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ABSTRACT

In this report an efficient method is presented for analyzing rectangular microstrip antennas with a dielectric cover, using a spectral domain moment method. The use of this moment method involves the numerical evaluation of slowly decaying Sommerfeld-type integrals. Several computational problems are involved with the evaluation of these integrals. Some methods to avoid these problems will be discussed in this report. The most interesting method that we developed is the one that accelerates the calculation of the infinite integrals over the slowly decaying (and strongly oscillating) integrands. This slow convergence is caused by the source singularity in the Green's function of the dielectric slab. To overcome this disadvantage, we developed a method of rewriting the integrals as a sum of a closed form expression and a quickly converging integral, resulting in a reduction of computer time by a factor 20. This is done for sinusoidal entire domain basis functions on the patch of the microstrip antenna. It should be noted that this method can also be applied for other types of basis functions.

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CONTENTS.

1. Introduction

In recent years, many different models for analyzing microstrip antennas have been developed and presented in literature. Varying from relatively simple and time efficient models (for example the cavity model) to sophisticated and time consuming models (for example the moment method). With the simple models it is difficult to predict the input impedance of a microstrip antenna in an accurate way. Therefore we have chosen the moment method to analyze microstrip antennas.

In [1,2,3] a moment method in the spectral domain was presented for analyzing rectangular microstrip antennas or microstrip arrays. In fig. 1 the geometry of the microstrip structure is shown. The patch is located at the z=z′ plane, so the antenna can have a dielectric cover. It is assumed that the patch is fed by a coaxial feed located at the point (x_s,y_s). ε_r is the (complex) permittivity of the substrate material.

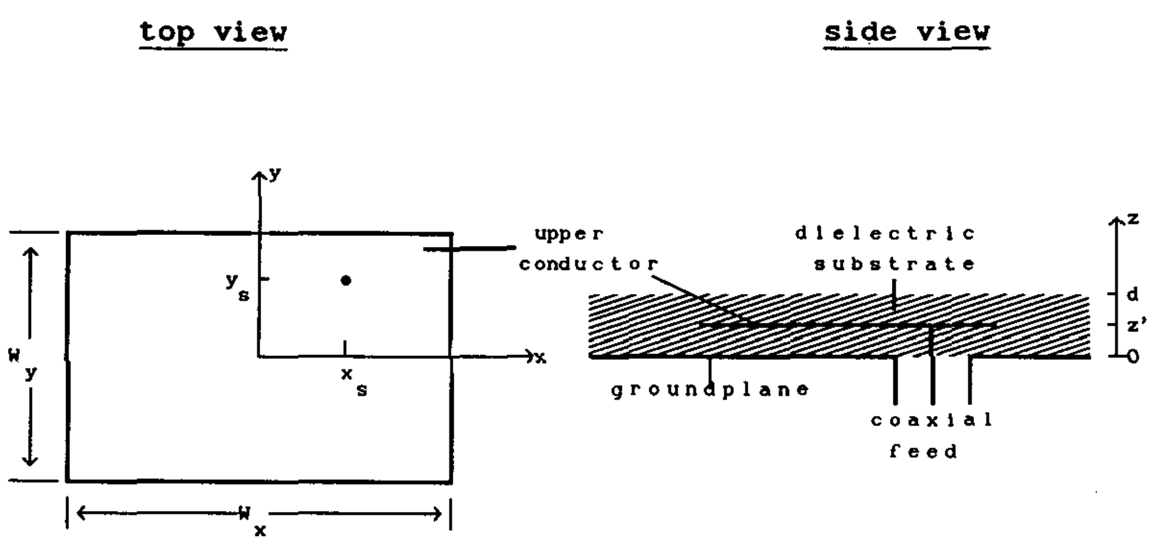


Fig 1.1: Geometry of a rectangular microstrip antenna

On the perfectly conducting patch surface of the microstrip antenna the total electric field vanishes, i.e.:

$$\vec{E}^{total}_{tan} = \vec{E}^{ex}_{tan} + \vec{E}^{s}_{tan} = \vec{0} \tag{1.1}$$

Here $\vec{E}^{ex}_{tan}$ and $\vec{E}^{s}_{tan}$ represents the excitation and scattered fields respectively. The scattered fields result from the induced currents on the patch and can be written in terms of the vector potential $\vec{A}$, with:

$$\vec{E}^{s}(\vec{r}) = -j\omega\vec{A}(\vec{r}) - \frac{j\omega}{k^2}\nabla(\nabla.\vec{A}(\vec{r}))$$

$$\vec{A}(\vec{r}) = \iint\limits_{patch} \bar{\bar{G}}(\vec{r},\vec{r}').\vec{J}_p(\vec{r}')\ dS' \tag{1.2}$$

where k is the wave number of the medium under consideration, $\bar{\bar{G}}(\vec{r},\vec{r}')$ is the dyadic Green's function. Applying the method of moments, integral equation (1.1) can be transformed into a set of linear equations by selecting appropriate expansion and test functions for the unknown current on the patch of the microstrip antenna. In chapter 2 a brief summary is given of the moment method formulation for coaxial-fed microstrip antennas . For more details about this formulation, one is referred to [1, chapter 2].

In chapter 3 and 4 the use entire domain sinusoidal basis functions is discussed. It will be shown that the symmetry properties of these functions can be used to reduce the number of integrals that have to be evaluated numerically.

Using the moment method formulation in the spectral domain as described in [1], it is necessary to evaluate numerically, time consuming infinite integrals over slowly decaying and oscillating integrands. Several computational problems are involved with the evaluation of these integrals. The problems due to TM- and TE-surface waves in the dielectric slab are already outlined in reference [1,chapter 3]. The other problems and the corresponding methods to avoid them, will be discussed in chapter 5 and 6 of this report. The most interesting method that we developed in order to facilitate the numerical evaluation of the integrals, is the method that accelerates the (numerical) calculation of the infinite integrals over the slowly decaying and oscillating integrands. This slow convergence is caused by the source singularity in the dyadic Green's function $\bar{\bar{G}}$. The slowly converging integrals are rewritten as a sum of a closed form expression and a quickly converging integral. This method is presented in this report for the case of entire domain sinusoidal basis functions to describe the unknown current density on the patch. Also a constant surface current density on the coaxial probe is assumed. The method can also be used if other types of basis functions are used on the patch or probe.

The software based on [1] is extended with the methods discussed here. The time needed to calculate the input impedance is now about 20 times less then it was before.

2. The moment method formulation for microstrip antennas in the spectral domain

In this chapter a short summary will be given of the moment method formulation of Reference [1].
The formulation is done in the spectral domain which means that all quantities used are Fourier transformed with respect to x and y. A general function f(x,y) and his corresponding Fourier transform $\tilde{f}(k_x,k_y)$ are defined as:

$$f(x,y) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \tilde{f}(k_x,k_y)\ e^{-jk_x x}\ e^{-k_y y}\ dk_x dk_y \qquad (2.1a)$$

$$\tilde{f}(k_x,k_y) = \iint_{-\infty}^{\infty} f(x,y)\ e^{jk_x x}\ e^{k_y y}\ dk_x dk_y \qquad (2.1b)$$

In the moment method formulation the Green's function, due to a unit current located at a arbitrarily point in the dielectric, is needed. Once the Green's function is known, a Galerkin type of moment method can be formulated, which results in the matrix equation:

$$[Z][I]=[V] \qquad (2.2)$$

Where the vector [I] contains the N unknown current coefficients of the patch current distribution. [V] is the excitation vector and [Z] is a NxN matrix. The elements of [Z] and [V] have the form [1]:

$$Z_{mn} = 4\pi^2 \iint_{patch} \vec{E}_{pn}.\vec{J}_{pm}\ dS \qquad (2.3)$$

$$V_m = 4\pi^2 \iint_{source} \vec{J}_s.\vec{E}_{pm}\ dS \qquad (2.4)$$

Where $\vec{J}_s$ is the surface current on the probe, which is assumed to be constant. The unknown surface current on the patch $\vec{J}_p$ is expanded in a set of N basis functions with unknown coefficients I_m:

$$\vec{J}_p = \sum_{m=1}^{N} I_m \vec{J}_{pm} \tag{2.5}$$

In this report entire domain sinusoidal basis functions are used. These basis functions are discussed in the next chapter. $\vec{E}_{pm}$ is the electric field in the dielectric slab due to a expansion current $\vec{J}_{pm}$ on the patch. An analytical expression for $\tilde{E}_{pm}$ in the spectral domain is known in closed form.
In [1] it was shown that Z_{mn} and V_m can also be written as:

$$\begin{aligned} Z_{mn} &= \iint_{-\infty}^{\infty} \vec{\tilde{E}}_{pn} \cdot \vec{\tilde{J}}^{*}_{pm} \Big|_{z=z'} dk_x dk_y \\ &= \iint_{-\infty}^{\infty} [\bar{\tilde{Q}} \cdot \vec{\tilde{J}}_{pn}] \cdot \vec{\tilde{J}}^{*}_{pm} \Big|_{z=z'} dk_x dk_y \end{aligned} \tag{2.6}$$

$$\begin{aligned} V_m &= \iint_{-\infty}^{\infty} \int_{0}^{z'} \vec{\tilde{E}}_{pm} \cdot \vec{\tilde{J}}^{*}_{s} \, dz \, dk_x dk_y \\ &= \iint_{-\infty}^{\infty} [\bar{\tilde{Q}}_v \cdot \vec{\tilde{J}}_{pm}] \cdot \vec{\tilde{J}}^{*}_{s} \, dk_x dk_y \end{aligned} \tag{2.7}$$

In (2.6) and (2.7) $\bar{\tilde{Q}}$ and $\bar{\tilde{Q}}_v$ are dyads in the spectral domain and are given by expression (2.3.8) resp. (2.4.17) of reference [1]. $\vec{\tilde{J}}_{pm}$ and $\vec{\tilde{J}}_s$ are the Fourier transforms of $\vec{J}_{pm}$ resp. $\vec{J}_s$, where $\vec{\tilde{J}}_s$ is given by (2.4.20) of reference [1].
Expression (2.6) and (2.7) are the final forms and will be used in

this report. In order to facilitate numerical evaluation of the two dimensional integrals, a change to polar coordinates is used, such that:

$$k_x = k_0\beta\cos\alpha \qquad (2.8)$$
$$k_y = k_0\beta\sin\alpha$$

with $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the free space wave number and ω is the radial frequency. Now an element of the [Z] matrix has the general form:

$$Z_{mn} = \int_{-\pi}^{\pi}\int_{0}^{\infty} [\bar{\tilde{Q}}.\vec{\tilde{J}}_{pn}].\vec{\tilde{J}}^*_{pm}\Big|_{z=z'} k_0^2\beta \; d\beta d\alpha \qquad (2.9)$$

In chapter 4 the evaluation of the α-integral is discussed for the case of sinusoidal entire domain basis functions on the patch. We will there use the even and odd properties of the α-integrand.
The numerical evaluation of the β integral is the most difficult one. The β integral can be divided in three intervals:

$$Z_{mn} = \int_{-\pi}^{\pi}\left(\int_{0}^{1} f(\beta,\alpha)d\beta + \int_{1}^{\sqrt{\varepsilon'_r}} f(\beta,\alpha)d\beta + \int_{\sqrt{\varepsilon'_r}}^{\infty} f(\beta,\alpha)d\beta \right) d\alpha \qquad (2.10)$$

with $f(\beta,\alpha) = [\bar{\tilde{Q}}.\vec{\tilde{J}}_{pn}].\vec{\tilde{J}}^*_{pm}\Big|_{z=z'}$ and ε'_r is the real part of the permittivity of the substrate.
In chapter 5 the evaluation of the three integrals in (2.10) will be discussed. In chapter 6 an similar method will be used for the calculation of [V] elements.

3. Entire domain sinusoidal basis functions

In the moment method presented in this report we shall use entire domain sinusoidal basis functions in order to describe the unknown surface current on the patch. It is assumed that a $\hat{x}$-directed basis function is y-independent and that $\hat{y}$-directed basis functions are x-independent. We shall use the coordinate system of fig. 3.1.

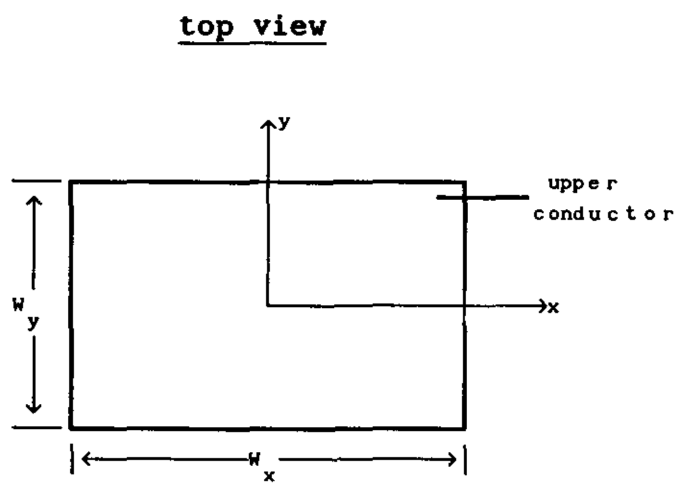


Fig. 3.1: Coordinate system

The basis functions now have the form:

i. $\hat{x}$-directed basis functions.

$$J_{pmx}(x) = \frac{1}{Wy}\sin\left[\frac{m\pi}{Wx}\left(x+\frac{Wx}{2}\right)\right] \quad \text{for} \quad \begin{cases} -Wx/2 \le x \le Wx/2 \\ -Wy/2 \le y \le Wy/2 \end{cases} \tag{3.1a}$$

In fig. 3.2 the first three basis functions are shown.

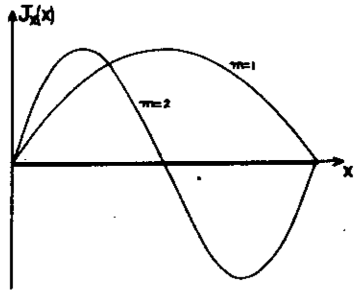


Fig. 3.2: Sinusoidal $\hat{x}$-directed basis functions

ii. $\hat{y}$-directed basis functions.

$$J_{pmy}(y) = \frac{1}{Wx}\sin\left[\frac{m\pi}{Wy}\left(y+\frac{Wy}{2}\right)\right] \qquad \text{for} \quad \begin{bmatrix} -Wx/2 \le x \le Wx/2 \\ -Wy/2 \le y \le Wy/2 \end{bmatrix} \tag{3.1b}$$

Because of the fact that the calculations are performed in the spectral domain, we need to know the Fourier transforms of (3.1). They are given by:

$$\tilde{J}_{pmx}(k_x,k_y) = F_s(m,k_x,W_x)\,F_c(m,k_y,W_y) \tag{3.2a}$$

$$\tilde{J}_{pmy}(k_x,k_y) = F_s(m,k_y,W_y)\,F_c(m,k_x,W_x) \tag{3.2b}$$

with:

$$F_s(m,k_x,W_x) = \begin{cases} \dfrac{2m\pi W_x \cos(k_x W_x/2)}{(m\pi)^2-(k_x W_x)^2} & m \text{ odd} \\[2ex] \dfrac{-j2m\pi W_x \sin(k_x W_x/2)}{(m\pi)^2-(k_x W_x)^2} & m \text{ even} \end{cases} \tag{3.2c}$$

$$F_c(m,k_y,W_y) = \frac{2\sin(k_y W_y/2)}{k_y W_y} \tag{3.2d}$$

It should be noted that expression (3.1) is a generalization of expression (2.5.2) of reference [1]. Because of the even and odd properties of the Fourier transforms of the basis functions, a great number of elements of the matrix [Z] are zero. This will be discussed in the next chapter.

4. Symmetry-properties of the integrands of the matrix elements

This chapter is a generalization of section 3.2 of reference [1]. We here shall use the same method to reduce the computation time needed to calculate the stiffness matrix [Z] and the excitation vector [V]. In [1] it was shown that using sub domain basis functions, many elements of the [Z] matrix have the same value. In this case, the value of a Z_{mn} matrix element depends on the distance between sub domain m en n. Using entire domain basis functions, this Toeplitz-like property of the [Z] matrix doesn't exists. Now we can use the even and odd properties of de Fourier transforms of the sinusoidal basis functions that are used. Because of this even and odd properties, a lot of elements of the [Z] matrix will be zero. In this paragraph we shall use these properties in order to reduce the computation time needed to evaluate the elements of [Z] and [V] and to show which elements of [Z] are zero.

If only $\hat{x}$- or $\hat{y}$-directed basis functions are used in the calculations, the matrix [Z] can be written in the following form:

$$[Z] = \begin{bmatrix} [Z^{xx}] & [Z^{xy}] \\ \\ [Z^{yx}] & [Z^{yy}] \end{bmatrix} \tag{4.1}$$

Where an element of the sub matrix $[Z^{ji}]$ represents the coupling between a $\hat{j}$- and a $\hat{i}$-directed basis function. Now we shall take a closer look to each of the four sub matrices.

i. $[Z^{xx}]$.

According to expression (3.9), an element of the sub matrix $[Z^{xx}]$ can be written as:

$$Z_{mn}^{xx} = \int_0^{\infty}\int_{-\pi}^{\pi} \tilde{Q}_{xx}(\beta,\alpha)\tilde{J}^*_{pmx}(\beta,\alpha)\tilde{J}_{pnx}(\beta,\alpha)k_0^2\beta d\alpha d\beta \qquad (4.2)$$

Where the substitution $\begin{bmatrix} k_x=k_0\beta\cos\alpha \\ k_y=k_0\beta\sin\alpha \end{bmatrix}$ is used. $\tilde{J}_{pmx}$ and $\tilde{J}_{pnx}$ are the Fourier transforms of the m-th resp. n-th $\hat{x}$-directed basis function. $\tilde{Q}_{xx}$ is given by expression (2.3.8) of [1]:

$$\tilde{Q}_{xx}(\beta,\alpha) = \frac{\omega\mu_0 \sin(k_1 z')}{\varepsilon_r \mathrm{TeTm} k_1}\Big[j(\beta^2\cos^2\alpha-\varepsilon_r)\mathrm{NeTm} - \beta^2\cos^2\alpha k_1^2(\varepsilon_r-1)\sin(k_1 z')\Big] \qquad (4.2a)$$

Where Te, Tm, Ne en k_1 are functions of β (see [1,Appendix 2A]). We shall now divide the problem in four different cases:

1. m is odd, n is odd
2. m is odd, n is even
3. m is even, n is odd
4. m is even, n is even.

ad 1.: m is odd, n is odd.

This situation is already discussed in [1,par.3.2]. So only the result will be given here:

$$Z_{mn}^{xx} = 4\int_0^{\infty}\int_0^{\pi/2} \tilde{Q}_{xx}\tilde{J}^*_{pmx}\tilde{J}_{pnx}\ k_0^2\beta d\alpha d\beta \qquad (4.3)$$

ad 2.: m is odd, n is even.

The α-integration is divided in two intervals:

$$Z_{mn}^{xx}=\int_0^\infty\left(\int_0^\pi \tilde{Q}_{xx}\tilde{J}^*_{pmx}\tilde{J}_{pnx}d\alpha + \int_{-\pi}^0 \tilde{Q}_{xx}\tilde{J}^*_{pmx}\tilde{J}_{pnx}d\alpha\right)k_0^2\beta d\beta$$

Use the substitution $\alpha'=-\alpha$ in the second α-integral and use the properties that:

$$\tilde{Q}_{xx}(-\alpha) = \tilde{Q}_{xx}(\alpha)$$

$$\tilde{J}^*_{pmx}(-\alpha) = \tilde{J}^*_{pmx}(\alpha) \text{ en } \tilde{J}_{pnx}(-\alpha) = \tilde{J}_{pnx}(\alpha)$$

This results in:

$$Z_{mn}^{xx} = 2\int_0^\infty\int_0^\pi \tilde{Q}_{xx}\tilde{J}^*_{pmx}\tilde{J}_{pnx}\ k_0^2\beta d\alpha d\beta$$

Substitute $\alpha'=\alpha-\pi/2$ in the above expression. This gives:

$$Z_{mn}^{xx} = 2\int_0^\infty\int_{-\pi/2}^{\pi/2} \tilde{Q}_{xx}(\alpha')\tilde{J}^*_{pmx}(\alpha')\tilde{J}_{pnx}(\alpha')\ k_0^2\beta d\alpha' d\beta$$

$$= 2\int_0^\infty\left(\int_0^{\pi/2} \tilde{Q}_{xx}(\alpha')\tilde{J}^*_{pmx}(\alpha')\tilde{J}_{pnx}(\alpha')\ d\alpha' + \int_0^{\pi/2} \tilde{Q}_{xx}(-\alpha')\tilde{J}^*_{pmx}(-\alpha')\tilde{J}_{pnx}(-\alpha')\ d\alpha'\right)k_0^2\beta d\beta$$

Now again use the odd and even properties:

$$\tilde{Q}_{xx}(-\alpha') = \tilde{Q}_{xx}(\alpha')$$

$$\tilde{J}^*_{pmx}(-\alpha') = \tilde{J}^*_{pmx}(\alpha') \text{ en } \tilde{J}_{pnx}(-\alpha') = -\tilde{J}_{pnx}(\alpha')$$

So apparently:

$$Z^{xx}_{mn} = 0$$

ad 3.: <u>m is even, n is odd</u>.
Analog to the previous case. Thus $Z^{xx}_{mn} = 0$.

ad 4.: <u>m is even, n is even</u>.
Analog to case 1, resulting in:

$$Z^{xx}_{mn} = 4\int_0^{\infty}\int_0^{\pi/2} \tilde{Q}_{xx}\tilde{J}^{*}_{pmx}\tilde{J}_{pnx}\; k_0^2\beta d\alpha d\beta \tag{4.3}$$

ii. $[Z^{xy}]$.

An element of this sub matrix has the form:

$$Z^{xy}_{mn} = \int_0^{\infty}\int_{-\pi}^{\pi} \tilde{Q}_{xy}(\beta,\alpha)\tilde{J}^{*}_{pmx}(\beta,\alpha)\tilde{J}_{pny}(\beta,\alpha)\; k_0^2\beta d\alpha d\beta \tag{4.4}$$

Where $\tilde{Q}_{xy}$ is given by expression (2.3.8) of [1]:

$$\tilde{Q}_{xy}(\beta,\alpha) = \frac{\omega\mu_0\beta^2\sin(k_1z')}{2\varepsilon_r TeTmk_1}\left[jNeTm-k_1^2(\varepsilon_r-1)\sin(k_1z')\right]\sin(2\alpha) \tag{4.4a}$$

Once again the problem is divided in four cases.

ad 1.: <u>m is odd, n is odd</u>.
Divide the α-integral in two parts:

$$Z_{mn}^{xy}=\int_{0}^{\infty}\left(\int_{0}^{\pi}\tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha + \int_{-\pi}^{0}\tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha\right)k_0^2\beta d\beta$$

Use the substitution $\alpha'=-\alpha$ in the second α-integral and make use of the odd and even properties:

$$\tilde{Q}_{xy}(-\alpha) = -\tilde{Q}_{xy}(\alpha)$$

$$\tilde{J}_{pmx}^{*}(-\alpha) = \tilde{J}_{pmx}^{*}(\alpha) \text{ en } \tilde{J}_{pny}(-\alpha) = \tilde{J}_{pny}(\alpha)$$

Which results in: $Z_{mn}^{xy} = 0$

ad 2.: <u>m is odd, n is even</u>.

Divide the α-integral in two parts:

$$Z_{mn}^{xy}=\int_{0}^{\infty}\left(\int_{0}^{\pi}\tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha + \int_{-\pi}^{0}\tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha\right)k_0^2\beta d\beta$$

Use the substitution $\alpha'=-\alpha$ in the second α-integral and use the properties that:

$$\tilde{Q}_{xy}(-\alpha) = -\tilde{Q}_{xy}(\alpha)$$

$$\tilde{J}_{pmx}^{*}(-\alpha) = \tilde{J}_{pmx}^{*}(\alpha) \text{ en } \tilde{J}_{pny}(-\alpha) = -\tilde{J}_{pny}(\alpha)$$

This results in:

$$Z_{mn}^{xy} = 2\int_{0}^{\infty}\int_{0}^{\pi}\tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}\ k_0^2\beta d\alpha d\beta$$

Substitute $\alpha'=\alpha-\pi/2$ in the above expression. The above expression then takes the form:

$$Z_{mn}^{xy} = 2\int_{0}^{\infty}\int_{-\pi/2}^{\pi/2} \tilde{Q}_{xy}(\alpha')\tilde{J}_{pmx}^{*}(\alpha')\tilde{J}_{pny}(\alpha')\ k_0^2\beta d\alpha' d\beta$$

$$= 2\int_{0}^{\infty}\left(\int_{0}^{\pi/2} \tilde{Q}_{xy}(\alpha')\tilde{J}_{pmx}^{*}(\alpha')\tilde{J}_{pny}(\alpha')\ d\alpha'\right.$$

$$\left.+\int_{0}^{\pi/2} \tilde{Q}_{xy}(-\alpha')\tilde{J}_{pmx}^{*}(-\alpha')\tilde{J}_{pny}(-\alpha')\ d\alpha'\right)k_0^2\beta d\beta$$

Once again use the even and odd properties:

$$\tilde{Q}_{xy}(-\alpha') = -\tilde{Q}_{xy}(\alpha')$$

$$\tilde{J}_{pmx}^{*}(-\alpha') = \tilde{J}_{pmx}^{*}(\alpha') \text{ en } \tilde{J}_{pny}(-\alpha') = \tilde{J}_{pny}(\alpha')$$

This implies that $Z_{mn}^{xy} = 0$

ad 3.: m is even, n is odd.

Analog to case 2. Thus $Z_{mn}^{xy} = 0$.

ad 4.: m is even, n is even.

The procedure is the same as in the previous cases. So Z_{mn}^{xy} is written as:

$$Z_{mn}^{xy}=\int_{0}^{\infty}\left(\int_{0}^{\pi} \tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha + \int_{-\pi}^{0} \tilde{Q}_{xy}\tilde{J}_{pmx}^{*}\tilde{J}_{pny}d\alpha\right)k_0^2\beta d\beta$$

Use the substitution $\alpha'=-\alpha$ in the second α-integral and use the properties that:

$$\tilde{Q}_{xy}(-\alpha) = -\tilde{Q}_{xy}(\alpha)$$

$$\tilde{J}^*_{pmx}(-\alpha) = \tilde{J}^*_{pmx}(\alpha) \text{ en } \tilde{J}_{pny}(-\alpha) = -\tilde{J}_{pny}(\alpha)$$

Resulting in:

$$Z^{xy}_{mn} = 2\int_0^\infty\int_0^\pi \tilde{Q}_{xy}\tilde{J}^*_{pmx}\tilde{J}_{pny} \; k_0^2\beta d\alpha d\beta$$

Substitute $\alpha'=\alpha-\pi/2$ in the above expression. This gives:

$$Z^{xy}_{mn} = 2\int_0^\infty\int_{-\pi/2}^{\pi/2} \tilde{Q}_{xy}(\alpha')\tilde{J}^*_{pmx}(\alpha')\tilde{J}_{pny}(\alpha') \; k_0^2\beta d\alpha' d\beta$$

$$= 2\int_0^\infty\left(\int_0^{\pi/2} \tilde{Q}_{xy}(\alpha')\tilde{J}^*_{pmx}(\alpha')\tilde{J}_{pny}(\alpha') \; d\alpha' + \int_0^{\pi/2} \tilde{Q}_{xy}(-\alpha')\tilde{J}^*_{pmx}(-\alpha')\tilde{J}_{pny}(-\alpha') \; d\alpha'\right) k_0^2\beta d\beta$$

Use the even and odd properties:

$$\tilde{Q}_{xy}(-\alpha') = -\tilde{Q}_{xy}(\alpha')$$

$$\tilde{J}^*_{pmx}(-\alpha') = -\tilde{J}^*_{pmx}(\alpha') \text{ en } \tilde{J}_{pny}(-\alpha') = \tilde{J}_{pny}(\alpha')$$

This finally gives:

$$Z^{xy}_{mn} = 4\int_0^\infty\int_0^{\pi/2} \tilde{Q}_{xy}\tilde{J}^*_{pmx}\tilde{J}_{pny} \; k_0^2\beta d\alpha d\beta \tag{4.5}$$

iii. $[Z^{yx}]$.

The matrix [Z] is symmetrical, i.e. $[Z^{yx}]=[Z^{xy}]$.

iv. $[Z^{yy}]$.

The elements of the sub matrix $[Z^{yy}]$ have the same form as the elements of $[Z^{xx}]$. Also in this case, an element of $[Z^{yy}]$ is not equal zero only if both m and n are odd or even.

The integrals involved in calculating the excitation vector [V] can also be reduced by using the odd or even properties of the sinusoidal entire domain basis functions. In general, all the elements of [V] are unequal zero.
If we use $\hat{x}$- and $\hat{y}$-directed basis functions, the excitation vector can be written in the form:

$$[V] = \begin{bmatrix} [V^x] \\ \\ [V^y] \end{bmatrix}$$

We will now take a closer look to the elements of the sub matrix $[V^x]$. The elements of $[V^y]$ are treated in a similar way.
According to expression (2.4.19) of [1] an V^x_m element can be written as:

$$V^x_m = \int_0^{\infty}\int_{-\pi}^{\pi} \tilde{Q}_{vx}(\beta,\alpha)\tilde{J}_{pmx}(\beta,\alpha)\tilde{J}^*_s(\beta,\alpha)k_0^2\beta d\alpha d\beta \qquad (4.6)$$

Where we used the substitution $\begin{cases} k_x = k_0\beta\cos\alpha \\ k_y = k_0\beta\sin\alpha \end{cases}$. Furthermore we shall assume that the surface current density on the coaxial feed with radius r_0 is constant, i.e. $\vec{J}_s = \vec{e}_z I_0/2\pi r_0$. It's Fourier transform has then the form [1,pp. 29]:

$$\vec{\tilde{J}}_s(\beta,\alpha) = \vec{e}_z J_0(r_0k_0\beta)e^{j(k_{xs}+k_{ys})} \qquad (4.6a)$$

with $k_{xs} = k_0\beta x_s\cos\alpha$
$k_{ys} = k_0\beta y_s\sin\alpha$

J_0 is the Besselfunction of the first kind, order zero. Note that we use the complex conjugate of $\vec{J}_s$ in (4.6).
Furthermore we have [1,pp. 29]:

$$\tilde{Q}_{vx}(\beta,\alpha)=\frac{j\omega\mu_0\beta\sin(k_1z')}{\varepsilon_r k_0 k_1 TeTm}\left[(\varepsilon_r-1)\sin(k_1z')k_0^2\beta^2+jNeTm\right]\cos\alpha \qquad (4.6b)$$

k_1,Te,Tm and Ne are functions of β (see [1,App. 2A]).
We shall divide the problem in two cases, i.e. odd m values and even m values.

ad 1. m odd.

This situation has already been analyzed in reference [1,par. 3.2]. It resulted in:

$$V_m^x = -4i\int_0^\infty k_0^2\beta J_0(r_0k_0\beta)\int_0^{\pi/2}\tilde{Q}_{vx}\tilde{J}_{pmx}\sin(k_{xs})\cos(k_{ys})\,d\alpha d\beta \qquad (4.7)$$

ad 2. m even.

We will use the same procedure as we did in the case of the elements of the [Z] matrix. So divide the α-integral in two parts and substitute $\alpha'=-\alpha$ in the second integral. Using the properties:

$$\tilde{Q}_{vx}(-\alpha)=\tilde{Q}_{vx}(\alpha)$$
$$\tilde{J}_{pmx}(-\alpha)=\tilde{J}_{pmx}(\alpha)$$

results in:

$$V_m^x = 2\int_0^\infty k_0^2\beta J_0(r_0k_0\beta)\int_0^{\pi}\tilde{Q}_{vx}\tilde{J}_{pmx}\cos(k_{ys})e^{-jk_{xs}}\,d\alpha d\beta$$

Substitute $\alpha'=\alpha-\pi/2$ in the above expression and divide the α-integral in two parts.

$$V_m^x=2\int_0^\infty k_0^2\beta J_0(r_0k_0\beta)\int_{-\pi/2}^{\pi/2}\tilde{Q}_{vx}(\alpha')\tilde{J}_{pmx}(\alpha')\cos(k_ys(\alpha'))e^{-jk_xs(\alpha')}d\alpha'd\beta$$

$$=2\int_0^\infty k_0^2\beta J_0(r_0k_0\beta)\left(\int_0^{\pi/2}\tilde{Q}_{vx}(\alpha')\tilde{J}_{pmx}(\alpha')\cos(k_ys(\alpha'))e^{-jk_xs(\alpha')}d\alpha'\right.$$

$$\left.+\int_0^{\pi/2}\tilde{Q}_{vx}(-\alpha')\tilde{J}_{pmx}(-\alpha')\cos(k_ys(-\alpha'))e^{-jk_xs(-\alpha')}d\alpha'\right)d\beta$$

Use the following odd and even properties:

$$\tilde{Q}_{vx}(-\alpha')=-\tilde{Q}_{vx}(\alpha')$$
$$\tilde{J}_{pmx}(-\alpha')=-\tilde{J}_{pmx}(\alpha')$$
$$k_{xs}(-\alpha')=-k_{xs}(\alpha') \qquad k_{ys}(-\alpha')=-k_{ys}(\alpha')$$

Finally one gets the expression:

$$V_m^x = 4\int_0^\infty k_0^2\beta J_0(r_0k_0\beta)\int_0^{\pi/2}\tilde{Q}_{vx}\tilde{J}_{pmx}\cos(k_{xs})\cos(k_{ys})\,d\alpha d\beta \qquad (4.8)$$

5.Calculation of the stiffness matrix [Z]

In the previous section the odd and even properties of the α-integrands where used to show which elements of the [Z] matrix were unequal zero. In this chapter it will be shown how these elements can be calculated in an efficient way. The two -dimensional integrals that have to be evaluated numerically have according to (2.9) the following general form:

$$Z_{mn} = \int_{-\pi}^{\pi}\int_{0}^{\infty} f(\beta,\alpha)\ d\beta d\alpha \tag{5.1}$$

De infinite β-integral is divided in three parts, with each part having its own numerical problems.

$$Z_{mn} = \int_{-\pi}^{\pi}\left(\int_{0}^{1} f(\beta,\alpha)d\beta + \int_{1}^{\sqrt{\varepsilon'_r}} f(\beta,\alpha)d\beta + \int_{\sqrt{\varepsilon'_r}}^{\infty} f(\beta,\alpha)d\beta \right) d\alpha \tag{5.2}$$

Where ε'_r is the real part of the permittivity of the substrate.
In the following three sections a method for evaluating the three β-integrals in a proper way, will be discussed. We will assume that the patch is located in the substrate, i.e. in the plane z=z' (see fig. 1.1). So the antenna may have a dielectric cover.

5.1 Calculation of the β-integral in the region $0\le\beta\le1$

The real part of the β-integrand has an infinite derivative at β=1, which implies that a lot of integration points are needed in the vicinity of this point in order to obtain a certain accuracy. Fortunately, this infinite derivative can be eliminated using a change of variables β=cost:

$$\int_{-\pi}^{\pi}\int_{0}^{1} f(\beta,\alpha)\,d\beta d\alpha = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\cos t,\alpha)\sin t\ dt d\alpha \tag{5.3}$$

The above integral can now be calculated very accurate using a simple fixed point integration rule.

5.2 Calculation of the β-integral in the region $1\le\beta\le\sqrt{\varepsilon_r'}$

We now have to consider the second β-integral in (5.2). In this interval, two specific numerical problems can be distinguished, namely one in which the integrand is almost singular due to surface waves and the other in which the imaginary part of the integrand has a infinite derivative at β=1. The first problem is already discussed in [1, chap. 3.1]. The infinite derivative can be eliminated by using a change of variables β=cosh(t), which then gives:

$$\int_{-\pi}^{\pi}\int_{1}^{\sqrt{\varepsilon_r'}} f(\beta,\alpha)\,d\beta = \int_{-\pi}^{\pi}\int_{0}^{\operatorname{arccosh}(\sqrt{\varepsilon_r'})} f(\cosh t,\alpha)\sinh t\ dt d\alpha \tag{5.4}$$

5.3 Calculation of the infinite β-integral: source term extraction

The infinite β integration is in practice of course terminated at a certain value $\beta=\beta_{max}$. In [1], it was already mentioned that the integrand converges very slowly to zero for large values of β. To avoid this problem, a method, called the source term extraction method, is proposed in which the asymptotic value of the integrand for large β-values is subtracted from the original integrand. This idea was first proposed in reference [4]. The integration over this new integrand converges much faster. The integration over the asymptotic value of the integrand can be calculated analytically. The method is called "source term extraction technique", because the asymptotic part of the β-integrand is due to the $\frac{1}{|\vec{r} - \vec{r}\,'|}$ singularity in the space domain Green's function [1, chapt. 3.3]. In [1, chap. 3.3] this technique was used for sub domain rooftop basis functions for the case that z'=d (thus patch on top of substrate). In this section another method, a more efficient method, is presented to calculate the integral over the asymptotic value of the original integrand. This new method can be used for both sub domain as entire domain basis functions and for an arbitrarily location of the patch in the substrate (thus z'≤ d). In this paragraph we will use entire domain sinusoidal basis functions in order to describe the unknown current density on the patch. The method can also be used for other types of basis functions (for example sub domain rooftop basis functions).
Using the subtracting technique, an element of the matrix [Z] is written as:

$$Z_{mn} = \int_0^\infty\int_{-\pi}^{\pi} [(\bar{\tilde{Q}}-\bar{\tilde{Q}}_h).\vec{\tilde{J}}_{pn}].\vec{\tilde{J}}^{*}_{pm}\; k_0^2\beta d\alpha d\beta$$

$$+ \int_0^\infty\int_{-\pi}^{\pi} [\bar{\tilde{Q}}_h.\vec{\tilde{J}}_{pn}].\vec{\tilde{J}}^{*}_{pm}\; k_0^2\beta d\alpha d\beta = [Z_{mn}-Z_{hmn}]+Z_{hmn} \qquad (5.5)$$

Where $\bar{\tilde{Q}}_h$ the asymptotic value of $\bar{\tilde{Q}}$ is. It can be easily shown that the asymptotic value of $\bar{\tilde{Q}}$, given by expression (2.3.8) of reference [1], takes the following form for large β:

$$\bar{\tilde{Q}}_h = \begin{bmatrix} \tilde{Q}_{hxx} & \tilde{Q}_{hxy} & 0 \\ \tilde{Q}_{hyx} & \tilde{Q}_{hyy} & 0 \\ \tilde{Q}_{hzx} & \tilde{Q}_{hzy} & 0 \end{bmatrix} \tag{5.5a}$$

z'=d:

$$\tilde{Q}_{hxx} = \frac{-j\omega\mu_0}{2k_0\beta}\left[1 - \frac{2\beta^2\cos^2\alpha}{(\varepsilon_r+1)}\right]$$

$$\tilde{Q}_{hyx} = \tilde{Q}_{hxy} = \frac{j\omega\mu_0\beta\sin(2\alpha)}{2k_0(\varepsilon_r+1)}$$

$$\tilde{Q}_{hyy} = \frac{-j\omega\mu_0}{2k_0\beta}\left[1 - \frac{2\beta^2\sin^2\alpha}{(\varepsilon_r+1)}\right]$$

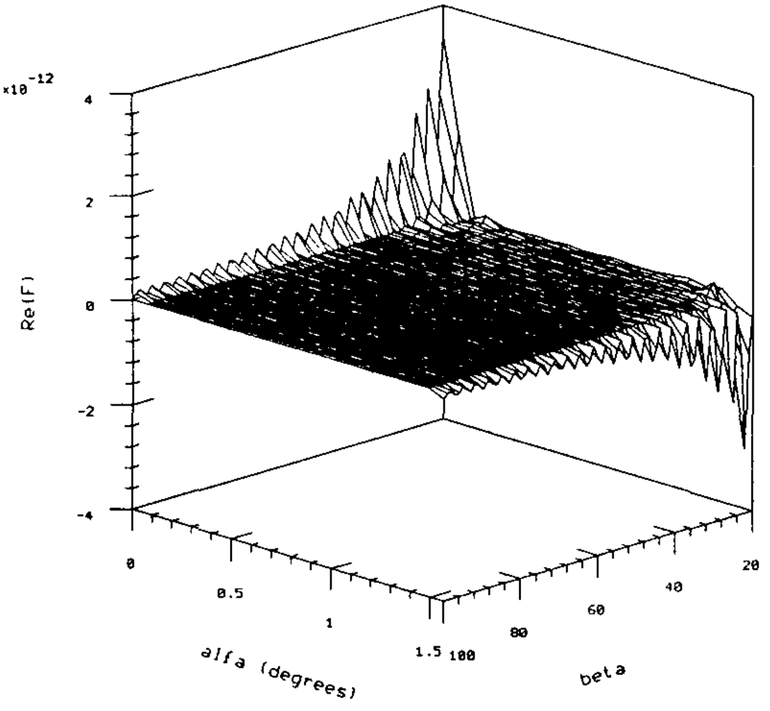
z'<d:

$$\tilde{Q}_{hxx} = \frac{-j\omega\mu_0}{2k_0\beta}\left[1 - \frac{\beta^2\cos^2\alpha}{\varepsilon_r}\right]$$

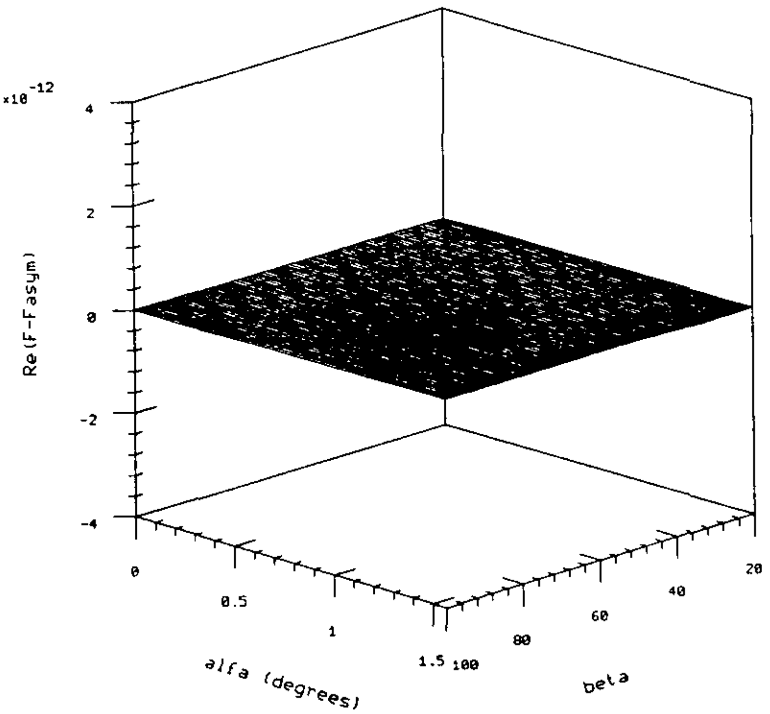
$$\tilde{Q}_{hyx} = \tilde{Q}_{hxy} = \frac{j\omega\mu_0\beta\sin(2\alpha)}{4k_0\varepsilon_r}$$

$$\tilde{Q}_{hyy} = \frac{-j\omega\mu_0}{2k_0\beta}\left[1 - \frac{\beta^2\sin^2\alpha}{\varepsilon_r}\right]$$

Where again $k_0=\omega\sqrt{\varepsilon_0\mu_0}$ is the free-space wave number and $\omega=2\pi f$ is the radial frequency. β and α are defined in (2.8). In figure 5.1 the effect of source term extraction on the β-integrand is shown for an Z_{11}^{xx} element. In figure 5.1a the original integrand is shown, i.e. an Z_{11}^{xx} element. In figure 5.1b the integrand with the subtracted asymptotic term is drawn, thus of an $(Z_{11}^{xx}-Z_{h11}^{xx})$ element. From this figure it is clear that using source term extraction, the number of integration points needed to evaluate the β-integral is much lower then the number of points needed for the evaluation of the original integral. It is also clear that the original integrand is a strongly oscillating function for large values of β, which implies that a very large number of integration points are required in order to obtain a certain level of accuracy for the infinite β-integral.



(a)



(b)

Fig. 5.1: (a) original integrand of an Z_{11}^{xx} element
(b) integrand of Z_{11}^{xx} using source term extraction
Antenna: Wx=Wy=20.1mm d=z'=1.59mm ε_r=2.55
tanδ=0.002 f=4.4 GHz (first resonance)

The derivation of Z_{hmn} in this chapter will be done for elements of the sub matrix $[Z^{xx}]$ for the case that $z'=d$. For the other elements of [Z] and for the case that $z'<d$, the source extraction technique can be applied in exact the same manner.
Now lets consider a Z^{xx}_{hmn} element, due to a $\hat{x}$-directed entire domain sinusoidal basis function m and a $\hat{x}$-directed entire domain basis function n, both on the patch. It will be shown that the infinite β integral in Z^{xx}_{hmn} can be calculated analytically using the theory of residues. Note that this is a complete other method then that was used in [1, chapt. 3.3]. The remaining α-integration can, in the most situations, also be done in closed form. In the cases where this is not possible, the numerical evaluation of the α-integral is relatively simple. The evaluation of the α-integral is not discussed here. Another advantage of the method used here, is the fact that the integrals of a Z^{xx}_{hmn}-element are frequency independent, so the only have to be evaluated once.

From chapter 4 it is clear that we only have to consider the cases when m and n are both odd or both even. Because the results for these two cases are usually the same, we will only consider the case that m and n are both odd. If the results for the case m and n both even are different, they will also be given.
We can distinguish two situations:

i. $m \neq n$

ii. $m=n$

Both situations are discussed in more detail.

i. $m \neq n$, m and n both odd.

Using expression (3.2), (5.5) and (5.5a), a Z^{xx}_{hmn} element takes the form for $z'=d$:

$$Z^{xx}_{hmn}=A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\int_0^{\infty}\left[1-\frac{2\beta^2\cos^2\alpha}{\varepsilon_r+1}\right]\frac{\cos^2(\beta k^x/2)\sin^2(\beta k^y/2)}{(n\pi-\beta k^x)(n\pi+\beta k^x)(m\pi-\beta k^x)(m\pi+\beta k^x)\beta^2}\,d\beta d\alpha$$

$$=A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\ I_\beta(\alpha)\,d\alpha \tag{5.6}$$

Where: $k^x=k_0\cos\alpha W x$ $\qquad A=\dfrac{-8j\omega\mu_0\pi^2 mnW_x^2}{k_0}$

$k^y=k_0\sin\alpha W y$

Note that k^x and k^y differ from the variables k_x and k_y which were used in chapter 2. In expression (5.6) we didn't make use of the symmetry in the α-integrand (see chapter 4), but this is of no relevance here. We may write:

$$\cos^2(\beta k^x/2)\sin^2(\beta k^y/2) = -\frac{1}{16}[f(\beta)+f(-\beta)] \tag{5.7}$$

$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{j\beta|k^x-k^y|}$$

Note that $k^x\ge 0$ and $k^y\ge 0$.
There are two different situations for which the problem has to be solved, namely $k^x\ge k^y$ and $k^x<k^y$.

a. $k^x\ge k^y$.

The complex function $f(\beta)$ can now be written in the form:

$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{j\beta(k^x-k^y)} \tag{5.8}$$

Because $\cos^2(\beta k^x/2)\sin^2(\beta k^y/2)$ is an even function with respect to β, the integral $I_\beta(\alpha)$ may be expressed as:

$$I_\beta(\alpha)=-\frac{1}{16}\int_{-\infty}^{\infty}\left[1-\frac{2\beta^2\cos^2\alpha}{\varepsilon_r+1}\right]\frac{f(\beta)}{(n\pi-\beta k^x)(n\pi+\beta k^x)(m\pi-\beta k^x)(m\pi+\beta k^x)\beta^2}\,d\beta$$

$$= \int_{-\infty}^{\infty} g(\beta)\,d\beta \tag{5.9}$$

The integrand in (5.9) has 5 single valued poles on the real β-axis. We can avoid these poles by choosing a proper integration path. This path is shown in figure 5.2. According to the Residue theorem of Cauchy [5], the integral over a closed integration path is zero if there are no poles within the area which is enclosed by the closed integration path. This theorem will be used here to calculate the infinite β-integrals. We shall use the notation β=Re(z). In formula form we now may write:

$$\int_{C_1} + \int_{C_{\delta_1}} + \int_{C_2} + \int_{C_{\delta_2}} + \int_{C_3} + \int_{C_{\delta_3}} + \int_{C_4} + \int_{C_{\delta_4}} + \int_{C_5} + \int_{C_{\delta_5}} + \int_{C_6} + \int_{C_\rho^+} = 0 \tag{5.10}$$

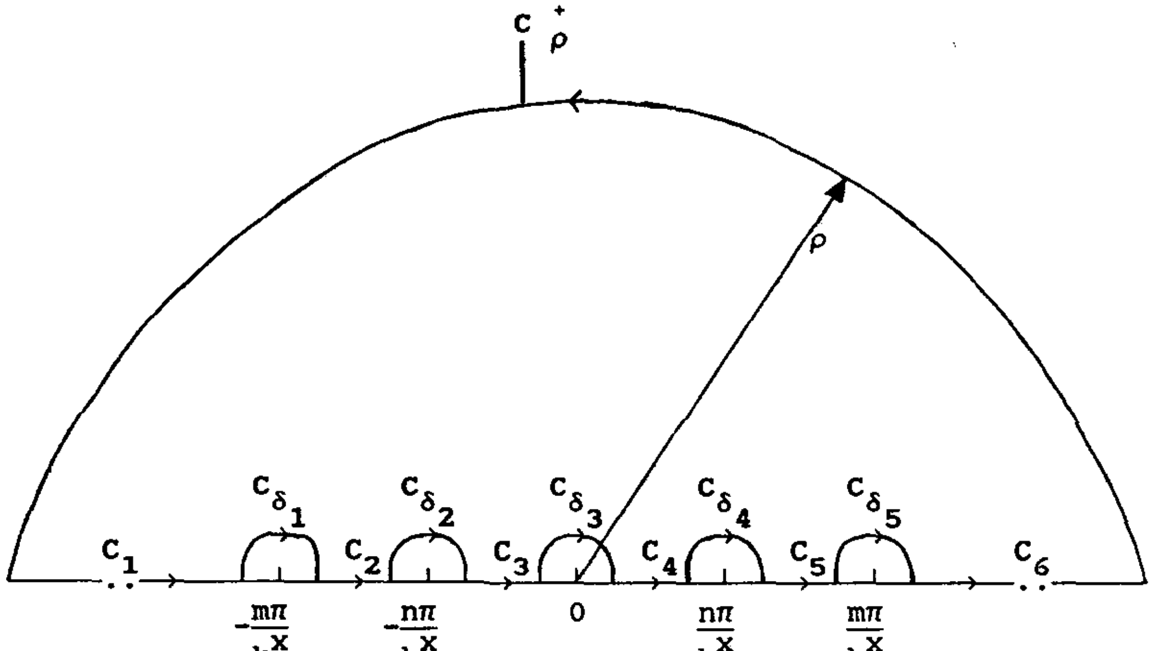


Fig. 5.2: Integration path in the complex z-plane, with β=Re(z)

In expression (5.8) all arguments are greater or equal zero. This implies that we can use Jordan's Lemma in order to determine the integral over C_ρ^+ [5,pp. 71]. This results in:

$$\lim_{\rho\to\infty} \int_{C_\rho^+} g(z)\,dz = 0 \tag{5.11}$$

The only thing left now, is to calculate the five integrals over $C_{\delta_1}, C_{\delta_2}, C_{\delta_3}, C_{\delta_4}$ and C_{δ_5}. If we take the zero-limit for each radius of the five half circles, we can write (5.10) as:

$$\begin{aligned} I_\beta(\alpha) &= \int_{-\infty}^{\infty} g(\beta)\,d\beta \\ &= -\lim_{\delta_i\to 0}\left(\int_{C_{\delta_1}} g(z)dz + \int_{C_{\delta_2}} g(z)dz + \int_{C_{\delta_3}} g(z)dz + \int_{C_{\delta_4}} g(z)dz + \int_{C_{\delta_5}} g(z)dz \right) \\ &= \pi j\left[\operatorname*{Res}_{z=-\frac{\pi m}{k^x}} g(z) + \operatorname*{Res}_{z=-\frac{\pi n}{k^x}} g(z) + \operatorname*{Res}_{z=0} g(z) + \operatorname*{Res}_{z=\frac{\pi n}{k^x}} g(z) + \operatorname*{Res}_{z=\frac{\pi m}{k^x}} g(z) \right] \end{aligned} \tag{5.12}$$

Where δ_i represents the radius of the half circle C_{δ_i}, $i=1,2,3,4,5$. The five integrals in (5.12) can be written in residual form, because the poles are single valued. Once these residues are known, $I_\beta(\alpha)$ is also known. The residues can be obtained very easily:

$$\begin{aligned} \operatorname*{Res}_{z=-\frac{\pi m}{k^x}} g(z) &= \lim_{z\to -\frac{\pi m}{k^x}} \frac{1}{k^x}(m\pi + zk^x)\,g(z) \\ &= \frac{jk^x}{16 m^3\pi^3\left[(n\pi)^2-(m\pi)^2\right]}\left[1-\frac{2(m\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y m\pi}{k^x}\right) \end{aligned} \tag{5.13a}$$

$$\operatorname*{Res}_{z=-\frac{\pi n}{k^x}} g(z) = \lim_{z\to-\frac{\pi n}{k^x}} \frac{1}{k^x}(n\pi+zk^x)g(z)$$

$$= \frac{jk^x}{16n^3\pi^3[(m\pi)^2-(n\pi)^2]}\left[1-\frac{2(n\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y n\pi}{k^x}\right) \quad (5.13b)$$

$$\operatorname*{Res}_{z=\frac{\pi m}{k^x}} g(z) = \lim_{z\to\frac{\pi m}{k^x}} -\frac{1}{k^x}(m\pi-zk^x)g(z)$$

$$= \frac{jk^x}{16m^3\pi^3[(n\pi)^2-(m\pi)^2]}\left[1-\frac{2(m\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y m\pi}{k^x}\right) \quad (5.13c)$$

$$\operatorname*{Res}_{z=\frac{\pi n}{k^x}} g(z) = \lim_{z\to\frac{\pi n}{k^x}} -\frac{1}{k^x}(n\pi-zk^x)g(z)$$

$$= \frac{jk^x}{16n^3\pi^3[(m\pi)^2-(n\pi)^2]}\left[1-\frac{2(n\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y n\pi}{k^x}\right) \quad (5.13d)$$

$$\operatorname*{Res}_{z=0} g(z) = \lim_{z\to 0} zg(z) = \frac{-jk^y}{8m^2n^2\pi^4} \quad (5.13e)$$

Together with (5.12) this finally results in:

$$I_\beta(\alpha) = \frac{-k^x}{8m^3\pi^2[(n\pi)^2-(m\pi)^2]}\left[1-\frac{2(m\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y m\pi}{k^x}\right)$$
$$+ \frac{-k^x}{8n^3\pi^2[(m\pi)^2-(n\pi)^2]}\left[1-\frac{2(n\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right]\sin\left(\frac{k^y n\pi}{k^x}\right) + \frac{k^y}{8m^2n^2\pi^3}$$
$$(5.14)$$

From expression (5.6) it is obvious that $\lim_{\alpha\to 0}\left[\frac{1}{\sin^2\alpha} I_\beta(\alpha)\right]$ must exist. It can be easily shown that the above expression fulfills this requirement.

a. $k^x<k^y$.

According to (5.7), $f(\beta)$ takes the form:

$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{-j\beta(k^x-k^y)} \tag{5.15}$$

Similar to the previous case, we may write $I_\beta(\alpha)$ in the form:

$$I_\beta(\alpha)=-\frac{1}{16}\int_{-\infty}^{\infty}\left[1-\frac{2\beta^2\cos^2\alpha}{\varepsilon_r+1}\right]\frac{f(\beta)}{(n\pi-\beta k^x)(n\pi+\beta k^x)(m\pi-\beta k^x)(m\pi+\beta k^x)\beta^2}\,d\beta$$

$$=\int_{-\infty}^{\infty} g(\beta)\,d\beta \tag{5.9}$$

In this case $g(\beta)$ has only 1 single valued pole at $\beta=0$. The procedure we have to follow in order to calculate $I_\beta(\alpha)$, is the same as in the case of $k^x\geq k^y$. We will use the integration path of figure 5.2. Using Jordan's Lemma for the integration path C_ρ^+, finally results in:

$$I_\beta(\alpha) = \int_{-\infty}^{\infty} g(\beta)\,d\beta = \pi j \operatorname*{Res}_{z=0} g(z) = \pi j \lim_{z\to 0} z g(z)$$

$$= \frac{-k^x+2k^y}{8m^2n^2\pi^3} \tag{5.16}$$

Note that for $k^x=k^y$ (5.14) and (5.16) are equal.

If we take m and n both even and m≠n the result differs from (5.16). $I_\beta(\alpha)$ now takes the form:

$$I_\beta(\alpha) = \frac{k^x}{4m^2n^2\pi^3} \qquad \text{m and n even m≠n} \tag{5.16a}$$

ii. m=n, m odd.

This situation is discussed apart from the previous one, because the β-integrand now has both single-valued poles as well as double valued poles. So we have to be careful in calculating the β-integral in the neighborhood of these double-valued poles. Like in the previous case (m≠n), we shall use Cauchy's theorem of residues in order to determine a closed form expression for $I_\beta(\alpha)$. For m equal to n, an Z^{xx}_{hmn} element takes the form:

$$Z^{xx}_{hmn} = A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\int_0^\infty\left[1-\frac{2\beta^2\cos^2\alpha}{\varepsilon_r+1}\right]\frac{\cos^2(\beta k^x/2)\sin^2(\beta k^y/2)}{(m\pi+\beta k^x)^2(m\pi-\beta k^x)^2\beta^2}\,d\beta d\alpha$$

$$= A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\,I_\beta(\alpha)\,d\alpha \tag{5.17}$$

with: $k^x=k_0\cos\alpha W_x$ $\qquad$ $A=\dfrac{-8j\omega\mu_0\pi^2m^2W_x^2}{k_0}$

$k^y=k_0\sin\alpha W_y$

We may write:

$$\cos^2(\beta k^x/2)\sin^2(\beta k^y/2) = -\frac{1}{16}[f(\beta)+f(-\beta)] \tag{5.7}$$
$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{j\beta|k^x-k^y|}$$

We can distinguish two situations, namely $k^x \geq k^y$ en $k^x<k^y$.

a. $k^x \geq k^y$.

This implies that:

$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{j\beta(k^x-k^y)} \tag{5.8}$$

We shall use the integration path of figure 5.3. The function $g(\beta)$ has two double valued poles at $\beta=\pm m\pi/k^x$ and one single valued pole at $\beta=0$.

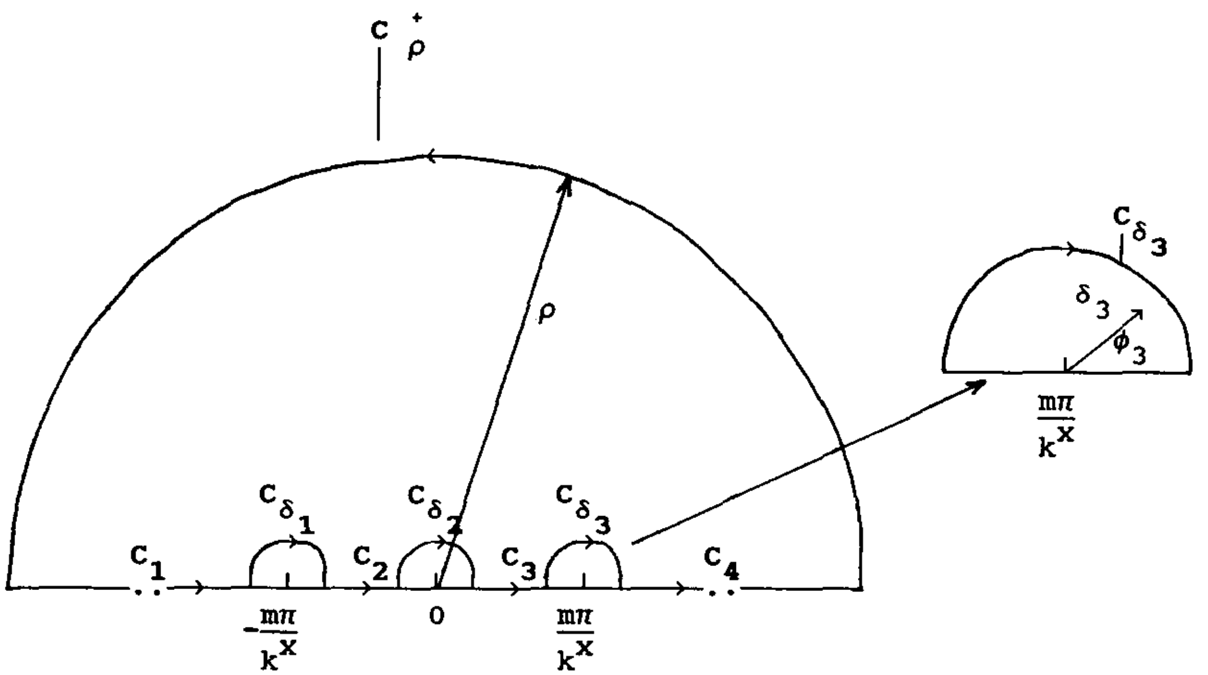


Fig. 5.3: Integration path in the complex z-plane, with $\beta=\mathrm{Re}(z)$

Using Cauchy's residual theorem and Jordan's Lemma $I_\beta(\alpha)$ reads:

$$I_\beta(\alpha) = \int_{-\infty}^{\infty} g(\beta)\,d\beta$$

$$= -\lim_{\delta_i \to 0} \left(\int_{C_{\delta_1}} g(z)dz + \int_{C_{\delta_2}} g(z)dz + \int_{C_{\delta_3}} g(z)dz \right)$$

$$= \pi j \operatorname*{Res}_{z=0} g(z) - \lim_{\delta_i \to 0} \left(\int_{C_{\delta_1}} g(z)dz + \int_{C_{\delta_3}} g(z)dz \right) \tag{5.18}$$

with:

$$\operatorname*{Res}_{z=0} g(z) = \lim_{z \to 0} zg(z) = \frac{-jk^y}{8m^4\pi^4}$$

and

$$I_\delta = \lim_{\delta_i \to 0} \left(\int_{C_{\delta_1}} g(z)dz + \int_{C_{\delta_3}} g(z)dz \right)$$

$$= \lim_{\delta_1 \to 0} \int_0^\pi g(-\frac{m\pi}{k^x} + \delta_1 e^{j\phi_1}) j\delta_1 e^{j\phi_1} d\phi_1 + \lim_{\delta_3 \to 0} \int_0^\pi g(\frac{m\pi}{k^x} + \delta_3 e^{j\phi_3}) j\delta_3 e^{j\phi_3} d\phi_3$$

where the substitutions $z=-\frac{m\pi}{k^x}+\delta_1 e^{j\phi_1}$ and $z=\frac{m\pi}{k^x}+\delta_3 e^{j\phi_3}$ are used. Combining the above two integrals and taking the limit for $\delta_1, \delta_3 \to 0$, we then find:

$$I_\delta = \int_0^\pi \lim_{\delta \to 0} \left[g(-\frac{m\pi}{k^x} + \delta e^{j\phi}) + g(\frac{m\pi}{k^x} + \delta e^{j\phi}) \right] j\delta e^{j\phi} d\phi$$

The limit for $\delta \to 0$ exists:

$$\lim_{\delta\to 0}\left[g(-\frac{m\pi}{k^x}+\delta e^{j\phi}) + g(\frac{m\pi}{k^x}+\delta e^{j\phi})\right]j\delta e^{j\phi} =$$

$$\left(\frac{1}{16m^4\pi^4} - \frac{\cos^2\alpha}{8\pi^2m^2(\varepsilon_r+1)k^{x2}}\right)\left(k^x + (k^y-k^x)\cos\left[\frac{\pi m k^y}{k^x}\right]\right)$$

$$-\left(\frac{3k^x}{16m^5\pi^5} - \frac{2\cos^2\alpha}{16\pi^3m^3(\varepsilon_r+1)k^x}\right)\sin\left[\frac{\pi m k^y}{k^x}\right]$$

Apparently the ϕ-integrand is independent of ϕ. So I_δ can now be easily obtained:

$$I_\delta = \pi \lim_{\delta\to 0}\left[g(-\frac{m\pi}{k^x}+\delta e^{j\phi}) + g(\frac{m\pi}{k^x}+\delta e^{j\phi})\right]j\delta e^{j\phi}$$

Using this result, $I_\beta(\alpha)$ then finally reads:

$$I_\beta(\alpha) = \int_{-\infty}^{\infty} g(\beta)\,d\beta$$

$$= \frac{k^y}{8\pi^3m^4} + \left(\frac{1}{16\pi^3m^4} - \frac{\cos^2\alpha}{8\pi m^2(\varepsilon_r+1)k^{x2}}\right)\left(k^x + (k^y-k^x)\cos\left[\frac{\pi m k^y}{k^x}\right]\right)$$

$$- \left(\frac{3k^x}{16\pi^4m^5} - \frac{\cos^2\alpha}{8\pi^2m^3(\varepsilon_r+1)k^x}\right)\sin\left[\frac{\pi m k^y}{k^x}\right] \qquad (5.19)$$

b. $k^x<k^y$.

Because of the fact that we want to use Jordan's Lemma on the integration path C_ρ^+ (see figure 5.3), we have to choose an $f(\beta)$ with all arguments greater of equal zero. Thus $f(\beta)$ has the form:

$$f(\beta) = -2-2e^{j\beta k^x}+2e^{j\beta k^y}+e^{j\beta(k^x+k^y)}+e^{-j\beta(k^x-k^y)} \tag{5.15}$$

The β-integrand has in this case three single-valued poles at $\beta=\pm m\pi/k^x$ and $\beta=0$. The procedure to follow is the same as it was in the previous situations. We shall choose the integration path of figure 5.3. Using Cauchy's theorem and Jordan's Lemma we finally get:

$$I_\beta(\alpha) = \int_{-\infty}^{\infty} g(\beta)\,d\beta$$

$$= \pi j\left[\operatorname*{Res}_{z=-\frac{\pi m}{k^x}} g(z) + \operatorname*{Res}_{z=0} g(z) + \operatorname*{Res}_{z=\frac{\pi m}{k^x}} g(z)\right]$$

$$\frac{2k^y-k^x}{8\pi^3 m^4} + \frac{k^x}{16\pi^3 m^4}\left[1-\frac{2(m\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right] \qquad \text{m odd} \tag{5.20}$$

Expression (5.20) was derived for the case that m (=n) is odd. In the case of an even m (=n) the result is slightly different for $k^x<k^y$. An element Z_{hmn}^{xx} has in this case the form:

$$Z_{hmn}^{xx} = A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\int_{0}^{\infty}\left[1-\frac{2\beta^2\cos^2\alpha}{\varepsilon_r+1}\right]\frac{\sin^2(\beta k^x/2)\sin^2(\beta k^y/2)}{(m\pi+\beta k^x)^2(m\pi-\beta k^x)^2\beta^2}\,d\beta d\alpha$$

$$= A\int_{-\pi}^{\pi}\frac{1}{\sin^2\alpha}\,I_\beta(\alpha)\,d\alpha \tag{5.21}$$

with: $k^x = k_o \cos\alpha W_x$ $\qquad A = \dfrac{-8j\omega\mu_o \pi^2 m^2 W_x^2}{k_o}$

$k^y = k_o \sin\alpha W_y$

The method for calculating $I_\beta(\alpha)$ is the same as for the situation where m was odd. The result for $k^x < k^y$ has the form:

$$I_\beta(\alpha) = \frac{k^x}{8\pi^3 m^4} + \frac{k^x}{16\pi^3 m^4}\left[1 - \frac{2(m\pi)^2\cos^2\alpha}{k^{x2}(\varepsilon_r+1)}\right] \qquad \text{m even} \qquad (5.22)$$

6. Calculation of the excitation vector [V]

The method for the efficient evaluation of the excitation vector [V], is practically equivalent to the case of calculating [Z]. However, due to the presence of a first order Besselfunction in the integrand, the calculation of the infinite integral is more complicated. The Besselfunction is introduced by the finite probe radius r_0 of the coaxial feed. A constant current density on the probe is assumed.

The two-dimensional integrals that have to be evaluated numerically, have according to (2.7) the following general form:

$$V_m = \int_{-\pi}^{\pi}\int_{0}^{\infty} f(\beta,\alpha)\ d\beta d\alpha \tag{6.1}$$

Where we used the change in variables (2.8)

The β-integration is divided in three parts:

$$V_m = \int_{-\pi}^{\pi}\left(\int_{0}^{1} f(\beta,\alpha)d\beta + \int_{1}^{\sqrt{\varepsilon'_r}} f(\beta,\alpha)d\beta + \int_{\sqrt{\varepsilon'_r}}^{\infty} f(\beta,\alpha)d\beta \right) d\alpha \tag{6.2}$$

In this paragraph the method for evaluating each of the three integrals will be discussed.

6.1 Calculation of the β-integral in the region $0\le\beta\le1$

Here we have the same situation as discussed in paragraph 5.1. Thus substituting β=cost in the integral results in:

$$\int_{-\pi}^{\pi}\int_{0}^{1} f(\beta,\alpha)d\beta d\alpha = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\mathrm{cos}t,\alpha)\mathrm{sin}t\ dtd\alpha \tag{6.3}$$

This integral can be evaluated numerically without any further problems.

6.2 Calculation of the β-integral in the region $1\le\beta\le\sqrt{\varepsilon'_r}$

Here we have the same problems as discussed in paragraph 5.2. Using the substitution β=cosht, the numerical problems at β=1 can be eliminated. This results in the expression:

$$\int_{-\pi}^{\pi}\int_{1}^{\sqrt{\varepsilon'_r}} f(\beta,\alpha)\,d\beta = \int_{-\pi}^{\pi}\int_{0}^{\operatorname{arccosh}(\sqrt{\varepsilon'_r})} f(\cosh t,\alpha)\sinh t\;dt\,d\alpha \qquad (6.4)$$

6.3 Calculation of the infinite β-integral: source term extraction

The method that we shall use in this paragraph is the same method as was previously discussed in paragraph 5.3 for the elements of the matrix [Z]. Again, the asymptotic part ($\beta\to\infty$) of the β-integrand is subtracted from the original integrand, ensuring a fast convergence. The infinite integration over the subtracted part is done analytically.
Using this source term extraction technique, an element of the vector [V] is written as (see (2.7)):

$$V_m = \int_0^{\infty}\int_{-\pi}^{\pi} [(\bar{\tilde{Q}}_v-\bar{\tilde{Q}}_{hv}).\vec{\tilde{J}}_{pm}].\vec{\tilde{J}}^{\,*}_s\; k_0^2\beta\,d\alpha\,d\beta$$

$$+\int_0^{\infty}\int_{-\pi}^{\pi} [\bar{\tilde{Q}}_{hv}.\vec{\tilde{J}}_{pm}].\vec{\tilde{J}}^{\,*}_s\; k_0^2\beta\,d\alpha\,d\beta = [V_m-V_{hm}]+V_{hm} \qquad (6.5)$$

Where $\bar{\tilde{Q}}_{hv}$ is the asymptotic value of $\bar{\tilde{Q}}_v$. It can be easily shown that the asymptotic value of $\bar{\tilde{Q}}_v$, given by expression (2.4.17) of reference [1], takes the following form for large β:

$$\bar{\tilde{Q}}_{hv} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{Q}_{hvzx} & \tilde{Q}_{hvzy} & 0 \end{bmatrix} \qquad (6.6)$$

<u>**z'=d**</u>:

$$\tilde{Q}_{hvzx} = \frac{-\omega\mu_0\cos\alpha}{k_0^2(\varepsilon_r+1)}$$

$$\tilde{Q}_{hvzy} = \frac{-\omega\mu_0\sin\alpha}{k_0^2(\varepsilon_r+1)}$$

<u>**z'<d**</u>:

$$\tilde{Q}_{hvzx} = \frac{-\omega\mu_0\cos\alpha}{2k_0^2\varepsilon_r}$$

$$\tilde{Q}_{hvzy} = \frac{-\omega\mu_0\sin\alpha}{2k_0^2\varepsilon_r}$$

The derivation of V_{hm} in this chapter will be done for elements of the sub vector $[V^x]$ for the case that $z'=d$ (thus $\hat{x}$-directed entire domain sinusoidal basis functions and the patch located on top of the substrate). For the other elements of [V] and for the case that $z'<d$, the source extraction technique can be applied in exact the same manner. According to (6.5), (4.7) (m odd) and (4.8) (m even), an element of $[V^x]$ can be written as:

<u>**i. m odd**</u>

$$V_m^x = -4i\int_0^\infty\int_0^{\pi/2} [\tilde{Q}_{vx}-\tilde{Q}_{hvx}]\tilde{J}_{pmx}\sin(\beta k_s^x)\cos(\beta k_s^y)J_0(r_0k_0\beta)k_0^2\beta d\alpha d\beta$$

$$- \quad 4i\int_0^\infty\int_0^{\pi/2} \tilde{Q}_{hvx}\tilde{J}_{pmx}\sin(\beta k_s^x)\cos(\beta k_s^y)J_0(r_0k_0\beta)\,k_0^2\beta\, d\alpha\, d\beta$$

$$= V_{hm}^x + [V_m^x - V_{hm}^x] \tag{6.7a}$$

ii. m even

$$V_m^x = 4\int_0^\infty\int_0^{\pi/2} [\tilde{Q}_{vx}-\tilde{Q}_{hvx}]\tilde{J}_{pmx}\cos(\beta k_s^x)\cos(\beta k_s^y)J_0(r_0k_0\beta)\,k_0^2\beta\, d\alpha\, d\beta$$

$$+ \quad 4\int_0^\infty\int_0^{\pi/2} \tilde{Q}_{hvx}\tilde{J}_{pmx}\cos(\beta k_s^x)\cos(\beta k_s^y)J_0(r_0k_0\beta)\,k_0^2\beta\, d\alpha\, d\beta$$

$$= V_{hm}^x + [V_m^x - V_{hm}^x] \tag{6.7b}$$

with:

$k_s^x = k_0 x_s \cos\alpha$

$k_s^y = k_0 y_s \sin\alpha$ $\qquad (x_s, y_s)$:excitation point probe (fig. 1.1)

J_0 is the Besselfunction of the first kind, order 0.
In order to determine the infinite β-integral in V_{hm}^x, we will make use of Cauchy's residue theorem and Jordan's Lemma [5]. The derivation is performed for the case that m is odd. For even values of m, only the final results are given.
Combining of (3.2) with (6.6) and (6.7a) results in:

$$V_{hm}^x = A\int_0^\infty \frac{\cos\alpha}{\sin\alpha}\int_0^{\pi/2} \frac{\cos(\beta k^x/2)\sin(\beta k^y/2)\sin(\beta k_s^x)\cos(\beta k_s^y)}{(\pi m - k^x\beta)(\pi m + k^x\beta)}\, J_0(r_0k_0\beta)\, d\beta\, d\alpha \tag{6.8}$$

with:

$k^x = k_0 \cos\alpha W_x$

$k^y = k_0 \cos\alpha W_y$ $\qquad A = \dfrac{16 j\pi m W_x \omega\mu_0}{k_0(\varepsilon_r + 1)}$

The Besselfunction J_0 will be used in integral form:

$$J_0(r_0k_0\beta) = \frac{1}{2\pi}\int_0^{2\pi} e^{j\beta k_\vartheta}\, d\vartheta \tag{6.9}$$

with: $k_\vartheta = r_0k_0\sin\vartheta$

Furthermore, the cos-sin product in (6.8) is expanded in a series of e-powers.

$$\cos(\beta k^x/2)\sin(\beta k^y/2)\sin(\beta k_s^x)\cos(\beta k_s^y) = -\frac{1}{16}[f(\beta)+f(-\beta)] \tag{6.10}$$

with:

$$\begin{aligned} f(\beta) &= e^{j\beta|k^x/2+k^y/2+k_s^x+k_s^y|} + e^{j\beta|k^x/2+k^y/2+k_s^x-k_s^y|} \\ &- e^{j\beta|k^x/2+k^y/2-k_s^x+k_s^y|} - e^{j\beta|k^x/2+k^y/2-k_s^x-k_s^y|} \\ &- e^{j\beta|k^x/2-k^y/2+k_s^x+k_s^y|} - e^{j\beta|k^x/2-k^y/2+k_s^x-k_s^y|} \\ &+ e^{j\beta|k^x/2-k^y/2-k_s^x+k_s^y|} + e^{j\beta|k^x/2-k^y/2-k_s^x-k_s^y|} \end{aligned}$$

Using the above expression, it is possible to extend the lower β-integration boundary to $-\infty$. Together with the integral representation of J_0, we then may write:

$$\begin{aligned} V_{hm}^x &= A\int_0^{\pi/2} \frac{\cos\alpha}{\sin\alpha} \int_{-\infty}^{\infty} \left[\frac{-\frac{1}{16}\, f(\beta)}{(\pi m-k^x\beta)\,(\pi m+k^x\beta)} \right] J_0(r_0k_0\beta)\, d\beta d\alpha \\ &= A\int_0^{\pi/2} \frac{\cos\alpha}{\sin\alpha}\, \frac{1}{2\pi} \int_0^{2\pi}\int_{-\infty}^{\infty} \left[\frac{-\frac{1}{16}\, f(\beta)\, e^{j\beta k_\vartheta}}{(\pi m-k^x\beta)\,(\pi m+k^x\beta)} \right] d\beta d\vartheta d\alpha \end{aligned}$$

$$= A\int_0^{\pi/2} \frac{\cos\alpha}{\sin\alpha}\,\frac{1}{2\pi}\int_0^{2\pi} I_{\beta v}(\alpha,\vartheta)\;d\vartheta d\alpha \tag{6.11}$$

Our aim is now to calculate the β-integral with infinite boundaries. We shall choose the integration path of figure 5.3 with $\delta_2=0$, because the β-integrand has no pole at $\beta=0$. The β-integrand has 2 single valued poles in this case at $\beta=\pm\pi m/k^x$. We may again use Jordan's Lemma for C_ρ^+ if the arguments of the e-powers in (6.11) all are greater or equal 0. The easiest way to calculate $I_{\beta v}$ is to divide the integral in 8 different parts, $I_{\beta v1},..,I_{\beta v8}$. This then results in:

$$I_{\beta vi}(\alpha,\vartheta) = \int_{-\infty}^{\infty}\left[\frac{g_i(\beta)}{(\pi m-k^x\beta)(\pi m+k^x\beta)}\right]d\beta \tag{6.12}$$

$$I_{\beta v}(\alpha,\vartheta) = \sum_{i=1}^{8} I_{\beta vi}(\alpha,\vartheta)$$

with:

$$g_1(\beta)=e^{j\beta|k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta|} \qquad g_5(\beta)=e^{j\beta|k^x/2-k^y/2+k_s^x+k_s^y+k_\vartheta|}$$
$$g_2(\beta)=e^{j\beta|k^x/2+k^y/2+k_s^x-k_s^y+k_\vartheta|} \qquad g_6(\beta)=e^{j\beta|k^x/2-k^y/2+k_s^x-k_s^y+k_\vartheta|}$$
$$g_3(\beta)=e^{j\beta|k^x/2+k^y/2-k_s^x+k_s^y+k_\vartheta|} \qquad g_7(\beta)=e^{j\beta|k^x/2-k^y/2-k_s^x+k_s^y+k_\vartheta|}$$
$$g_4(\beta)=e^{j\beta|k^x/2+k^y/2-k_s^x-k_s^y+k_\vartheta|} \qquad g_8(\beta)=e^{j\beta|k^x/2-k^y/2-k_s^x-k_s^y+k_\vartheta|}$$

As an example we shall now calculate $I_{\beta v1}$. The other 7 integrals $I_{\beta v2},..,I_{\beta v8}$, can be evaluated using exact the same strategy. In the case of $I_{\beta v1}$, we can distinguish two situations:

i. $k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta\geq 0$

ii. $k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta<0$

i. $k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta \geq 0$.

According to expression (6.12), $g_1(\beta)$ has now the form:

$$g_1(\beta)=e^{j\beta(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}$$

Because the argument of the above e-power is always greater or equal zero, we may use Jordan's Lemma on C_ρ^+.

$I_{\beta v1}$ can be written as:

$$I_{\beta v1}(\alpha,\vartheta) = -\frac{1}{16}\int_{-\infty}^{\infty}\left[\frac{e^{j\beta(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}}{(\pi m-k^x\beta)(\pi m+k^x\beta)}\right]d\beta = \int_{-\infty}^{\infty} h(\beta)\,d\beta$$

Choosing the integration path of figure 5.3 and using Jordan's Lemma for C_ρ^+, leads to:

$$I_{\beta v1}(\alpha,\vartheta) = \pi j\left[\operatorname*{Res}_{z=-\frac{\pi m}{k^x}} g(z) + \operatorname*{Res}_{z=\frac{\pi m}{k^x}} g(z)\right]$$

with: $$\operatorname*{Res}_{z=-\frac{\pi m}{k^x}} g(z) = \frac{-1}{32\pi m k^x}\left[e^{-j\frac{\pi m}{k^x}(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}\right]$$

$$\operatorname*{Res}_{z=\frac{\pi m}{k^x}} g(z) = \frac{1}{32\pi m k^x}\left[e^{j\frac{\pi m}{k^x}(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}\right]$$

This finally gives:

$$I_{\beta v1}(\alpha,\vartheta) = \frac{-1}{16mk^x}\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right] \tag{6.13}$$

ii. $k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta<0$.

In order to fulfill all conditions concerning Jordan's Lemma, $g_1(\beta)$ is written as (see (6.12)):

$$g_1(\beta)=e^{-j\beta(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}$$

$I_{\beta v1}$ then reads:

$$I_{\beta v1}(\alpha,\vartheta) = -\frac{1}{16}\int_{-\infty}^{\infty}\left[\frac{e^{-j\beta(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)}}{(\pi m-k^x\beta)(\pi m+k^x\beta)}\right]d\beta$$

Following the same procedure as in the previous case finally results in:

$$I_{\beta v1}(\alpha,\vartheta) = \frac{1}{16mk^x}\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right] \tag{6.14}$$

Note that (6.14) and (6.13) only differ by a minus sign.

Once the remaining 7 integrals, $I_{\beta v2}$ ... $I_{\beta v8}$, are also known, $I_{\beta v}$ can be determined. For the case that m is odd this gives:

$$\begin{aligned}
I_{\beta v}(\alpha,\vartheta)=\frac{-1}{16mk^x}\Bigg[&\mathrm{sgn}(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right]\\
&+\mathrm{sgn}(k^x/2+k^y/2+k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x-k_s^y+k_\vartheta\right)\right]\\
&-\mathrm{sgn}(k^x/2+k^y/2-k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2-k_s^x+k_s^y+k_\vartheta\right)\right]\\
&-\mathrm{sgn}(k^x/2+k^y/2-k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2-k_s^x-k_s^y+k_\vartheta\right)\right]\\
&-\mathrm{sgn}(k^x/2-k^y/2+k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right]
\end{aligned}$$

$$- \operatorname{sgn}(k^x/2-k^y/2+k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2+k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$+ \operatorname{sgn}(k^x/2-k^y/2-k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2-k_s^x+k_s^y+k_\vartheta\right)\right]$$

$$+ \operatorname{sgn}(k^x/2-k^y/2-k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2-k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$\mathbf{m\ odd} \qquad (6.15)$$

For even values of m, the result is slightly different. According to (6.7b), (3.2) and (6.6), V_{hm}^x is in this case written as:

$$V_{hm}^x = A\int_0^\infty \frac{\cos\alpha}{\sin\alpha}\int_0^{\pi/2} \frac{\sin(\beta k^x/2)\sin(\beta k^y/2)\cos(\beta k_s^x)\cos(\beta k_s^y)}{(\pi m-k^x\beta)(\pi m+k^x\beta)} J_0(r_0 k_0\beta)\,d\beta\,d\alpha \qquad (6.16)$$

with:

$$k^x=k_0\cos\alpha W_x \qquad A=\frac{16j\pi m W_x\omega\mu_0}{k_0(\varepsilon_r+1)}$$

$$k^y=k_0\cos\alpha W_y$$

J_0 is given by its integral representation (6.9). Using the same method as described above for the case of odd m values, the resulting expression for $I_{\beta v}$ becomes:

$$I_{\beta v}(\alpha,\vartheta)=\frac{-1}{16mk^x}\Bigg[\operatorname{sgn}(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right]$$

$$+ \operatorname{sgn}(k^x/2+k^y/2+k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2+k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$+ \operatorname{sgn}(k^x/2+k^y/2-k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2-k_s^x+k_s^y+k_\vartheta\right)\right]$$

$$+ \operatorname{sgn}(k^x/2+k^y/2-k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2+k^y/2-k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$- \operatorname{sgn}(k^x/2-k^y/2+k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2+k_s^x+k_s^y+k_\vartheta\right)\right]$$

$$- \operatorname{sgn}(k^x/2-k^y/2+k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2+k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$- \operatorname{sgn}(k^x/2-k^y/2-k_s^x+k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2-k_s^x+k_s^y+k_\vartheta\right)\right]$$

$$- \operatorname{sgn}(k^x/2-k^y/2-k_s^x-k_s^y+k_\vartheta)\sin\left[\frac{\pi m}{k^x}\left(k^x/2-k^y/2-k_s^x-k_s^y+k_\vartheta\right)\right]$$

$$m \text{ even} \qquad (6.17)$$

7. Conclusion

The time consuming infinite integrals associated with the spectral domain moment method solution for microstrip antennas can be computed efficiently by using the methods proposed in the report:

* use the symmetry of the basis functions to reduce the number of integrals that have to be evaluated (chapter 4)
* eliminate the infinite derivative in the integrands by using a proper change of variables (par. 5.1,5.2,6.1 and 6.2)
* use the source term extraction method, where the asymptotic part of the slowly converging integrand is subtracted from the original integrand. The integration over the asymptotic part can be done in closed form, resulting in a reduced computation time.

Applying these methods, results in a reduction of computation time by a factor 20.

8. References

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