T.M. III (sterkteleer) : aanvulling knik van staven

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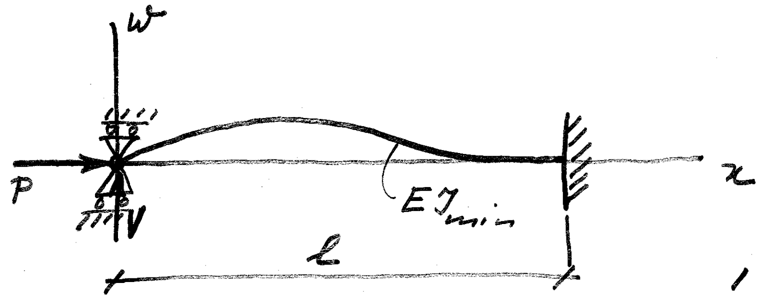
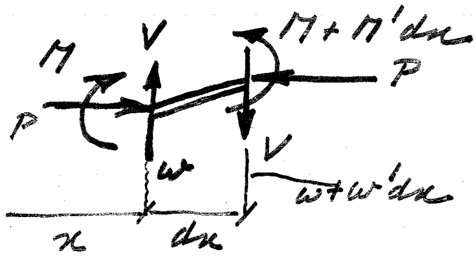
Aanvulling Knik van staven



$' = \frac{d}{dx}$



$$V\,dx - P w'\,dx - M'\,dx = 0$$

$$\boxed{M' = V - P w'} \quad \text{Evenw.}$$

$$\boxed{EI w'' = M} \quad \text{elast.}$$

Hieruit $\quad EI w^{IV} + P w'' = 0$

$$\alpha^2 = \frac{P}{EI}$$

$$w = A \cos \alpha x + B \sin \alpha x + C x + D$$

4 Randvoorwaarden $\Rightarrow$ homogene vergelijkingen in A, B, C en D.

Voorbeeld boven:

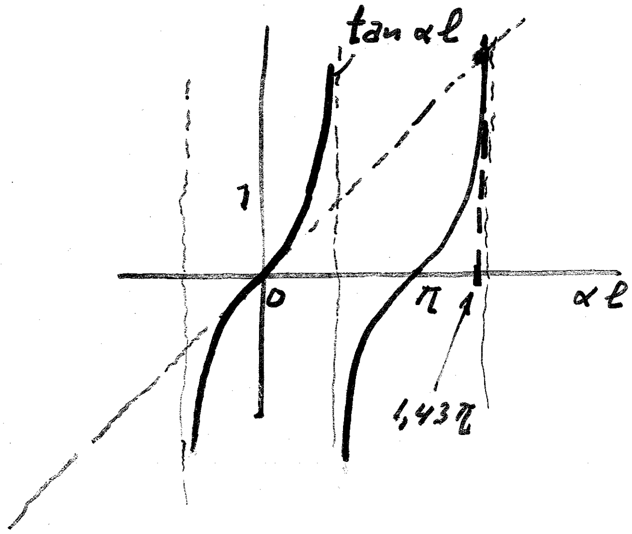
$$\left.\begin{array}{lllr} x=0 & \begin{cases} w = 0 \longrightarrow & A + D = 0 \\ w'' = 0 \longrightarrow & -\alpha^2 A = 0 \end{cases} & & (1) \\ & & & (2) \\ x=\ell & \begin{cases} w = 0 \longrightarrow & A\cos\alpha\ell + B\sin\alpha\ell + C\ell + D = 0 \\ w' = 0 \longrightarrow & -A\alpha\sin\alpha\ell + B\alpha\cos\alpha\ell + C = 0 \end{cases} & & (3) \\ & & & (4) \end{array}\right\}$$

Uit (1) en (2) : $A = D = 0$

Uit ③ en ④

$$\begin{vmatrix} \sin\alpha l & l \\ \alpha\cos\alpha l & 1 \end{vmatrix} = 0$$

Hieruit knikvoorwaarde: $\alpha l = \tan\alpha l$

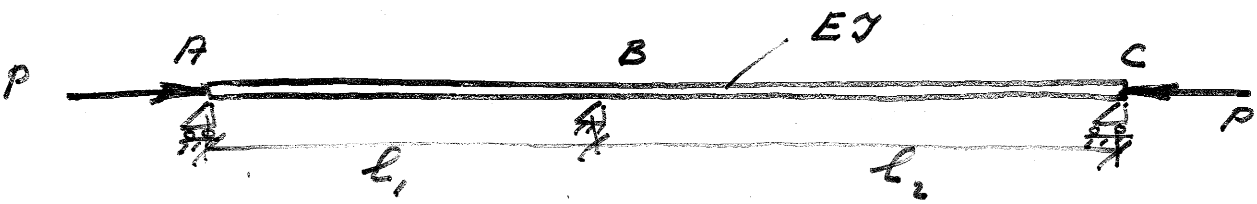


Kleinste oplossing:

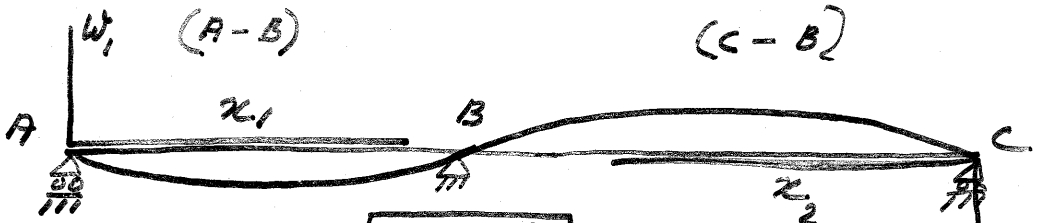
$$\alpha l = 1{,}43\,\pi$$

$$\boxed{P_k = 2{,}04\ \frac{\pi^2 EJ}{l^2}}$$

Voorbeeld:



$$l_1 + l_2 = l$$



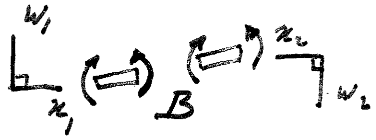
$$\boxed{\alpha^2 = \frac{P}{EJ}}$$

$0 \leq x_1 \leq l_1$; $w_1 = A_1 \cos\alpha x_1 + B_1 \sin\alpha x_1 + C_1 x_1 + D_1$

$0 \leq x_2 \leq l_2$; $w_2 = A_2 \cos\alpha x_2 + B_2 \sin\alpha x_2 + C_2 x_2 + D_2$

$$\left.\begin{array}{lll} x_1 = 0 & & w_1'' = 0 \text{ en } w_1 = 0 \\ & \text{dan:} & \\ x_2 = 0 & & w_2'' = 0 \text{ en } w_2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} A_1 = 0 = D_1 \\ A_2 = 0 = D_2 \end{array}$$

Aansluiting bij B :



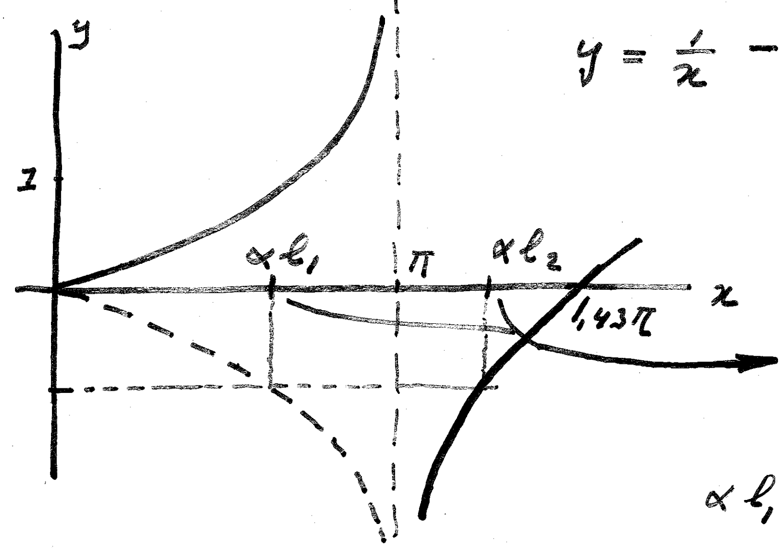
$$x_1 = \ell_1 \qquad x_2 = \ell_2 \; ; \quad \left\{ \begin{array}{l} \dfrac{dw_1}{dx_1} = \dfrac{dw_2}{dx_2} \quad \text{en} \quad \dfrac{d^2w_1}{dx_1^2} = -\dfrac{d^2w_2}{dx_2^2} \\[2ex] w_1 = w_2 = 0 \qquad\qquad (4 \text{ relaties}) \end{array} \right.$$

$$\left. \begin{array}{lllll} B_1 \sin\alpha\ell_1 + C_1\ell_1 & & & & = 0 \\ & & B_2 \sin\alpha\ell_2 + C_2\ell_2 & & = 0 \\ B_1\alpha\cos\alpha\ell_1 + C_1 & & -B_2\alpha\cos\alpha\ell_2 - C_2 & & = 0 \\ B_1\alpha^2\sin\alpha\ell_1 & & +B_2\alpha^2\sin\alpha\ell_2 & & = 0 \end{array} \right\}$$

$$\left\{ \begin{array}{l} B_1\left(\cos\alpha\ell_1 - \dfrac{\sin\alpha\ell_1}{\alpha\ell_1}\right) - B_2\left(\cos\alpha\ell_2 - \dfrac{\sin\alpha\ell_2}{\alpha\ell_2}\right) = 0 \\[2ex] B_1 \sin\alpha\ell_1 \qquad\qquad + B_2 \sin\alpha\ell_2 \qquad\qquad = 0 \end{array} \right\}$$

$$\left(\cos\alpha\ell_1 - \frac{\sin\alpha\ell_1}{\alpha\ell_1}\right)\sin\alpha\ell_2 + \left(\cos\alpha\ell_2 - \frac{\sin\alpha\ell_2}{\alpha\ell_2}\right)\sin\alpha\ell_1 = 0$$

$$\boxed{-\left[\frac{1}{\alpha\ell_1} - \operatorname{cotan}\alpha\ell_1\right] = \left[\frac{1}{\alpha\ell_2} - \operatorname{cotan}\alpha\ell_2\right]}$$



$$y = \frac{1}{x} - \frac{1}{\tan x} = \frac{\tan x - x}{x\tan x}$$

$x \to 0$ dan $y \approx \frac{1}{3}x$

$\alpha\ell_1$ en $\alpha\ell_2$ voldoen aan $\square$; dan $\alpha\ell_1 + \alpha\ell_2 = \alpha\ell$ (getal)

en $\boxed{P_k = \dfrac{(\alpha\ell)^2 EJ}{\ell^2}}$

$$P_e = \frac{\pi^2 EJ}{\ell^2}$$