Modelling of the fuel consumption of a fuel cell powered car

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Traineeship report

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Preface

For my trainee ship of fourteen weeks, I've been working on the PAC-car project at the Measurement and Control Laboratory (IMRT) of the Swiss Federal Institute (ETH) in Zürich. This was really a great experience for me. I have had the opportunity to develop myself both professionally and personally. I would like to thank Lino Guzzella for the opportunity to work at the ETH, Brigitte Rohrbach und Claudia Wittwer for the administrative part, Christopher Onder for coaching me during this period and Antonio Sciarretta, Gino Paganelli, Jean-Jaques Santin, Alexander Schilling, Paul Rodatz, Erik Müller and Daniel Brand for the help with all kinds of problems with my project.

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Introduction

The PAC-car is an ultra light three wheeler test car, powered by a fuel cell, which has been developed especially for the Shell Eco Marathon. The aim at this Eco-challenge is to drive at the Nogaro circuit (France) with a minimum average speed of 30 km/h and use as less fuel as possible. To minimize the fuel consumption during this challenge, one strategy is to develop an optimal control strategy such that at each position, the fuel optimal speed can be determined over a given time horizon. There is already a model that can describe the fuel consumption while driving on the Nogaro circuit. Because this model is a very complicated Simulink flow chart model, it can not be used to implement into an optimal control strategy.

The aim of this research is to model the PAC-car system and find a simple function of the hydrogen mass flow as a function of speed and accelerator angle. This function must be usable to implement in an optimal control strategy, such that the fuel optimal speed profile can be found.

First, a short description of the Shell Eco Challenge will be given. Afterwards, the working principle of a fuel cell will be explained. Then the PAC-car vehicle will be analyzed and described. Some relationships of the PAC-car will be described by fitting equations on measurement data. By combining all these equations, a control oriented model can be formed which describes the mass flow of hydrogen as a function of speed and accelerator angle. This formula will be used, together with the vehicle dynamics, to implement in an optimal control strategy and in this way, the fuel optimal speed profile can be found.

Chapter 1

The Shell Eco Challenge

The PAC ("Pile a Combustible"which means fuel cell)– car is a project of Measurement and Control Laboratory (IMRT) of the Swiss Federal Institute of Technology (ETH) Zurich and the Université de Valenciennes et du Hainaut Cambresis (UVHC) in France. A part of the car (the carbon-fiber frame, mechanical parts, wheels and chassis) has been developed in France while the complete power train has been developed in Switzerland.
The PAC-car (see Fig. 1.1) has especially been designed to drive at the Shell Eco challenge [8]. This challenge is a competition in which different universities and companies participate by building a car that drives at the Nogaro circuit (France) using as less fuel as possible. It is allowed to use all kind of fuels and new technologies. The fuel of the PAC-car is hydrogen which will be converted into energy by a fuel cell. This energy powers all auxiliaries and two electro motors.

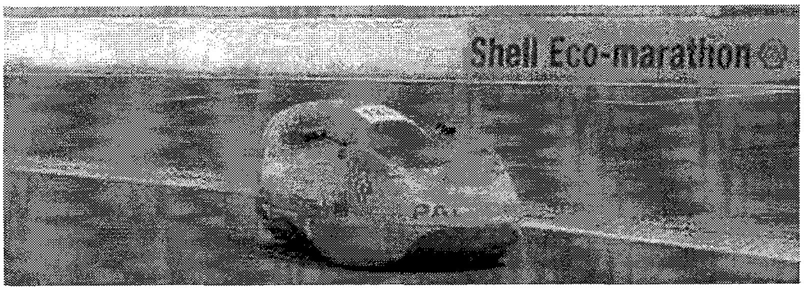

Figure 1.1: The PAC car driving on the Nogaro circuit

Every participating car has to drive 7 laps at the Nogaro circuit in a maximum time of 50 minutes and 34 seconds. Each lap is 3.636 km which means that the minimum average speed has to be 30 km/h. The fuel consumption during these

7 laps will be measured. Depending on the kind of fuel that has been used, the total consumption will be recalculated to obtain an equivalent consumption of petrol with the help of the heat value of hydrogen (= $119.93\ MJ/kg$) and the heat value of petrol (= $44\ MJ/kg$). The petrol equivalent consumption of the PAC-car was $1694\ km/l_{petrol}$!
With this consumption, the final ranking of the PAC car was the 11^{th} position. It has won the 2^{nd} prize for the alternative energy category as well.

The Nogaro circuit contains different slopes(see Fig. 1.2). This means that if one tries to drive the average speed of $30\ km/h$ continuously, it will not lead to the optimal fuel consumption. There is some speed profile, depending on the slope profile, that will lead to the optimal fuel consumption. This speed profile has to be found.

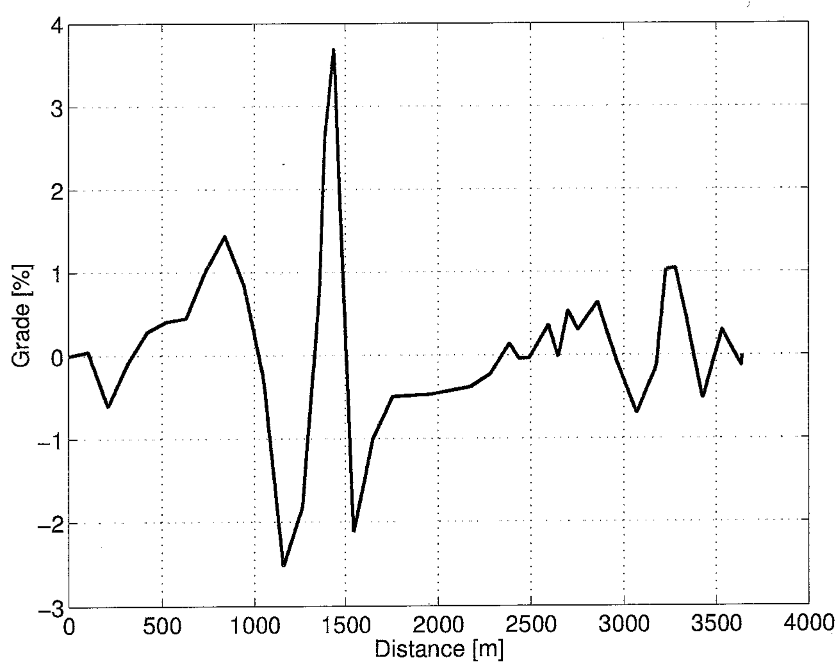

Figure 1.2: The grade profile of the Nogaro circuit as a function of covered distance

Chapter 2

The Fuel cell

In this chapter, the working principle of a fuel cell will be explained [5]. There are different kinds of fuel cells with different fuels, working temperatures, efficiencies, advantages and disadvantages. The fuel cell used in PAC-car is a Proton Exchange Membrane (PEM) fuel cell. These kind of fuel cells are also often called Polymer Electrolyte Membrane fuel cells. The properties of the fuel cell used in the PAC-car can be seen in Appendix A

The PEM fuel cell is powered by hydrogen. Properties of hydrogen can be found in Appendix C. Big advantages of the PEM are that no pollution is produced and the only resulting products are water and heat. The PEM fuel cell has a high efficiency ($50-60\%$), a high power density and operates at fairly low temperatures ($40-80°C$), which means they warm up quickly and do not require expensive containment structures. Another advantage of the PEM fuel cell is that air can be used as cathode gas such that an O_2 tank is not necessary.

2.1 The hydrogen storage system

The hydrogen is stored in a hydrogen tank with the metal hydrides storage system. The metal hydrides are made of an alloy material that can absorb gaseous hydrogen. The hydrogen will become solid then. This is a exothermic chemical reaction and therefor heat will be released when the hydrogen is absorbed. If heat is supplied to the hydrides, the hydrogen will become gaseous again and can be used for the fuel cell. This type of storage tank is a very safe way to store the hydrogen.

2.2 The construction of a PEM fuel cell

The construction of a PEM exists of various parts and is some kind of a sandwich construction. The outer parts (the anode and cathode) have small canals in which the hydrogen and air are brought into contact with the catalyst to facilitate the splitting reaction. In the middle of the construction the electrolyte (the proton exchange

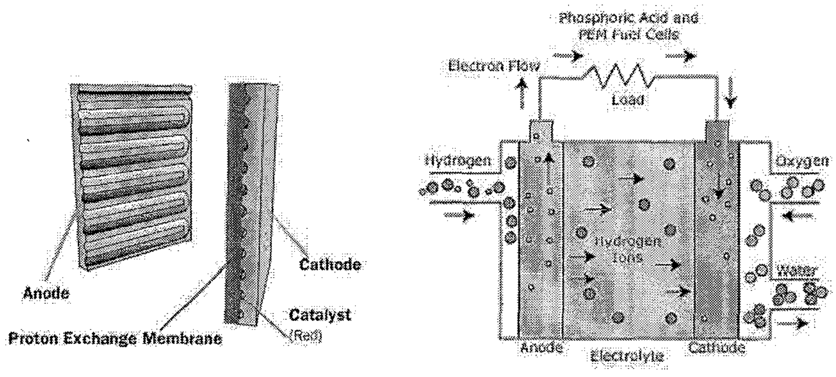

Figure 2.1: **a.** Composition of the different parts of a fuel cell **b.** Schematic picture of the working principle of a fuel cell.

membrane) has been placed. A schematic picture of the different parts of a PEM fuel cell can be seen in figure Fig. 2.1a.

In Fig. 2.1b a schematic picture of the working principle of a fuel cell can be seen. At the anode of the fuel cell, H_2 is split in two hydrogen protons and two electrons ($2H^+$ en $2e^-$) with the help of a platinum catalyst. The catalyst is a special material that facilitates the reaction of oxygen and hydrogen. It is usually made of platinum powder very thinly coated onto carbon paper or cloth. The catalyst is rough and porous so that the maximum surface area of the platinum can be exposed to the hydrogen or oxygen. After this splitting reaction the H^+ ions diffuse through the electrolyte that is only permeable for hydrogen and then go to the cathode. The electrons that exist during the splitting reaction of hydrogen diffuse through the anode and are going to the external electrical circuit and afterwards to the cathode. This is the current I, that can be used to power the system. Meanwhile, on the cathode side of the fuel cell, oxygen gas (O_2) is being forced through the catalyst, where it forms two oxygen atoms. Each of these atoms has a strong negative charge. This negative charge attracts the two H^+ ions through the membrane, where they combine with an oxygen atom and two of the electrons from the external circuit to form a water molecule (H_2O). In this way, hydrogen fuel's natural tendency to oxidize and form water is utilized to produce electricity and useful work.

The next three equations describe the reactions at the anode, cathode and the total reaction in the fuel cell.

Anode

$$2H_2 \longrightarrow 4H^+ + 4e^- \quad (2.1)$$

Cathode

$$4H^+ + 4e^- + O_2 \longrightarrow 2H_2O \quad (2.2)$$

Fuel cell

$$2H_2 + O_2 \longrightarrow 2H_2O \quad (2.3)$$

This reaction in a single fuel cell produces only about 1.2 volts. To get this voltage up to a reasonable level, many separate fuel cells must be combined to form a fuel-cell stack. A picture of a fuel cell stack can be seen in Fig. 2.2.

Figure 2.2: More fuel cells are set parallel such that they form a fuel cell stack that can produce a higher voltage

2.3 Voltage of a fuel cell

The maximum voltage of a fuel cell depends on the energy W that is produced by the free electrons. In every $kMol$ hydrogen, there are N_o (Number of Avogadro) molecules. Every molecule hydrogen has two electrons (n_e). The energy W that is produced by one $kMol$ hydrogen is

$$W = n_e \cdot q \cdot N_0 \cdot V = n_e \cdot F \cdot V \quad (2.4)$$

in which

$$F = 96.49 \cdot 10^6 C/kMol (\text{Faraday Constant})$$
$$q = 1.6 \cdot 10^{-19} C$$

When the system is reversible (this means that the free entropy of hydrogen ($\Delta G = 228.16 MJ/kmol$) will be completely converted into work W) and after rewriting Eq. 2.4 such that U will be freed, the maximum voltage can be described by Eq. 2.5

$$V_{th} = \frac{\Delta G}{n_e \cdot F} = \frac{228.6}{2 \cdot 96.48} = 1.185V \tag{2.5}$$

Because this maximum voltage is too low for most of the systems, fuel cells can be set in serial connection such that higher voltages can be produced. This is called a fuel cell stack. The voltage of a stack that exists of N fuel cells can be described by

$$V_{st} = N \cdot V_{fc} \tag{2.6}$$

The power of a fuel cell stack can than be given by:

$$P_{fc} = N \cdot I_{fc} \cdot V_{fc} \tag{2.7}$$

The characteristic static polarization curve of the voltage and current of a fuel cell can be seen in Fig. 2.3

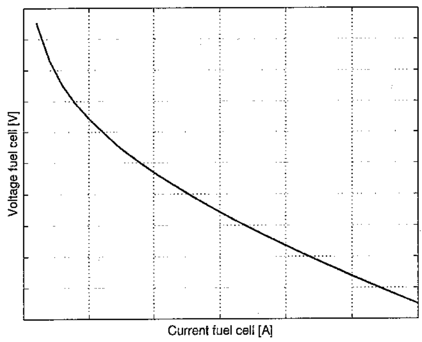

Figure 2.3: Characteristic curve of the voltage and current of a fuel cell

2.4 Current of a fuel cell

The current of a fuel cell can be described by

$$I_{fc} = \frac{\dot{m}_{H_2} \cdot n_e \cdot F}{M_{H_2}} \tag{2.8}$$

in which $\dot{m}_{H_2}$ is the mass flow of the hydrogen and M_{H_2} is the mol mass of hydrogen. For the current of a fuel cell stack, Eq. 2.8 should be multiplied by the factor $1/N$.

$$I_{fc} = \frac{\dot{m}_{H_2} \cdot n_e \cdot F}{M_{H_2} \cdot N} \tag{2.9}$$

This means that the mass flow is a linear function of the current drawn from the fuel cell.

$$\dot{m}_{H_2} \sim I_{fc} \tag{2.10}$$

2.5 Efficiency of a fuel cell

As in all systems, the efficiency of a fuel cell can not be 100%. The highest efficiency that can be reached with a fuel cell is called the Electrical Carnot efficiency. This maximum exists because there will always be a difference in the energy that can be released (ΔG, the free entropy) and the entropy (ΔH) of the fuel. The energy Q that can not be used will be transformed into heat ($Q = T \cdot \Delta S$). The maximum efficiency of a fuel cell is given by Eq. 2.11.

$$\eta_{max} = \frac{\Delta G}{\Delta H} = 1 - T \cdot \frac{\Delta S}{\Delta H} \tag{2.11}$$

With the values of $\Delta G = -228.6\ MJ/kMol$ and $\Delta H = -241.8\ MJ/kMol$ for hydrogen gas and Eq. 2.11, the maximum efficiency of a fuel cell becomes

$$\eta_{max} = \frac{-228.6}{-241.8} = 94.5\% \tag{2.12}$$

Besides the maximal efficiency, the voltage efficiency has been defined. This efficiency is based on the maximum voltage that theoretically can be reached (V_{th}) with respect to δG and the voltage that is accomplished by the fuel cell (V_{fc}).

$$\eta_V = \frac{V_{fc}}{V_{th}} \tag{2.13}$$

The curve of the efficiency will be descending because the fuel cell voltage drops at higher current.This is a characteristic polarization curve for a fuel cell and can be seen in Fig. 2.4.

A system efficiency can be defined as well. Because the fuel cell stack needs the auxiliaries to be powered, this can be seen as a loss. So when the power that really can be used for the system will be compared with the power that theoretically can come out of the fuel cell, the fuel cell system efficiency can be defined.

$$\eta_{fc,stack} = \frac{P_{el}}{N \cdot V_{th} \cdot I_{fc}} = \frac{V_{fc} \cdot I_{fc} - P_{aux}}{N \cdot V_{th} \cdot I_{fc}} \tag{2.14}$$

This curve can be seen in Fig. 2.4 as well. What can be seen is that in the low current region, the efficiency is very bad because the power that will be produced will go to the auxiliaries and therefor this will be seen as a loss.

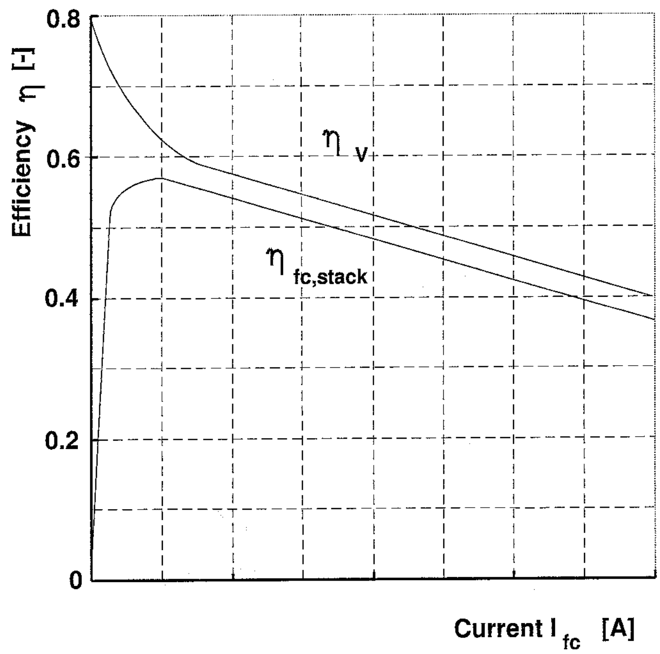

Figure 2.4: The voltage efficiency and the system efficiency of a fuel cell.

Chapter 3

The PAC-car

The system of the PAC-car exists of different subsystems, which can be seen in Fig. 3.1. Al these systems will be described in the next paragraphs. The working principle of the hydrogen tank and the fuel cell have already been explained in Chapter 2. In this chapter, the auxiliaries to drive the fuel cell, the driver, the power electronics, the electro motor, and the transmission will be explained. The vehicle dynamics will be described in chapter 7.

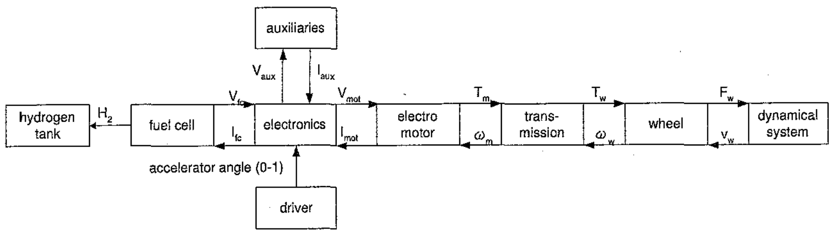

Figure 3.1: The schematic layout of the PAC car

3.1 The auxiliaries

To drive the fuel cell, auxiliaries such as different valves and pumps, flow meters, sensors, vessels etc are needed. In Fig. 3.2 the schematic structure of the fuel cell connected to the auxiliaries can be seen. There are three circuits: the air circuit, the water circuit and the hydrogen circuit.
The air circuit takes normal air from the environment. This air will go through an air filter and will be brought to a sudden pressure before it enters the fuel cell. The air accumulator takes care that the air has a constant pressure instead of the pulsating pressure that comes from the air pump. The valves in the circuit will take care of the hight of the pressure. In the fuel cell the oxygen will be used and the

rest of the air will flow back to the environment.
The water circuit has the purpose to cool down the fuel cell and to heat up the hydrogen tank such that hydrogen will be released.
In the hydrogen circuit, hydrogen will come from the tank and will be brought to a sudden pressure controlled by valves. Then the hydrogen can enter the fuel cell and most of the hydrogen will react with oxygen into water. The hydrogen that will not be used will go through the water separator and will be extracted from the water. This hydrogen will be lead back to the fuel cell. The water that has been formed will go to the environment.

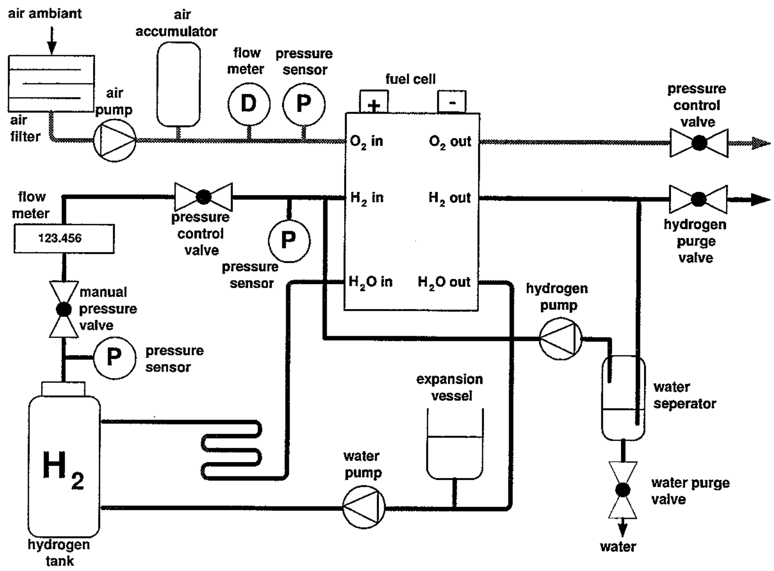

Figure 3.2: The water circuit, the hydrogen circuit and the oxygen circuit are needed to run the fuel cell.

3.2 The driver

The driver of the PAC-car can control the system with the help of an accelerator peddle. The accelerator is a potentiometer and the angle of the accelerator will be measured and multiplied by a factor 30. This signal will be used as a setpoint for the current control of the electro motor and will be called I_{motset}. In the next paragraph will be described how this signal will be used.

$$I_{motset} = 30\alpha \tag{3.1}$$

3.3 The power electronics

The auxiliaries of the fuel cell are powered by the fuel cell itself. Only to start up the fuel cell, a battery will be used. In the electronics part, the current and voltage from the fuel cell will be transformed into a current and voltage that can be used to power the auxiliaries. Also the voltage and current going to the electro motor will be transformed such that the tracking force and speed required can be accomplished. The scheme can be seen in Fig. 3.3.
The current (I_{fc}) coming from the fuel cell will be split up in 2 parts. One part (I_{aux}) is going to the auxiliaries and the other part (I_m) can be used by the motor. To realize the actual current I_{mot} used to drive the electro motor (which should be equal to the setpoint), I_{mot} will be measured and compared with the signal I_{motset} coming from the accelerator angle. The difference of this signal is going through a PI controller and a signal generator to the switch that will downsize the current I_m to the requested current I_{mot}.

The relationships between the currents can be described by the next equations.

$$I_{fc} = I_m + I_{aux} \quad (3.2)$$

$$I_m = \frac{I_{mot} \cdot V_{mot}}{V_{fc}} \quad (3.3)$$

$$(3.4)$$

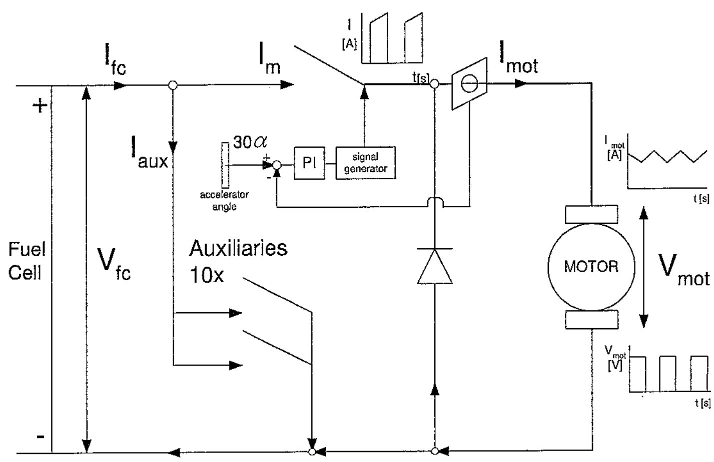

Figure 3.3: The electrical circuit to power the electro motor and the auxiliaries

The DC/DC converter has a constant efficiency η_c of 95%

3.4 The electro motor

Fig. 3.3 can be simplified to Fig. 3.4. The left part of Fig. 3.3 is replaced by the DC/DC box and in the electro motor has been split up in the inner resistance(R_i) of the motor and the voltage that can be used to drive the motor(V_i) [9].

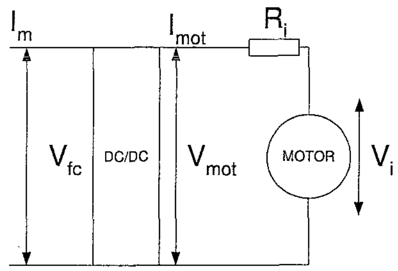

Figure 3.4: The electromotor

From this figure the next equation can be derived, where we assume the inductance $L\frac{di}{dt}$ to be small.

$$V_{mot} = I_{mot} \cdot R_i + V_i \tag{3.5}$$

For the conversion from electrical energy to mechanical energy, two electro motors are set parallel such that a higher torque can be occurred. This means that the inner R_i in Eq. 3.5 has to be multiplied with a factor 0.5.
The relation between torque T and current I is described by Eq. 3.6 and the relation between radial velocity ω and voltage V is in Eq. 3.7 in which $k_n = 569\ rpm/V$ and $k_m = 16.8\ mNm/A$ are motor constants.

$$T_{mot} = I_{mot} \cdot k_m \tag{3.6}$$
$$\omega_{mot} = V_i \cdot k_n \tag{3.7}$$

The motor used is a DC Micro motor and its specifications are in Table C.

3.5 The transmission

To get a useful speed and torque, a transmission has been used with an differential ratio $i = 306/15$. The losses by friction in this part of the system are not known but are assumed they can be neglected. So the torque and radial speed at the wheel can be described by

$$\omega_w = \frac{1}{i} \cdot \omega_{mot} \tag{3.8}$$

$$T_w = i \cdot T_{mot} \tag{3.9}$$

To get the speed v and traction force F_t of the vehicle, the wheel radius $r_w = 0.25\ m$ must be taken into account.

$$v = \frac{r_w}{i} \cdot \omega_{mot} \tag{3.10}$$

$$F_t = \frac{i}{r_w} T_{mot} \tag{3.11}$$

Chapter 4

Modelling of the hydrogen consumption

The final goal of this study is to obtain a formula that describes the mass flow of the system as a function of accelerator angle and speed. In the chapters before, formulas for the different subsystems have been found. By combining these formulas, a formula for the hydrogen mass flow can be found.

4.1 The fuel cell current

From Eq. 2.9 follows that the mass flow of the fuel cell stack is a linear function of the current drawn from the fuel cell. After rewriting this equation, the mass flow can be described by

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \cdot I_{fc} \tag{4.1}$$

in which

$$I_{fc} = I_m + I_{aux} \tag{4.2}$$

Because M_{H_2}, n_e, N and F in Eq. 4.1 are constants, only the fuel cell current should be known to describe the mass flow. This term I_{fc} can be split in two parts: one for the motor current I_m and one for the current used by the auxiliaries I_{aux}. The first term can be derived analytically. The motor that is used is a DC micro motor. Because the DC/DC converter has an efficiency η_c of 95 %, Eq. 3.3 will change to Eq. 4.3.

$$I_m = \frac{I_{mot} \cdot V_{mot}}{\eta_c \cdot V_{fc}} \tag{4.3}$$

in which

$$I_{mot} = 30\alpha \tag{4.4}$$

$$V_{mot} = 30\,\alpha \cdot 0.5\,R_i + V_i \tag{4.5}$$

$$V_i = \frac{\omega_{mot}}{k_n} \tag{4.6}$$

$$\omega_{mot} = \frac{v \cdot i}{r_w} \tag{4.7}$$

Combining (4.3)- (4.7) leads to a formula that describes the motor current(assumed is that the current controller works perfect).

$$I_m = \frac{30\alpha \cdot (30\alpha \cdot 0.5\,R_i + \frac{i}{r_w} \cdot k_{mot} \cdot v)}{\eta_c \cdot V_{fc}} \tag{4.8}$$

in which the motor constant $1/k_n = 0.0168$ will be renamed k_{mot}.

For the second term of I_{fc}(the current that is going to the auxiliaries I_{aux}) the only thing that is known is that the current depends mainly on the mass flow rate of hydrogen. It is also dependent on the fuel cell temperature and the air flow rate, but the main part of the auxiliaries current is going to the hydrogen compressor.

$$I_{aux} = f(\dot{m}_{H_2}) \tag{4.9}$$

The current going to the auxiliaries will be measured and afterwards an function will be fitted such that can be used in the mass flow model. This will be done in Chapter 5.

4.2 The mass flow formula

Putting (4.1),(4.2) and (4.8) together will lead to the analytical formula for the mass flow.

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\,0.5\,R_i + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot V_{fc}(\dot{m})} + I_{aux}(\dot{m}) \right) \tag{4.10}$$

At this point the mass flow is described as a implicit function of α, v in which I_{aux} and V_{fc} are a function of the mass flow.

$$\dot{m}_{H_2} = f(\alpha, v) \tag{4.11}$$

$$V_{fc} = f(\dot{m}_{H_2}) \tag{4.12}$$

$$I_{aux} = f(\dot{m}_{H_2}) \tag{4.13}$$

In this formula V_{fc} is the second term that will be measured and fitted as well. This will also be done in Chapter 5.

In Fig. 4.1 a scheme can be seen in which the origin of the different parts of the mass flow formula can be seen.

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\,R + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot V_{fc}} + I_{aux} \right)$$

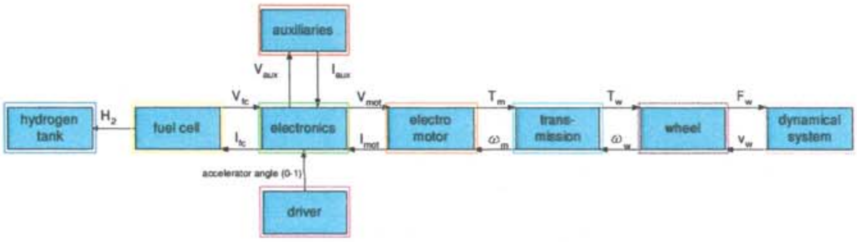

Figure 4.1: The different colors show which terms in the formula are corresponding to different parts of the system

Chapter 5

Measurements

To gain some data from the PAC-Car, measurements have been done. In those measurements different quantities have been measured. The setpoint of each measurement was the motor current (I_{mot}) that could be adjusted by the accelerator angle. At each measurement, the current was increased by 1 A. The two electro motors have been replaced by two resistances that were set parallel. The voltage of the motor (V_{mot}) was measured with a multi meter in Amperes. This value had to be converted from Amperes into Volts ($1A = 13.33V$). The current of the fuel (I_{fc}) cell and electro motor (I_{mot}) were measured with multi meters as well. The pressure of the hydrogen could be read from the pressure sensor, the temperature of the fuel cell and the voltage of the fuel (V_{fc}) cell could be read from the display from the PAC car. The hydrogen mass flow has been measured by a flow meter.The measurements are included in Appendix B.

From these measurements some other information can be computed. The motor current I_m that is requested by the motor can be calculated by Eq. 4.3. Afterwards, the auxiliaries current can be calculated by Eq. 3.2. Because the flow meter was very inaccurate, the mass flow has been computed from the measurement of the fuel cell current (I_{fc}) together with Eq. 4.1. This has been done because the measurement of the current of the fuel cell is much more accurate and Eq. 4.1 is valid.
Measurements from a fuel cell can change +/- 10% from one day to another. Especially when the system has not functioned for a long time, the measurements can differ from the system that has been working for a long time. In this case the PAC-car had not worked for half a year. This means that the measurements will probably not be fully representative concerning the values of the measurements. The reason that the measurements have been used after all in the model is that the characteristic behavior of the system will not change, only the values will be something different. The measurements are only a small part of the model and therefor the model will probably still have a good characteristic behavior.

5.1 Measurements vs Nogaro model

The measurement plots can be seen as solid lines in Fig. 5.1. The dots in Fig. 5.1 come from a Simulink model that describes the complete PAC-Car system including the dynamical model and the properties of the slopes and distances of the Nogaro circuit characteristics. Because this model can be used to compute the hydrogen consumption during a race on the Nogaro circuit, this model will be called "the Nogaro model". This model can not be used to implement into the optimal control strategy because it is a Simulink flow chart model. For the optimal control strategy an analytical formula is needed. Therefor the Nogaro model will only be used as a reference.

In the left upper figure from Fig. 5.1 the polarization curve of the fuel cell can be seen. This figure can be compared with Fig. 2.3. Combing the two figures below will lead to a the characteristic efficiency of a fuel cell that can be seen in Fig. 2.4. In the right upper figure the measurements are exactly the same as the model. This is because the mass flow has been computed with the help of Eq. 4.1 instead of using the measured data for the hydrogen mass flow. In Fig 5.1 can be seen that the auxiliaries current is higher than the values of the Nogaro model, what can be explained by losses which have not been modelled. The motor current is more or less the same as the values coming from the model.

5.2 Fitting curves

Because analytical formulas are needed for the current of the auxiliaries and the voltage of the fuel cell as a function of the mass flow, those two plots will be fitted. For the current of the auxiliaries an exponential function will be used.

The measured data can be fitted with a exponential curve of the form

$$I_{aux} = a + b * (1 - e^{(-c*\dot{m})}) \tag{5.1}$$

Fitting the measurements with the help of the matlab command fminsearch in Matlab will lead to the next a, b, c

$$a = 1.6061 \tag{5.2}$$
$$b = 3.0452 \tag{5.3}$$
$$c = 0.5259 \tag{5.4}$$

This fitted function can be seen in Fig. 5.2.

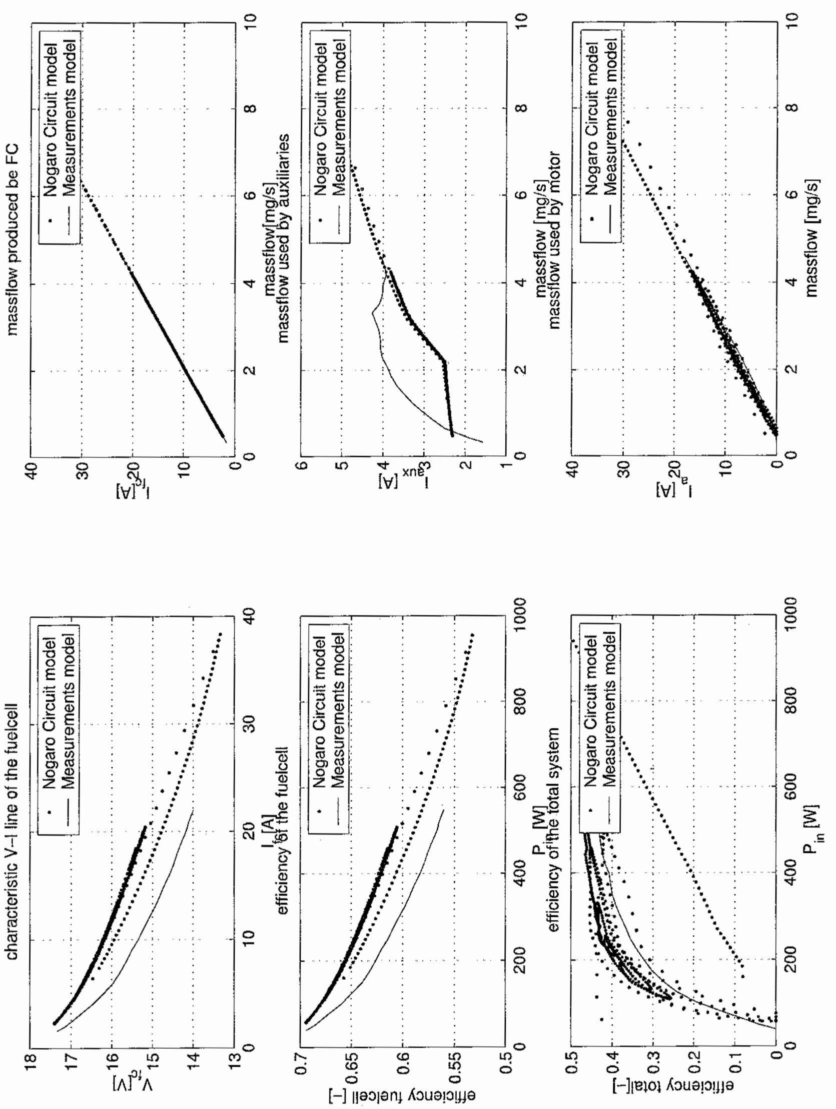

Figure 5.1: Measurements from the PAC-Car system compared with data obtained by the Nogaro circuit model

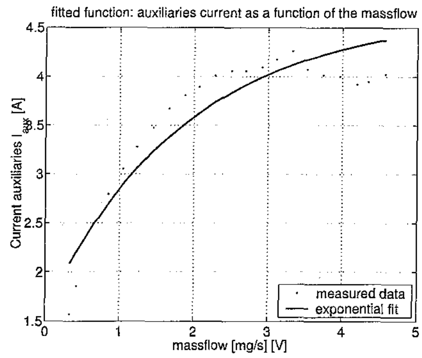

Figure 5.2: The exponential fit function of the auxiliaries current

From [6] follows that the voltage of the fuel cell can be fitted by a logarithmical fit to extrapolate the function to higher currents. Afterwards a more simple second order fit will be done on the logarithmic curve (see fig 5.2). This second fit has been done because the logarithmic fit had some integrating problems in Simulink model in Matlab.

The logarithmic fit will have the form

$$V_{fc} = a - b \cdot log(I_{fc}) - c \cdot I_{fc} \tag{5.5}$$

in which

$$a = 17.826 \tag{5.6}$$
$$b = 0.81136 \tag{5.7}$$
$$c = 0.060809 \tag{5.8}$$

The quadratic fit will have the form

$$V_{fc} = a \cdot {I_{fc}}^2 + b \cdot I_{fc} + c \tag{5.9}$$

in which

$$a = 0.000955 \tag{5.10}$$
$$b = -0.15697 \tag{5.11}$$
$$c = 17.033 \tag{5.12}$$

Both logarithmic and quadratic fits can be seen in Fig. 5.3. The quadratic function has its biggest error in the low current region. Because the auxiliaries will always use current, this part of the graphic will not be used and therefor this function is valid for the region in which it will be used.

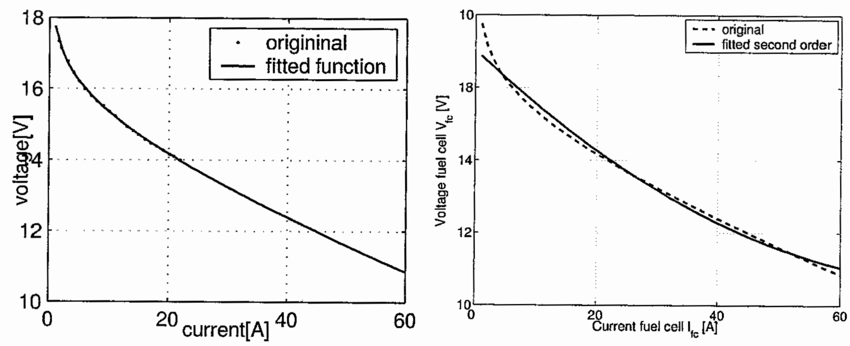

Figure 5.3: **a.** The logarithmic fit function **b.** The quadratic fit function.

Chapter 6

The control oriented model

6.1 The measurement model

The exponential function for the auxiliaries current Eq. 5.1 and the quadratic function for the voltage of the fuel cell Eq. 5.2 will be substituted into Eq. 4.10 and the resulting equation will be called the measurement model formula(Eq. 6.1).

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\,0.5\,R_i + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot (a \cdot {I_{fc}}^2 + b \cdot I_{fc} + c)} + a + b * (1 - e^{(-c * \dot{m})}) \right) \tag{6.1}$$

In Eq. 6.1, I_{fc} is a function of the mass flow. Therefor this formula depends only on the mass flow, speed and accelerator angle. This formula is too complicated to bring $\dot{m}$ to the left side of the equation and use in a optimal control strategy afterwards. If V_{fc} will be approached by a constant value (Eq. 6.2) and if the I_{aux} will be approached by a affine function (Eq. 6.3), $\dot{m}$ can be brought to the left side of the equation and written as an equation with α and v as variables which will not be too complicated for optimal control.

$$V_{fc} = c \tag{6.2}$$

$$I_{aux} = s + t\,\dot{m} \tag{6.3}$$

This will be done in the next paragraph.

Eq. 6.1 can be used in the Nogaro model to compute the mass consumption while driving on the Nogaro circuit. The plots of the hydrogen mass flow and the total hydrogen consumption can be seen in Fig. 6.1.

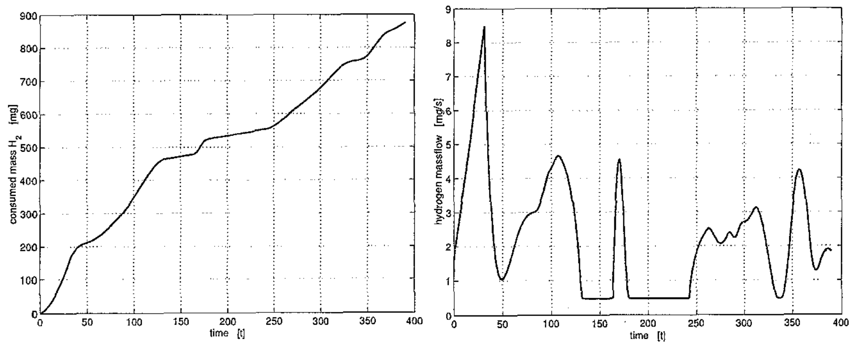

Figure 6.1: **a.** The total mass consumption of the measurement model driving on the Nogaro circuit. **b.** The mass flow of the measurement model during the race.

6.2 The three parameter model

The model that will be described in this chapter will be called the three parameter model. This model has been obtained by using Eq. 6.2 and Eq. 6.3 in Eq. 4.10.

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\, 0.5\, R_i + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot V_{fc}} + s + t\,\dot{m} \right) \tag{6.4}$$

To obtain the parameters V_{fc}, s, and t in this equation, a Simulink file has been build that can be implemented into the Nogaro model as well. This Simulink model can be seen in Fig. 6.2. The α and v in this figure are coming from the Nogaro model itself. These data will be inserted into Eq. 6.4 that can be found in the subsystem. The mass flow will come out of this subsystem. After integrating these data, the mass consumption can be computed as well. The subsystem can be seen in the Fig. 6.3. In this subsystem the 3 parameters V_{fc}, s and t will be fitted such that the mass flow at each time will be closest to the mass flow from the measurement model(see fig 6.1). In the block 'formula mass flow', Eq. 4.10 has been inserted. The mass flow will be computed and the value that comes from this block will be led back to the formula for $I_{aux}$with a unit time delay. After running the model, the values for the fitted parameters are

$$V_{fc} = 15.766 \tag{6.5}$$
$$s = 1.8441 \tag{6.6}$$
$$t = 0.9158 \tag{6.7}$$

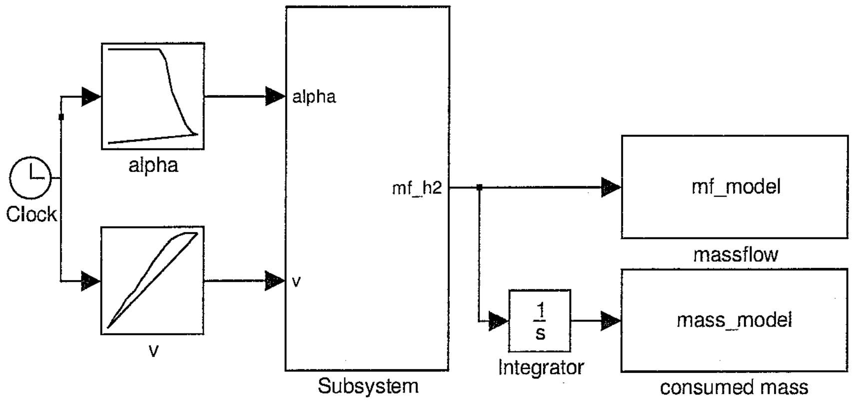

Figure 6.2: The Simulink file that has been implemented into the Nogoro model.

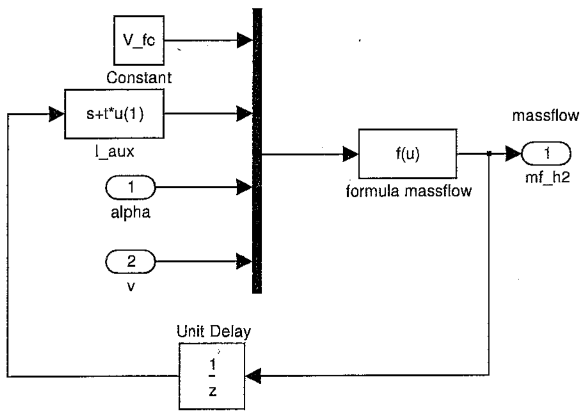

Figure 6.3: The subsystem of the Simulink model

These fits can be seen in Fig. 6.4 together with the quadratic fit for the V_{fc} and the exponential fit of I_{aux}.

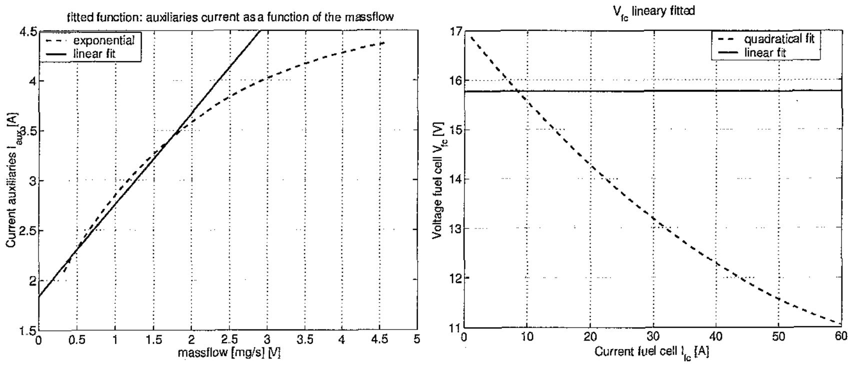

Figure 6.4: **a.** The affine function for the auxiliaries current together with the exponential fit. **b.** The constant value for the voltage of the fuel cell together with the quadratic fit.

Substituting the parameters that have been found into Eq. 6.4 will lead to an implicit function for the mass flow.

$$\dot{m} = \frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\,0.5\,R + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot 15.766} + 1.8441 + 0.9158 \cdot \dot{m} \right) \tag{6.8}$$

This is a formula for the mass flow which is only dependent on α, v and itself. Bringing the term with $\dot{m}$ to the left side will lead to

$$\dot{m} = \frac{\frac{M_{H_2} \cdot N}{n_e \cdot F} \left(\frac{30\,\alpha(30\,\alpha\,0.5\,R + \frac{k \cdot i}{r} v)}{\eta_{conv} \cdot 15.766} + 1.8441 \right)}{1 - 0.9158 \cdot \frac{M_{H_2} \cdot N}{n_e \cdot F}} \tag{6.9}$$

Filling in all the known values that can be found in (Appendix C) will lead to the final equation that can be used in the optimal control strategy

$$\dot{m} = 1.2376\,\alpha^2 + 0.70693\,\alpha\,v + 0.47476 \tag{6.10}$$

The mass flow during the race can be computed with the α and v coming from the Nogaro model. Comparing the measurement model and the three parameter model, the total mass consumption of seven laps only differs 0.06%(see Fig. 6.5). This small difference occurs in the first thirty seconds in which the car has to accelerate from 0 to 30 km/h. Because this acceleration traject will only be driven one time in seven laps, this small deviation can be neglected and the model can be validated as a good model.

In Fig. 6.6 the mass flow of the three parameter model can be seen as a function of accelerator angle and speed.

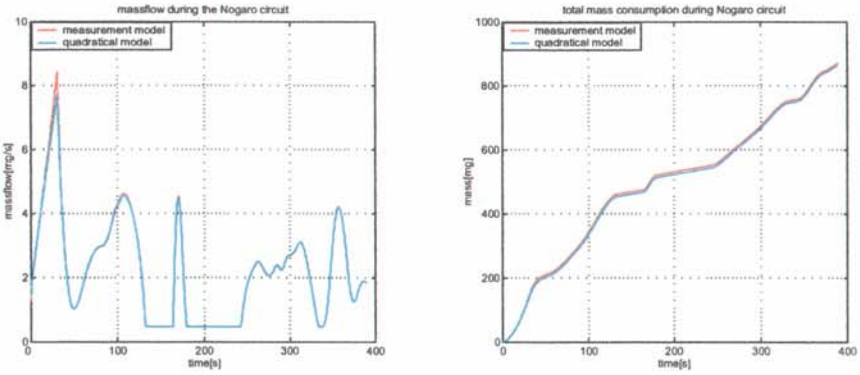

Figure 6.5: The measurement model compared to the quadratic model

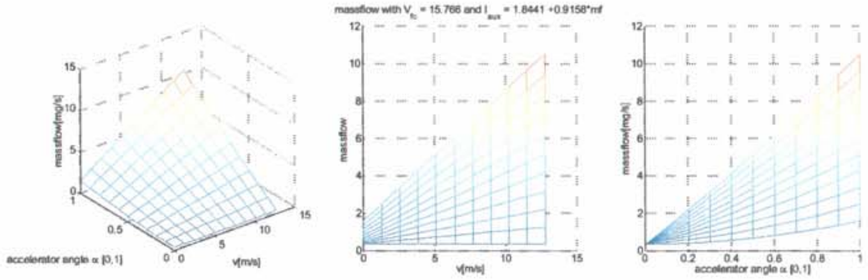

Figure 6.6: The mass flow as a function of α and v for the three parameter model

A good, simple and usable analytical formula for the hydrogen mass flow has been derived and will be used in the Chapter 8.

Chapter 7

The vehicle dynamics

In the chapters before, the PAC car system has been described. In this chapter, the system in which the PAC-car has to operate will be described. This is important because the speed from the PAC-car (that results from the different forces) has to be inserted in the mass flow formula.

Different forces are acting on the system [7] such as aerodynamic drag force (F_a), tracking force(F_t), grade resistance force (F_γ) and the rolling resistance force (F_r) which can be seen in Fig. 7.1.

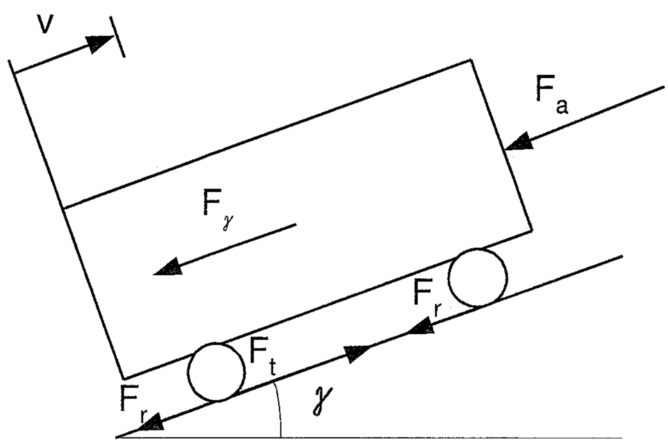

Figure 7.1: Forces acting on the PAC-car related to the speed of the car.

massfactor	m_f	1.1
mass	m	115 kg
radial velocity	ω	$[0-50]$ rad/s
velocity	v	$[0-12.5]$ m/s
wheel radius	r_w	0.25 m
density air	ρ_{air}	1.29 kg/m^3
frontal area	A_f	0.3 m^2
airresistance coefficient	c_d	0.3
gravitation constant	g	9.81 m/s^2
road grade	γ	$[-4-4]$ %
rolling resistance coefficient	c_r	0.002

Table 7.1: numerical values

Together with the second law of Newton, the next state space system can be defined

$$\dot{v} = \frac{1}{m_f \cdot m} \cdot (F_t - F_a - F_\gamma - F_r) \quad (7.1)$$

$$\dot{x} = v \quad (7.2)$$

in which m is the vehicle mass and m_f is an additional mass fraction. The forces that act on the vehicle mass are:

$$F_t = k_{mot} \cdot 30\alpha \cdot \frac{i}{r} \quad (7.3)$$

$$F_a = \frac{\rho_{air}}{2} \cdot A_f \cdot c_d \cdot v^2 \quad (7.4)$$

$$F_\gamma = mg \cdot \sin(\gamma) \quad (7.5)$$

$$F_r = mg \cdot c_r \cdot \cos(\gamma) \quad (7.6)$$

The numerical values used in these equations are given in table 7.1. The different forces are computed with those values and can be seen in figure 7.2. The tractive force is a linear function of α, the grade resistance force is approximately linear to grade γ and the rolling resistance force a constant function ($F_r \approx 2.25N$). The air resistance force is quadratic with the speed v of the vehicle.

The system is connected with the PAC-car by the torque at the wheel and the speed of the vehicle. Because the driver will define a sudden signal I_{mot} by changing α, a torque will occur at the wheels. This torque accomplishes an acceleration and the acceleration changes the speed v which should be inserted into the mass flow formula.

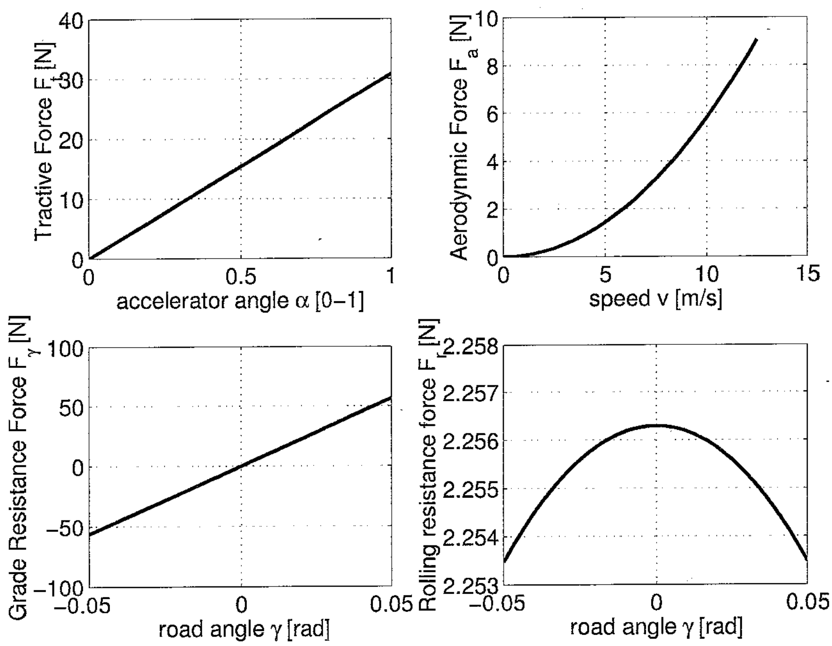

Figure 7.2: Forces acting on the PAC-car

Chapter 8

Optimal control

To optimize the fuel consumption during this challenge, an optimal control strategy has to be designed such that the optimal control vector u^o for α will be found. Using this vector u^o together with the information of the grade profile and the vehicle dynamics, the optimal speed profile can be found as well.In this chapter, the optimal control problem will be stated.

8.1 Hamiltonian function and necessary conditions

The theory of optimal control can be found in [4], [2], [3], The Hamiltonian function H will look like

$$\begin{aligned}
H(x(t), u(t), t, \lambda(t)) &= L(x(t), u(t), t) + \lambda^T(t)\, f(x(t), u(t), t) \qquad (8.1)\\
\text{in which} & \\
u(t) &= \alpha \qquad (8.2)\\
x_2(t) &= v \qquad (8.3)\\
L(x(t), u(t), t) &= \dot{m} \qquad (8.4)\\
f(x(t), u(t), t) &= [\dot{x_1} \quad \dot{x_2}]^T \qquad (8.5)
\end{aligned}$$

and $\lambda(t) = [\lambda_1, \lambda_2]$ are the Lagrange multipliers. Inserting L and f in the Hamiltonian function gives

$$\begin{aligned}
H(x(t), u(t), t, \lambda(t), \lambda_0) &= \lambda_0\, (a \cdot u(t)^2 + b \cdot u(t) \cdot x_2(t) + c + \ldots \\
&\quad + \lambda_1(t)\, x_2(t) + \ldots \qquad (8.6)\\
&\quad + \lambda_2(t) \Big(C1 \cdot u(t) - C2 \cdot x_2(t)^2 - C3 - C4 \cdot F_\gamma(x_1, \gamma) \Big)
\end{aligned}$$

The problem to find a vector u^o that minimizes the fuel consumption can be formulated as an optimal control problem with fixed final state and fixed end time.

Problem definition
Find a optimal control vector u^0 for α, such that the next differential equations

$$\begin{aligned} \dot{x_1} &= x_2 && (8.7) \\ \dot{x_2} &= C1 \cdot u(t) - C2 \cdot x_2^o(t)^2 - C3 - C4 \cdot F_\gamma(x_1, \gamma) && (8.8) \\ & && (8.9) \end{aligned}$$

for all $t \in [t_0, t_e]$, in which:

$$\begin{aligned} C1 &= \frac{k}{m_f \cdot m} && (8.10) \\ C2 &= \frac{\frac{\rho_{air}}{2} \cdot A_f \cdot c_d}{m_f \cdot m} && (8.11) \\ C3 &= \frac{2.25}{m_f \cdot m} && (8.12) \\ C4 &= \frac{1}{m_f \cdot m} && (8.13) \\ F_\gamma(x_1, \gamma) &= mg \cdot \sin(\gamma) && (8.14) \\ & && (8.15) \end{aligned}$$

and the boundary conditions

$$\begin{aligned} x_1^o(t_0) &= 0 && (8.16) \\ x_2^o(t_0) &= 0 && (8.17) \\ x_1^o(t_e) &= 3636 && (8.18) \\ x_2^o(t_e) &= [7 - 10]m/s && (8.19) \\ t_0 &= 0s && (8.20) \\ t_e &= 436.32s && (8.21) \\ & && (8.22) \end{aligned}$$

will be satisfied and the Gute index(fuel optimization)

$$J(u) = \int_{t_0}^{t_e} \dot{m} \, dt \qquad (8.23)$$

will be minimized.

8.2 Results

Dr. Antonio Sciarretta from IRMT, ETH Zürich has calculated this problem and he has found an improvement of fuel consumption of 8%. In [1] can be read how he has found this result.

In Fig. 8.1 the results for the case of simulating 7 laps can be seen. The reference line is the simulation of a PID controller that tries to drive the Nogaro circuit at constant speed. The first figure shows the solution of the optimal control vector u_o. What can be seen is that the optimal controller has a much more smooth way of controlling the system. The 3^{rd} figure shows the mass flow consumption. The mass flow consumption from the optimal control is much smoother as the PID controller as well.

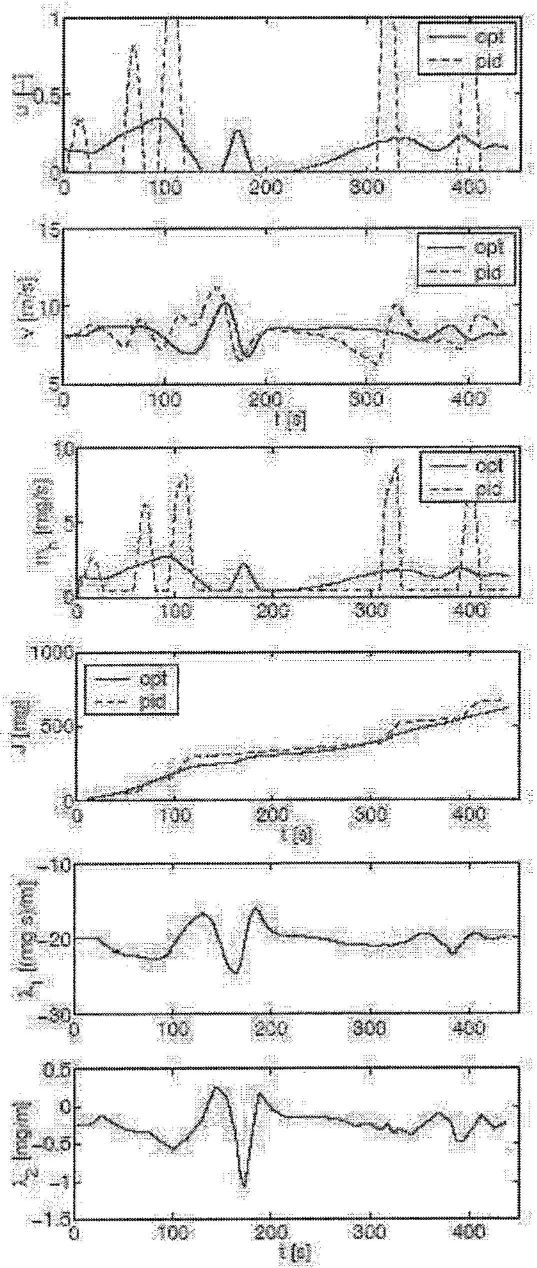

Figure 8.1: Top to bottom: control law, speed trajectory, hydrogen mass flow rate, hydrogen mass consumption, Lagrange multipliers as a function of time, periodic route case

Conclusion

The PAC-car has been modelled analytically. For the variables that were not known, measurements have been made and formulas have been fitted to describe the variables as a function of the mass flow of hydrogen. Using these formulas in the analytical model, the measurement model has been created. Because this model is too complex to insert in an optimal control strategy, some mathematical simplifications had to be made. Assumed is that the voltage of the fuel cell will be a constant value and that the current of the auxiliaries will be an affine function of the hydrogen mass flow. By using these simplifications, a formula for the hydrogen mass flow has been found that can be used in an optimal control strategy and is only dependent on the variables accelerator angle α and speed v. This simplified model that will be called the three parameter model, differs only a little bit of the final mass consumption computed by the measurement model.

Dr. Antonio Sciarretta has used the three parameter model in an optimal control strategy. He has obtained a fuel consumption of 8%.

If more realistic parameters for the mass flow formula are needed, a recommendation can be to make measurements at the next Shell Eco Challenge. The variables hydrogen mass flow, speed and accelerator angle as a function of time will have to be measured. In this way a good reference can be obtained to calibrate the parameters of the model. Although the parameters of the three parameter model derived in this report are not correct compared to the real world, the characteristic format of the formula can be used to derive the new parameters.

Summary

Every year, the Shell Eco Challenge takes place. The goal of this contest is to drive at the Nogaro circuit as fuel optimal as possible. The department IMRT of the Swiss Federal Institute (ETH)Zurich has designed and builded a fuel cell powered car especially for this challenge. All the hardware has been designed to drive fuel optimal, but by implementing extra controllers, the fuel consumption can be decreased even more. Because the circuit has different slopes, the speed of the vehicle will change. This means that the fuel consumption will change as well. By calculating the speed profile for an optimal fuel optimal consumption, the fuel consumption can probably be decreased by +/-10%. To do this, a formula for the mass flow has to be found and implemented in an optimal control strategy. In this report a formula will be derived to calculate the mass flow of hydrogen as a function of the speed of the vehicle and the accelerator angle. The biggest part of this formula will be derived analytically. For some variables it isn't possible to do this. These variables will be measured and and fitted into a function. The final mass flow function is too complex to insert in an optimal control strategy. The fitted functions for the measured variables in this model will be approximated by more simple functions. In this way a very simple but reliable function of the hydrogen mass flow has been derived to implement in the optimal control problem. This problem has been solved by Antonio Sciarretta (see [1]) and he has found a speed profile which saves 8% of the total fuel consumption.

Samenvatting

Elk jaar vindt de Shell Eco Marathon plaats. Het doel van deze wedstrijd is om op het Nogaro circuit zo zuinig mogelijk te rijden. Het departement IMRT van de Eidgenössische Technische Hochschule (ETH) Zürich heeft een voertuig ontwikkeld en gebouwd, speciaal om aan deze wedstrijd mee te doen. Dit voertuig (de PAC-car) ontleent zijn energie aan een brandstof cel. Alle delen van de PAC-car zijn zo ontwikkeld dat hij brandstof optimaal kan rijden. Door het toevoegen van een regel strategie kan er zonder iets aan de hardware te veranderen, het brandstof verbruik toch nog omlaag gebracht worden. Omdat er verschillende hellingen in het circuit zitten, zal de voertuig snelheid en het verbruik hiervan afhankelijk zijn. Met de implementatie van een optimal control strategie kan berekend worden welke snelheid er gereden moet worden als functie van de afgelegde afstand, om een optimaal verbruik van waterstof te verkrijgen. Om dit te kunnen berekenen moet er een formule zijn die het brandstof verbruik kan weergeven als functie van snelheid van het voertuig en de gaspedaal stand. Deze formule wordt in dit verslag afgeleid. Dit zal in eerste instantie analytisch gebeuren. Omdat enkele variabelen in deze formule niet analytisch kunnen worden weergegeven in verband met complexiteit, zullen deze worden gemeten en in een functie gefit worden. Na implementatie van deze functies in de analytische formule kan het brandstof verbruik goed worden weergegeven. Echter, deze formule is te complex om in de optimal control strategie toe te passen. Daarom worden de variabelen nogmaals op een andere manier gefit. Hierdoor ontstaat een zeer simpele formule die het brandstof verbruik goed weergeeft. De probleemstelling van het optimal control vraagstuk wordt vervolgens samen gesteld uit de voertuig dynamica en deze formule van het brandstof verbruik. Antonio Sciarretta heeft dit vraagstuk opgelost en kwam tot een brandstof besparing rond de 8% (zie [1]))

Bibliography

[1] Lino Guzzella Antonio Sciarretta. Fuel optimal trajectories of a fuel cell vehicle. 2004.

[2] Falb P.L. Athans, M. *Optimal Control*. New York: McGraw-Hill Book Company, 1966.

[3] O. Föllinger. *Optimierung dynamischer Systeme*. Munchen: Oldenbourg Verlag, 1985.

[4] Geering H.P. Guzella, L. *Optimale Regelung*. Institut fur Mess- und Regeltechnik, ETH-Zurich.

[5] L. Guzzella. *Fahrzeugantriebssysteme*. ETH Zurich, Measurement and Control Laboratory(IMRT), 2001.

[6] S. Srinivasan C.E. Chamberlin J. Kim, S.-M-Lee. Modeling of proton exchange membrane fuel cell performance with an empirical equation. *J. Electrochem. Soc.*, Vol.142(No.8):pp.2670, Aug. 1995.

[7] Naunheimer H. Lechner, G. *Automotive Transmissions*. Berlin: Springer, 1999.

[8] Alexander Shilling. *Fuel efficiency optimization of the PAC-Car vehicle*. Measurement and Control Laboratory(IMRT), ETH Zurich, 2003.

[9] A. Vandenput. *Elektromechanica*. Technical University of Eindhoven, 1999.

Appendix A

Properties of the PEM fuel cell

Producer	PSI
Type	PEM
Number of cells	20
Active surface area per cell	$120cm^2$
Maximum power	$900w$
Nominal voltage	$18V$
Maximum operational pressure	$2bar$
Nominal operational pressure	1.5 *bar*
Operational temperature	$2 - 70\,^{\circ}$C.
Dimensions	180 x 200 x 160 *mm*
Weight	$5kg$

Appendix B

Measurement data

Voltage motor [V]	0	1.07	2.38	3.42	4.31	4.97	5.67	6.28
Current motor [A] *13.33	0	0.143	0.305	0.431	0.529	0.609	0.694	0.768
Voltage fuel cell [V]	17.34	17.14	16.77	16.46	16.21	15.99	15.81	15.65
Current fuel cell [A] *13.33	0.118	0.149	0.233	0.304	0.377	0.445	0.523	0.600
Temperature fuel cell [C]	39	39	39	39	40	41	42	42
Mass flow H_2[SLPM]	0.327	0.413	0.647	0.844	1.047	1.236	1.452	1.666
Pressure hydrogen [bar]	0.69	0.71	0.7	0.69	0.68	0.68	0.68	0.66

Voltage motor [V]	6.86	7.36	7.82	8.33	8.77	9.23	9.57	9.94
Current motor [A]*13.33	0.839	0.9	0.966	1.029	1.081	1.136	1.180	1.225
Voltage fuel cell [V]	15.52	15.36	15.23	15.08	14.95	14.82	14.72	14.59
Current fuel cell [A] *13.33	0.676	0.746	0.823	0.903	0.972	1.052	1.121	1.198
Temperature fuel cell [C]	42	44	46	47	47	49	50	51
Mass flow H_2[SLPM]	1.877	2.072	2.286	2.508	2.699	2.921	3.113	3.326
Pressure hydrogen [bar]	0.67	0.67	0.66	0.65	0.65	0.67	0.65	0.65

Voltage motor [V]	10.38	10.76	11.12	11.5	11.73	12.03
Current motor [A]*13.33	1.279	1.330	1.376	1.422	1.450	1.488
Voltage fuel cell [V]	14.48	14.39	14.28	14.16	14.11	14.0
Current fuel cell [A] *13.33	1.271	1.348	1.428	1.51	1.565	1.648
Temperature fuel cell [C]	52	54	54	56	57	57
Mass flow H_2[SLPM]	3.529	3.743	3.965	4.193	4.346	4.576
Pressure hydrogen [bar]	0.64	0.63	0.61	0.6	0.55	0.5

Appendix C

List of Symbols

Quantity	Symbol	Value	Dimension
Hydrogen			
Density	ρ_{H_2}	0.08988	kg/m^3
Heat value	H	119.93	MJ/kg
Entropy	δH	-241.8	$MJ/kmol$
Enthalpy	δG	-228.6	$MJ/kmol$
Molecular weight	M	2.016	$kg/kmol$
Number of cells	N	20	[-]
Free electrons	n_e	2	[-]
Electro motor			
Motor constant	K_{mot}	0.0168	V/rmp
Inner resistance	R	0.16	Ω
Efficiency	η_{max}	0.95	[-]
Nominal Voltage	V_n	12	V
Resistance	R	0.16	Ω
No load speed	ω	6500	ω
Max recommended torque	T_{max}	110	Nm
PAC-car			
Transmission	i	$\frac{306}{15}$	[-]
Mass	m	115	kg
Additional mass factor	m_f	1.1	[-]
Wheel radius	r	0.25	m
frontal area	A_f	0.3	m^2
Air resistance coefficient	c_d	0.3	[-]
Rolling resistance coefficient	c_r	0.002	[-]
Constants			
Faraday constants	F	$96.49 \cdot 10^{-19}$	$C/kmol$
Density air	ρ_{air}	1.2	kg/m^3
Gravitation constant	g	9.81	m/s^2