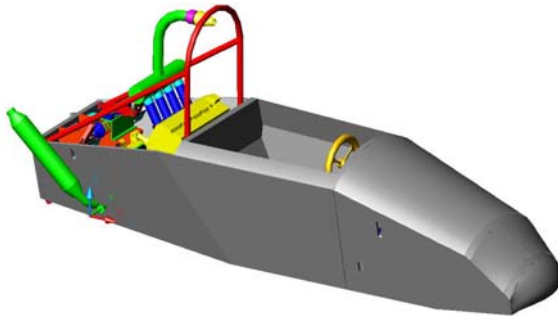


figure 9: the vehicle body

### 2.1.3. The rear axle

#### Introduction

The rear axle consists of 4 parts. In the centre, an intermediate body connects both suspensions to the vehicle body. The right and left side of the car are symmetric around the x-axis also. The suspension consists of an upper and lower A-arm, the upright with the axle connected to the tyres and a tie rod and body-spring-damper subsystem. Furthermore, the tyres are actuated by the engine- and brake torque and a reaction torque is put on the intermediate body. The layout of the rear axle can be seen in figure 10.

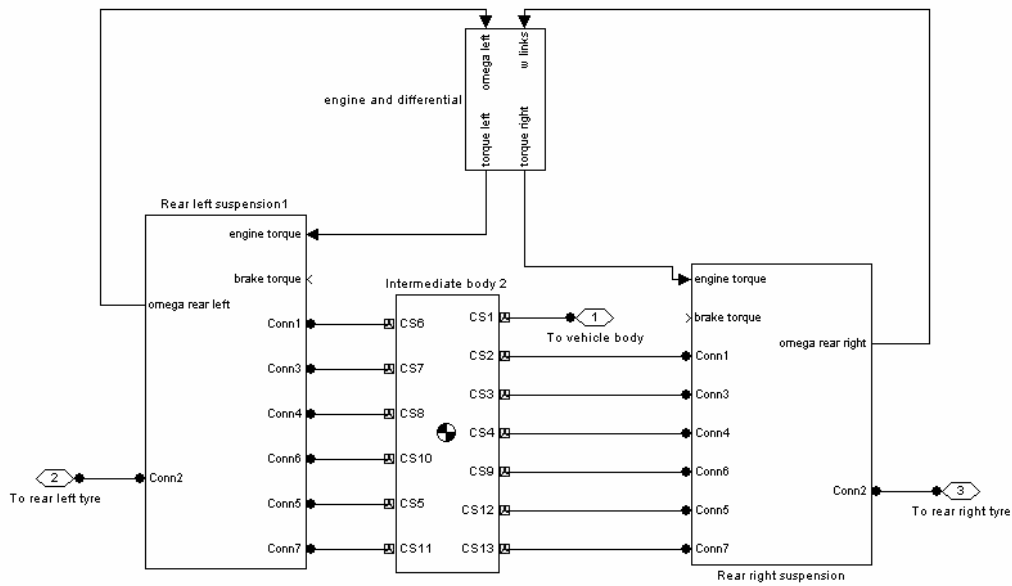


figure 10: the rear axle

### The rear suspension subsystem

The rear left and rear right suspensions are basically the same, only the y-coordinates differ, because of symmetry. In figure 11, one can see how the suspension is built up.

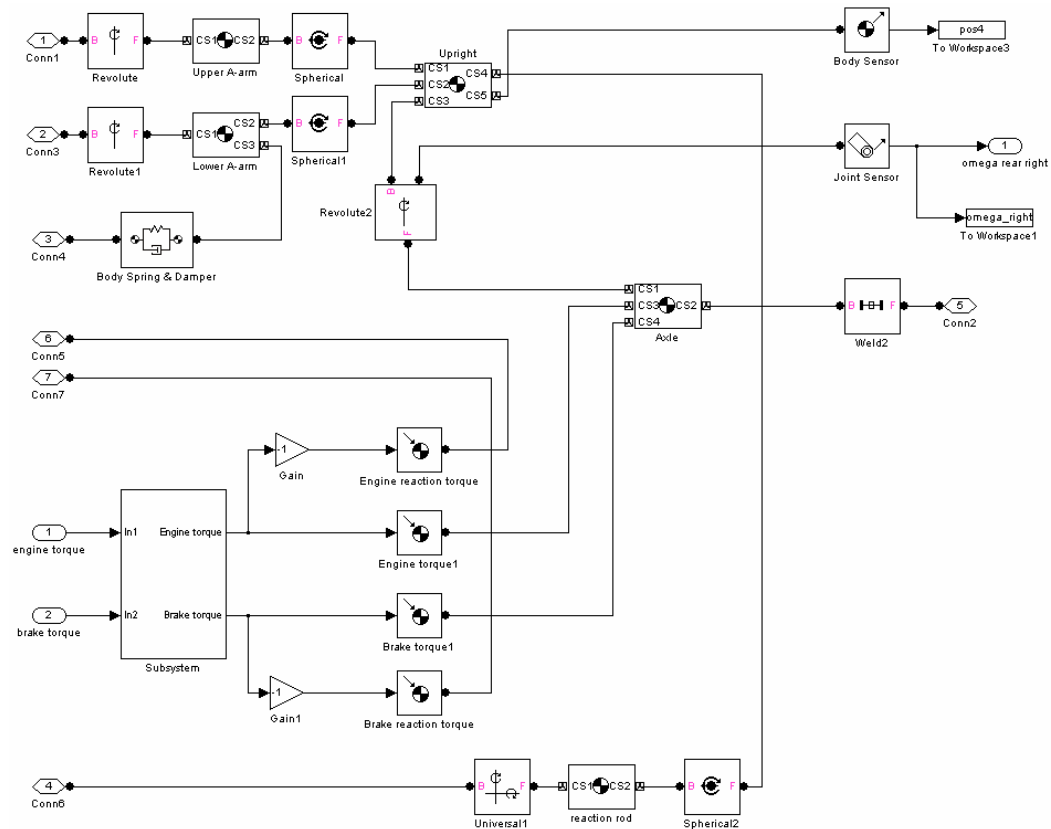


figure 11: the rear suspension subsystem

The suspension subsystem describes the double wishbone, upright, body-spring-damper subsystem and reaction rod. These subsystems are positioned by coordinate systems relative to the world. The original system was over constrained also, but this has been overcome in the same way as the front suspension. There are also some differences in comparison with the front suspension:

- 1) Because the upright does not have to rotate, the tie rod is replaced by a reaction rod.
- 2) The tyre is connected to the upright via an axle
- 3) The torques (that is the engine as well as the brake torque) actuates the tyre via the axle body. The reaction torque is sensed by the vehicle body at the height of the wheel axis. This connection is made at the intermediate body, so its height does not alter when the wheels displace vertically, which is not physically correct.

The rotational velocity is sensed by a joint sensor and put in "omega\_right". Rotation of the axle body around the y-axis hereby is not taken into account.

### ***The differential***

In physical reality, the configuration of a differential is as follows (see figure 12):

- 1) Crown wheel with differential casing
- 2) Left axle
- 3) Right axle
- 4) Pinions

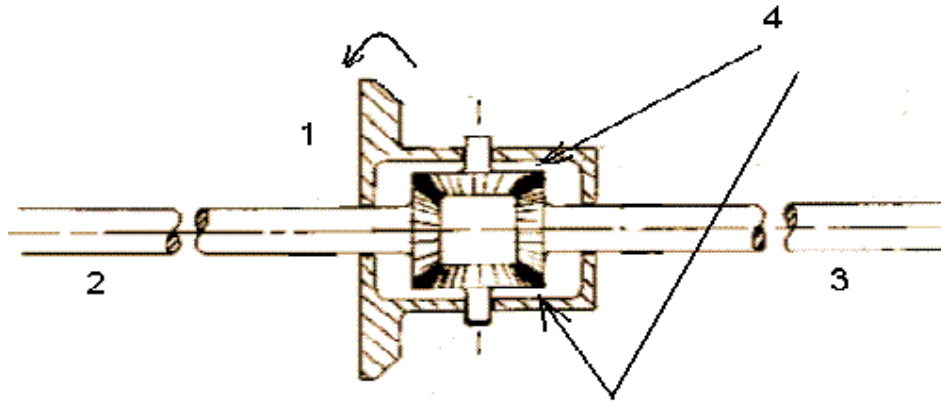


figure 12: “standard” differential

### ***The engine and differential subsystem***

In this subsystem the engine and differential are placed. The differential is used to compensate for the different traveled lengths the tyres cover when the car is cornering. The Formula Student car has a torsen differential, but because only a first analysis has to be done, a standard differential is used. An analytical approach is chosen. Figure 13 shows the layout of the differential and engine.

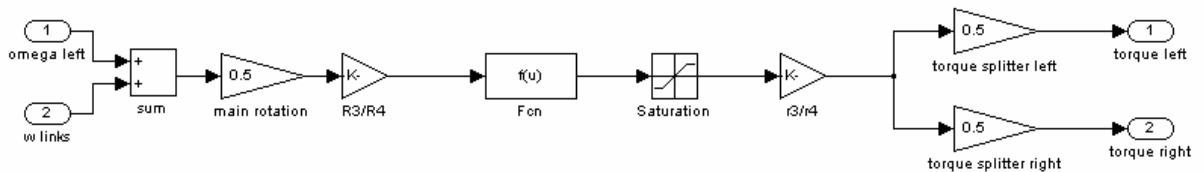


figure 13: the differential and engine subsystem

At the left side, the rotational velocities of the rear left and rear right tyres are summed and divided by two, to get the main rotation of the differential casing. After that, the rotation of the differential casing is converted to (a higher) rotational velocity of the engine. The engine provides a torque via the function:

$$P = T * \omega \quad (1)$$

which states that the torque is a linear function of the constant power of the engine and the rotational velocity. This provides a hyperbolic relationship. Because the torque would be infinite when the car stands still, the starting omega is about 1/100.

First, the connection of the engine to the tyres was made with rods, but it showed that this slowed the simulation down a lot. Therefore an analytical approach has been chosen. Now the shaft connected to the wheel is directly actuated with both a brake and an engine torque. The reaction torque is led through an intermediate body which in his case is welded to the vehicle body.

## 2.2. Other important subsystems

### *Introduction*

In the original model, not a body-spring-damper is used, but a push rod, which connects the lower A-arm to a rocker. In between the left – and right rockers (see [2]) a spring and damper are placed, as well as an anti roll mechanism. This model has none of them, but the body-spring-damper connects the lower A-arm and the vehicle body with each other. The properties of the front and rear tyres are described in “fronttyre.tpf” and ‘reartyre.tpf’ respectively. The properties have been altered to fit the Formula Student characteristics.

### *Spring and damper subsystem*

In order to make the model less complex and upgrade the performance in calculation speed, the spring and damper set has been made less complex. The lower A-arm is no longer connected via a push rod and a rocker to the spring and damper, but now in between the lower A-arm and the vehicle body, a body-spring damper subsystem is placed. The connection point of that subsystem to the chassis is the same as the connection point of the rocker. The values for the spring constant and the damper constant are kept the same as can be seen in table 1 and altered later on, but the spring natural length is the distance between both connections. The roll movement of the car is now also covered by the body spring damper system, whereas in the real world, the bump and roll movement is separated. In figure 14, one can see how the new configuration is realised.

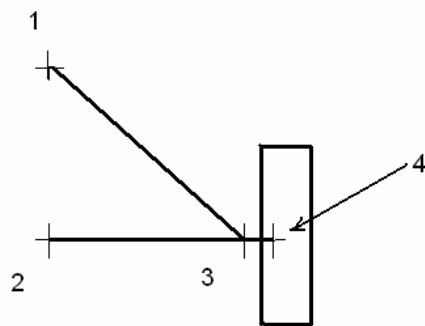


figure 14: new A-arm design

- 1: body spring damper connection point at chassis
- 2: lower A-arm connection point at chassis
- 3: body spring damper connection point at lower A-arm
- 4: upright connection to tyre

<b>Property [dimension]</b>	<b>Value (Front)</b>	<b>Value (Rear)</b>
Spring constant [N/m]	68000	68000
Damper constant [Ns/m]	1000	1000
Spring natural length [m]	0.415	0.437

Table 1: properties of the body spring damper subsystem

The spring natural length is taken as the distance in between both connection points. In the real world, prestress is also taken into account which is delivered by a larger spring natural length. The spring constants do also differ as can be seen in Appendix C. Because these calculated values did not give the expected results, the car has been “tuned” to run in a straight line until a speed of 40 m/s. Tuning means that the layout of the suspension and e.g. wheelbase are as in reality, but the spring and damper constants are altered.

### *Tyre properties*

The tyre properties are described in separate files for the front and rear tyres. Because the tyres have to distribute the engine- and brake torque to the wheels, its characteristics are very important. The model comes from TNO, which uses other (non important for this case) values for the different characteristics as named in table 2. Now, the properties have to be altered, such as the vertical stiffness and vertical damping to fit with the tyres used under the Formula Student racing car. The values have been retrieved from [3] and are summed in table 2.

<b>Property</b>	<b>Front</b>	<b>Rear</b>	<b>Dimension</b>
Vertical stiffness	15.4	18.1	kNm <sup>-1</sup>
Vertical damping	50	50	Nsm <sup>-1</sup>
Nominal vertical force	665	810	N
Longitudinal slip coefficient	1,3	1,4	-
Lateral slip coefficient	1,3	1,4	-
Rolling resistance coefficient	0,015	0,015	-

Table 2: tyre properties



### 3. Vehicle simulations

#### *Introduction*

To get a first impression of the vehicle behaviour, some basic vehicle simulations have been done. In that way, the model can be validated and properties like over- and understeer can be made visible. This can be done in two ways;

One way is to simulate the model and show the results by means of the visualisation option in SimMechanics, but this is only done to check the overall layout and get a basic impression of the car's behaviour.

Another option is to gain data from the model, for example forces and positions and plot these against time. That approach is chosen in this chapter.

#### *3.1. Simulation types*

Three simulations have been done, which will be explained in the following sections.

- 1) Static equilibrium
- 2) Accelerating
- 3) Constant velocity and cornering

#### *3.1.1. Static equilibrium*

#### *Introduction*

With the static equilibrium test, one can check whether the weight distribution of the car is correct. Vertical forces on the tyres can be plotted against time.

Because the Centre of Gravity (cog) of the car and the distance of the front and rear axle to this cog) are known, one can manually calculate the weight distribution and compare this to the analyses done by the computer. This manual calculation can be seen in appendix D. In figure 15, one can see the forces acting on the car.

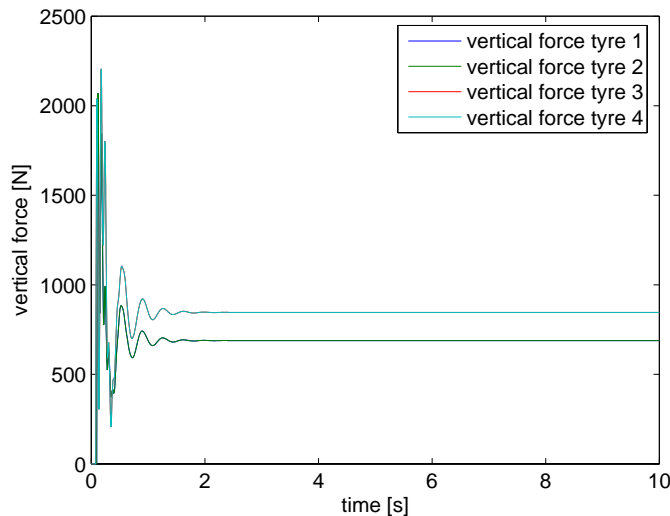


figure 15: forces on the tyres

First, the car must find its equilibrium position and it can be seen that the forces on the tyres damp out, due to the damping of the tyre and the body-spring-damper subsystem. The forces in the first couple of seconds are higher than those in equilibrium, because the car initially is not placed on the ground.

In figure 16, one can see that the difference in forces between the right and left tyres is very small, in the order of 0.001 Newton, at the front- as well as at the rear axle, which is negligible. The tyre force on the front tyre is about 688 N and that on the rear tyre 846 N.

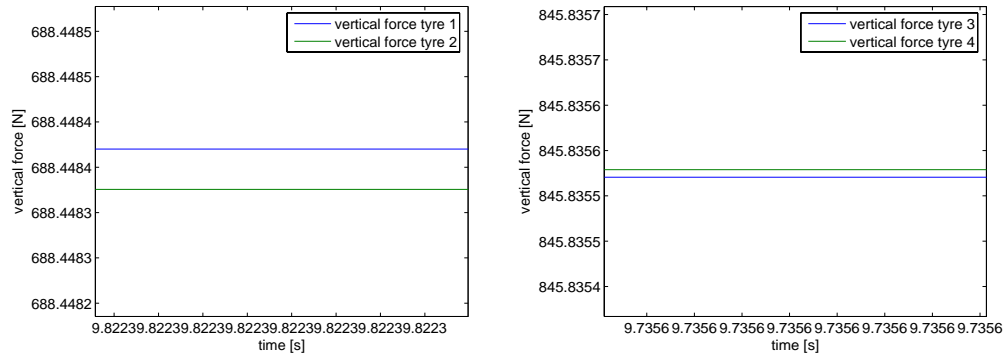


figure 16: a) vertical forces on front tyres b) vertical forces on rear tyres

### 3.1.2. Accelerating

#### Introduction

Under acceleration, the Center of Gravity of the car feels inertia forces. Because the cog is placed above the axis of rotation of the tyres, the weight distribution is altered from the front to the rear tyres. In figure 17, the forces acting on the car under acceleration can be seen.

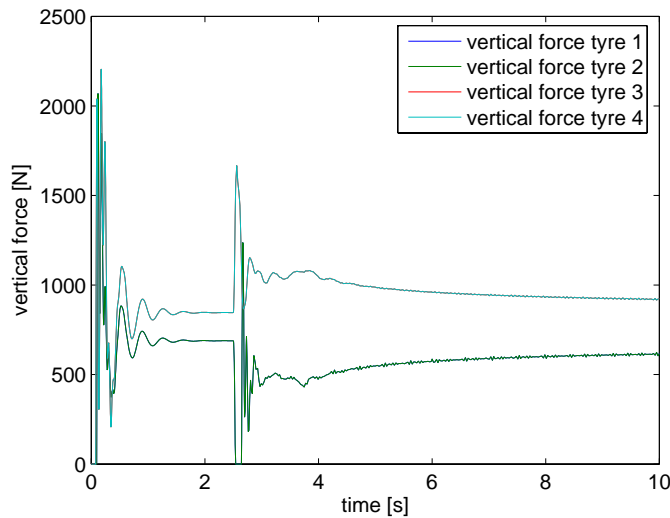


figure 17: weight distribution on front and rear

From figure 17, it follows that under constant acceleration the weight is first distributed from the front to the rear of the car and then takes in an equilibrium position. The peaks of the first seconds can be explained with the fact that the car must first find its equilibrium position and the peaks at 2.5 seconds follow from the fact that at that time, the throttle is put to the metal. From figure 18, it follows that the connections of the tyres on the upright at the rear go down and at the front go up

under acceleration. At  $t = 2.5$  seconds, one can see that front goes up a little and the back goes down. Because behind the engine a CVT (Continuously Variable Transmission) is placed, the car accelerates until the engine can not provide enough torque to accelerate in relation with the rotational velocity. It has to cope with rolling resistances of the tyres and although air resistance is not taken into account, finally the acceleration will stop.

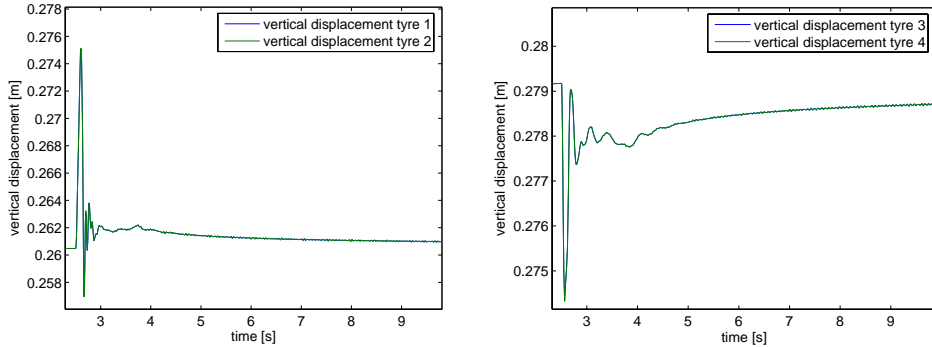


figure 18: displacement of uprights under acceleration, a) front b) rear

### 3.1.3. Steady speed and cornering

#### Introduction

Not only the straight line driving behaviour such as bump movement is important, cornering is maybe even more important than straight driving. For example, how fast is it possible to go round corners and not have to correct the car with the steering wheel? Has the car a safe understeer behaviour, or does it viciously oversteers?

In figure 19 can be seen what happens when the car first gets at a steady 20 m/s and then, at six seconds simulation time, the steering wheel is turned over to the left at 30 degrees.

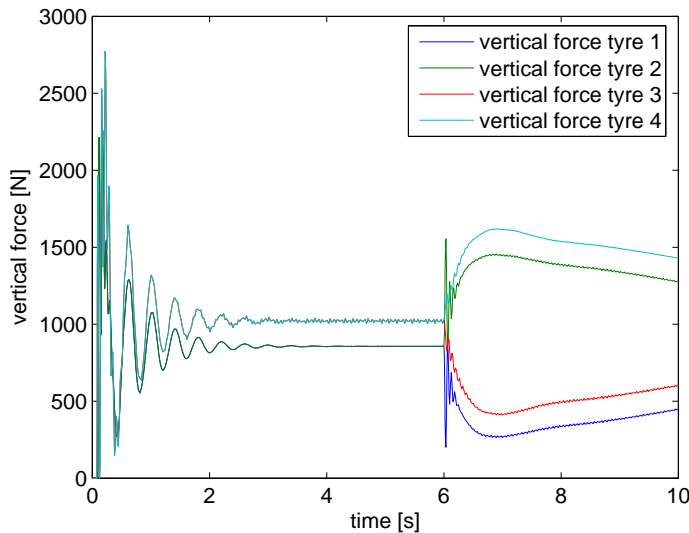


figure 19: the car during cornering

One can see that after initialisation, the vehicle reaches steady state behaviour. At  $t=6$  sec. the steering wheel is turned and an instantaneous weight distribution from the left to the right side can be seen. In figure 20, there has been zoomed on the steering moment of the previous picture.

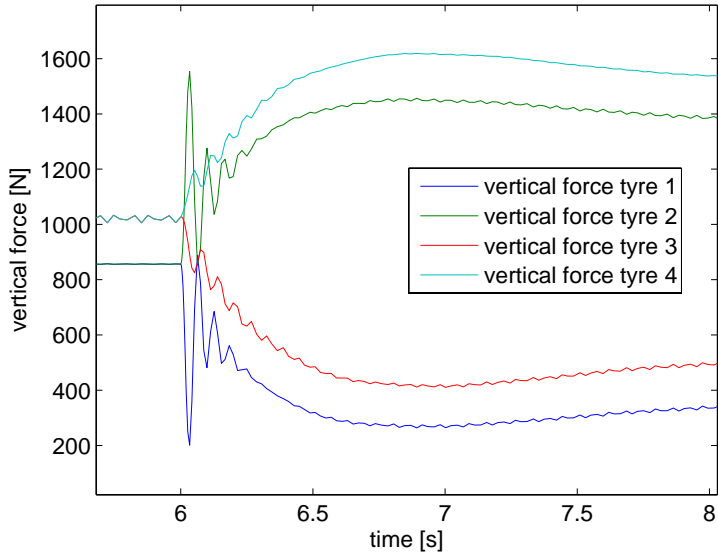


figure 20: zoom in of cornering behaviour

During the left corner, just after applying the steer input, the weight at the front goes from the left to the right tyre and some extra weight of the cog is also placed on the front right tyre. The rear tyres both stay on the ground and the weight on the rear right tyre is also larger than the removal of weight on the rear left tyre because of the displacement of the cog.

### 3.2. Vehicle handling

To predict whether the car under- or oversteers or even rolls over, the side slip angle  $\alpha$  has to be checked. That angle is defined as can be seen in figure 21.

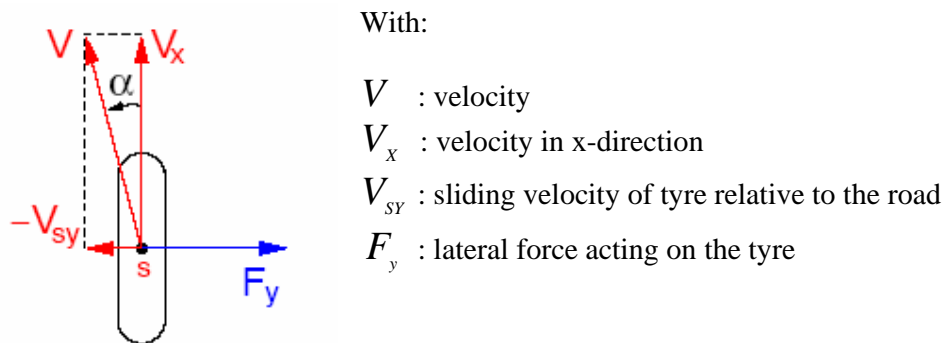


figure 21: definition of the side slip angle  $\alpha$

When the side slip angle of the front tyres is smaller than that of the rear tyres, the car is understeered, vice versa, the car is oversteered. To check this behaviour, a test has

been done at 20 m/s and a steering angle of 90 degrees. In figure 22, one can see the vertical wheel forces on the tyres during this test.

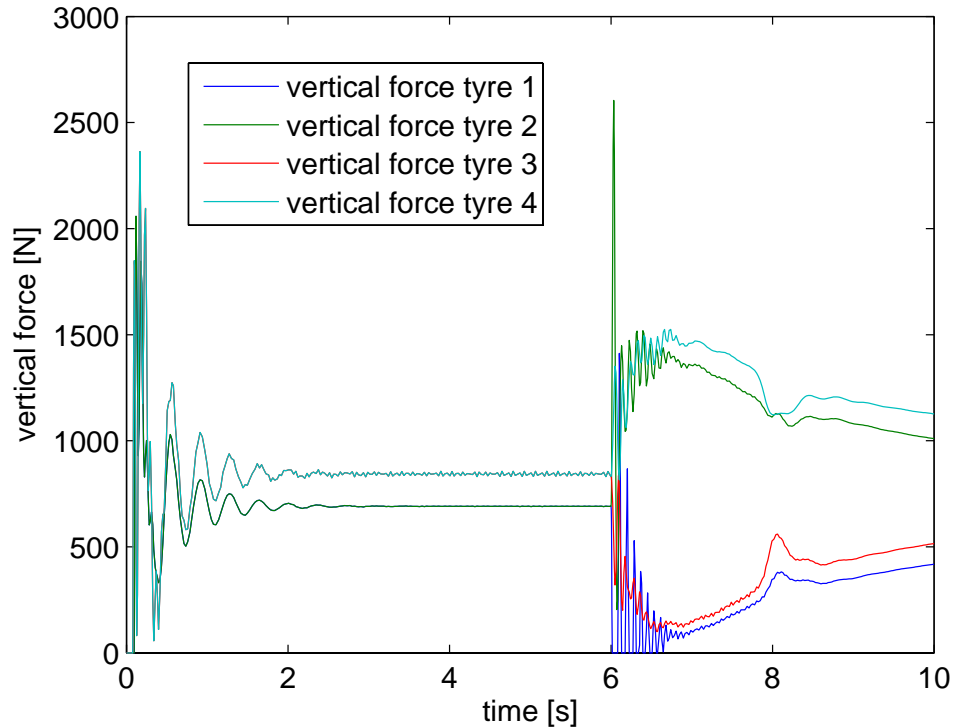


figure 22: vertical tyre forces under cornering (steering angle 90 degrees)

From figure 22 it shows that the front left tyre actually loses contact with the ground, but then stabilises and contact is regained. This means that the car only lifts the tyre, but it will not roll over. To check whether the car over- or understeers, the side slip angles of the four wheels have to be checked. Those angles can be seen in figure 23.

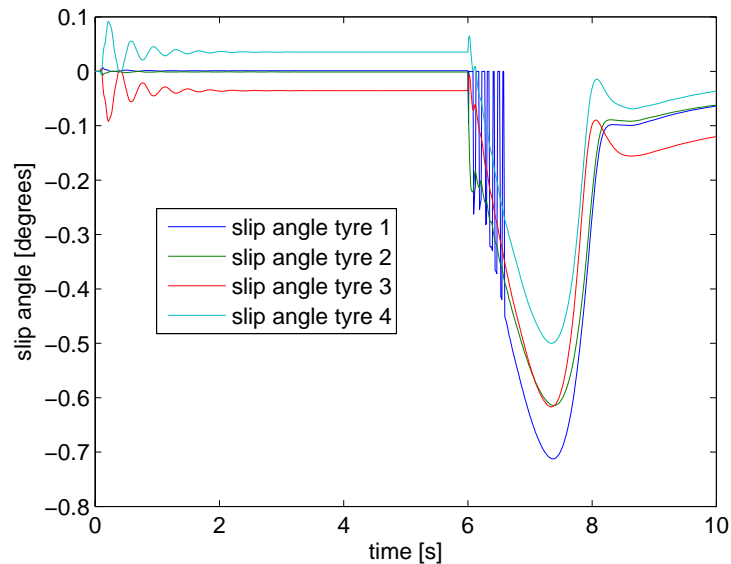


figure 23: side slip angles  $\alpha$  under cornering

As can be seen in figure 23, especially the side slip angle of tyre 4 (that is the rear right tyre) is smaller than that of both front tyres. Also the angle of the rear left tyre is larger, but not that significantly. This means that the car behaves understeered, so when a corner is turned in with a velocity which was too high, the car will slide over its front tyres, making it easier to compensate for this movement in comparison to an oversteered vehicle. With an oversteered vehicle, the chance to lose control over the car is larger.

## **4. Conclusion**

A new, better performing model is made. The layout is less complex and the design of the different components is simplified. Due to that reason, the calculation time has been improved with about a factor 10 also. A drawback on this new design is that it does not exactly represent reality, because it lacks a mechanism to control the roll-movement. Cornering behaviour is now dealt with by a body-spring-damper subsystem where in reality bump- and roll movements can be controlled separately. The new vehicle has a simple model of an engine coupled to a Continuously Variable Transmission (CVT) and a differential to provide a more realistic cornering behaviour. To check whether the model behaves like one would expect from physical reality, some tests have been done. Static equilibrium is satisfied and cornering with a speed of 20 m/s and a steering angle of 90 degrees shows a safe understeered behaviour.

## 5. Recommendations

### 5.1. *The model*

After 10 weeks of work, there are still some flaws at the model. The model still does not perfectly represent the physical reality. There fore some recommendations can be done:

- 1) The body-spring-damper subsystem has to be altered to a combination of a spring and an anti-roll mechanism. To get a better fit with reality, rockers could also be added. One can choose for a non linear spring and damping system as in Appendix E, but the Formula Student team used a linear system so far.
- 2) At the beginning of the simulation, the car is not placed on the ground. The center of gravity initial position is not correct, so has to be altered to.
- 3) The engine now follows a simple relation to calculate the torque, but this is not the way the torque curve looks like in reality. With a look-up-table, a good representation can be made
- 4) The simple actuation of the model, like the input of the velocity, torque (engine as well as brake) has to be provided. The functions are available, but not in one interface.
- 5) Some tests can be done, like a severe lane change
- 6) Also, the configuration parameters of the 2006 model have to be put in, when the behaviour of next year's car has to be predicted.
- 8) Make the model symmetric around the x-axis in a way that when you alter a coordinate on one side, the same coordinate at the other side is altered also
- 9) Make the suspension dependent, in a way that the wheelbase can be altered very easily.
- 10) Add a throttle in a way that the power can be set on the car at any moment and can be put off at any time.

### 5.2. *The design of the FS race car*

With studying the car, some faults of the design of the car came clear.

Some recommendations on this topic:

- 1) Add a damper in the anti-roll mechanism to damp out the roll of the vehicle.
- 2) A device which puts tension on the drivechain has to be added also, to make sure the chain does not run off the cogwheel.
- 3) For the stability and performance of the vehicle, some electronic measures could be taken, such as a stability program, anti lock and anti spin for the wheels



## Appendices

### *Appendix A: Layout of the vehicle model*

#### *Introduction*

The model consists of 3 main components, the front and rear axle and the vehicle body. The front axle consists of an intermediate body to which the left and right suspension is connected. The suspension subsystem at the front consists of an upper and lower A-arm and the upright in between them. The lower A-arm is connected to the vehicle body with a body-spring-damper subsystem. Furthermore, the wheels can be steered via a tie rod connected to the upright. In the steering subsystem the steering action is done. The rear axle has the same built up, but the suspension subsystem has been designed in a slightly different way. The upper and lower A-arm as well as the upright and the body-spring-damper subsystem are the same. The tie rod has been replaced by a reaction rod and at the upright an actuated axis is connected. In that way, the wheels can be actuated by a brake- and engine torque.

#### *Coordinates*

##### *A.1. Front right suspension*

<b>Function (name)</b>	<b>Connection method</b>	<b>Coordinates [mm]</b>	<b>Mass [kg]</b>	<b>Inertia [kg*m2]</b>
Upper A-arm	Rev Sph.	CG: [1592 -429 339] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.4 1/1000* eye(3)
Lower A-arm	Rev Sph. Bo-spr-da	CG: [1608 -426 153] CS1: [0 0 0] CS2: [0 0 0] CS3: [1615 -571 180]	Wo Ad Ad Wo	0.4 1/1000* eye(3)
Upright	Sph. Sph. Rev(y) Sph. Body sensor	CG: [1600 -598 262] CS1: [1585 -586 372] CS2: [1615 -610 153] CS3: [0 0 0] CS4: [1565 -590 181] CS5: [0 0 0]	Wo Wo Wo CG Wo CG	2 1/100* eye(3)

### A.2. Front left suspension

Function (name)	Connection method	Coordinates [mm]		Mass [kg]	Inertia [kg*m2]
Upper A-arm	Rev Sph.	CG: [1592 429 339] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.4	1/1000* eye(3)
Lower A-arm	Rev(x) Sph. Bo-spr-da	CG: [1608 426 153] CS1: [0 0 0] CS2: [0 0 0] CS3: [1615 571 180]	Wo Ad Ad Wo	0.4	1/1000* eye(3)
Upright	Sph. Sph. Rev(y) Sph. Body sensor	CG: [1600 598 262] CS1: [1585 586 372] CS2: [1615 610 153] CS3: [0 0 0] CS4: [1565 590 181] CS5: [0 0 0]	Wo Wo Wo CG Wo CG	2	1/100* eye(3)

### A.3. Steering subsystem

Function (name)	Connection method	Coordinates [mm]		Mass [kg]	Inertia [kg*m2]
Left tie rod	Univ(x,z) Sph.	CG: [1533 422 181] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.1	1/1000* eye(3)
Right tie rod	Sph. Univ(x,z)	CG: [1533 -422 181] CS1:[0 0 0] CS2:[0 0 0]	Wo Ad Ad	0.1	1/1000* eye(3)
<b>Steering rack</b>					
Basis block	Rev(z) Rev(y) Weld Cu(no rot y)	CG: [1500 0 181] CS1:[1500 0 281] CS2:[1500 -100 181] CS3:[0 0 0] CS4:[0 0 0]	Wo Wo Wo Ad CG	0	0* eye(3)
Gear	Rev(z) Rev(y)	CG: [1500 0 281] CS1:[0 0 0] CS2:[0 0 0]	Wo Ad CG	0	0* eye(3)
Gear 1	Rev(y) Gear constr. screw	CG: [1500 -100 181] CS1:[0 0 0] CS2:[0 0 0] CS3:[0 0 0]	Wo Ad CG CG	0	0 * eye(3)
Rack	Screw	CG: [1500 0 181] CS1:[0 0 0]	Wo Ad	1	1*eye(3)

	Left tie rod	CS2:[1500 254 181]	Wo		
	Right tie rod	CS3:[1500 -254 181]	Wo		
	Cu(no rot y)	CS4:[0 0 0]			
		CG			

#### *A.4. Rear right suspension*

<b>Function (name)</b>	<b>Connection method</b>	<b>Coordinates [mm]</b>		<b>Mass [kg]</b>	<b>Inertia [kg*m2]</b>
Upper A-arm	Rev Sph.	CG: [49 -439 354] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.4	1/1000* eye(3)
Lower A-arm	Rev Sph. Bo-spr-da	CG: [50 -412 173] CS1: [0 0 0] CS2: [0 0 0] CS3: [11 -564 179]	Wo Ad Ad Wo	0.4	1/1000* eye(3)
Upright	Sph. Sph. Rev(y) Sph Body sensor	CG: [0 -600 265] CS1: [0 -600 377] CS2: [0 -599 152] CS3: [0 0 0] CS4: [-55 -600 283] CS5: [0 0 0]	Wo Wo Wo CG Wo CG	2	1/100* eye(3)
Axle	Rev(y) Weld Body act. Body act.	CG: [0 -600 265] CS1: [0 0 0] CS2: [0 0 0] CS3: [0 0 0] CS4: [0 0 0]	Wo CG CG CG CG	0.1	1/1000* eye(3)
Reaction rod	Univ(x,z) Sph	CG: [-80 -424 264] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.1	1/1000* eye(3)

#### *A.5. Rear left suspension*

<b>Function (name)</b>	<b>Connection method</b>	<b>Coordinates [mm]</b>		<b>Mass [kg]</b>	<b>Inertia [kg*m2]</b>
Upper A-arm	Rev Sph.	CG: [49 439 354] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.4	1/1000* eye(3)
Lower A-arm	Rev Sph.	CG: [50 412 173] CS1: [0 0 0] CS2: [0 0 0]	Wo Ad Ad	0.4	1/1000* eye(3)

	Bo-spr-da	CS3: [11 564 179]	Wo		
Upright	Sph.	CG: [0 600 265]	Wo	2	1/100* eye(3)
	Sph.	CS1: [0 600 377]	Wo		
	Sph.	CS2: [0 599 152]	Wo		
	Rev(y)	CS3: [0 0 0]	CG		
	Sph	CS4: [-55 600 283]	Wo		
	Body sensor	CS5: [0 0 0]	CG		
Axle		CG: [0 600 265]	Wo	0.1	1/1000* eye(3)
	Rev(y)	CS1: [0 0 0]	CG		
	Weld	CS2: [0 0 0]	CG		
	Body act.	CS3: [0 0 0]	CG		
	Body act.	CS4: [0 0 0]	CG		
Reaction rod		CG: [-80 424 264]	Wo	0.1	1/1000* eye(3)
	Univ(x,z)	CS1: [0 0 0]	Ad		
	Sph	CS2: [0 0 0]	Ad		

#### ***A.6. Calculation of the axis of rotation of the A-arms***

To provide a certain path when the wheel travels, the A-arms are made to rotate around an axis. Because in this report, the “A-arms” are modeled by a single body, this axis has to be defined. The axis of rotation is the difference between both connection points on the vehicle body of the A-arms.

The axis of rotation can be seen in the following table:

<b>Place (Front/rear)</b>	<b>Upper/lower arm</b>	<b>Axis of rotation relative to world</b>
Front (right)	Upper	[-373.2 -13.9 -0.6]
	Lower	[376.5 27.9 -69.3]
Front (left)	Upper	[-373.2 13.9 -0.6]
	Lower	[376.5 -27.9 -69.3]
Rear (right)	Upper	[408.264 -27.503 -0.507]
	Lower	[411.908 -41.618 55.427]
Rear (left)	Upper	[408.264 27.503 -0.507]
	Lower	[411.908 41.618 55.427]

### A.7. Intermediate bodies

The purpose of these massless and inertialess bodies is to provide “hardpoints” at which the suspension can be connected to. In the tables, those points are summed.

#### A.7.1. Front intermediate body

Function (name)	Connec-tion method	Coordinates [mm]	Mass [kg]	Inertia [kg*m2]
Interme-diate body rear	weld	CG: [1600 0 250] Wo CS1: [0 0 0] Ad CS2: [1599 -272 305] Wo CS3: [1601 -242 153] Wo CS4: [1615 -230 416] Wo CS5: [1600 200 250] Wo CS6: [1599 272 305] Wo CS7: [1601 242 153] Wo CS8: [1615 230 416] Wo	none	none

#### A.7.2. Rear intermediate body

Function (name)	Connec-tion method	Coordinates [mm]	Mass [kg]	Inertia [kg*m2]
Interme-diate body rear	weld	CG: [0 0 0] CS1 CS1: [0 0 0] Ad CS2: [98 -279 331] Wo CS3: [100 -243 194] Wo CS4: [110 -230 442] Wo CS5: [0 0 265] Wo CS6: [98 279 331] Wo CS7: [100 243 194] Wo CS8: [110 230 44] Wo CS9: [-106 -247 245] Wo CS10: [-106 247 245] Wo CS11: [0 0 0] CS5 CS12: [0 0 0] CS5 CS13: [0 0 0] CS5	none	none

### A.8. Dependency

The left- and right side of the car are exactly the same. The coordinates at the right y-axis are negative, where the coordinates at the left y-axis are the same, but positive. One could think of a mask, where the y-dependency is put in and altering the one side, automatically results in an altered other side.

### ***Appendix B: Calculation of the pitch***

Because in SimMechanics it is not possible to make a physical correct steering rack, another solution has to be provided. The choice was made to use a gear constraint connected to a screw. The first one of the gears is the cogwheel connected with the steering wheel, the second one is in fact the bolt on the screw. The pitch is calculated as follows (using [3]):

Largest displacement of the tie rods: 0.021 m

Extreme steering wheel positions: -180 and 180 degrees

This means that per rotation of the steering wheel, 0.042 meters of tie rod displacement should be reached, so this will be the pitch of the screw.

## Appendix C: Calculation of spring properties

### Prestress and natural length

The weight of the car makes the springs become shorter and in that way, they counteract to the weight. The suspension is made in such a way that the springs on the front as well as at the rear need some prestress to provide that to the wheels. That prestress is provided by a longer spring natural length (not only the distance between both connection points). In figure 21, one can see the front wheel stiffness.

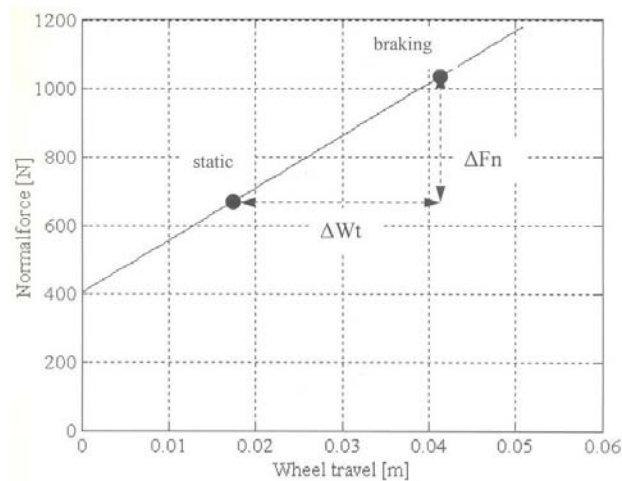


figure 24: front wheel stiffness

One can see that the line does not go through the point (0,0) and prestress is needed. The compression of the spring which is needed to provide this is approximately 0.026 meters. Hereby it is assumed that the spring is put in a vertical way. That extra length of the spring is calculated as follows. When one has the slope of the line, it is also known what length is needed to overcome the prestress. The same can be done with the rear suspension, as can be seen in figure 22:

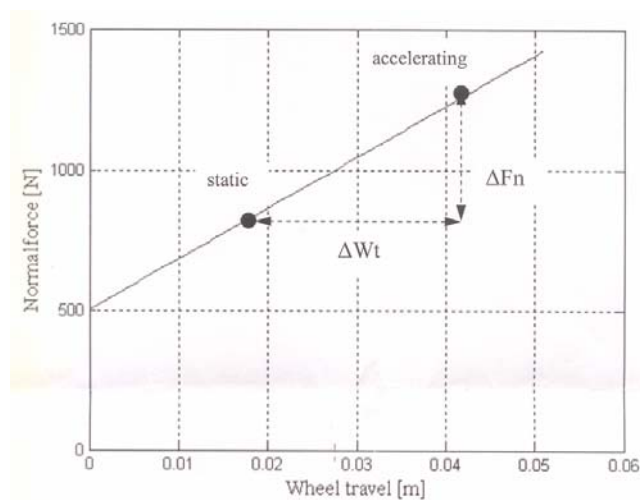


figure 25: rear wheel stiffness

For the rear wheels, the same story applies, but this time the line goes through (0,500). That means an unloaded spring length of 0.028 meters extra.

### ***Spring stiffness***

The tyres provide a certain vertical stiffness and damping and the spring stiffness is derived from that stiffness. First of all, the angle of orientation of the body-spring-damper subsystem has to be calculated. This is simplified by the distance in y-direction between the connection point of that subsystem and the projection of the connection point at the vehicle chassis on the lower A-arm. The height is the difference between the upper and lower connection point of the body-spring-damper subsystem. It follows that the front-angle is about 34.7 degrees and with a front tyre stiffness of  $1.53 \cdot 10^4$  [N/m], the front spring stiffness is about 26.800 [N/m]. The same applies for the rear, an angle of about 38.3 degrees and with a rear tyre stiffness of  $1.81 \cdot 10^4$  [N/m], the rear spring stiffness is about 29.200 [N/m].

### ***Implementation***

Because after discussion the approach to calculate the spring stiffness showed not to be correct and the vehicle did not show good behaviour with a larger spring natural length and the spring stiffness mentioned before, the properties have been tuned to drive in a straight line until a speed of about 40 m/s. Therefore the spring stiffness and damper properties have been set to other values as can be seen in table 1 on page 15.



## ***Appendix D: Static weight distribution***

### ***Introduction***

To check the results of the simulation done with static equilibrium, some simple hand calculations can be done. The weight of the car is known, as well as the location of its Center of Gravity and the distance of the cog to the front and rear axle. With these data, one can also calculate the expected forces on the tyres.

### ***Calculation***

The Center of Gravity is positioned at [0.72 0 0.33]. The distance from the front axle to the cog is 0.88 meters and from the cog to the rear axle is 0.72 meters.

This gives us the following formula, the moment equilibrium around the rear tyres:

$$\sum M_R = 0 \Rightarrow 0.72 * m * g - 1.6 * F_{front} = 0 \quad (2)$$

Translational equilibrium:

$$\sum F = 0 \Rightarrow -m * g + F_{front} + F_{rear} = 0 \quad (3)$$

With a weight of 230 kg and the weight of the front and rear tyre as 15 kg and 20 kg respectively, this gives us the forces on the tyres, being:

$$F_{fronttyre} = 655N$$

$$F_{reartyre} = 817N$$

From the simulations it follows that there are some differences between the analytical results and the values calculated by the computer. The simulations show a front tyre force of 688 N and a force on one rear tyre of about 846 N. Compared to the, analytically calculated, 655 N and 817 N for the front and rear tyre respectively, a constant deviation of about 30 N can be seen. These 3 kilograms can be explained by the weight of the suspensions.

## Appendix E: Spring-damper subsystem

### Introduction

Because the roll is only controlled by a spring, with an “elch-test” manoeuvre, the roll of the car will not damp out. When the car is excited in its eigenfrequency it can eventually turn over.

### Non linear suspension and anti-roll

Furthermore, a non linear suspension can be added. Although the Formula Student team at Eindhoven University has not made calculations with a non linear spring and damper, the physical reality is non linear. The control of the bump-movement and the suspension are separated. In figure 26, one can see the anti-roll mechanism.

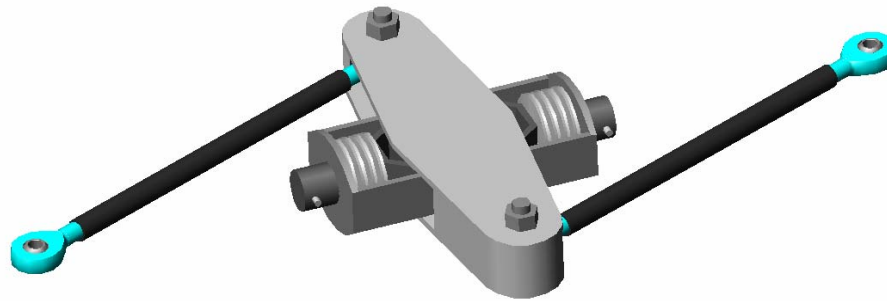


figure 26: the anti-roll mechanism

The bump-movement is controlled by means of a (non) linear spring and as input the displacement of a rocker. When both rockers move as a parallelogram, this is “seen” by an interval tester which compares this value with 0. Then an “enable-block” is enabled and the bump movement is reduced. In figure 27, one can see how this is done in Simulink.

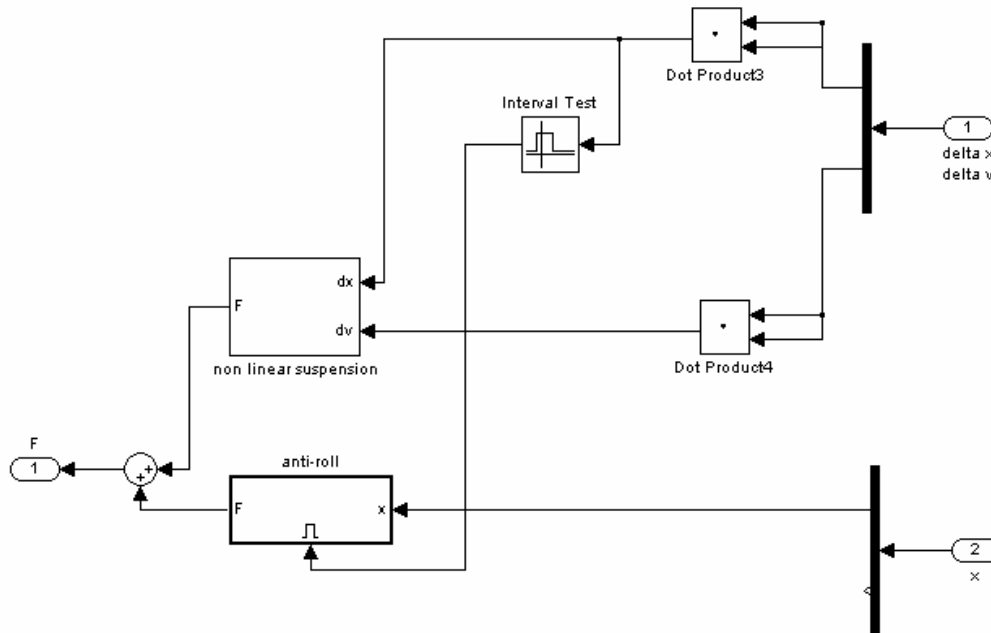


figure 27: the spring/damper subsystem

In contrary, the suspension for compression for example, is a combination of a spring and damper. Here, the difference in displacement and velocity is used. Now, the spring is split up in a bump stop and the (non) linear spring itself. This can be seen in the following figure:

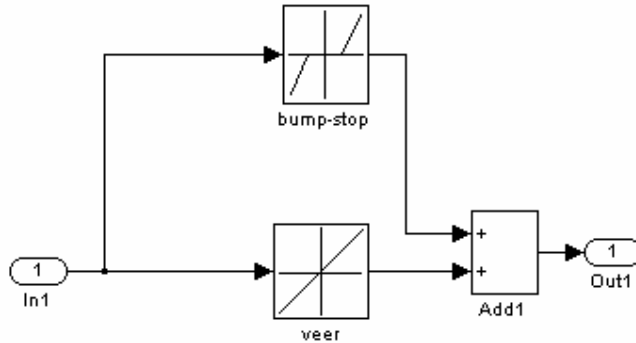


figure 28: non linear spring with bump stop

There is however another way how this could be implemented in SimMechanics. At the middle of the vehicle body, a massless connection rod (green) is placed, which can rotate around a revolute joint. At the top of this rod, a rod is connected which also contains the body-spring-damper (light blue) and is used to reduce the bump movement. At the latter rod, a body-spring-damper (red) is placed to reduce the roll movement. One has to take the proportion of transmission into account in the stiffness of the spring. The system should be as showed in figure 29:

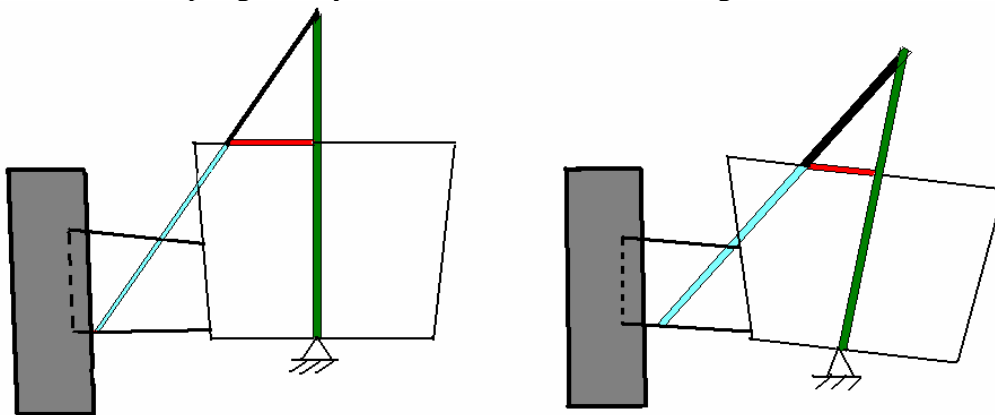


figure 29: the anti-roll mechanism; a) straight line driving b) cornering

blue: body-spring-damper (anti-bump)  
 red : body-spring-damper (anti-roll)  
 green: massless connection rod

### *Appendix F: Symbols*

<b>Symbol</b>	<b>Description</b>	<b>Dimension</b>
<b>F</b>	<b>Force</b>	<b>[N]</b>
<b>G</b>	<b>Gravitational constant</b>	<b>[m/(s*s)]</b>
<b>M</b>	<b>Moment</b>	<b>[Nm]</b>
<b>M</b>	<b>Mass</b>	<b>[kg]</b>
$\theta$	<b>Pitch angle/ rotation angle tyre</b>	<b>[rad]</b>
$\omega$	<b>Rotational velocity</b>	<b>[rad/s]</b>
$\dot{\omega}$	<b>Rotational acceleration</b>	<b>[rad/(s*s)]</b>
$\varphi$	<b>Yaw angle</b>	<b>[rad]</b>
$\phi$	<b>Roll angle</b>	<b>[rad]</b>
<b>P</b>	<b>Power</b>	<b>[W]</b>
<b>T</b>	<b>Torque</b>	<b>[Nm]</b>
<b>x</b>	<b>displacement</b>	<b>[m]</b>

## *Appendix G: References*

- [1] <http://www.imeche.org.uk/formulastudent/>
- [2] Bram de Jong, Modelling of a Formula Student race car in Matlab/Simulink with Simmechanics” report no. 2004.106
- [3] Wouter Berkhout, Design for a Formula Student race car, Eindhoven October 2004