Experimental development of an anisotropic viscoelastic vessel model

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Contents

Chapter 1

Introduction

1.1 Atherosclerosis

Atherosclerosis is a major cause of death in the western world. In the last decades research has been done to find the cause of this disease. The main topic of this research is to understand when and how atherosclerotic lesions are formed. There are several hypotheses regarding this. One hypothesis may be that atherosclerosis correlates with low or oscillating wall shear rates, as well as with low or compressive principal stresses in the vessel wall.
At the Eindhoven University of Technology a project on mechanical aspects of blood-wall interaction is running. Aim of the study is to investigate the correlation between the local wall shear stress distribution and the local wall strain distribution. Because it is impractical to perform experiments on human vessels, in-vitro models have to be made. Using these models, velocity profiles and wall displacements can be measured by means of Laser-Doppler experiments and video techniques.

1.2 Method

The project on atherosclerosis running at the Eindhoven University of Technology can be divided into two parts. The first part concerns complex vessel wall behaviour together with a Newtonian fluid and the second part concerns the mechanical behaviour of a complex fluid near the vessel wall.
An attempt has been made to make a vessel model which has the same complex mechanical behaviour as a human vessel, see Rutten (1995) and Caimmi (1995a,b). Although the results were qualitatively good, they did not match those of a human vessel.
To make a vessel model which has the right characteristics requires knowledge of the material used. Research has to be done to obtain that knowledge.

1.3 Outline

In this report the derivation of the mechanical characteristics of a two component orthotropic model is described. After an introduction to vessel wall behaviour a mathematical model is made. Then the mechanical properties of the two components of the vessel model are determined.

The next step is to determine what the mechanical properties of the model should be. Then a tubular model with the desired properties can be made.

Chapter 2

Mechanical behaviour of the vessel wall

2.1 Components

The constituents of vascular tissue are elastin, collagen fibres and smooth muscle cells. The mechanical behaviour is mainly determined by these three components.

Elastin Elastin is a biological material with an almost linear stress-strain relationship. It has a Young's modulus of approximately 0.6 MPa and remains elastic up to draw ratios λ of approximately 1.6. The material shows hardly any hysteresis. (Fung, 1993a)

Collagen Collagen is a basic structural protein. It consists of three helically wound chains of amino-acids. These helices are collected together in micro-fibrils, which in their turn form subfibrils and fibrils. The fibres are normally arranged in a wavy form. Due to this waviness the stress-strain relationship shows a very low stiffness at small stretch ratios. The stiffness increases fast once the fibres are deformed to straight lines. (Fung, 1993a)

Vascular smooth muscle There is little known about the effect of passive vascular muscle on the stress-strain relationship of vessels. However Cox (1978) reported the effect of active vascular muscle on the stress-strain behaviour of vessels as a whole.

2.2 Mechanical behaviour

Vascular tissue shows complex mechanical behaviour. There are three main characteristics: a non-linear stress-strain relationship, viscoelastic and orthotropic behaviour. Each of these three characteristics call for special attention when building a vessel model.

2.3 Vessel models

The mechanical behaviour of vascular tissue can be described using a phenomenological model. These models only fit experimental data and therefore the predictive capabilities for other loading situations are often poor. Fung (1993b); Chuong and Fung (1983); Han and Fung

(1991) showed that the reference state of an artery (axially unstretched and no transmural pressure present) is not stress-free. The vessel model used in Rutten (1995) also has a non stress-free reference state, because the fibres are wound with a prestrain on top of the isotropic tube.

Chapter 3

Vessel model

3.1 Introduction

In Caimmi (1995a) and Rutten (1995) a composite of fibres and rubber was used as a vessel model. The base of the composite is an isotropic tube of viscoelastic EPDM rubber. To make the tube anisotropic and giving it a nonlinear stress-strain relationship, several layers of fibres are applied.

In this chapter a mathematical model for the nonlinear, anisotropic tube is derived. With the help of a mathematical model an estimate of several production parameters can be given. The production parameters are the parameters, such as the winding angle, the number of layers of fibres, the prestrain applied to the fibres and their volume fraction, which can be altered to change the mechanical properties of the tube.

3.2 Manufacturing of the model

The rubber is dissolved in xylene and 1% by weight dibenzoylperoxide is added to enable crosslinking. The solution is applied is several layers to a teflon-coated mandrel of 18 mm diameter by means of dip-coating. The wall thickness has to be as small as possible to reduce the influence of the rubber on the total mechanical behaviour of the tube. The smallest wall thickness that can be reached is 0.25 mm. After application of each layer, the mandrel rotates until all the solvent has evaporated. Then the mandrel and the rubber are put in an oven at a temperature of 120^0 for three hours. The crosslinked tube can then be removed and put on another mandrel to enable removing after applying two plies of lycra fibres on the isotropic tube. The fibres are wound at a specific angle to obtain the right mechanical properties of the tube and are prestrained to obtain a more pronounced nonlinear behaviour. After application of the fibres, the model is put in an oven again to acquire a firm bond between the fibres and the rubber. The final result is an orthotropic tube.

3.3 Mathematical model

The base of the model is the constitutive equation for orthotropic behaviour:

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\varphi\varphi} \\ \sigma_{zz} \\ \sigma_{rz} \end{bmatrix} = \begin{bmatrix} \frac{E_r}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{\mu_{r\varphi}E_\varphi}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{\mu_{rz}E_z}{1-\mu_{z\varphi}\mu_{\varphi z}} & 0 \\ \frac{\mu_{\varphi r}E_r}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{E_\varphi}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{\mu_{\varphi z}E_z}{1-\mu_{z\varphi}\mu_{\varphi z}} & 0 \\ \frac{\mu_{zr}E_r}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{\mu_{z\varphi}E_\varphi}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{E_z}{1-\mu_{z\varphi}\mu_{\varphi z}} & 0 \\ 0 & 0 & 0 & G_{rz} \end{bmatrix} \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\varphi\varphi} \\ \varepsilon_{zz} \\ \varepsilon_{rz} \end{bmatrix} \tag{3.1}$$

with

$$E_r = E_r(\dot{\lambda}, \lambda, \lambda_{w,0}, \theta) \tag{3.2}$$

$$E_\varphi = E_\varphi(\dot{\lambda}, \lambda, \lambda_{w,0}, \theta) \tag{3.3}$$

$$E_z = E_z(\dot{\lambda}, \lambda, \lambda_{w,0}, \theta) \tag{3.4}$$

E_r, E_φ, E_z are the stifnesses of the tube in respectively radial, circumferential and longitudinal direction. The axisymmetric coordinate system is shown in figure 3.1.

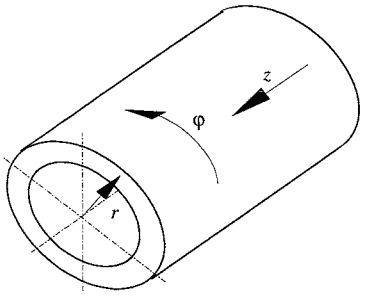

Figure 3.1: *Axisymmetric coordinate system*

Due to nonlinearity the Young's moduli are not constant. Use of nonlinear parameters in a linear constitutive equation may lead to inaccurate results.
Because the thickness of the wall of the tube is small, axisymmetric plane stress is applicable and equation (3.1) reduces to:

$$\begin{bmatrix} \sigma_{\varphi\varphi} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \frac{E_\varphi}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{\mu_{\varphi z}E_z}{1-\mu_{z\varphi}\mu_{\varphi z}} \\ \frac{\mu_{z\varphi}E_\varphi}{1-\mu_{z\varphi}\mu_{\varphi z}} & \frac{E_z}{1-\mu_{z\varphi}\mu_{\varphi z}} \end{bmatrix} \begin{bmatrix} \varepsilon_{\varphi\varphi} \\ \varepsilon_{zz} \end{bmatrix} \tag{3.5}$$

Equation (3.5) describes the macroscopic behaviour of the tube. The Young's moduli of the tube consist of two components. To determine the relationship between the Young's moduli and the Young's modulus of the fibres and the EPDM rubber, laminate theory is used.
The laminate theory presumes linear mechanical behaviour. The mechanical behaviour of the fibres is strongly nonlinear. Therefore an inaccuracy will occur.
For the k^{th} layer the stiffnessmatrix is denoted as $\underline{Q}_k$. The 1- and 2-direction are the main axes of the layer, with direction 1 the direction of the fibres and direction 2 perpendicular to direction 1. Because the main axes of the layer differ from the main axis of the composite, the stiffness matrix has to be transformed. The stiffness matrix related to the circumferential and

longitudinal direction of the tube of the k^{th} layer is according to Halpin (1992) and Agarwal and Broutman (1980):

$$\underline{Q}_k^* = \underline{T}_k^{-1} \underline{Q}_k \underline{T}_k \tag{3.6}$$

with $\underline{Q}_k$ the stiffness matrix in the 1- and 2-direction (the main axis of the layer) and

$$\underline{T}_k = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}_k \tag{3.7}$$

with θ the angle between the fibres and the circumferential direction of the tube. The total stiffness of the fibre layers $\underline{Q}$ is the sum of stiffness matrices of each layer, so

$$\underline{Q} = \sum_{k=1}^{n} \underline{Q}_k^* \tag{3.8}$$

with n the number of layers.
Because the mechanical behaviour of the fibre is one-dimensional, the stiffness of the fibres can be written in circumferential and longitudinal direction using equations (3.6) to (3.8):

$$E_{f\varphi} = nE_f(\lambda)\cos^4\theta \tag{3.9}$$
$$E_{fz} = nE_f(\lambda)\sin^4\theta \tag{3.10}$$

with E_f the stiffness of the fibres. If one layer is applied with an angle of θ, the next one will be applied with an angle of $-\theta$.
Because a finit number of fibres is applied, the Halpin-Tsai relation is used to compute the stiffness of the tube (Halpin, 1992):

$$E_\varphi = E_{f\varphi} v_f + E_m(1 - v_f) \tag{3.11}$$
$$E_z = E_{fz} v_f + E_m(1 - v_f) \tag{3.12}$$

with v_f the volume fraction of the fibres and E_m the stiffness of the EPDM matrix.
The Poisson ratios for the fibre and the matrix can be derived in a similar way and are:

$$\mu_{f\varphi} = 0.5 \tag{3.13}$$
$$\mu_{fz} = 0.5 \tag{3.14}$$
$$\mu_m = 0.5 \tag{3.15}$$

Analog to equation (3.11) an equation for the Poisson ratio of the tube can be found. Halpin (1992) gives for $\mu_{\varphi z}$ and $\mu_{z\varphi}$:

$$\mu_{\varphi z} = \mu_{f\varphi} v_f + \mu_m(1 - v_f) \tag{3.16}$$
$$\mu_{z\varphi} = \mu_{fz} v_f + \mu_m(1 - v_f) \tag{3.17}$$

Now every unknown in the constitutive equation is determined, except the stiffness of the fibre E_f and the modulus of the rubber E_m. Experiments are required to determine these material parameters.

Chapter 4

Material

4.1 Lycra fibres

4.1.1 Introduction

Lycra fibres are polyurethane fibres (DuPont de Nemours, Dordrecht (NL)). One fibre of 78 dtex (1 dtex= 1 kg / 10km) is a bundle of six fibres of 13 dtex each. Lycra fibres are highly elastic. The draw ratio at which the fibre is ruptured is approximately 8.
The lycra fibres are subjected to preconditioning. The preconditioning consists of stretching the fibre and subsequently heating it. Because of preconditioning the mechanical behaviour of the fibres changes. It is necessary to introduce a λ-axis transformation which links the various states of the fibre to the original state before preconditioning.
After the preconditioning, a tensile test is performed and the Young's modulus can be determined.

4.1.2 λ-axis transformation

During the experimental characterization of the lycra fibres, there are 2 different states of the fibre which can be used as reference state:

1. Untreated, undeformed. This is the state of the fibre as is.
2. Treated, undeformed. This is the state of the fibre after preconditioning when it is loosened.

Throughout this section the second index of a parameters determines the state to which that parameter is related. The index $\cdot_{,0}$ refers to the untreated state, the index $\cdot_{,t}$ refers to the treated state.

Prestrain

Originally the prestrain is applied to the untreated, undeformed state of the fibre. After heating and releasing the fibre it does not return to its reference state: some deformation remains. This state can be used as another reference state. So the prestrain related to this state (treated, undeformed) is:

$$\lambda_{w,t} = \frac{\lambda_{w,0}}{\lambda_{0,t}} \tag{4.1}$$

with $\lambda_{0,t}$ the remaining strain in the treated, undeformed state.

Tensile test

The treated, undeformed state is considered the reference state for the tensile test, so all draw ratios during the tensile test are related to this state. The strain of the fibre can be related to the untreated, undeformed state with the following equation:

$$\lambda_{f,0} = \lambda_{0,t}\lambda_t \tag{4.2}$$

with λ_t the applied draw ratio during the tensile test.

The composite

The fibres have a prestrain of $\lambda_{w,t}$ related to the treated, undeformed state. If the composite made of EPDM rubber and lycra fibres is stretched, the draw ratio of the fibres related to the treated, undeformed state is:

$$\lambda_{f,t} = \lambda_{w,t}\lambda_c \tag{4.3}$$

with λ_c the applied draw ratio. The draw ratio of the fibres related to the untreated, undeformed state is:

$$\lambda_{f,0} = \lambda_{w,t}\lambda_{0,t}\lambda_c = \lambda_{w,0}\lambda_c \tag{4.4}$$

4.1.3 Preconditioning

One fibre is stretched to a certain draw ratio $\lambda_{w,0}$. Then the fibre is heated to 100 ^{0}C during 30 minutes. The reference length marked on the fibre will be longer after the heat treatment compared with the reference length before heating. The remaining stretch, noted as $\lambda_{0,t}$, can be calculated by loosening the fibre and measuring the reference length after the heat treatment. $\lambda_{0,t}$ is derived by dividing the length after stretching and heating by the reference length after heating. The relation between the prestrain and the remaining stretch in the treated, undeformed state is shown in table 4.1.

fibre	$\lambda_{w,0}$ [-]	$\lambda_{0,t}$ [-]
1	1.2	1.2
2	1.6	1.4
3	2.3	1.8
4	2.6	1.8
5	3.0	1.9
6	3.7	2.0
7	4.0	2.1
8	4.9	2.3

Table 4.1: *Applied prestrain and remaining stretch after heat treatment.*

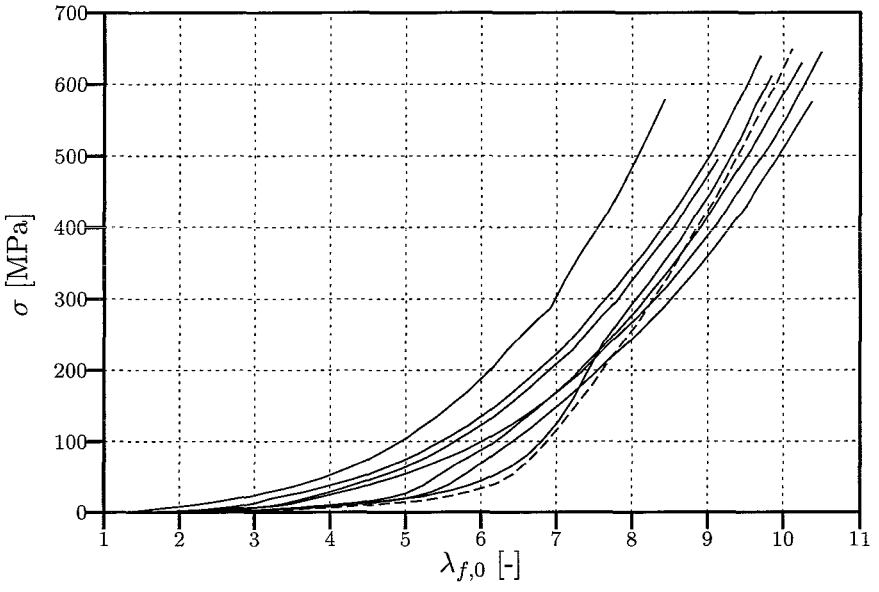

Figure 4.1: *Experimental data of 8 lycra fibres with different prestrain as function of the stretch related to the untreated, undeformed state. The left most curve represents the fibre with the lowest prestrain. The dashed curve represents the fibre with the highest prestrain.*

4.1.4 Tensile test

To determine the mechanical behaviour of the material, tensile tests are performed. The state of the fibres after preconditioning is stated as the undeformed state, so the applied elongation λ_t of the fibre is related to the treated, undeformed state. The strain called true strain throughout this section, is related to the untreated, undeformed state of the fibre, so equation (4.2) can be applied:

$$\lambda_{f,0} = \lambda_{0,t}\lambda_t \tag{4.5}$$

with $\lambda_{f,0}$ the draw ratio of the fibre, λ_t the draw ratio applied during the tensile test and $\lambda_{0,t}$ the remaining strain in the treated, undeformed state.
The modulus of the fibre relates the true strain with the Cauchy stress. The Cauchy stress is the applied force divided by the momentanuous cross-sectional area, which can be calculated by using the fact that the fibre can be considered incompressible, i.e. the volume of the fibre remains constant. The area A of the fibre can be computed by using the following equation:

$$A = A_0 \frac{1}{\lambda_{f,0}} \tag{4.6}$$

with A_0 the area of the fibre in the untreated, undeformed state, which is $8.7 \cdot 10^{-9}$ m^2 (see Caimmi (1995a)). Figure 4.1 shows the results of the tensile test. The effect of preconditioning on the stress-strain relationship is shown in the figure. Stretching and subsequently heating the fibre gives it a more pronounced nonlinear stress-strain relationship. For a large prestrain this means that the stiffness of the fibre increases rapidly at a draw ratio of approximately 6.5.

4.1.5 Derivation of the stiffness of the fibre

To obtain a relationship between the Young's modulus and the strain, a function is fitted to the tensile test data. If the relationship between the stress and the strain is known, the relationship for the stiffness of a 1-D material can be derived by applying:

$$E_f = \frac{\partial \sigma}{\partial \varepsilon} \tag{4.7}$$

Fitting the stress-strain relationship

The lycra fibre shows rubber-like behaviour, which means that the elastic deformation is mainly entropic. This behaviour can be described by using a neo-Hookean constitutive equation. In figure 4.1 it is clearly shown that the stress-strain relation is exponential. The function that will be used for fitting the testdata is a exponential function with a neo-Hookean strain-scale. This yields for σ_{11}:

$$\sigma_{11} = a_1 e^{(a_2 \lambda_{f,0}^2 - a_3 \frac{1}{\lambda_{f,0}})} \tag{4.8}$$

with $\lambda_{f,0}$ the draw ratio of the fibre related to the untreated, undeformed state. The testdata are fitted with a least squares method. The fitting process gives the results presented in table 4.2.

fibre	$\lambda_{w,0}$ [-]	$\lambda_{0,t}$ [-]	a_1 [MPa]	a_2 [-]	a_3 [-]
1	1.2	1.2	$6.3 \cdot 10^2$	$1.7 \cdot 10^{-2}$	11.00
2	1.6	1.4	$1.1 \cdot 10^3$	$1.1 \cdot 10^{-2}$	14.38
3	2.3	1.8	$1.5 \cdot 10^3$	$0.9 \cdot 10^{-2}$	16.71
4	2.6	1.8	$1.4 \cdot 10^3$	$0.8 \cdot 10^{-2}$	17.36
5	3.0	1.9	$7.1 \cdot 10^3$	$1.9 \cdot 10^{-2}$	26.97
6	3.7	2.0	$6.7 \cdot 10^3$	$0.6 \cdot 10^{-3}$	29.04
7	4.0	2.1	$3.7 \cdot 10^5$	$-1.3 \cdot 10^{-2}$	50.95
8	4.9	2.3	$3.3 \cdot 10^5$	$-1.1 \cdot 10^{-2}$	51.88

Table 4.2: *Applied prestrain and remaining stretch after heat treatment and results after fitting the experimental data.*

Derivation of the Young's modulus

The Young's modulus can be derived by using equation (4.7) and (4.8). Because the strain ε equals $\varepsilon = \lambda_{f,0} - 1$, the equation for the Young's modulus becomes:

$$E_f = \frac{\partial \sigma}{\partial \varepsilon} = \frac{\partial \sigma}{\partial \lambda_{f,0}} \frac{\partial \lambda_{f,0}}{\partial \varepsilon} = \frac{\partial \sigma}{\partial \lambda_{f,0}} \tag{4.9}$$

Application of equation (4.9) to equation (4.8) gives:

$$E_f = a_1 e^{(a_2 \lambda_{f,0}^2 - a_3 \frac{1}{\lambda_{f,0}})} (2 a_2 \lambda_{f,0} + a_3 \frac{1}{\lambda_{f,0}^2}) \tag{4.10}$$

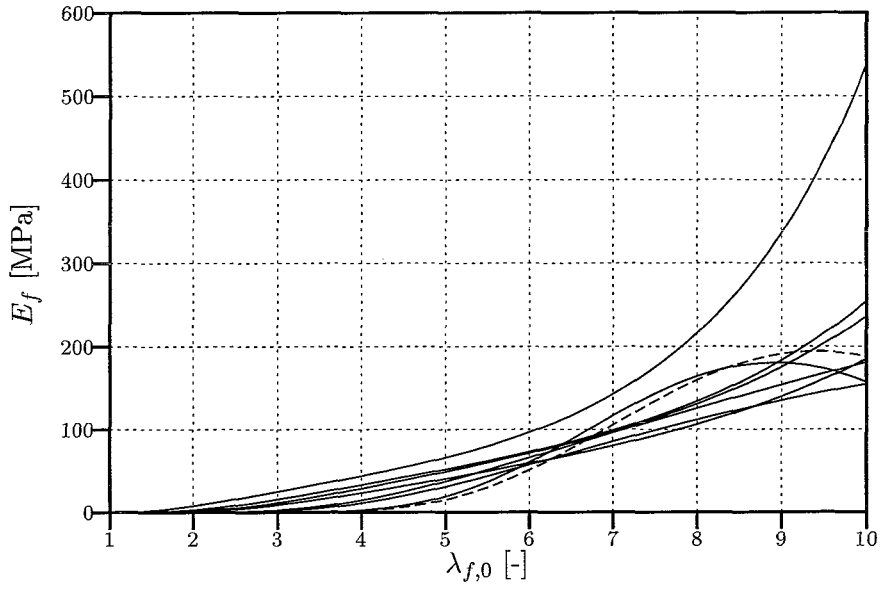

Figure 4.2: *Young's modulus derived from the fitted stress-strain relationship for 8 values of the prestrain. The left most curve represents the fibre with the lowest prestrain. The dashed curve represents the fibre with the highest prestrain.*

The results of the fitting process is shown in figure 4.2. At high values of $\lambda_{f,0}$ the Young's modulus drops for fibres with a high prestrain. This could be due to the fitting process. Because other functions could not describe the experimental data better over the whole range of draw ratios, equation (4.10) gives the best results and the possible inaccuracy can not be avoided. Therefore equation (4.10) is only applicable for draw ratios lower than $\lambda_{f,0} = 8$.

4.2 EPDM rubber

4.2.1 Introduction

EPDM rubber belongs to the family of ethylene-propylene rubber and is made by DSM, Geleen (NL). The EPDM rubber that is used for experiments is Keltan 320. An extensive description of EPDM and the production process is given in Caimmi (1995a).

4.2.2 Relaxation experiments

EPDM rubber is viscoelastic. Therefore relaxation experiments are done to determine a relaxation modulus. These experiments can also be used to see if the rubber is linear viscoelastic or nonlinear viscoelastic.
Figure 4.3 shows the data derived from the relaxation experiments. The vertical axis shows the stress divided by the applied step in the strain ε_0. Because it is hard to see if the stress keeps decreasing for a wide time range, the test data can be plotted on a log-log scale, which is shown in figure 4.4.
Because the relaxation modulus of the rubber does not depend on the applied step in the strain, the rubber can be considered linear viscoelastic. The third specimen shows a relaxation

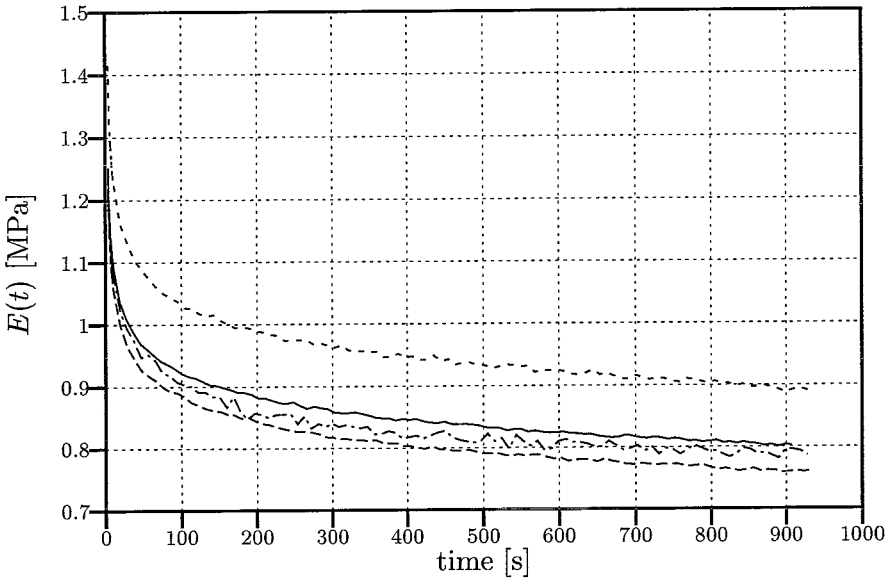

Figure 4.3: *The Young's modulus as function of the time after relaxation experiments.* $\varepsilon_0 = 0.25$ *(solid),* $\varepsilon_0 = 0.2$ *(dashed),* $\varepsilon_0 = 0.15$ *(dotted),* $\varepsilon_0 = .05$ *(dash-dotted)*

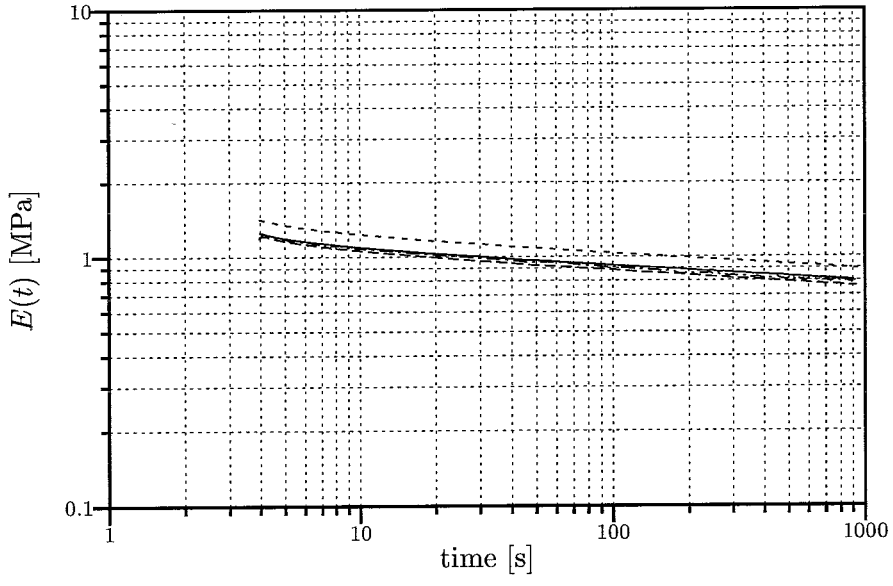

Figure 4.4: *The Young's modulus as function of the time after relaxation experiments on a log-log scale.*

modulus which is translated in the vertical direction. This may be the result of measurement inaccuracies.

4.2.3 Viscoelastic behaviour

EPDM rubber is linear viscoelastic. A phenomenological constitutive equation for the Young's modulus is a power-law model. The Cauchy stress σ decreases linearly with a negative power of the time when a step in the strain is applied. The constitutive equation for the stress is:

$$\sigma = E(t)\varepsilon_0 = \left[c_1 t^{-n}\right] \varepsilon_0 \tag{4.11}$$

with ε_0 the strain applied stepwise to the material. The dimension of c_1 is MPa s^n. The material parameters c_1 and n can be derived from a relaxation test. If a relaxation experiment is performed, the rubber is stretched to a certain strain stepwise and the stress is measured over time. Fitting the experimental data with equation (4.11) gives $c_1 = 1.3$ MPa s^n and $n = 0.078$. The low value of n shows that the rubber is nearly elastic. Only when loads with frequencies higher than 10 Hz are applied, the viscoelasticity will play an important role.

Superposition

If the load changes as a function of the time, the stress can be computed by using the superposition theory. This theory is applicable because the EPDM rubber shows linear viscoelastic behaviour. Using superposition, the equation for the stress becomes:

$$\sigma(t) = \int_0^t E(t-\tau)\dot{\varepsilon}(\tau)d\tau \tag{4.12}$$

For a tensile test, the strain rate $\dot{\varepsilon}$ is constant, so equation (4.12) can be solved analytically. Because the strainrate is constant, equation (4.12) becomes:

$$\sigma(t) = \dot{\varepsilon} \int_0^t E(t-\tau)d\tau \tag{4.13}$$

If a new variable $\psi = t - \tau$ is introduced, equation (4.13) can be rewritten as:

$$\sigma(t) = \dot{\varepsilon} \int_0^t E(\psi)d\psi \tag{4.14}$$

Solving this equation gives:

$$\sigma(t) = \dot{\varepsilon}\frac{c_1}{1-n}t^{1-n} \tag{4.15}$$

Dynamic behaviour

A second way to describe viscoelastic behaviour is by means of the complex modulus. The complex modulus $\hat{E}$ can be determined by Laplace transformation of equation (4.11):

$$\hat{\sigma} = \hat{H}\hat{\varepsilon} = \frac{\hat{E}}{s}\varepsilon_0 \tag{4.16}$$

Together with equation (4.11) this yields for the complex modulus:

$$\hat{E}(s) = c_1\Gamma(1-n)s^n \tag{4.17}$$

with Γ the gamma function. In terms of frequency the complex modulus is:

$$\hat{E}(j\omega) = c_1\Gamma(1-n)\omega^n e^{jn\frac{\pi}{2}} \tag{4.18}$$

Chapter 5

Results

5.1 Introduction

In chapter 3 a mathematical vessel model is presented. Several experiments have been done to require the necessary material parameters. Now the experiments and the contitutive equations can be combined to estimate the production parameters that will lead to a vessel model with the right properties.
There are three parameters which can be altered to obtain the right properties. These three parameters are the winding angle θ, the volumefraction of fibres v_f and the number of applied layers of fibres n and the applied prestrain. First the mechanical behaviour as function of the three parameters is given, then the influence of the production parameters on the stress-strain relationship is discussed.

5.2 Mechanical behaviour

The moduli of the tube in circumferential and longitudinal direction are:

$$E_\varphi = nv_f \cos^4\theta A + B(1 - v_f) \tag{5.1}$$

$$E_z = nv_f \sin^4\theta A + B(1 - v_f) \tag{5.2}$$

with

$$A = a_1 e^{\left(a_2\lambda_{w,0}^2\lambda_{c,\varphi}^2 - a_3\lambda_{w,0}^{-1}\lambda_{c,\varphi}^{-1}\right)}(2a_2\lambda_{w,0}\lambda_{c,\varphi} + a_3\lambda_{w,0}^{-2}\lambda_{c,\varphi}^{-2}) \tag{5.3}$$

which is the Young's modulus of the fibre and

$$B = c_1 t^{-n} \tag{5.4}$$

which is the relaxation modulus of the rubber. a_1 to a_5 are the parameters derived from the fibre fitting process at a certain value of $\lambda_{w,0}$, $\lambda_{w,0}$ is the applied prestrain, $\lambda_{c,\varphi}$ and $\lambda_{c,z}$ are the draw ratios of the tube in respectively circumferential and longitudinal direction and c_1 and n the material parameters for the EPDM rubber.
Because the relaxation modulus of the EPDM rubber is used, the equations for the moduli of the tube are only applicable if a step in the strain is applied. The way the stresses are calculated depends on the load scheme. If a step in the strain ε_0 is applied, the stress can

be calculated by multiplying the modulus with ε_0. If a load scheme other than a step in the strain is used then the superposition theory or the dynamic modulus has to be applied.
The Poisson moduli can also be calculated and equal to:

$$\mu_{\varphi z} = 0.5v_f + 0.5(1 - v_f) = 0.5 \tag{5.5}$$

$$\mu_{z\varphi} = 0.5v_f + 0.5(1 - v_f) = 0.5 \tag{5.6}$$

5.2.1 General stress-strain relationship

Using equations (3.5) and (5.1) to (5.4) give for the Cauchy stress in the model:

$$\sigma_{\varphi\varphi} = \frac{1}{1-\mu_{z\varphi}\mu_{\varphi z}} \left[(nv_f \cos^4\theta A)\,(\lambda_{c,\varphi} - 1) + C(1 - v_f) + \right. \tag{5.7}$$

$$\left. \mu_{\varphi z}\,(nv_f \sin^4\theta A)\,(\lambda_{c,z} - 1) + \mu_{\varphi z}C(1 - v_f)\right] \tag{5.8}$$

$$\sigma_{zz} = \frac{1}{1-\mu_{z\varphi}\mu_{\varphi z}} \left[\mu_{z\varphi}\,(nv_f \cos^4\theta A)\,(\lambda_{c,\varphi} - 1) + \mu_{\varphi z}C(1 - v_f) + \right. \tag{5.9}$$

$$\left. (nv_f \sin^4\theta A)\,(\lambda_{c,z} - 1) + C(1 - v_f)\right] \tag{5.10}$$

where C depends on the load scheme.
Using the superposition theory for tensile test with a constant strainrate $\dot{\lambda}$, C can be written as:

$$C = \dot{\lambda}\frac{c_1}{1-n}t^{1-n} \tag{5.11}$$

5.3 Influence of the parameters

To produce a model which has the same characteristics as a real human vessel, an estimation of the parameters is needed.

5.3.1 Winding angle θ

By changing the angle at which the fibres are wound on the tube, the anisotropy of the tube can be altered. The range of θ lies between $\theta = 0^0$ and $\theta = 90^0$. A winding angle $\theta = 45^0$ will result in a tube which has the same characteristics in φ and z-direction. In figure 5.1 the stress-strain relationship is given for increasing values of θ from $\theta = 45^0$ to $\theta = 90^0$. If θ is smaller than 45^0, the modulus in longitudinal direction becomes lower than the stress in cicumferential direction.

5.3.2 Volumefraction v_f

The higher the volumefraction of fibres, the more nonlinear the stress-strain relationship will be. The volumefraction varies between 0 and 1. If the volumefraction is chosen very small, the influence of the fibres on the mechanical behaviour of the tube is very small. Therefore a high value for v_f is needed to match the characteristics of a real vessel.
In figure 5.2 the stress-strain relationship is given for a volume fraction from 0.01 to 0.1.

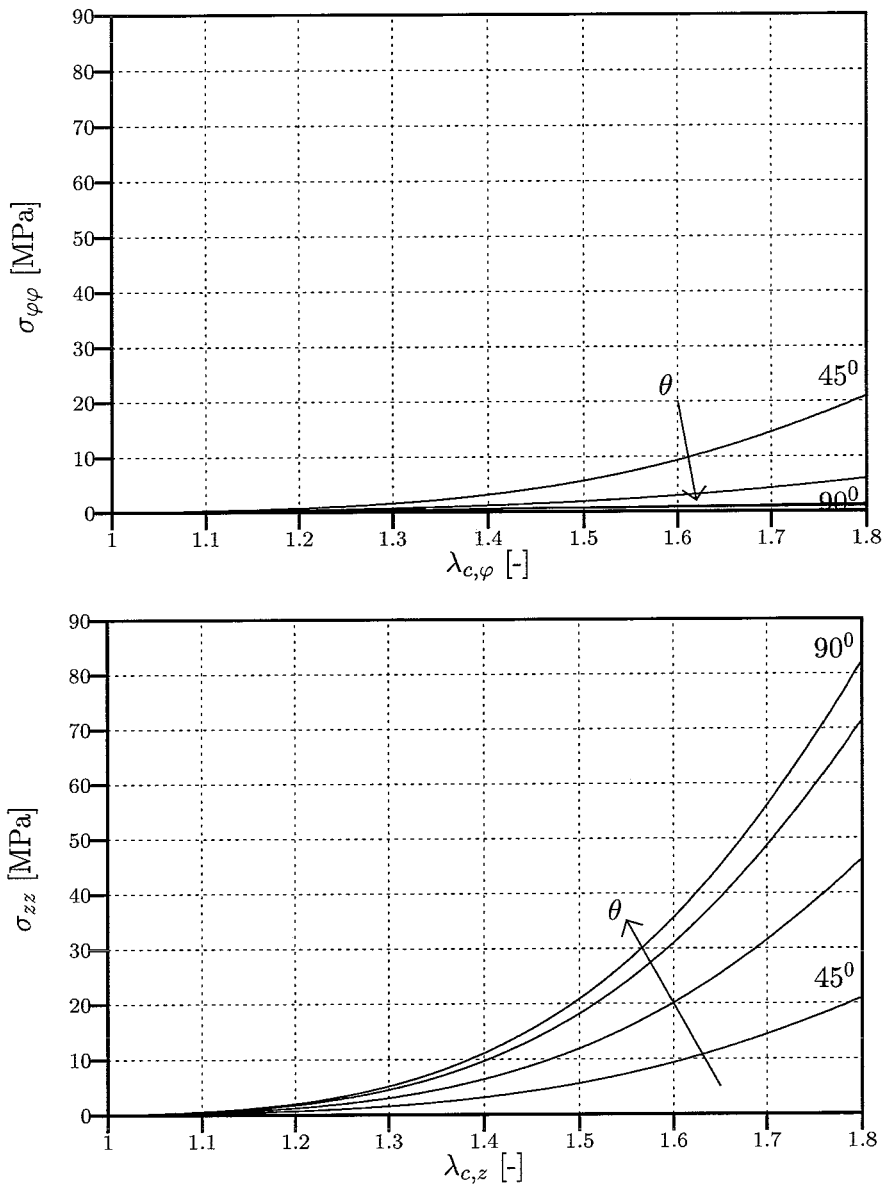

Figure 5.1: *Stress-strain relationship for increasing winding angle θ in circumferential direction (top plot) and longitudinal direction (bottom plot).* $v_f = .1; \dot{\lambda} = 0.01$

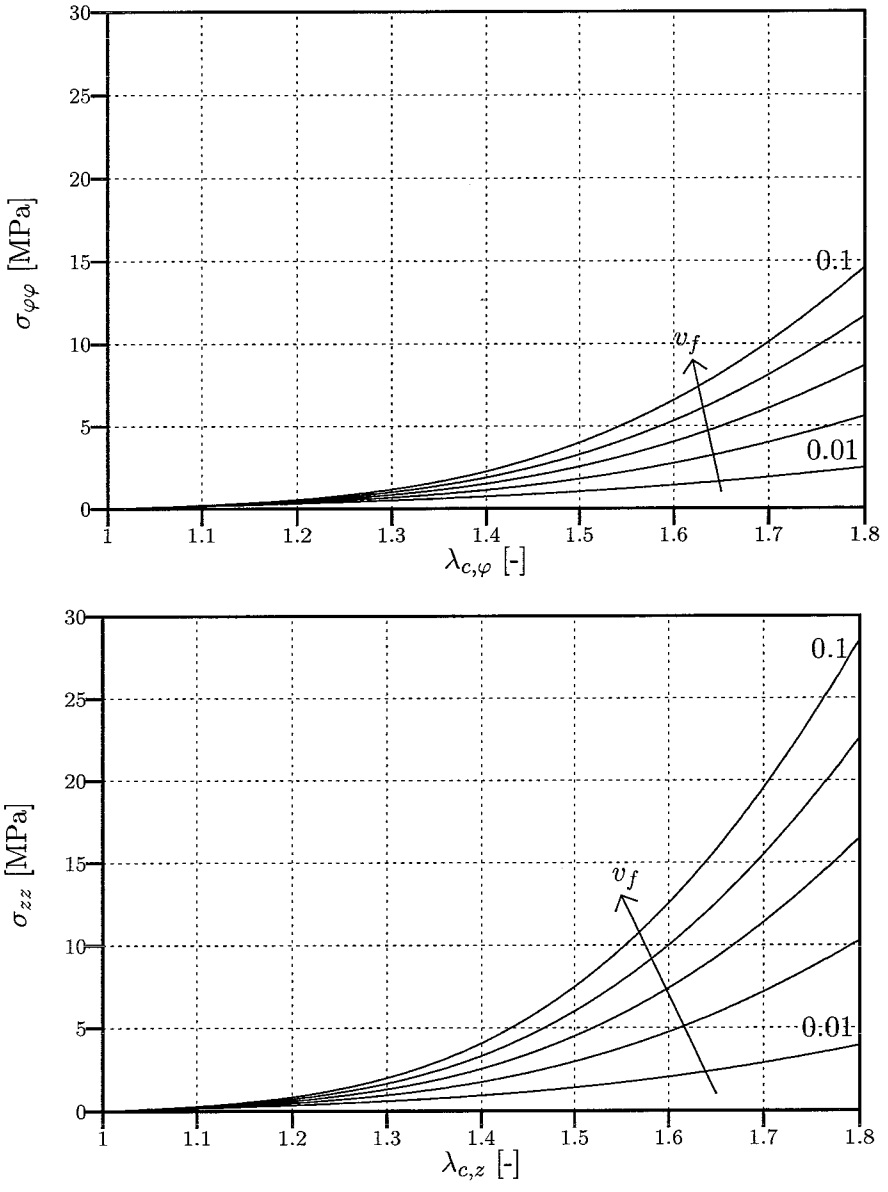

Figure 5.2: *Stress-strain relationship for increasing volumefraction in circumferential direction (top plot) and longitudinal direction (bottom plot).* $\dot{\lambda} = 0.01; \theta = 50^0$

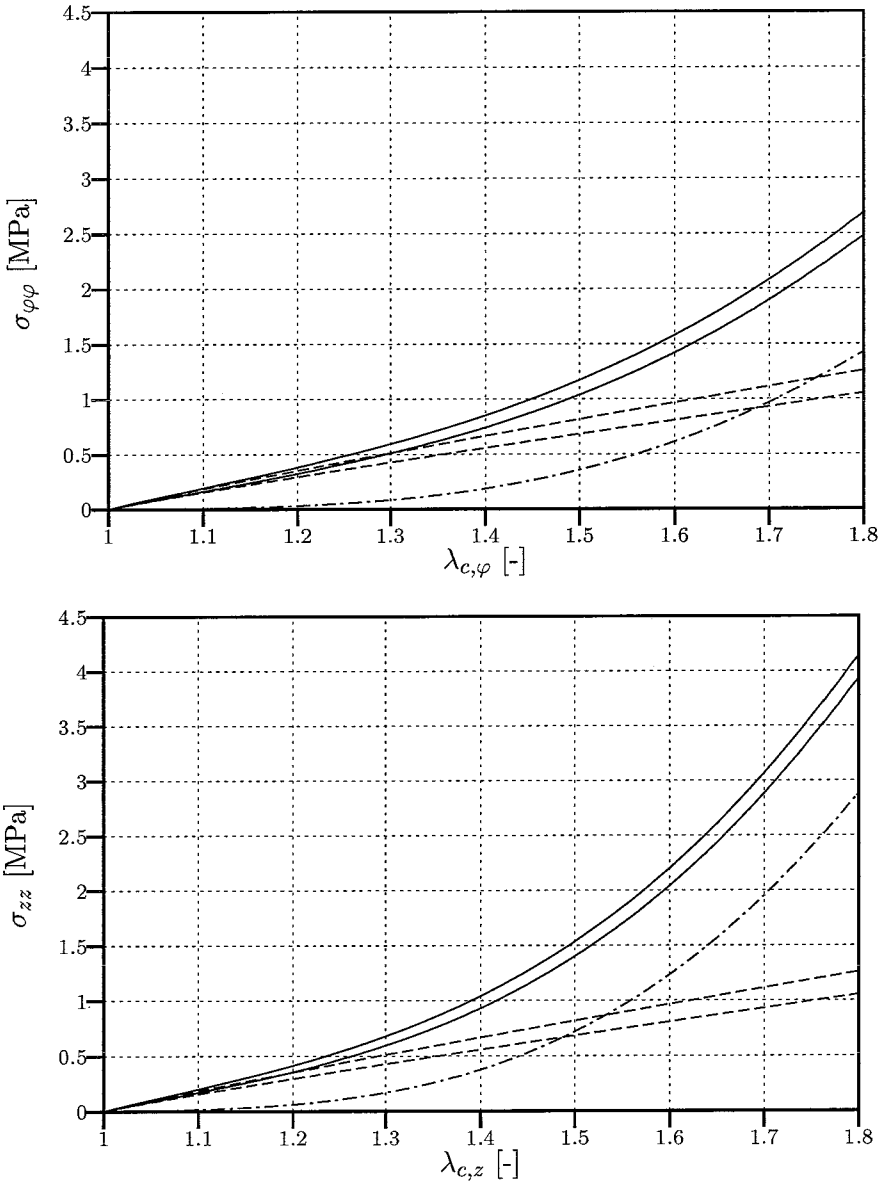

Figure 5.3: *Stress-strain relationship for increasing strainrate in circumferential direction (top plot) and longitudinal direction (bottom plot) with the stress in the rubber (dashed), the fibres (dash-dotted) and the tube (solid).* $v_f = 0.01; \theta = 50^0$

5.3.3 Strainrate

The strainrate determines the stress-strain relationship of the EPDM rubber. If the strainrate increases the rubber will become stiffer. This can be seen in figure 5.3. The strainrate is increased from 0.01 to 0.1. The stress in the rubber increase with increasing strainrate.

5.4 Vessel model parameters

To obtain a correct vessel model, the parameters have to be estimated. The strainrate $\dot{\lambda}$ is taken 0.1, which can differ from an in-vivo situation. It is yet unknown what the strainrate of a vessel wall is. Because the stiffness in longitudinal direction is greater than the stiffness in circumferential direction, the winding angle has to be greater than 45^0.
One parameter is not included in the model: the wall thickness of the tube. It is assumed to be 1. The stress-strain relationship of the vessel can be scaled with the wall thickness to match the characteristics of a human vessel, which is found e.g. in Kas'yanov and Knet-s (1974).
If $\theta = 48^0$ and the volumefraction $v_f = 0.1$ the stress-strain relationship of the vessel model needs to be scaled with 0.1 to match Kas'yanov and Knet-s (1974). In figure 5.4 both relationships are plotted. It can be seen that the vessel model is not as nonlinear as a human vessel.
The working area for the model is from $\lambda = 1$ to $\lambda = 1.5$. Within this range the obtained vessel model describes human vascular tissue.

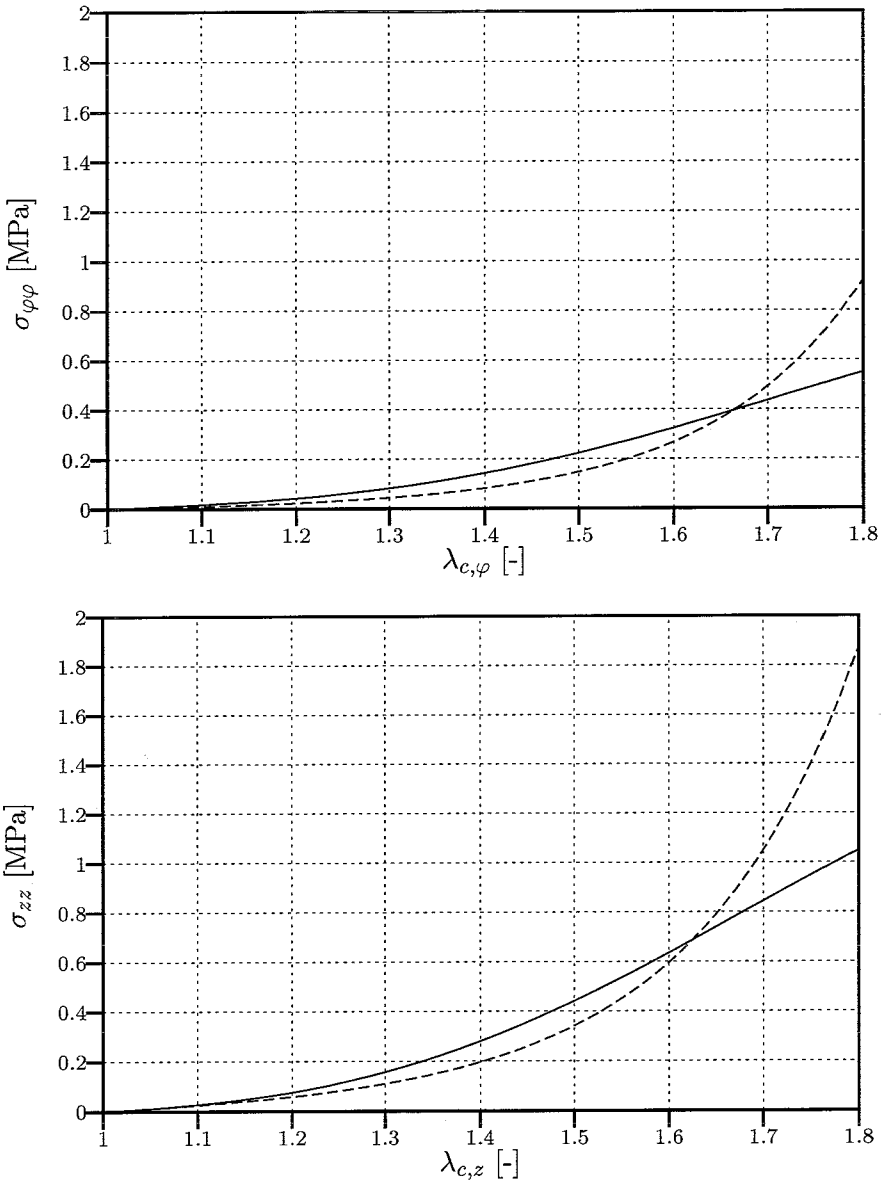

Figure 5.4: *Stress-strain relationship of the vessel model (solid) and the human vessel from Kas'yanov and Knet-s (1974) (dashed).* $v_f = 0.1$; $\theta = 48^0$; $\dot{\lambda} = 0.1$; $h = 0.1$

Chapter 6

Remarks and conclusions

6.1 Remarks

The basic constitutive equation for the vessel wall describes orthotropic behaviour. It implies that the material properties in r-direction do not change across the cross sectional area of the tube and along its whole length. The tube made of EPDM rubber and lycra fibre is orthotropic, so that constitutive equation may be used.
The introduced constitutive equation is only applicable for geometrical linear mechanical behaviour. The vessel model shows nonlinear behaviour, so an inaccuracy is introduced. When the tube is subjected to a tensile test, the angle of the fibres will change. Because the winding angle of the fibre determines the anisotropy of the tube, the anisotropy will change during the tensile test. This introduces geometrical nonlinear behaviour.
Because the cross-sectional area of the fibres is very small, it is difficult to measure a marked reference length. A small error will be increased due to the λ-axis transformation.
To make a tube of EPDM rubber requires dissolving it in xylene and adding dibenzoylperoxide to enable crosslinking. The amount of dibenzoylperoxide and the time during which the tube is put in the oven, determine the rate of crosslinking. Because the amount of added benzoylperoxide is small the rubber is not crosslinked entirely. Small fluctuations in the amount of benzoylperoxide results in fluctuations in the crosslink-density. Every time a tube is made, relaxation experiments have to be done, because the mechanical behaviour of the rubber can be different, because of the poor reproducebility of the crosslinking process.

6.2 Conclusions

Lycra fibres and EPDM rubber can be used to make a vessel model. By preconditioning the fibres, they gain properties which are useful for the vessel model. The EPDM rubber shows little viscoelasticity, which results in viscoelastic behaviour when a load with high frequencies (higher than 10 Hz) is applied.
A nonlinear vessel model shows physical and geometrical nonlinear behaviour.
The physical nonlinear behaviour is a result of the used material and is modelled using EPDM rubber and lycra fibres. If the fibres are preconditioned (prestrained and heated) they show a more pronounced nonlinear behaviour.
Because large strains are applied, the deformation is nonlinear. Geometrical nonlinear behaviour can not be modelled with the linear constitutive equation presented in this report.

An estimation for the winding angle, the volume fraction fibres and the wall thickness can be made. The volume fraction is approximately 0.1, the winding angle is 48^0. The wall thickness has to be 0.1 mm. Then the stress-strain relationship of the model matches the stress-strain relationship of a human vessel measured in Kas'yanov and Knet-s (1974) for small strains. Unfortunately it is impossible to make a tube with a wall thickness of 0.1 mm with the present production method.
A better method has to be developed to obtain the mechanical behaviour of the fibres.

6.3 Suggestions for further research

A method to determine a reference state for the lycra fibres has to be developed. A possibility is adding a preload, which forces the fibre to stretch. Then a reference length can be marked and measured. Because the fibre is stretched, it remains straigth and the measurement can be more accurate than without adding a preload.
The model presented in chapter 3 can not be used to compute the mechanical behaviour of the vessel model when a large strain is applied. Because pressure-wave propagation experiments (which cause inhomogenuous deformation) are done on the nonlinear vessel model, a FEM model would be practical. The developed FEM model could be used in FEM computations regarding pressure waves.
Only if the vessel model has a wall thickness of 0.1 mm, the stress-strain relationship describes vascular tissue well (for small strains). At present it is impossible to remove a tube with a wall thickness of 0.1 mm from the mandrel without breaking it. So another production method has to be developed to enable making a tube which has a very low wall thickness.

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