

Chua's circuit

Citation for published version (APA):

van den Broek, T. H. A. (2004). *Chua's circuit: synchronisation*. (DCT rapporten; Vol. 2004.072). Technische Universiteit Eindhoven.

Document status and date:

Published: 01/01/2004

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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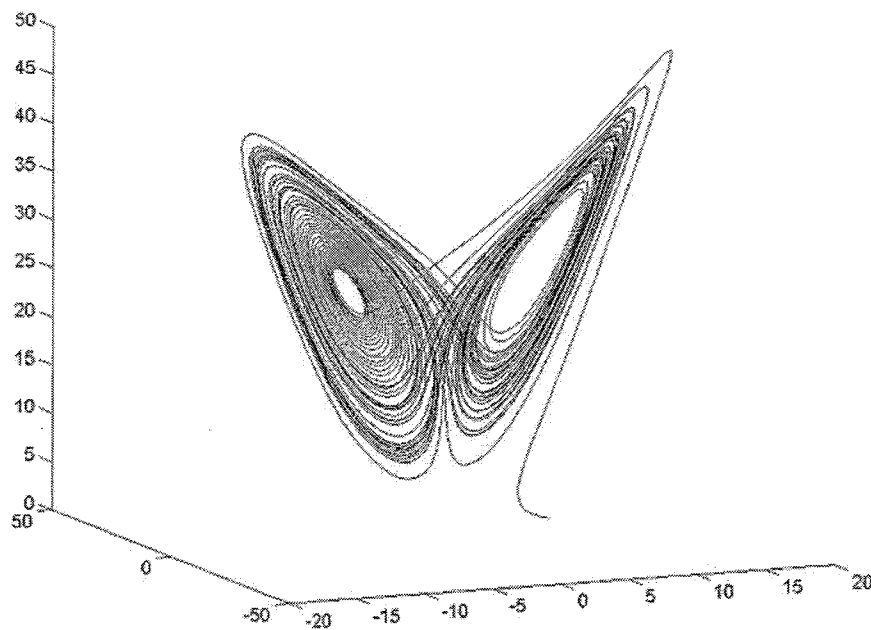
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Chua's Circuit: Synchronisation



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DCT 2004.72

Eindhoven, 5 Mai 2004

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Introduction

What is chaos? When engineers use the word chaos, they normally mean that a predictable dynamic system can give unpredictable results. The easiest way to observe chaos is in electronic circuits. This is because of its simplicity, inexpensive and because electronic devices are well understood.

Chua's circuit is an example of a chaotic circuit. But because of its simplicity and universality, this circuit is bit more special. A lot of questions can be asked about this system. In this report an answer will be given to the question whether it is possible to synchronise two chua's circuits. The two chua's circuits will have different starting values and/or different values for the components.

First there will be looked at the history of chaos. After that the theory of the chua's circuit will be explained, with experimental results. When it is understood how the circuit works there will be explained to what extend the circuit is controlled and how it can be synchronised.

Chapter 1: History of Chaos

The first time that chaos is found in history is in 1885 in Sweden. King Oscar II had a question about a 'three-body problem'. Three planets in space, which move in regard to each other by mutual gravitation forces. The person who would solve this question would receive a price. The same question, but then with two planets, was years before, solved by Newton. But this case is much more difficult.

A mathematic named Henri Poincaré sends in an opinion of what he thinks is the answer. Moments before publication he noticed he has made a mistake. He explained that the problem in some way is unsolvable. The reason why this problem is unsolvable is that a very small difference in the start position or velocity can make a huge different in the final result. With this conclusion Poincaré had the chaos principle.

Edward Lorenz has also an important place in the history of chaos. In the sixties he developed computer models. One of these models is of the earth atmosphere at the northern hemisphere. He made a lot of calculations to six decimal places. When he rounded off these numbers to three decimal places, the result was astonishing. In the beginning the old and new calculations coincided, but after some time the results were totally different. The conclusion is that the results of his calculation are strongly depended of the starting point. A small different in the start position can lead to large differences in the final result. When he repeated the process many times, by which he used the results as starting points. The differences in the results are plotted in a three-dimensional graph, a butterfly appears then. This graph is also called the Lorenz attractor. See figure 1.

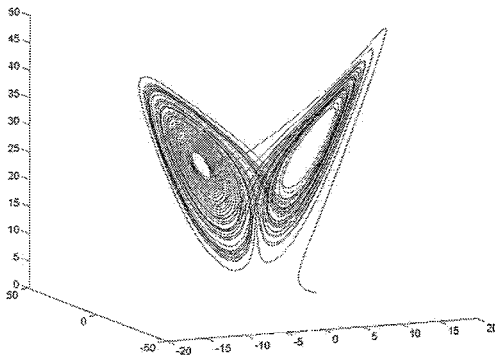


Figure 1: The Lorenz attractor

The first observations of chaos in electrical circuits are in 1927. By Van der Pol and Van der Mark. They see it in nonlinear oscillators. In 1980 Ueda and Akamatsu find chaos in negative resistance oscillators. But the circuit of Chua is a bit more special. Because of its simplicity and universality. Chua's circuit is dated from 1993. This circuit will be used in the experiments of controlling chaos.

Chapter 2: The theory of Chua's circuit

Chaotic dynamics can be described in many different differential equations. That can be build into a electrical circuit. An example of an electrical circuit, which is used in this research, is Chua's circuit.

The differential equations of Chua's circuit are here below:

$$\begin{aligned} C_1 \frac{dv_1}{dt} &= \frac{1}{R}(v_2 - v_1) - f(v_1) \\ C_2 \frac{dv_2}{dt} &= \frac{1}{R}(v_1 - v_2) + i_L \\ L \frac{di_L}{dt} &= -v_2 - R_0 i_L \end{aligned} \quad (1)$$

In the figure below it is shown how Chua's circuit looks like. Where C_1 and C_2 are the parameters of the conductors. R is the changeable resistance, R_0 is the internal resistance of the coil and L is the constant value of the coil.

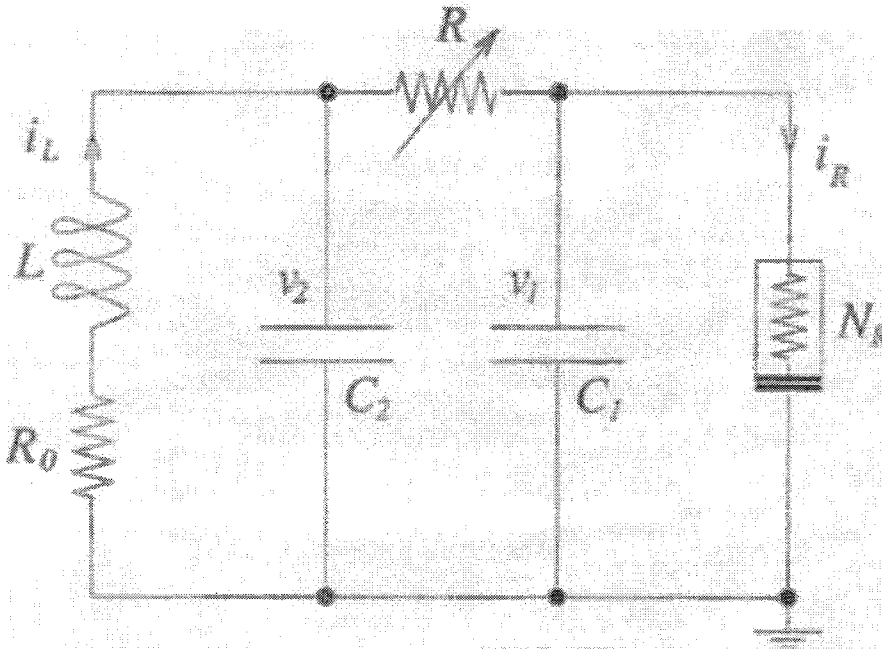


Figure 2: The schematic representation of Chua's circuit

N_R is the value of the negative resistance. The measured values are v_1 , v_2 and i_L . These values can be measured, which will give experimental results. But they can also theoretical be calculated by the differential equations above.

The value of N_R depends of $f(v_1)$. Equation (2) is $f(v_1)$. The values of G_a , G_b and B_p are explained in figure 3.

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + B_p| - |v_1 - B_p|) \quad (2)$$

Equation (2) can also be written according to the following equation, this is because G_b and B_p can have different values. The steepness of the graph and the breakpoints can have different values (see figure 3):

$$f(v_1) = G_b(v_1)v_1 + (G_a - G_b(v_1))f_1(v_1) \quad (3)$$

For $f_1(v_1)$ is:

$$f_1(v_1) = \begin{cases} v_1 & \text{if } |v_1| < B_p(v_1) \\ \text{sign}(v_1)B_p(v_1) & \text{otherwise} \end{cases} \quad (4)$$

Where:

$$G_b(v_1) = \frac{1}{2} [G_b^+(1 + \text{sign}(v_1)) + G_b^-(1 - \text{sign}(v_1))] = \begin{cases} G_b^+, & \text{if } v_1 > 0 \\ G_b^-, & \text{if } v_1 < 0 \end{cases} \quad (5)$$

And:

$$B_p(v_1) = \frac{1}{2} [B_p^+(1 + \text{sign}(v_1)) + B_p^-(1 - \text{sign}(v_1))] = \begin{cases} B_p^+, & \text{if } v_1 > 0 \\ B_p^-, & \text{if } v_1 < 0 \end{cases} \quad (6)$$

These functions are implemented in MATLAB. The script for the equation $f(v_1)$ is shown in appendix 2. The values for the parameters are the same as Santaboni has chosen. He also has worked with Chua's circuit for the Department of Mechanical Engineering. The values for C_1 , C_2 , L , R_L , G_a , G_b^+ , G_b^- , B_p^+ and B_p^- are respectively $10.0nF$, $100.0nF$, $22.0mH$, 10.0Ω , $-0.757mS$, $-0.410mS$, $-0.410mS$, $1.70V$ and $1.70V$. With matrices $fv11$ and $v11$ is the figure plotted. The figure is shown below, the value of G_a , G_b and B_p are also shown in the figure, as said before.

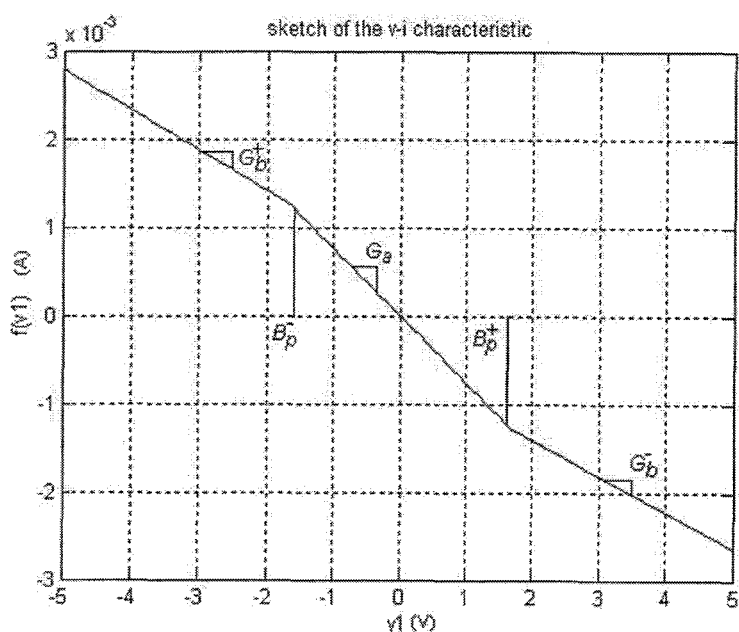


Figure 3: The sketch of the $v-i$ characteristic

The range of v_1 is between the 5 and -5 Volt. This is because the breakpoints B_p^+ and B_p^- are clearly shown. And also, as will be shown in later experimental results, the output value of v_1 and v_2 will lay in that area.

When the parameters of the electrical components are known, the differential equations can be implemented in MATLAB. The script is divided into two parts. One part solves the differential equation by using ode45. Therefore it has to call up a function that will be solved by the ode45. The most important parts of the script are given below. The entire file is also listed in appendix 1.

```
[t,v] = ode45(@dvdt, Tspan, v0, options);
```

The solver for the differential equations is ode45. MATLAB has a lot of different solver. Ode45 is one the best. Which solvers MATLAB has, and what the different accuracies the solvers are, is shown in table 1. The table is copied of the MATLAB help file.

Table 1: The MATLAB help file about the different solvers

Solver	Problem Type	Order of Accuracy	When to Use
ode45	Nonstiff	Medium	Most of the time. This should be the first solver you try.
ode23	Nonstiff	Low	If using crude error tolerances or solving moderately stiff problems.
ode113	Nonstiff	Low to high	If using stringent error tolerances or solving a computationally intensive ODE file.
ode15s	Stiff	Low to medium	If ode45 is slow because the problem is stiff.
ode23s	Stiff	Low	If using crude error tolerances to solve stiff systems and the mass matrix is constant.
ode23t	Moderately Stiff	Low	If the problem is only moderately stiff and you need a solution without numerical damping.
ode23tb	Stiff	Low	If using crude error tolerances to solve stiff systems.

The solver first calls up the function, then the time range, with time step, and after that the starting value and the different tolerances. The data that will be calculated is a matrix with a couple of columns. One column has the time (t). In the other is v_1 , v_2 and v_3 . With v_3 is i_L mentioned.

The function that is used is written in a matrix. Again a part of script is shown. This time it is the function.

```
if abs(v(1)) < Bpv1
    dv = [ (1/R*(v(2)-v(1))-(Gbv1*v(1)+(Ga-Gbv1)*v(1)))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L ];
else
    dv = [ (1/R*(v(2)-v(1))-(Gbv1*v(1)+(Ga-Gbv1)*sign(v(1))*Bpv1))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L ];
end
```

As shown in this little part, the function of $f(v_1)$ is directly implemented into the function. This is easier programming.

With this script it is possible to calculate the out coming signals as v_1 , v_2 and i_L for different starting values and a different value for the adjustable parameter R . Plots can

be made, in which time is set to one of the out coming signals. It is also feasible to plot v_1 to v_2 . Then there can be made a difference between a chaotic output and a non-chaotic output. An example is shown.

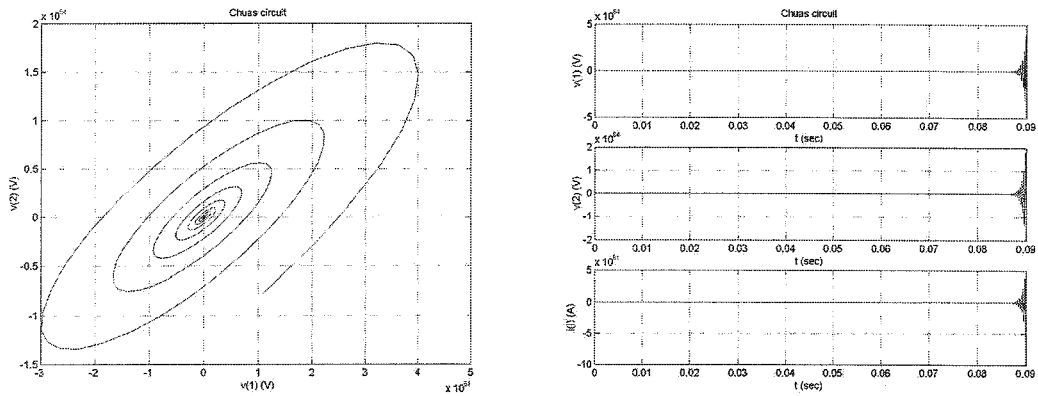


Figure 4: Chaos, with R is $1.55e3$ ohm

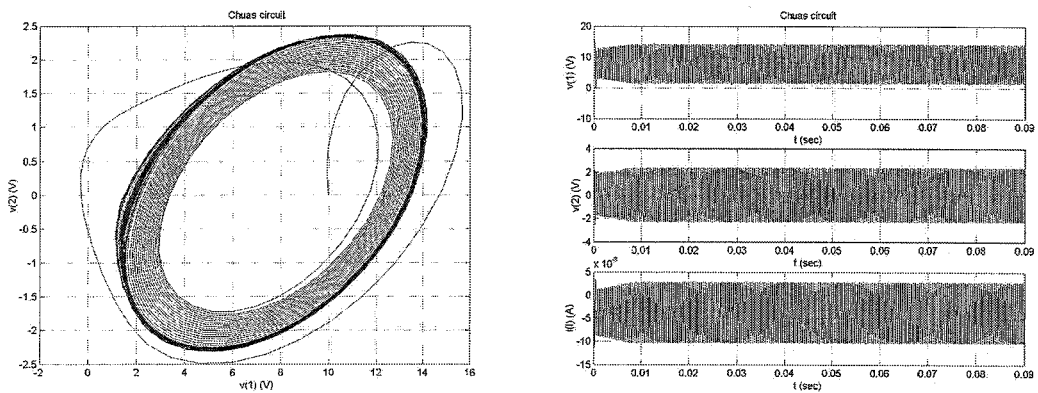


Figure 5: Non-chaos with R is $2.05e3$ ohm

The upper one is a chaotic output and the lower one is a non-chaotic output. Clearly it is shown that the non-chaotic output is in some way stable. It repeats a certain movement around a point. In the right plot of figure 5 is shown there is a constant period that repeats itself. Instead of the chaotic output, which goes to infinite values. Of course these values are calculated from the differential equations, in reality there is a maximum output voltage of 15 volt.

Chapter 3: Experimental results

The theory of Chua's circuit is known now. The circuit is also build from different electronically components, so that the theory can be compared with the experiments. The inductor and negative resistor are simulated by different operational amplifiers (OpAmps), resistors and condensators. How the electric circuit looks like, see appendix 6. The builded Chua circuit has the same values as the theoretical Chua circuit in figure 2.

With the use of the computer programme Siglab, the v_1 , v_2 and i_L can be measured. In the program two plots are shown. One of the plots is v_1 to v_2 and the other plot is v_1 to time and v_2 to time. The data that is saved has the extinction -.vos. Another computer program can recall this data. MATLAB is used for that. It loads the structure and in this structure all matrices are implanted. In the Siglab map exist a 'Measure Structure' file, to find the proper matrices with data. With this help a MATLAB script is made. See appendix 3.

The adjustable resistance R can vary between zero and one thousand ohm. But because another resistor of one thousand ohm is in serie, the adjustable resistor varies between one thousand and two thousand ohm.

First it is analysed how the outcoming voltage v_1 and v_2 change when the resistor changes. The system has al lot of stable points. In a stable point there is a periodic solution for v_1 and v_2 . An example of a stable point is given below. Here the system repeats itself. The v_1 - v_2 plot is of a double loop.

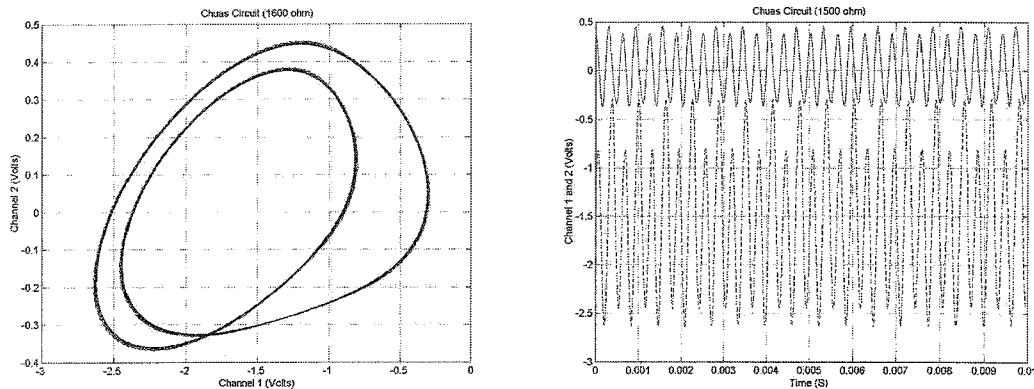


Figure 5: A stable double loop with adjustable R is 1600 ohm

The striped graph represents voltage one in respect of the time and the other graph is voltage two. For each period there are two amplitudes shown with different heights. They are the two loops that are plotted in the first figure.

In figure 6 is an example of a non-stable point. It is not possible to determine a single period. That is why this working point is unstable. Again the voltages in regard of the time are shown.

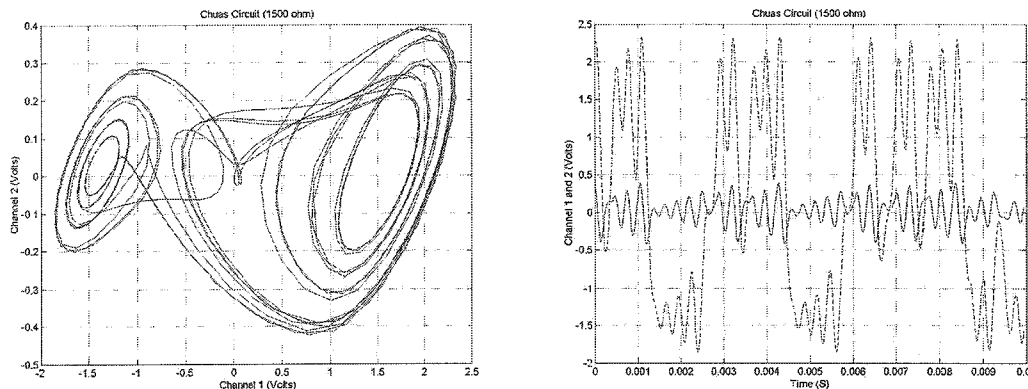


Figure 6: An non-stable point with adjustable R is 1500 ohm

It becomes clear that the system is chaotic, because a little variation of the adjustable resistor, can give high difference in the voltages.

First it is examined what the behaviour is when the adjustable resistor is changed upwards or downwards. Each time a step of one hundred ohm is taken. It is expected that the system will give the same results for v_1 , v_2 and i_L by the same value for the adjustable resistor, but this is not true. In appendix 7 is shown what the outcoming voltages are by changing the resistor from one thousand ohm to two thousand ohm. And in appendix 8 are the plots from two thousand ohm to one thousand ohm. The plot of eighteen hundred, nineteen hundred and two thousand ohm of both appendices are not interesting. This is because the resistor is too high and the outcoming voltages do not really vary any more. In most cases taking steps of one hundred ohm is accurate enough. The reason for this experiment is to see if there are differences when the adjustable resistor is increased or decreased. And the result is clearly shown in figure 7.

Mainly the plots of both appendices are the same, but by fourteen hundred ohm there is a big different. The graphs are shown below.

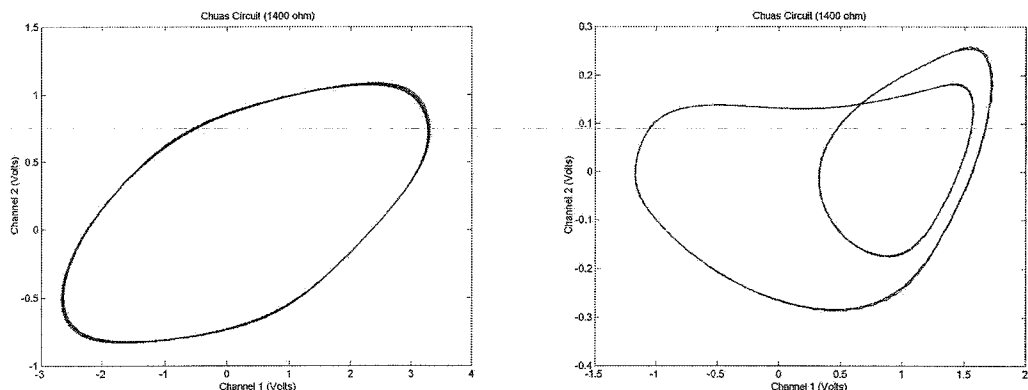


Figure 7: The outcoming voltages with adjustable R is 1400 ohm

The left figure is when the adjustable resistor is changed from one thousand ohm to two thousand ohm and the right figure is when the resistor is changed from two thousand ohm to one thousand ohm.

The difference between the two figures can be explained because the system is chaotic. Small difference in the start values can result in huge difference in the ending values.

The second strange behaviour is that the system has different stable points, when the adjustable resistor is changed. When the resistor is changed from one thousand ohm to two thousand ohm it has seven stable points, but when it is changed from two thousand ohm to one thousand ohm there are fourteen stable points. See the appendices 9 and 10 for the different stable points.

Another behaviour of the system is shown below.

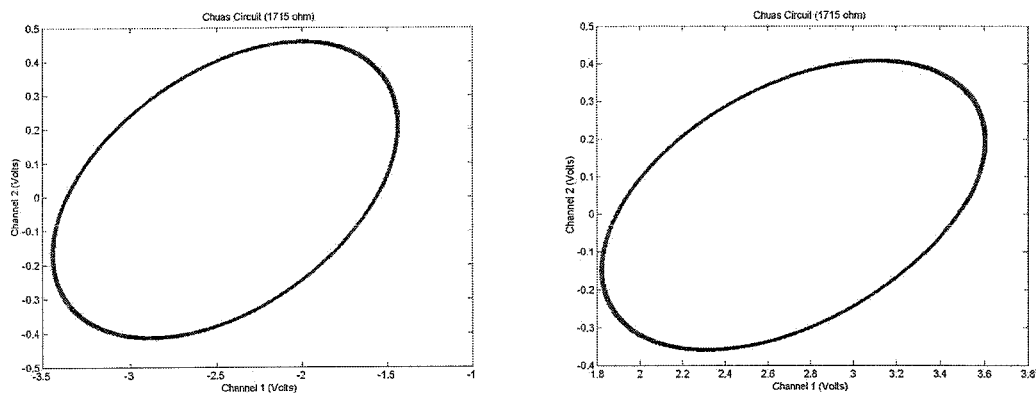


Figure 8: Both plots with adjustable R is 1715 ohm

There are two plots, both by 1715 ohm. The difference is very clear in the figures. In the left figure is at each time a negative v_1 , while in the right figure v_1 is positive at each moment. To create the different plots the adjustable resistor is kept constant. The only thing that is done, is taken the power off the system and then again on the system. The difference can occur because the starting values can vary when the power is set back on the system.

Chapter 4: Control of Chua's-circuit

To control the system it is important to know where errors occur. The errors that occur can come from the tolerances of the resistor, conductor and operational amplifiers. And because this system is chaotic, the small tolerances can result in large differences. If all resistors and conductors have their maximum tolerance offset there will be a worst-case scenario. For a resistor a tolerance of one percent is acceptable. For the conductor that is ten percent. The maximum output voltage of the operational amplifier varies between twelve and fourteen volt for a resistor of more than ten thousand ohm and varies between ten and thirteen volt for a resistor of more than two thousand ohm.

First the inductor will be explained. In figure 9 the inductor is shown, build from different resistors and a conductor.

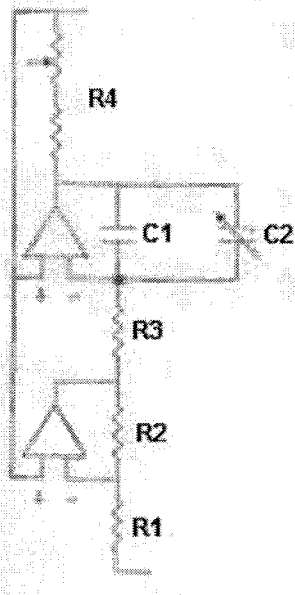


Figure 9: The conductor

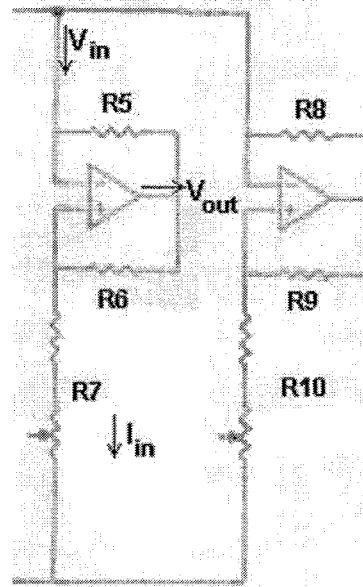


Figure 10: The negative resistor

The conductor and adjustable conductor will be regarded as one. That conductor will be called C . Equation 7 explains how it can be calculated. At the top of the conductor there is a resistor in serie with an adjustable resistor. Those resistors added up are called R_4 .

$$C = C_1 + C_2 \quad (7)$$

The inductor can be calculated by the following equation:

$$L = \frac{R_1 R_3 R_4}{R_2} C \quad (8)$$

The value of the negative resistor is a bit harder to determine. It has two systems working parallel of each other. Each system with three resistors and one operational amplifier. The values of the two resistors in serie are added up. They represent R_7 and R_{10} . The two systems, of one operational amplifier and three resistors, will be added up in the following way:

$$\frac{1}{N_R} = \frac{1}{R_{eq1}} + \frac{1}{R_{eq2}} \quad (9)$$

The equation for R_{eq1} and R_{eq2} depends on the output voltage of the operational amplifier. When none of the output voltages are reached, both systems are negative and the graph of figure 3 is the steepest. The part between B_p -min and B_p -plus represent that. When one of the operational amplifiers has reached his maximum output voltage, the graph will be less steep. This is after B_p -min and B_p -plus. When the output voltage is as high as both maximum output voltage of the operational amplifiers the graph is positive steep. This part is not shown in figure 3. The maximum output voltages will be too high.

For the system of R_5 , R_6 and R_7 the equations will be explained. Because it is working exactly the same for the other system.

The maximum output voltage has a relation with the maximum incoming voltage. See equation 11:

$$I_1 = \frac{V_{in}}{R_7} \quad (10)$$

And for the output voltage:

$$V_{out} = I_1(R_6 + R_7) = V_{in} \frac{R_6 + R_7}{R_7} \quad (11)$$

The maximum output voltage for the operational amplifier is known. So the maximum V_{in} is also known.

As long as the maximum is not reached, the following equation for V_{in} is used:

$$V_{in} = I_{in}R_5 + V_{out} = I_{in}R_5 + V_{in} \frac{R_6 + R_7}{R_7} \quad (12)$$

Multiply (12) with R_7 to simplify it:

$$V_{in}R_7 = I_{in}R_5R_7 + V_{in}R_7 + V_{in}R_6 \quad (13)$$

And this can be rewritten to:

$$\frac{V_{in}}{I_{in}} = -\frac{R_5R_7}{R_6} \quad (14)$$

Now it is known what the equations are when the maximum output voltage of the operational amplifier is not yet reached. But when the maximum is reached, V_{out} is not the same as equation 11, but a constant value.

$$V_{in} = I_{in}R_5 + V_{out,max} \quad (15)$$

$$\frac{\partial V_{in}}{\partial I_{in}} = R_5 \quad (16)$$

These equations can be implemented in N_R . There will be three equations for N_R . The first one is when both maximum output voltages are not yet reached. That is between B_p -plus and B_p -min.

$$N_{R1} = \frac{1}{\frac{R_6}{R_5 R_7} - \frac{R_9}{R_8 R_{10}}} \quad (17)$$

When the first output voltage is reached:

$$N_{R2} = \frac{1}{-\frac{R_6}{R_5 R_7} + \frac{1}{R_8}} \quad (18)$$

When both the output voltages are reached:

$$N_{R3} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_8}} \quad (19)$$

But in the equations of Chua's circuit, not the value of the resistance is asked, but the stiffness of the graph. Namely G_a , G_b -plus and G_b -min. These values can be calculated like this:

$$G_a = \frac{1}{N_{R1}} \quad (20)$$

This goes exactly the same way for G_b -plus and G_b -min. The maximum input voltages represent B_p -plus and B_p -min.

For the adjustable conductor a minimum value of five picofarad is taken. For the adjustable resistor the maximum value of two thousand ohm is taken. Beneath here is a table in which the components are stated three times. One time with no tolerance errors. The other two times is a worst-case scenario. The parameters are calculated according to the equations above.

Table 2: The parameters in three conditions

Parameters	No tolerance error	Worst-Case (below)	Worst-Case (over)
C_1	10.0 nF	9.0 nF	11.0 nF
C_2	100 nF	90 nF	110 nF
L	16.7 mH	14.4 mH	19.1 mH
R_L	10.0 Ω	9.9 Ω	10.1 Ω
G_a	-0.474 mS	-0.460 mS	-0.489 mS
G_b	-0.255 mS	-0.247 mS	-0.264 mS
B_p	1.80 V	1.63 V	1.98 V

G_b -plus and G_b -min have the same value as G_b . And B_p -plus and B_p -min also have the same value as B_p . When these values are implemented in MATLAB, three different plots occur. The time span runs till 0.01 seconds. The input voltages are for v_1 , v_2 and i_L respectively one volt, one volt and zero ampere. And the adjustable resistor parameter is sixteen hundred ohm.

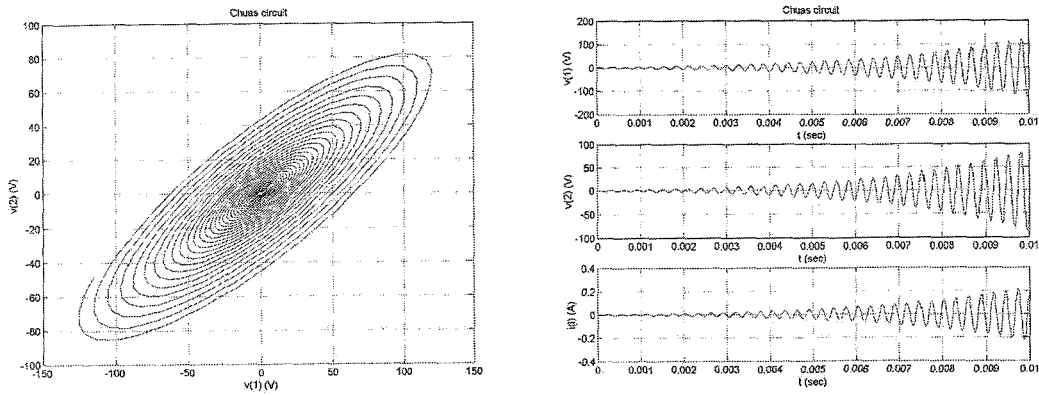


Figure 11: No tolerance

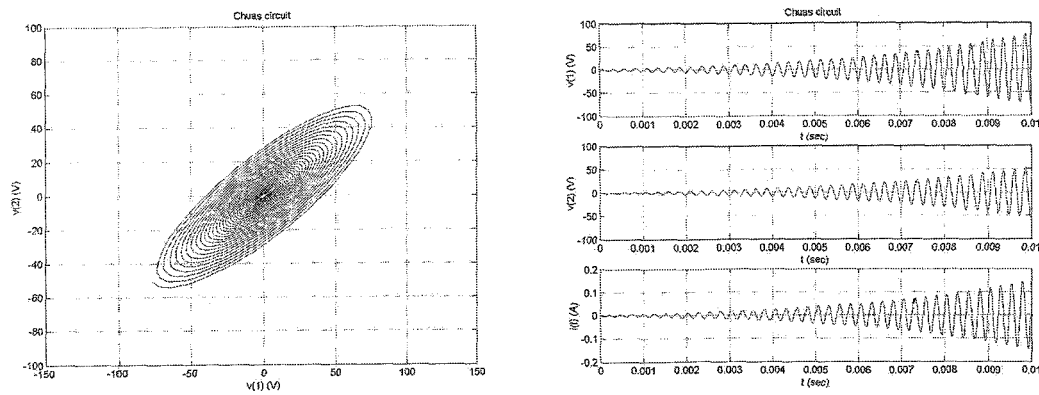


Figure 12: Worst-case below

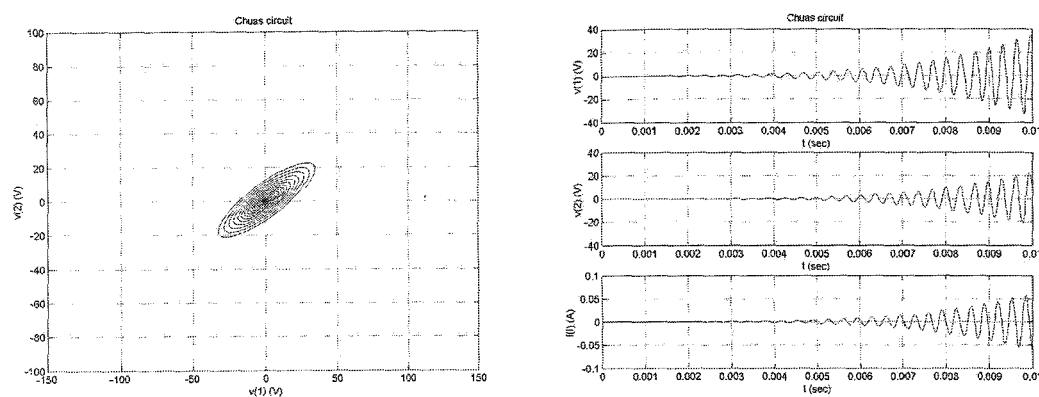


Figure 13: Worst-case over

The small tolerances in the separate components result in big difference as shown in the three figures. While the start voltages are the same, and for the adjustable resistor no error is taken.

If two Chua's circuits are built with the same parameters for the electrical components, there is big chance that there are significant differences between the output signals.

But is it possible, when two Chua's circuits are built, with a possible error by the components and possible difference in the starting voltage, to synchronise it? So that the same output signals are withdrawn.

Chapter 5: Synchronisation of Chua's circuit

Continuous control is used to synchronise Chua's circuit. (Kapitaniak, 2000) The two chaotic circuits can be schematically represented as:

$$\frac{dx}{dt} = f(x) \tag{21}$$

$$\frac{dy}{dt} = f(y)$$

The outgoing signals of $x_i(t)$ and $y_i(t)$ are coupled so that the difference $D(t)$ is used as a control signal. See also (22).

$$K(x_i(t) - y_i(t)) = KD(t) \tag{22}$$

The schematically illustration is below. One of the two chaotic systems is synchronised. The parameter K will be determined by using trial and error.

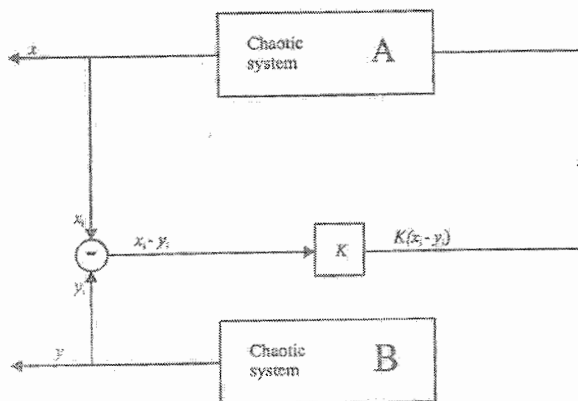


Figure 14: Schematic representation of two chaotic circuits synchronised.

This is also possible with Chua's circuits. Because there are three outgoing parameters, all three of them have to be separately coupled. In figure 15 is shown how v_2 is synchronised.

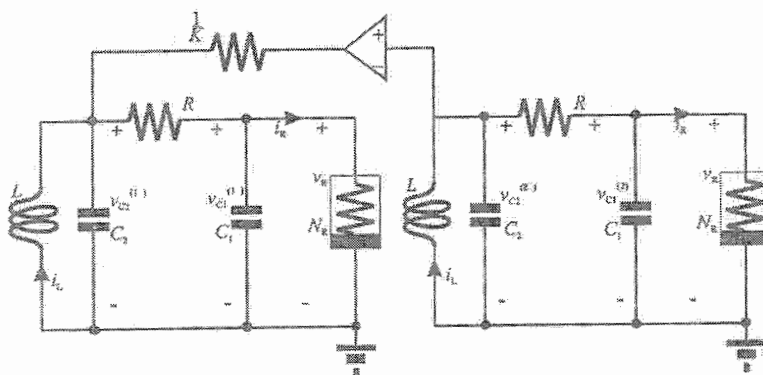


Figure 15: Two identical coupled Chua's circuits.

The differential equation is also a little bit different. An extra term is added up. Nondimensional, the equations are as (23). For the precise equations see appendix 4.

$$\begin{aligned}
\frac{dx^{(1)}}{dt} &= \alpha(y^{(1)} - x^{(1)} - f(x^{(1)})) \\
\frac{dy^{(1)}}{dt} &= x^{(1)} - y^{(1)} + z^{(1)} + K(y^{(2)} - y^{(1)}) \\
\frac{dz^{(1)}}{dt} &= -\beta y^{(1)} \\
\frac{dx^{(2)}}{dt} &= \alpha(y^{(2)} - x^{(2)} - f(x^{(2)})) \\
\frac{dy^{(2)}}{dt} &= x^{(2)} - y^{(2)} + z^{(2)} \\
\frac{dz^{(2)}}{dt} &= -\beta y^{(2)}
\end{aligned}
\tag{23}$$

By this equation only v_2 is coupled. Eventually all parameters are coupled, so that the chua's circuits are synchronised. Now the differential equation is adapted, K is changed. When the error, $y^{(2)} - y^{(1)}$, becomes zero, the proportional amplifier works. By using trial and error eventually K is known.

First it is tried to synchronise both chua's circuits with different starting values. The component parameters are kept constant. A value for K of 10000 gives good results. In the figure below the errors of v_1 , v_2 and i_L are shown.

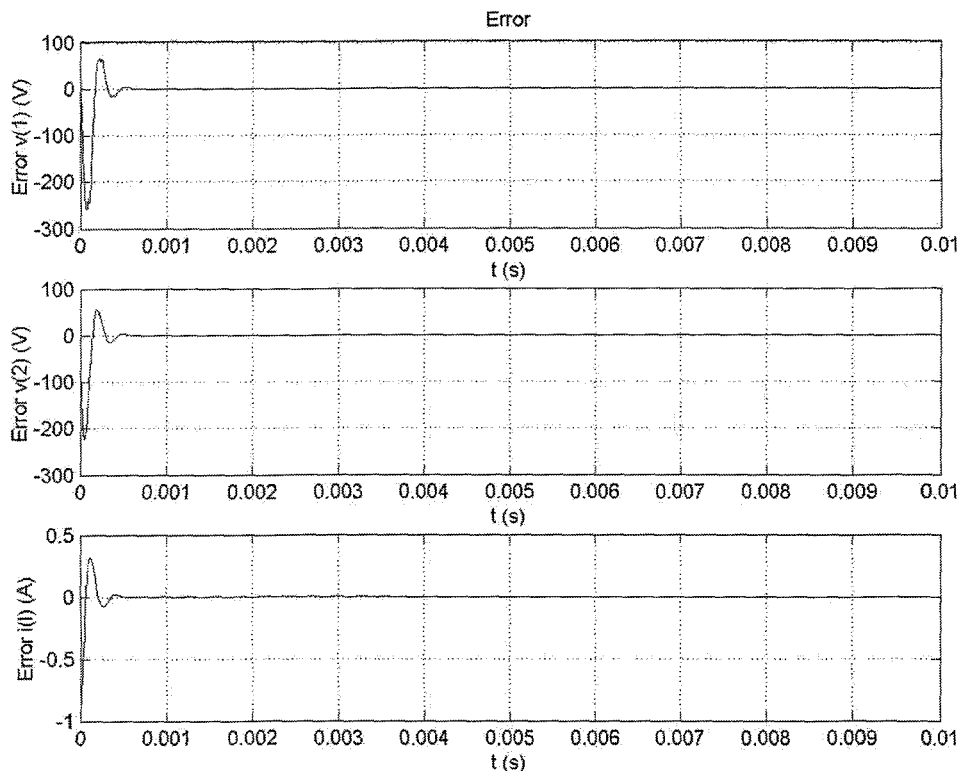


Figure 16: The error of v_1 , v_2 and i_L .

It is obvious that the amplitude of the error decreases very fast and eventually becomes zero. So the coupled chua's circuits are synchronised with different starting values.

It is also possible to synchronise all three signals, with only one type of error coupled. The same Matlab-script is used, but K is coupled with $v_2^{(1)}-v_2^{(2)}$ by all three differential equations. With this synchronisation the error will not be exactly zero, therefore it is better to couple all signals back, as done before and as done in the appendix 4.

The chua's circuits can be synchronised with different starting values this way, but it is not possible to synchronise them when the parameters of the components are different. All worst case scenarios for the individual parameters are tried, but it was never possible to synchronise the circuits.

Conclusion

There are a lot of conclusions that can be drawn. It is shown that the output signals of v_1 and v_2 are different when the adjustable resistor is changed from one thousand to two thousand ohm and turned round. There is also a difference in the quantity of stable points. When the resistor is changed from two thousand ohm downwards to one thousand ohm there are more stable points.

Chua's circuit is build from different components. Each component has a certain tolerance. But because it is a chaotic circuit, a little difference in the component, can lead to large differences in the outcoming voltages. This is also by the different starting values.

In theory it is possible to synchronise chua's circuit. But only when all the components are exactly the same and only the starting values differ. It is not possible to synchronise it with continuous control when the parameters of the components are different.

List of literature

Kapitaniak, T., 2000, *Chaos for Engineers: Theory, Applications and Control*. Second revised edition, Berlin: Springer.

Santoboni, G, *Adaptive observer-based synchronization*. Eindhoven: Department of Mechanical Engineering, TU/e

Santoboni, G. a.o., 1999, *Experimental definition of the basin of attraction for Chua's circuit*. Third revised edition, London.

Appendix 1: Matlab script of solving the differential equation

```
function rundvdt

clear all
close all

% time, starting voltage and tolerance
Tspan = [0 0.1];
v0 = [1; 1; 0];
options = odeset('RelTol',1e-3,'AbsTol',1e-6);

% solving the differential equation
[t,v] = ode45(@dvdt, Tspan, v0, options);

% plotting the figures
figure
plot(v(:,1),v(:,2))
xlabel('v(1)'); ylabel('v(2)'); title('Chuas circuit')
grid on

figure
subplot(3,1,1)
plot(t(:,1), v(:,1))
xlabel('t'); ylabel('v(1)'); title('Chuas circuit')
grid on

subplot(3,1,2)
plot(t(:,1), v(:,2))
xlabel('t'); ylabel('v(2)')
grid on

subplot(3,1,3)
plot(t(:,1), v(:,3))
xlabel('t'); ylabel('i(l)')
grid on

% -----
% the differential equation%

function dv = dvdt(t,v)

%constant values
C1 = 10e-9;
C2 = 100e-9;
L = 22e-3;
R0 = 10;
Ga = -0.757e-3;
Gb = -0.410e-3;
Bp = 2.002;
R = 2.05e3;

Gbplus = -0.432e-3;
Gbmin = -0.432e-3;
Bpplus = 2.002;
Bpmin = 2.002;

% calculating Bpv1 and Gbv1
if v(1) > 0
    Bpv1 = Bpplus;
    Gbv1 = Gbplus;
elseif v(1) < 0
    Bpv1 = Bpmin;
    Gbv1 = Gbmin;
else
    Bpv1 = 0;
```

```

    Gbv1 = 0;
end

% the differential equation
if abs(v(1)) < Bpv1
    dv = [ (1/R*(v(2)-v(1))-(Gb*v(1)+(Ga-Gbv1)*v(1)))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L ];
else
    dv = [ (1/R*(v(2)-v(1))-(Gb*v(1)+(Ga-Gbv1)*sign(v(1))*Bpv1))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L ];
end
%-----

```


Appendix 2: Matlab script of the v – i characteristic

```
clear all
close all

% constant values
C1 = 10e-9;
C2 = 100e-9;
L = 22e-3;
R0 = 10;
Ga = -0.474e-3;
Gb = -0.41e-3;
Bp = 1.7;
R = 1.8e3;

Gbplus = -0.255e-3;
Gbmin = -0.255e-3;
Bpplus = 1.66;
Bpmin = 1.66;

% defining the matrices
Bpv11 = [];
Gbv11 = [];
fv11 = [];
v11 = [];

% calculating fv1 by different v1
for v1 = -5:0.1:5
    if v1 > 0
        Bpv1 = Bpplus;
        Gbv1 = Gbplus;
    elseif v1 < 0
        Bpv1 = Bpmin;
        Gbv1 = Gbmin;
    else
        Bpv1 = 0;
        Gbv1 = 0;
    end

    if abs(v1) < Bpv1
        fv1 = Gbv1*v1 + (Ga -Gbv1)*v1;
    else
        fv1 = Gbv1*v1 + (Ga -Gbv1)*sign(v1)*Bpv1;
    end

    Bpv11 = [Bpv11, Bpv1];
    Gbv11 = [Gbv11, Gbv1];
    fv11 = [fv11, fv1];
    v11 = [v11, v1];
end

Bpv1 = Bpv11;
Gbv1 = Gbv11;
fv1 = fv11;
v1 = v11;

% plotting the figures
figure
plot(v1,fv1)
xlabel('v1'); ylabel('f(v1)'); title('sketch of the v-i characteristic')
grid on
```

Appendix 3: Loading data into Matlab

```
clear all
close all

% loading the data
load test.vos -mat

whos;

SLm.tdxvec;

channel_1 = 1; %Voltage 1
channel_2 = 2; %Voltage 2

% plotting the figures

% voltage 1 to voltage 2
figure
plot(SLm.scmeas(channel_1).tdmeas,SLm.scmeas(channel_2).tdmeas)
title('Chuas Circuit'); xlabel('Channel 1 (Volts)'); ylabel('Channel 2 (Volts)')

% voltage 1 to time and voltage 2 to time
figure
plot(SLm.tdxvec, SLm.scmeas(channel_1).tdmeas,'r')
hold on
plot(SLm.tdxvec, SLm.scmeas(channel_2).tdmeas,'b')
grid
title('Chuas Circuit'); xlabel('Time (S)'); ylabel('Channel 1 and 2 (Volts)');
```

Appendix 4: Matlab script of synchronisation of Chua's circuits

function controlling_all

```
clear all
close all
```

```
%time, starting voltage and tolerance
Tspan = [0 0.05];
v0 = [5; 3; 2; 10; 5; 1];
options = odeset('RelTol',1e-3,'AbsTol',1e-6);
```

```
%solving the differential equation
[t,v] = ode45(@dvdt, Tspan, v0, options);
```

```
%plotting the figures
figure
plot(v(:,1),v(:,2))
xlabel('v(1)'); ylabel('v(2)'); title('Chuas circuit (1)')
grid on
```

```
figure
plot(v(:,4),v(:,5))
xlabel('v(1)'); ylabel('v(2)'); title('Chuas circuit (2)')
grid on
```

```
figure
subplot(3,1,1)
plot(t,v(:,1)-v(:,4))
xlabel('t (s)'); ylabel('Error (V)'); title('Error')
grid on
```

```
subplot(3,1,2)
plot(t,v(:,2)-v(:,5))
xlabel('t (s)'); ylabel('Error (V)')
grid on
```

```
subplot(3,1,3)
plot(t,v(:,3)-v(:,6))
xlabel('t (s)'); ylabel('Error (V)')
grid on
```

```
%-----
%the differential equation%
```

```
function dv = dvdt(t,v)
```

```
%constant values
C1 = 10e-9;
C2 = 100e-9;
L = 16.7e-3;
R0 = 10;
Ga = -0.474e-3;
Gb = -0.255e-3;
Bp = 1.80;
R = 1.50e3;
```

```
Bpv1 = Bp;
Gbv1 = Gb;
```

```
C1x = 10.0e-9;
C2x = 100e-9;
Lx = 16.7e-3;
R0x = 10.0;
Gax = -0.474e-3;
Gbx = -0.255e-3;
```

```

Bpx = 1.80;
Rx = 1.50e3;

Bpv1x = Bpx;
Gbv1x = Gbx;

K = 10000;

%the differential equation
if abs(v(1)) < Bpv1
    dv = [ (1/R*(v(2)-v(1))-(Gbv1*v(1)+(Ga-Gbv1)*v(1)))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L
          (1/Rx*(v(5)-v(4))-(Gbv1x*v(4)+(Gax-Gbv1x)*v(4)))/C1x + K*(v(1)-v(4))
          (1/Rx*(v(4)-v(5))+v(6))/C2x + (v(1)-v(4)) + K*(v(2)-v(5))
          (-v(5)-R0x*v(6))/Lx + K*(v(3)-v(6))];
else
    dv = [ (1/R*(v(2)-v(1))-(Gbv1*v(1)+(Ga-Gbv1)*sign(v(1))*Bpv1))/C1
          (1/R*(v(1)-v(2))+v(3))/C2
          (-v(2)-R0*v(3))/L
          (1/Rx*(v(5)-v(4))-(Gbv1x*v(4)+(Gax-Gbv1x)*sign(v(4))*Bpv1x))/C1x + K*(v(1)-v(4))
          (1/Rx*(v(4)-v(5))+v(6))/C2x + (v(1)-v(4)) + K*(v(2)-v(5))
          (-v(5)-R0x*v(6))/Lx + K*(v(3)-v(6)) ];
end

```

Appendix 5: Schematic representation of Chua's circuit

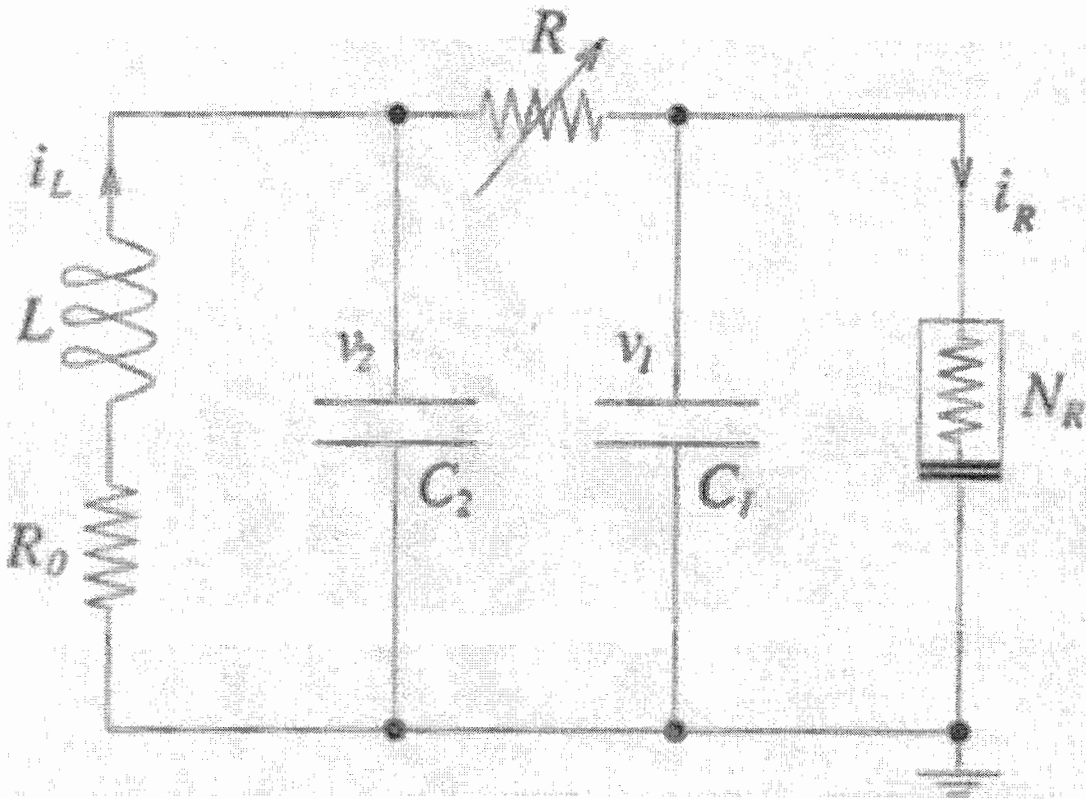


Figure 17: Schematic representation of Chua's circuit

Appendix 6: Chua's circuit with electrical components

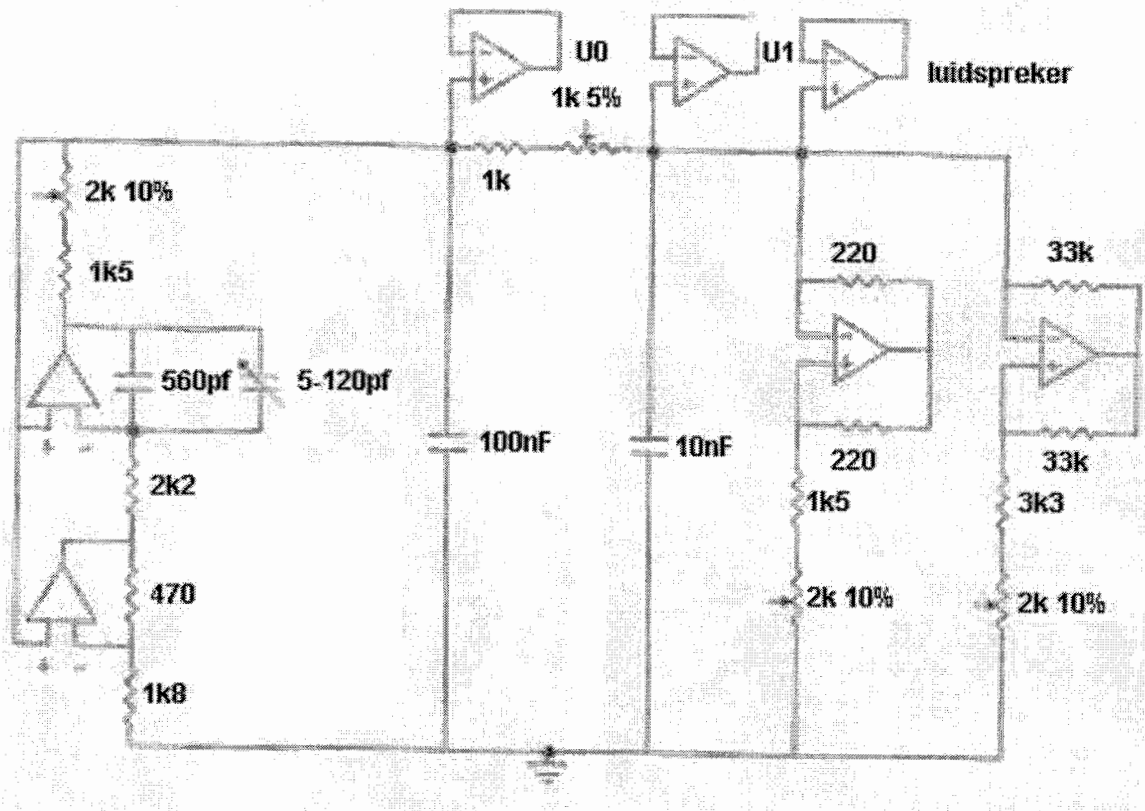


Figure 18: Representation of Chua's circuit with electronically component

Appendix 7: Changing adjustable resistor from 1000 to 2000 ohm

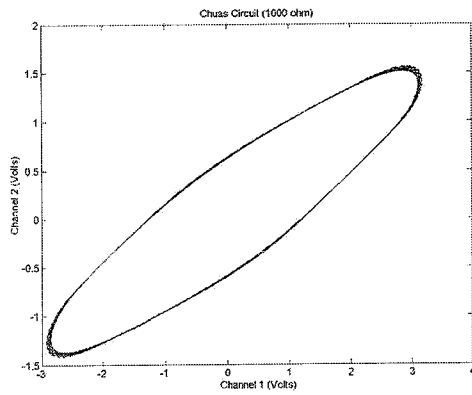


Figure 19: With R is 1000 ohm

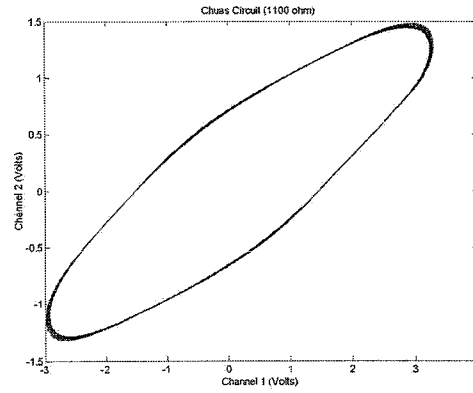


Figure 20: With R is 1100 ohm

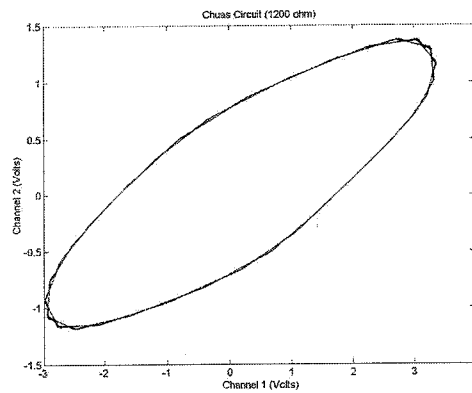


Figure 21: With R is 1200 ohm

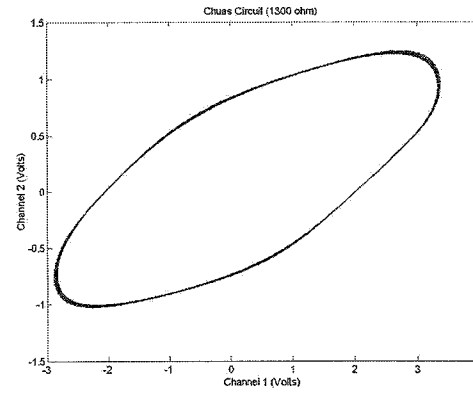


Figure 22: With R is 1300 ohm

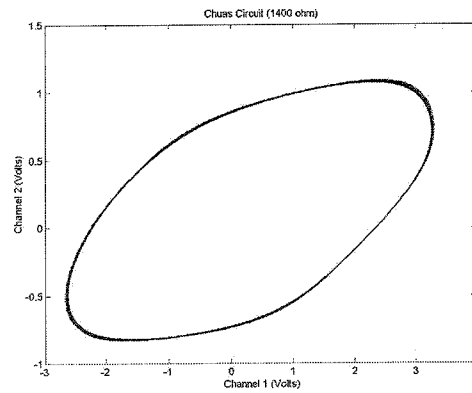


Figure 23: With R is 1400 ohm

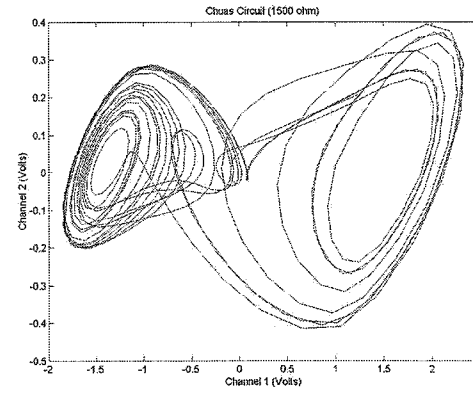


Figure 24: With R is 1500 ohm

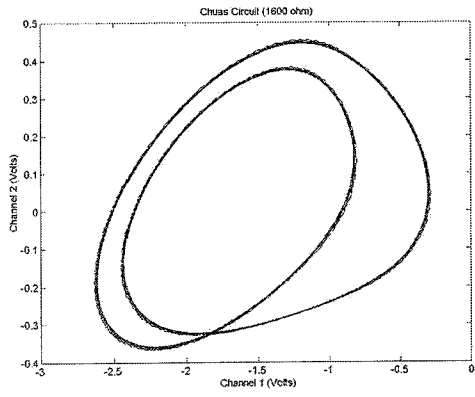


Figure 25: With R is 1600 ohm

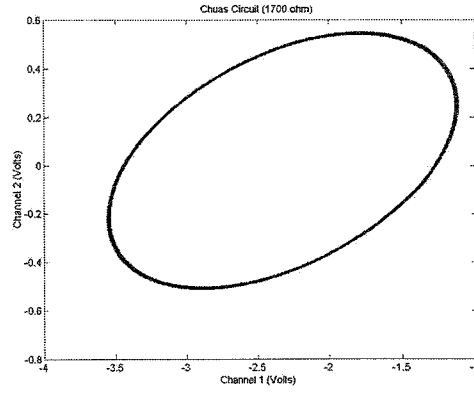


Figure 26: With R is 1700 ohm

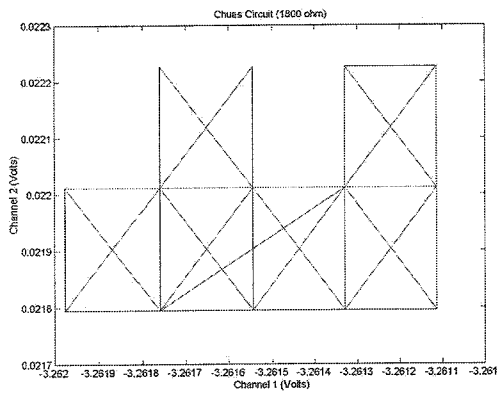


Figure 27: With R is 1800 ohm

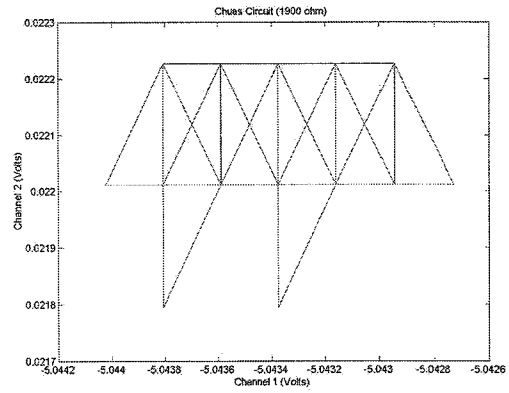


Figure 28: With R is 1900 ohm

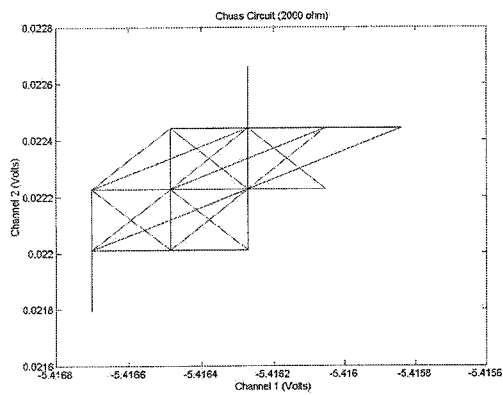


Figure 29: With R is 2000 ohm

Appendix 8: Changing adjustable resistor from 2000 to 1000 ohm

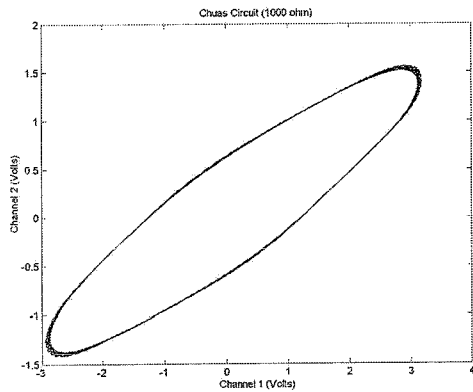


Figure 30: With R is 1000 ohm

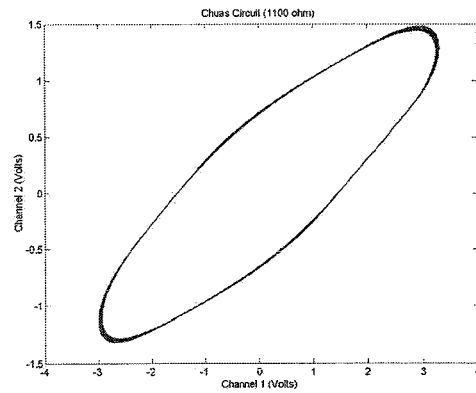


Figure 31: With R is 1100 ohm

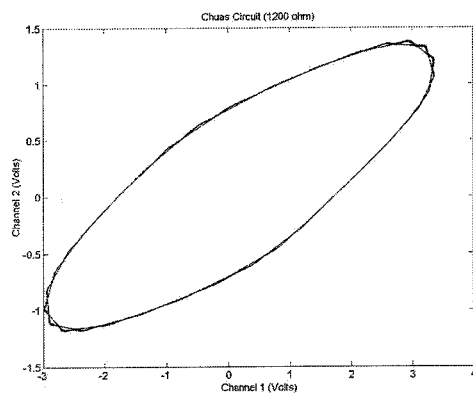


Figure 32: With R is 1200 ohm

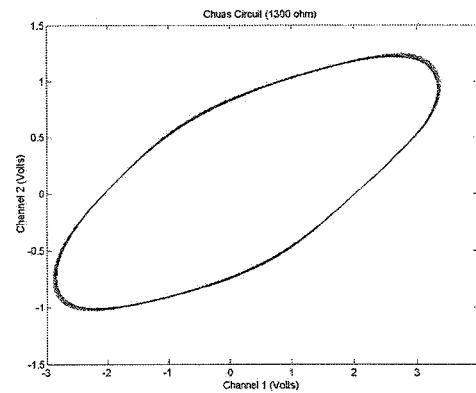


Figure33: With R is 1300 ohm

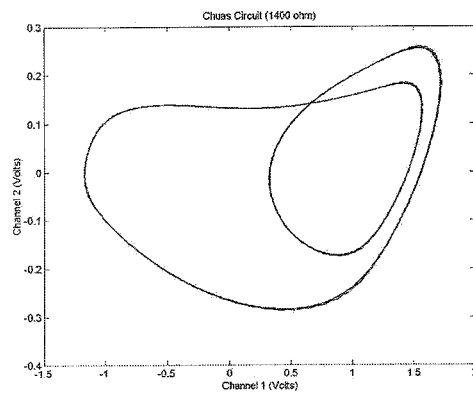


Figure 34: With R is 1400 ohm

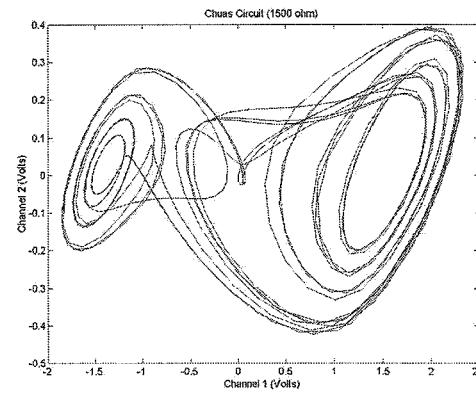


Figure 35: With R is 1500 ohm

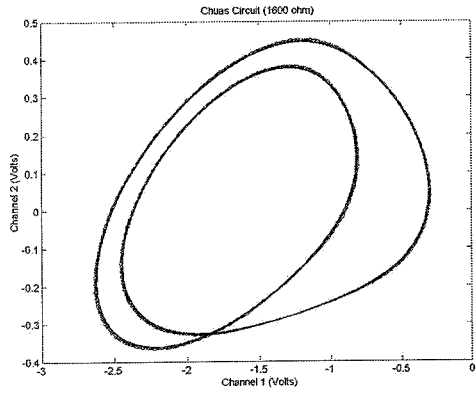


Figure 36: With R is 1600 ohm

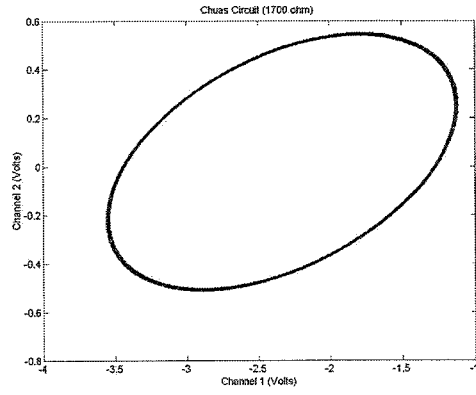


Figure 37: With R is 1700 ohm

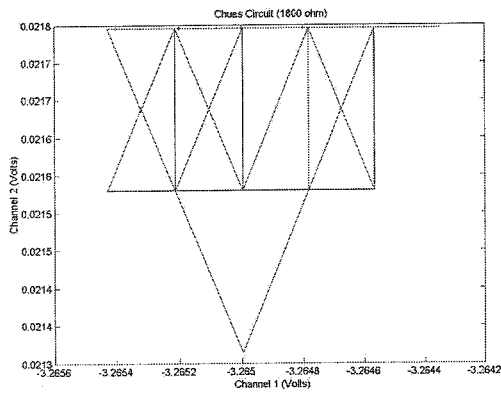


Figure 38: With R is 1800 ohm

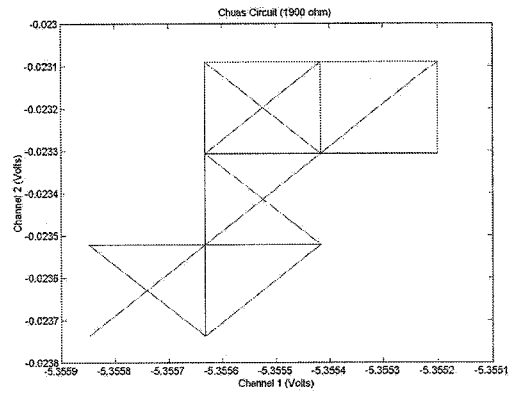


Figure 39: With R is 1900 ohm

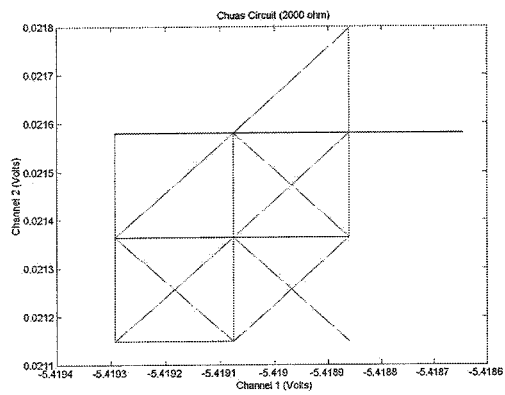


Figure 40: With R is 2000 ohm

Appendix 9: Stable points from 1000 to 2000 ohm

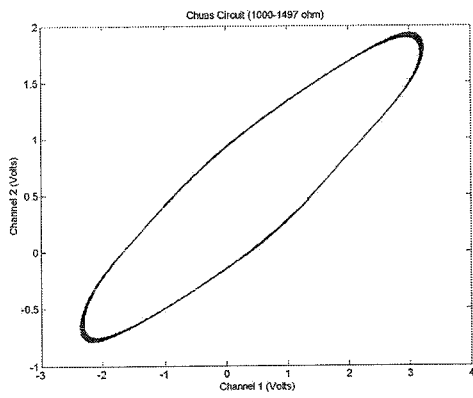


Figure 41: With R is 1000-1497 ohm

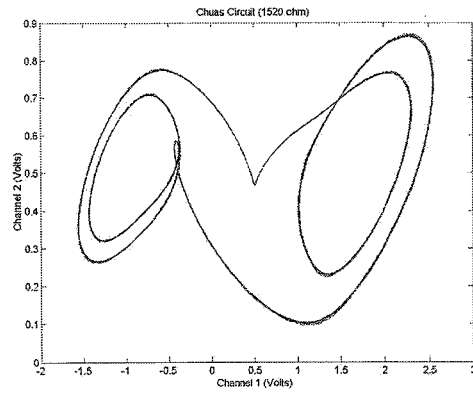


Figure 42: With R is 1520 ohm

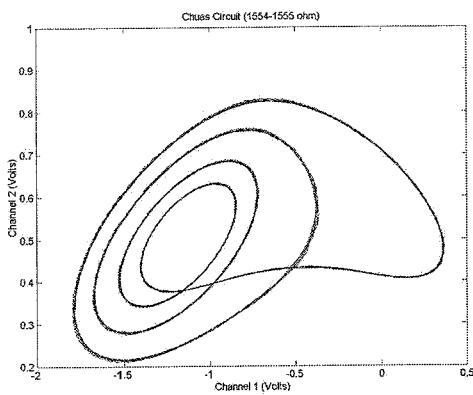


Figure 43: With R is 1554-1555 ohm

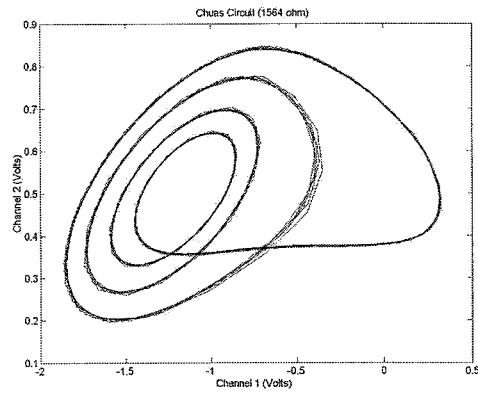


Figure 44: With R is 1564 ohm

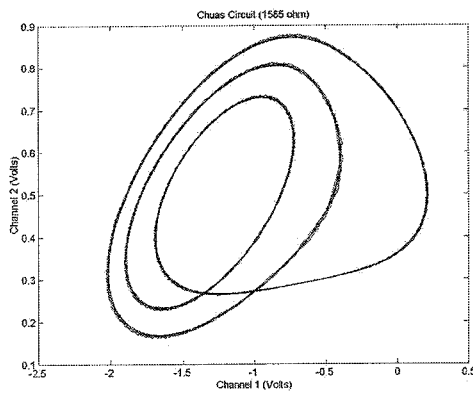


Figure 45: With R is 1585 ohm

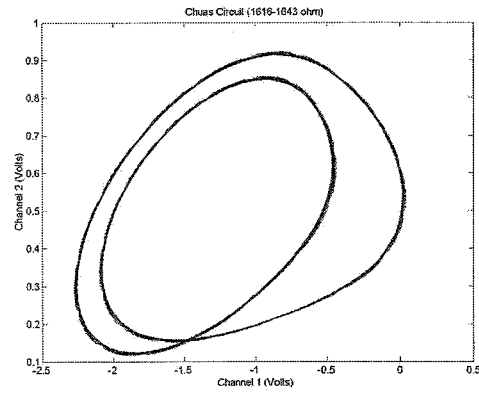


Figure 46: With R is 1616-1643 ohm

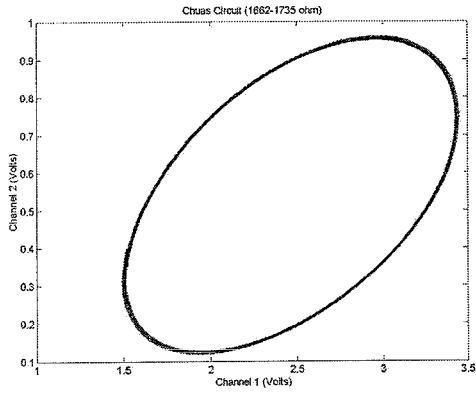


Figure 47: With R is 1662-1735 ohm

Appendix 10: Stable points from 2000 to 1000 ohm

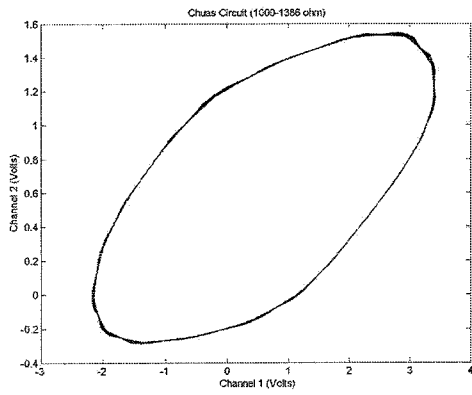


Figure 47: With R is 1000-1386 ohm

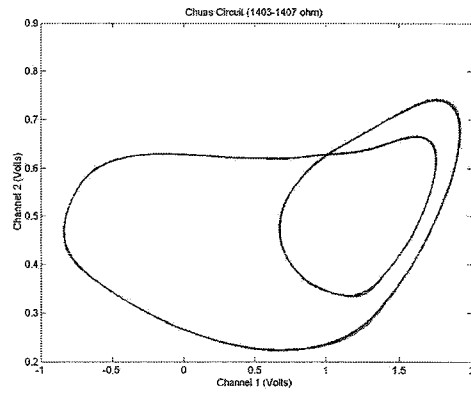


Figure 48: With R is 1403-1407 ohm

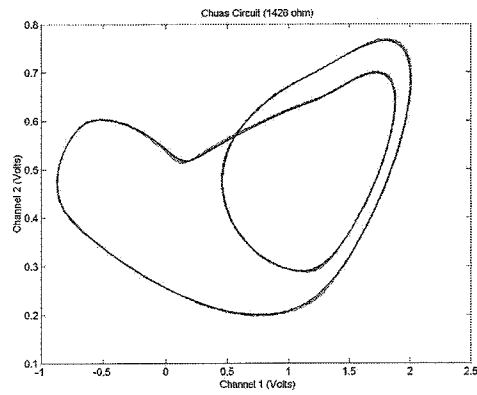


Figure 49: With R is 1426 ohm

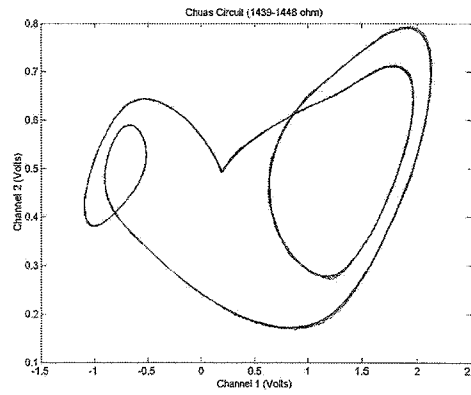


Figure 50: With R is 1439-1448 ohm

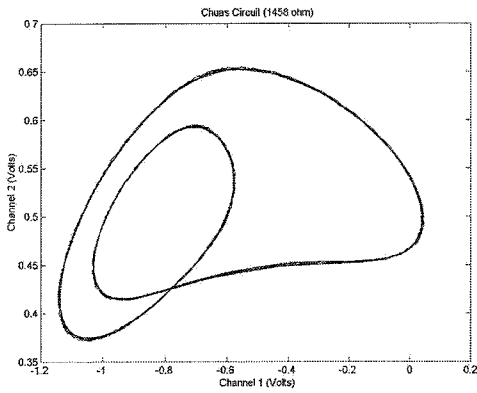


Figure 51: With R is 1458 ohm

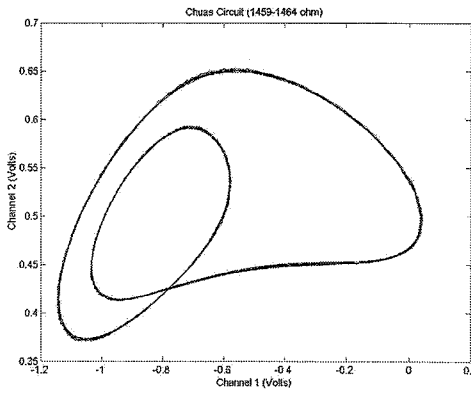


Figure 52: With R is 1459-1464 ohm

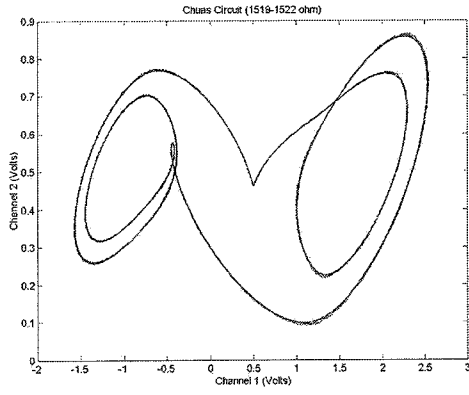


Figure 53: With R is 1519-1522 ohm

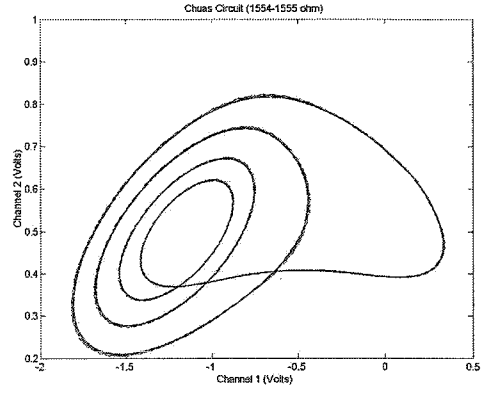


Figure 54: With R is 1554-1555 ohm

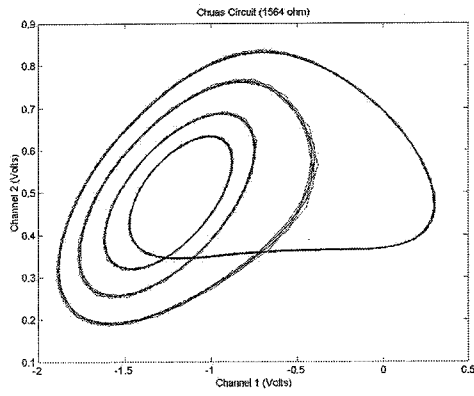


Figure 55: With R is 1564 ohm

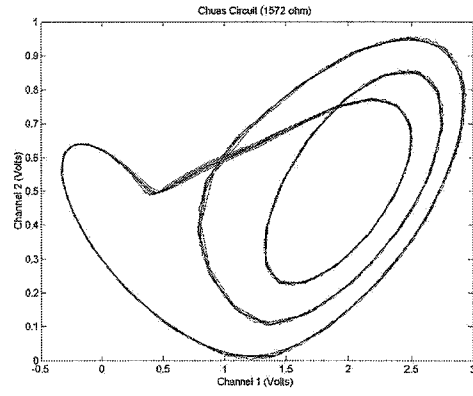


Figure 56: With R is 1572 ohm

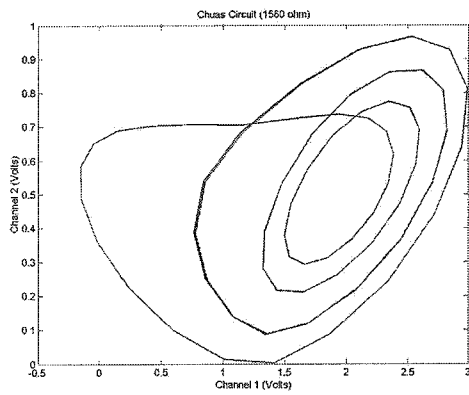


Figure 57: With R is 1580 ohm

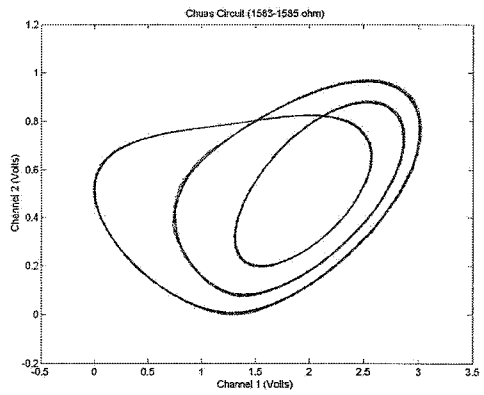


Figure 58: With R is 1583-1585 ohm

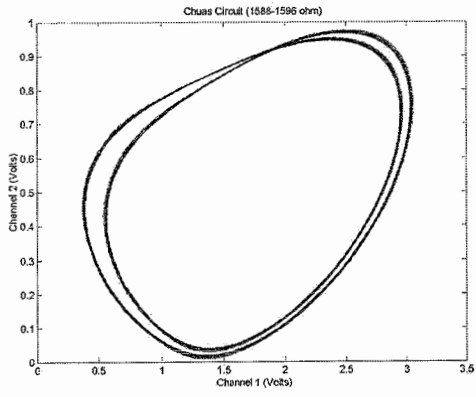


Figure 59: With R is 1588-1596 ohm

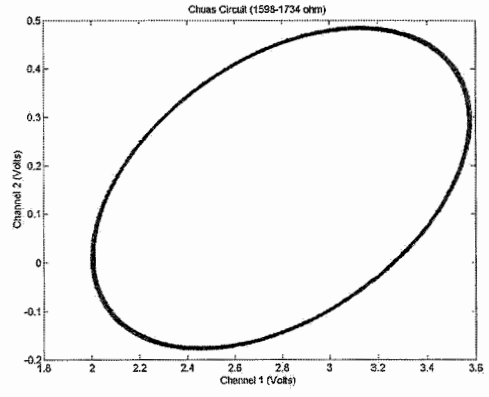


Figure 60: With R is 1598-1734 ohm