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A GENERAL EXCITATION MODEL FOR THE SATURATION PHASE

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It is known from experiments, that for a certain range of plasmaparameters the population densities of excited atomic states are independent of the electron density (n_e). This regime is called the excitation saturation phase. It is an intermediate stage between corona (low n_e) and (P)L.T.E. (high n_e). Essential for the saturation phase is that both excitation and deexcitation by electron collisions are more important than radiative processes and recombination. It turns out, that in this regime the functional dependence of the atomic level population on the ionization energies can be described in a good approximation by a power law and not by the Boltzmann distribution. This power law was found for e.g. the systems ArI, HeII and NaI and therefore seems to be independent of the atomic system.

These experimental facts can be used as a touchstone for the semi-empirical (S.E.) approximations for electron collision rate coëfficients. We do this analogous to Fujimoto [1] and we will give analytical approximations for the excitation flow (ladderlike flow leading finally to ionization) and the ionization- and recombinationflow. We use hydrogen formulas. Generalization for other atomic species is obtained by using an effective principal quantumnumber $p = \sqrt{Ry/E_i}$ (Ry = 13,6 eV; E_i = ionization energy).

Due to the fact that for the saturation phase radiative processes are negligible, the population balance of an H-level with principal quantumnumber p is quite simple :

$$n_e \sum_{q\neq p} C(q,p)n_q - n_e n_p \sum_{q\neq p} C(p,q) = S(p)n_p n_e - Q(p)n_e^3 \quad (1)$$

We assume a quasi-steady state and we neglect transport of excited particles. The number density of atoms in states with quantumnumber p is denoted by n(p); C(q,p) is the excitation rate coëfficient by electronimpact from q to p; S(p) and Q(p) are resp. the ionization and three body recombination rate coëfficients.

We use the S.E. approximation for C(p,q) in which the excitation cross section is proportional to $f_{pq} \cdot E_{pq}$. The quantity f_{pq} is the oscillatorstrength for an optical allowed transition, E_{pq}^{-1} the energy difference between the levels p and q : $E_{pq} = (E_p - E_q)$. For a Maxwellian electron distribution we obtain with Drawin [2] for C(pq), using Unsölds expressions for the oscillator strength f_{pq} and $E_{pq} = Ry(p^{-2} - q^{-2})$

$$C(p,q) = C\, p^4\, k^{-4}$$

with $k = q-p$ ($k \ll p$) and

$$c = 4 \cdot 10^{-14}\, \hat{T}_e^{\frac{1}{2}}\, \psi(u_p, p+k) \; ; \; [m^3/sec]$$

$\hat{T}_e$ is expressed in eV.

At a constant temperature c is only a weak function of the principal quantumnumber p. It is obvious that $C(p,p+1) \gg \sum_{q>p+1} C(p,q)$.

Thus the step $\Delta p = 1$ is the most probable one.

For ionization we use [2] to obtain

$$S(p) = sp^2$$

where

$$S = 2.14\ 10^{-13}\, \hat{T}_e^{-\frac{1}{2}}\, \psi(u_p) \quad [m^3/sec]$$

is again a weak function of p; $u_p = E_p\ (kT_e)^{-1}$ and $E_p = Ry/p^2$.

Detailed balancing (D.B.) gives us the expression for the recombination :

$$Q(p) = 2\, q\, sp^4 \text{ with}$$
$$q = 1{,}66\ 10^{-28}\, g(+)^{-1}\, \hat{T}_e^{3/2} \exp u_p/kT_e\, u_p) \quad (3)$$

Note, that $g(p)q\, n_e^2$ is the Sahapopulation. Now we define the "stepflow" J in the excitationspace at the "location" p as the excitation processes leading from p to p+1 minus the number of deexcitation processes from p+1 back to p.

$$J = n_e\, n(p)\, C(p,p+1) - n_e\, n(p+1)\, C(p+1,p)$$

Introducing $\eta(p) = n(p)/g(p)$ and using D.B. we obtain for large enough T_e and p :

$$J(p) = n_e\, g(p)\, C(p,p+1)(\eta(p) - \eta(p+1))$$

If we use a continuous description and infinitesimalization of the energy step with $\eta = d\eta/dp$, we can write :

$$J(p) = -2c\, n_e p^6\, \eta' \qquad (4)$$

We define the ionization flow I as the number of ionization processes $m^{-3}s^{-1}$ from "location" p. We obtain :

$$I(p) = 2s\, n_e p^4\, \eta(p) \qquad (5)$$

The recombination flow in the excitation space at location p will be :

$$R = 2q\, s\, n_e^3\, p^4 \qquad (6)$$

Now we write down the continuity equation (1) in the form :

$$\frac{dJ}{dp} = R - I \qquad (7)$$

With the analytical expressions (2)-(4) we get the simple second-order differential equation :

$$c\, p^2\, \eta'' + 6cp\eta' = s\eta - sq\, n_e^2$$

with the solution :

$$\eta(p) = b\, p^{-5.5} + q\, n_e^2 \qquad (8)$$

The last term at the r.h.s. is the Sahapopulation. It is the boundary condition for $p = \infty$. The constant b comes from the boundary condition for p=1 and is equal to the neutral density. In general, the exponent x will be : $x = -2.5 - 0.5\sqrt{25+4.s/c}$ and is dependent of the quotient s/c i.c. of the function ψ. The exponent -5.5 is obtained by putting s/c = 2.75 according to the tables of [2] for p=6 and T_e=3.5 eV. If we neglect the ionization we find $\eta \propto p^{-5}$; if we neglect the deexcitation from high to low p-values the solution will be $\eta \propto p^{-6}$. This is what Fujimoto found.

In fig. 1 we show experimental results of ArI [3], HeII [4] and Na [5]. The slope of $p^{-5.5}$ is also indicated. The agreement is good. There is no difference between the various systems. They all behave H-like and the S.E. approximation turns out to be a good description. As mentioned in [3] we succeeded in keeping the argon neutral density constant in the experiment. So the values of b are independent of n_e. At $n_e = 1.5\ 10^{19} m^{-3}$ the $2p_2$ level (p=2.3) is not yet in the saturation phase. At higher densities this happens as shown.

It is interesting to compare the stepflow with the ionization- and recombinationflow. For this purpose we substitute (8) in (4), (5) and (6) and we find :

$$J = 11\, bc\, n_e\, p^{-0.5}$$
$$I = 2\, bs\, n_e\, p^{-1.5} + 2\, qs\, n_e^3\, p^4$$
$$R = 2\, qs\, n_e^3\, p^4$$

These curves are plotted in fig. 2.

It is important to remark that even when the population density has reached the P.L.T.E. value within e.g. 1% the stepflow remains nearly a slowly decaying value. The recombination is very fast growing function of p as well as the second term of the ionization. But I-R is never greater than the stepflow. The small surplus over the Sahapopulation has to be removed by stepwise excitation even far in P.L.T.E. The conclusion from the analysis and the experiments is, that even for complicated atomic systems a relatively simple excitation population exists which can be characterized by a simple power law, provided that the excitation saturation phase is reached.

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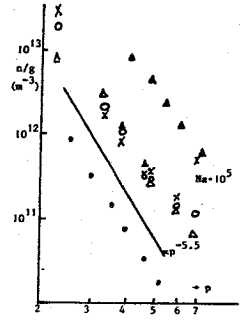

Fig. 1 Level densities as functions of the effective quantum p; ▲ Na, ref 5; ● Be II, ref. 4; △ n_e = 1.5 $10^{19} m^{-3}$; ○ n_e = 6.8 $10^{19} m^{-3}$; × n_e = 16.$10^{19} m^{-3}$, Ar I, ref. 3

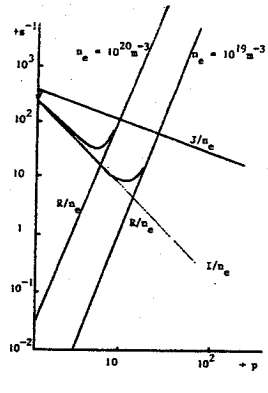

Fig. 2 The various streams J/n_e (excitation), I/n_e (ionization) and R/n_e (recombination) as functions of the effective quantum number p.