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A Behavioral Approach to Optimal Model Reduction

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Abstract

The problem of optimal state reduction under minimization of the angle between system behaviors is investigated. This problem is analysed for linear time-invariant dynamical systems, in a behavioral setting. In particular, no assumptions are made on input/output decompositions and stability properties of systems. The class of all optimal approximate systems of state dimension one less than the state dimension of the to-be-approximated system is characterized. Generalizations of this result are discussed and a simulation example is given.

Keywords

Optimal model reduction, linear systems, ℓ_2-systems, behavioral theory.

1 Introduction

' The general aim of model approximation is to replace a complex dynamical system by a simpler, less complex system without undue loss of accuracy. Model approximation techniques have found widespread applications and are of paramount interest in engineering and in areas where modeling, control and system identification are the key elements in the analysis and synthesis of dynamical systems.

Many techniques exist for approximating a complex system by a simpler one. However, few of these techniques provide quantitative insight in the question of the accuracy of the approximate system with respect to the original, complex one. On the other hand, frequently used model approximation techniques such as balanced truncations, optimal Hankel norm reductions, Padé approximations, assume stability of the system, which constitutes a severe limitation for many practical situations.

In this paper we investigate a model approximation problem for the class of linear time-invariant systems on discrete

time. We address the question of synthesizing a linear time-invariant dynamical system whose state dimension is strictly smaller than the one of a given system, and such that the angle between the two systems is minimized. The angle between systems is similar to the gap [1-4] and defined in a worst-case sense, as the largest angle that can occur between a system trajectory and its optimal approximation in the reduced order model. Throughout this paper we refrain from making assumptions on the stability of the system. Here, the common transfer function or input-output formulation of the model approximation problem is replaced by a set-theoretic behavioral formalism.

2 Notation and preliminaries

Integers, and the real and complex numbers are denoted by $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$, respectively. $\mathbb{Z}_+$ and $\mathbb{Z}_-$ denote the nonnegative and negative elements of $\mathbb{Z}$, respectively. For $T \subseteq \mathbb{Z}$ and $(W, \| \cdot \|)$ a normed vector space we define $\ell(T, W) := W^T$ and $\ell_2(T, W) := \{w \in \ell(T, W) \mid \sum_{t \in T} \|w_t\|^2 < \infty\}$. The latter space is equipped with its usual inner product, $\langle \cdot, \cdot \rangle$, and is also denoted as ℓ_2 or ℓ_2^q if the dimension q of the signal space W is relevant for the context. Further, $\ell_2^- := \ell_2(\mathbb{Z}_-, W)$ and $\ell_2^+ := \ell_2(\mathbb{Z}_+, W)$. For $k \in \mathbb{Z}$, $\sigma^k : \ell_2 \to \ell_2$ denotes the k-shift $(\sigma^k w)_t = w_{t+k}$. Let $w, w' \in \ell_2^q$ be two multivariate time series. shifts(w) denotes the collection of all shifts of w, i.e., shifts$(w) := \{\sigma^k w \mid k \in \mathbb{Z}\}$. The operator span is used to generate the linear span of its arguments. Restrictions of sequences to a subset $F \subset \mathbb{Z}$ are denoted as w_F. The symbols $\perp$ and $\perp\!\!\!\perp$ are defined as $w \perp w' :\Leftrightarrow \langle w, w' \rangle = 0$ and $w \perp\!\!\!\perp w' :\Leftrightarrow \langle \text{shifts}(w), \text{shifts}(w') \rangle = 0$. The symbol $\bigwedge_t$ denotes the concatenation product of time series at time t, i.e. $w \bigwedge_t w'$ denotes the time series $\{\ldots, w_{t-2}, w_{t-1}, w'_t, w'_{t+1}, \ldots\}$. We write $\wedge$ for $\bigwedge_t$ if the concatenation instant t is obvious from the context.

3 Problem formulation

The behavioral theory [9, 10] has advocated a definition of dynamical systems, or *systems* for short, as sets of mappings $w : T \to W$ defined on a *time set* T and taking values in a *signal space* W. Formally, a system is any subset $\mathcal{B} \subseteq \ell(T, W)$. In this paper we exclusively consider systems with discrete time set $T = \mathbb{Z}$ and finite dimensional real signal spaces $W = \mathbb{R}^q$. We will further focus on ℓ_2-systems which are those systems whose behavior $\mathcal{B}$ is a subset of ℓ_2. A system $\mathcal{B}$ is called *linear* if $\mathcal{B}$ is a linear subspace of $\ell(T, W)$. It is called *time-invariant* if $w \in \mathcal{B}$ implies that the k-shifted trajectory $\sigma^k w$ belongs to $\mathcal{B}$ for any integer $k \in \mathbb{Z}$. Further, a subset $\mathcal{B} \subseteq \ell_2$ is called ℓ_2-*complete* if a trajectory w belongs to $\mathcal{B}$ whenever $w \in \ell_2$ and its restrictions $w_F \in \mathcal{B}_F$ for all intervals[1] $F \subset \mathbb{Z}$. Linearity, time-invariance and ℓ_2-completeness are the properties that define the system class which we study in this paper. For this, let $\mathbb{L}^q$ denote the set of all linear, time-invariant and ℓ_2-complete systems $\mathcal{B} \subseteq \ell_2(\mathbb{Z}, \mathbb{R}^q)$.

Associate with any system $\mathcal{B} \in \mathbb{L}^q$ the unique pair of integers (m, n) such that

$$\dim \mathcal{B}|_{[1,N]} = mN + n$$

for all $N \geq n$. Then m and n are well defined [6, 7], m is called the *rank* of the system, n its *degree*, and the pair (m, n) will be refered to as the *complexity* of the system $\mathcal{B}$.

As an approximation criterion for dynamical systems we consider the *angle* between two systems. This is defined as follows. Let $w, w' \in \ell_2$ be two non-zero time series and define the angle between these trajectories by putting

$$\theta(w, w') := \arccos \left(\frac{|\langle w, w' \rangle|}{\|w\| \|w'\|} \right).$$

The angle is defined to be $\pi/2$ if either w or w' is zero. The angle between a time series and a closed linear subspace of ℓ_2 is defined (with some abuse of notation) as

$$\theta(w, \mathcal{B}) := \min_{w' \in \mathcal{B}} \theta(w, w').$$

This minimum exists and it is easy to see that it is attained for the orthogonal projection of w onto $\mathcal{B}'$, i.e., $\theta(w, \mathcal{B}) = \theta(w, w')$ with w' the orthogonal projection of w on $\mathcal{B}$.

Definition 3.1 (Angle) The angle between two systems $\mathcal{B}, \mathcal{B}'$ in $\mathbb{L}^q$ is defined as

$$\theta(\mathcal{B}, \mathcal{B}') := \max \left\{ \sup_{w \in \mathcal{B}, w \neq 0} \theta(w, \mathcal{B}'), \sup_{w' \in \mathcal{B}', w' \neq 0} \theta(w', \mathcal{B}) \right\}.$$

The angle is called *flat* if $\theta(\mathcal{B}, \mathcal{B}') = \theta(w, \mathcal{B}') = \theta(w', \mathcal{B})$ for all nonzero $w \in \mathcal{B}$ and $w' \in \mathcal{B}'$

Some basic properties of the angle are summarized in the following lemma.

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[1] In this paper an *interval* is by definition a finite interval.

Lemma 3.2 *Let $\mathcal{B}$ and $\mathcal{B}'$ be elements of $\mathbb{L}^q$ of rank m and m', respectively. Then*

1. $0 \leq \theta(\mathcal{B}, \mathcal{B}') \leq \pi/2$.

2. $\theta(\mathcal{B}, \mathcal{B}') = 0$ if and only if $\mathcal{B} = \mathcal{B}'$.

3. If $\widetilde{\mathcal{B}} = \mathcal{B}^\perp$ and $\widetilde{\mathcal{B}}' = [\widetilde{\mathcal{B}}']^\perp$ denote the orthogonal complement (in ℓ_2) of $\mathcal{B}$ and $\mathcal{B}'$ then $\theta(\mathcal{B}, \mathcal{B}') = \theta(\widetilde{\mathcal{B}}, \widetilde{\mathcal{B}}')$.

4. $\theta(\mathcal{B}, \mathcal{B}') = \pi/2$ if $m \neq m'$.

The notions introduced so far lead to the following problem formulation.

Definition 3.3 (Optimal Model Reduction Problem) Given a system $\mathcal{B} \in \mathbb{L}^q$ with rank m and degree n, determine a system $\mathcal{B}' \in \mathbb{L}^q$ with the same rank m and degree $n' < n$, such that the angle $\theta(\mathcal{B}, \mathcal{B}')$ is minimized.

The class of all such systems is denoted by $\mathbb{L}(\mathcal{B}, n')$. Precisely, if $\mathbb{L}_{m,n}$ denotes the set of systems in $\mathbb{L}^q$ with rank m and degree n then, with $\mathcal{B} \in \mathbb{L}_{m,n}$ and $n' < n$, we define

$$\mathbb{L}(\mathcal{B}, n') := \{\mathcal{B}' \in \mathbb{L}_{m,n'} \mid \theta(\mathcal{B}, \mathcal{B}') \leq \theta(\mathcal{B}, \mathcal{B}'') \\ \text{for all } \mathcal{B}'' \in \mathbb{L}_{m,n'}\}. \quad (3.1)$$

Systems in $\mathbb{L}(\mathcal{B}, n')$ are called *optimal approximants* of $\mathcal{B}$ of degree n'. Note that in Definition 3.3 the rank of the approximant is required to be the same as the rank of the to-be-approximated system. This requirement is motivated by statement 4 in Lemma 3.2 from which we infer that an approximant $\mathcal{B}'$ of $\mathcal{B}$ with different rank will necessarily have maximum angle. In view of the angle criterion it is therefore of little interest to also reduce the rank m of $\mathcal{B}$.

4 Cutting links between past and future

In this section we introduce the system structures that are relevant for the model approximation problem formulated in Defintion 3.3. Let $\mathcal{B} \in \mathbb{L}^q$ be a given system and define its past and future behavior as

$$\mathcal{B}^- := \mathcal{B}_{\mathbb{Z}_-} \\ \mathcal{B}^+ := \mathcal{B}_{\mathbb{Z}_+}. \quad (4.1)$$

Trajectories $w^- \in \mathcal{B}^-$ and $w^+ \in \mathcal{B}^+$ are said to be *compatible* (or *linked*) if their concatenation $w^- \wedge w^+ \in \mathcal{B}$. For any such compatible pair, w^+ is said to be a *minimal future* of w^- if its norm, $\|w^+\|$, is minimal among all compatible futures of w^-. The notion of a *minimal past* of $w^+ \in \mathcal{B}^+$ is similarly defined.

Definition 4.1 (Past-future links) A past-future link of a system $\mathcal{B}$ is a system trajectory $w = w^- \wedge w^+ \in \mathcal{B}$ in which w^- is a minimal past of w^+ and w^+ a minimal future of w^-. The set of all past-future links of $\mathcal{B}$ is denoted by $\mathcal{B}^\leftrightarrow$. The set of all minimal futures of trajectories in $\mathcal{B}^-$ is denoted by $\mathcal{B}^\rightarrow$. Similarly, $\mathcal{B}^\leftarrow$ denotes the set of all minimal pasts.

Note that $\mathcal{B}^{\rightarrow} = [\mathcal{B}^{\leftrightarrow}]^{-}$ and $\mathcal{B}^{\leftarrow} = [\mathcal{B}^{\leftrightarrow}]^{+}$. Clearly, a past trajectory may or may not be compatible with a zero future. Similarly, futures (i.e. trajectories in $\mathcal{B}^{+}$) may or may not be compatible with a zero past. To distinguish between these trajectories we introduce what we will call the *left*- and *right-part* of the system.

$$\mathcal{B}^{\leftarrow 0} := \{w \in \mathcal{B} \mid w_t = 0 \text{ for } t \in \mathbb{Z}_+\} \text{ and}$$
$$\mathcal{B}^{0\rightarrow} := \{w \in \mathcal{B} \mid w_t = 0 \text{ for } t \in \mathbb{Z}_-\} \quad (4.2)$$

The idea is that these sets reflect pasts that bring the system into its equilibrium, or futures that can emerge from rest. In the next proposition we summarize some basic properties of past-future links.

Proposition 4.2 (Past-future links) *Let $\mathcal{B} \in \mathbb{L}^q$. Then*

1. $\sigma^j \mathcal{B}^{\rightarrow} \subseteq \mathcal{B}^{\rightarrow}$ *and* $\sigma^{-j} \mathcal{B}^{\leftarrow} \subseteq \mathcal{B}^{\leftarrow}$ *for all* $j \in \mathbb{Z}_+$.

2. $\mathcal{B}^{-} = (\mathcal{B}^{\leftarrow 0})^{-} \oplus \mathcal{B}^{\leftarrow}$ *and* $\mathcal{B}^{+} = \mathcal{B}^{\rightarrow} \oplus (\mathcal{B}^{0\rightarrow})^{+}$.

3. $\mathcal{B} = \mathcal{B}^{\leftarrow 0} \oplus \mathcal{B}^{\leftrightarrow} \oplus \mathcal{B}^{0\rightarrow}$.

4. $\dim(\mathcal{B}^{\leftrightarrow})$, $\dim(\mathcal{B}^{\leftarrow})$ *and* $\dim(\mathcal{B}^{\rightarrow})$ *are all finite and equal to the degree of $\mathcal{B}$.*

5. *A past $w^{-} \in \mathcal{B}^{-}$ is compatible with a future $w^{+} \in \mathcal{B}^{+}$ if and only if $w^{\leftarrow} \wedge w^{\rightarrow} \in \mathcal{B}^{\leftrightarrow}$, with $w^{\leftarrow}$ and $w^{\rightarrow}$ the orthogonal projections of, resp., w^{-} onto $\mathcal{B}^{\leftarrow}$ and w^{+} onto $\mathcal{B}^{\rightarrow}$.*

The *weakest forward* and *weakest backward gain* of a system $\mathcal{B} \in \mathbb{L}^q$ is defined as, respectively,

$$\rho_f := \min\left\{\frac{\|f\|}{\|p\|} \mid 0 \neq (p \wedge f) \in \mathcal{B}^{\leftrightarrow}\right\}$$

$$\rho_p := \min\left\{\frac{\|p\|}{\|f\|} \mid 0 \neq (p \wedge f) \in \mathcal{B}^{\leftrightarrow}\right\}$$

Weakest forward and backward links in $\mathcal{B}$ are past-future links that achieve the ratios ρ_f and ρ_p, respectively. The *weakest gain*, ρ, of $\mathcal{B}$ is the minimum of ρ_f and ρ_p, and *weakest links* are weakest forward or backward links that achieve this ratio.

The weakest backward and forward gain of a system determine the bounds for all past-future ratios in past-future links $\|p\|/\|f\|$: they are in between ρ_p and ρ_f^{-1}. We refine these notions of extreme ratios to a set of increasing past-future ratios

$$\rho_p =: \rho_1 \leq \ldots \leq \rho_i \leq \ldots \rho_n := \rho_f^{-1}$$

with n equal to the degree of the system. These numbers are called the *canonical past-future ratios* of $\mathcal{B} \in \mathbb{L}^q$ with degree n and are defined as the unique sequence of n positive non-decreasing real numbers $\{\rho_i\}_{i=1,\ldots,n}$ with $\rho_1 := \rho_p$ and

$$\rho_k := \min\{\frac{\|p\|}{\|f\|} \mid 0 \neq (p \wedge f) \in \mathcal{B}^{\leftrightarrow} \text{ and}$$
$$(p \wedge f) \perp (p_{(j)} \wedge f_{(j)}), \text{ for } j = 1, \ldots, k-1\}.$$

Here, $(p_{(j)} \wedge f_{(j)})$ is the element in $\mathcal{B}^{\leftrightarrow}$ that achieves the j-th past-future ratio $\{\frac{\|p_{(j)}\|}{\|f_{(j)}\|} = \rho_j$.

Before returning to the model approximation problem, we discuss how dynamical systems in the class $\mathbb{L}^q$ can be generated from time series. This is achieved by a process called *completion*. For a subspace $\mathcal{S} \subseteq \ell_2^q$ its completion is defined as

$$\text{comp}(\mathcal{S}) := \{w \in \ell_2^q \mid w_F \in \mathcal{S}_F \text{ for all intervals } F \subset \mathbb{Z}\}.$$

It follows that $\text{comp}(\mathcal{S})$ is the smallest ℓ_2-complete set that contains $\mathcal{S}$.

Definition 4.3 The system generated by a time series $w \in \ell_2^q$ is

$$\mathcal{B}(w) := \text{comp}(\text{span}[\text{shifts}(w)]).$$

For any $w \in \ell_2^q$, the generated system $\mathcal{B}(w)$ actually belongs to $\mathbb{L}^q$. Note also that $w \in \mathcal{B}(w)$ for any $w \in \ell_2$. Obviously, $\mathcal{B}(w)$ is the smallest (in the sense of set inclusions) dynamical system in $\mathbb{L}^q$ containing w. $\mathcal{B}(w)$ is also refered to as the *most powerful unfalsified model* [9] of w in the model class $\mathbb{L}^q$. There exist an obvious generalization of Definition 4.3 where systems are generated by finite sets W of time series in ℓ_2^q. Interestingly, every system $\mathcal{B} \in \mathbb{L}^q$ can be generated by a *finite number* of time series:

Proposition 4.4 *Every system in $\mathbb{L}^q$ can be generated by a finite set W of trajectories in ℓ_2^q. $\mathcal{B}(W)$ has rank at most rank m if the generating set W consists of m time series of finite degree, that is, if the dimensions of the sets $\{(\sigma^k w)_{\mathbb{Z}_+}\}_{k>0}$ and $\{(\sigma^k w)_{\mathbb{Z}_-}\}_{k<0}$ are both finite for all $w \in W$.*

5 Optimal reductions

A solution of the optimal model reduction problem (Definition 3.3) for degree one reductions (i.e. for $n' = n-1$) is the main result of this paper and is given in the following theorem.

Theorem 5.1 *Let $\mathcal{B} \in \mathbb{L}^q$ be a system of rank $m = 1$ and degree n, and weakest gain ρ. Let $(p \wedge f) \in \mathcal{B}^{\leftrightarrow}$ denote a weakest link in $\mathcal{B}$. Define*

$$\mathcal{B}' := \begin{cases} \mathcal{B}(p \wedge 0) & \text{if } \|p\| > \|f\| \\ \mathcal{B}(0 \wedge f) & \text{if } \|p\| \leq \|f\| \end{cases} \quad (5.1)$$

Then $\mathcal{B}'$ is an optimal reduced order approximation of $\mathcal{B}$. Furthermore, the angle $\theta(\mathcal{B}, \mathcal{B}') = \arctan(\rho)$, and this angle is flat. Conversely, all systems $\mathcal{B}' \in \mathbb{L}^q$ with degree at most $n-1$ that achieve the angle $\arctan(\rho)$ are of this form, i.e., they are given by (5.1) where $(p \wedge f) \in \mathcal{B}^{\leftrightarrow}$ is a weakest link in $\mathcal{B}$.

Theorem 5.1 therefore implies that for any $\mathcal{B} \in \mathbb{L}^q$ of rank $m = 1$, the optimal approximants

$$\mathbb{B}(\mathcal{B}, n-1) = \{\mathcal{B}' \mid \mathcal{B}' \text{ satisfies (5.1) with}$$
$$(p \wedge f) \in \mathcal{B}^{\leftrightarrow} \text{ a weakest link in } \mathcal{B}\}.$$

6 Generalizations

We have found the optimal approximation of reduced degree for rank one systems. The result asks for a generalization in two respects: for systems of arbitrary rank, and for reduction with more than one degree. The generalization to arbitrary rank is rather straightforward and involves a generalization of Definition 4.3 to finite sets of generating trajectories. The generalization of Theorem 5.1 to the case where $n' < n - 1$ is more involved and is the topic of further investigations. We list some partial results on these generalizations in the following proposition.

Proposition 6.1 (Reduction of higher rank systems) *Let $\mathcal{B} \in \mathbb{L}^q$ with rank m and degree n, with weakest gain ρ. Let $\mathcal{B}' \in \mathbb{L}^q$ denote a system of the same rank m and lower degree $n' < n$. Then*

1. $\theta(\mathcal{B}, \mathcal{B}') \geq \arctan(\rho)$

2. $\theta(\mathcal{B}, \mathcal{B}') = \arctan(\rho)$ *implies that $\mathcal{B}'$ contains either the time series $(p \wedge 0)$ or $(0 \wedge f)$ where $(p \wedge f)$ is a weakest link of $\mathcal{B}$.*

3. If $m = q - 1$ and $(p \wedge f) \in \mathcal{B}^{\leftrightarrow}$ is a weakest link in $\mathcal{B}$, then

$$\mathcal{B}' := \begin{cases} [\mathcal{B}(p \wedge 0)]^{\perp} & \text{if } \|p\| \leq \|f\| \\ [\mathcal{B}(0 \wedge f)]^{\perp} & \text{if } \|p\| > \|f\| \end{cases} \quad (6.1)$$

is an optimal approximant of $\mathcal{B}$ of rank $m = q - 1$ and degree $n' = n - 1$, i.e., $\mathcal{B}' \in \mathbb{L}_{m,n-1}$.

7 Simulation example

We illustrate the model reduction approach by a numerical example. For a third order system we determine its second order optimal approximation in the sense of Definition 3.3. For the numerical computations we made use of the Mathematica package.

Consider the second order system in two variables u and y satisfying

$$y_t - \frac{1}{3}y_{t-1} = u_t - u_{t-1} + \frac{1}{2}u_{t-2}. \quad (7.1)$$

Formally, this defines the ℓ_2-system

$$\mathcal{B} = \{w = \left(\begin{smallmatrix} u \\ y \end{smallmatrix}\right) \in \ell_2^2 \mid (7.1) \text{ holds}\}. \quad (7.2)$$

The system has rank $m = 1$ and degree $n = 2$. If y is regarded as the output of the system and u as its input, then the system corresponds to the transfer function

$$G_1(z) := \frac{z^2 - z + \frac{1}{2}}{z^2 - z/3}.$$

Similarly, if u is taken as output and y as input then the transfer function associated with (7.1) is given by

$$G_2(z) = G_1^{-1} = \frac{z^2 - z/3}{z^2 - z + \frac{1}{2}}.$$

A numerical algorithm for the computation of the optimal approximant of degree $n' = 1$ of $\mathcal{B}$ makes use of isometric state space representations [7] of the system $\mathcal{B}$, but is not detailed in this paper for reasons of space. The optimal approximation is given by the solution set of the equation

$$y_t - \frac{73 - 13\sqrt{229}}{206}y_{t-1} = $$
$$= \frac{274 + 2\sqrt{229}}{309}(u_t - \frac{17 - \sqrt{229}}{20}u_{t-1}). \quad (7.3)$$

Specifically, let $\mathcal{B}'$ be defined by

$$\mathcal{B}' :== \{w = \left(\begin{smallmatrix} u \\ y \end{smallmatrix}\right) \in \ell_2^2 \mid (7.3) \text{ holds}\}$$

Then $\mathcal{B}'$ is the unique first order system with minimal angle with respect to $\mathcal{B}$. The angle equals

$$\theta(\mathcal{B}, \mathcal{B}') = \arcsin\sqrt{\frac{41 - 2\sqrt{229}}{595}}$$

which is about 7.7 degrees. Moreover, this is the angle with respect to $\mathcal{B}$ of every element of the approximation, and, conversely, every system trajectory in $\mathcal{B}$ has this angle with respect to this system. We finally remark that the angle between $\mathcal{B}$ and the approximant $\mathcal{B}'$ is flat.

8 Conclusions

We formalized an optimal model approximation problem in the behavioral setting and provided a complete solution for systems of rank one and reductions of the degree of the to-be-approximated system with one. Generalizations to higher rank systems are straightforward. The algorithm for the calculations of optimal reductions makes use of isometric state space representations and will be detailed elsewhere [8].

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