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***Citation for published version (APA):***

Mullen, van der, J. J. A. M., Sijde, van der, B., & Schram, D. C. (1981). A general excitation model for the saturation phase. In *ICPIG : International Conference on Phenomena in ionized gases : proceedings, 15th, Minsk, July 14-18, 1981, vol. 1* (pp. 441-442).

***Document status and date:***

Published: 01/01/1981

***Document Version:***

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

***Please check the document version of this publication:***

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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A GENERAL EXCITATION MODEL FOR THE SATURATION PHASE

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It is known from experiments, that for a certain range of plasmaparameters the population densities of excited atomic states are independent of the electron density ( $n_e$ ). This regime is called the excitation saturation phase. It is an intermediate stage between corona (low  $n_e$ ) and (P)L.T.E. (high  $n_e$ ). Essential for the saturation phase is that both excitation and deexcitation by electron collisions are more important than radiative processes and recombination. It turns out, that in this regime the functional dependence of the atomic level population on the ionization energies can be described in a good approximation by a power law and not by the Boltzmann distribution. This power law was found for e.g. the systems ArI, HeII and NaI and therefore seems to be independent of the atomic system.

These experimental facts can be used as a touchstone for the semi-empirical (S.E.) approximations for electron collision rate coefficients. We do this analogous to Fujimoto [1] and we will give analytical approximations for the excitation flow (ladder-like flow leading finally to ionization) and the ionization- and recombinationflow. We use hydrogen formulas. Generalization for other atomic species is obtained by using an effective principal quantum-number  $p = \sqrt{Ry/E_i}$  ( $Ry = 13,6$  eV;  $E_i =$  ionization energy).

Due to the fact that for the saturation phase radiative processes are negligible, the population balance of an H-level with principal quantumnumber  $p$  is quite simple :

$$n_e \sum_{q \neq p} C(q,p) n_q - n_e n_p \sum_{q \neq p} C(p,q) = S(p) n_p n_e - Q(p) n_e^3 \quad (1)$$

We assume a quasi-steady state and we neglect transport of excited particles. The number density of atoms in states with quantumnumber  $p$  is denoted by  $n(p)$ ;  $C(q,p)$  is the excitation rate coefficient by electron impact from  $q$  to  $p$ ;  $S(p)$  and  $Q(p)$  are resp. the ionization and three body recombination rate

coefficients.

We use the S.E. approximation for  $C(p,q)$  in which the excitation cross section is proportional to  $f_{pq} \cdot E_{pq}^{-1}$ . The quantity  $f_{pq}$  is the oscillator strength for an optical allowed transition,  $E_{pq}^{-1}$  the energy difference between the levels  $p$  and  $q$  :  $E_{pq} = (E_p - E_q)$ . For a Maxwellian electron distribution we obtain with Drawin [2] for  $C(pq)$ , using Unsöld's expressions for the oscillator strength  $f_{pq}$  and  $E_{pq} = Ry(p^2 - q^2)$

$$C(p,q) = C p^4 k^{-4}$$

with  $k = q-p$  ( $k \ll p$ ) and

$$c = 4 \cdot 10^{-14} \hat{T}_e^{1/2} \psi(u_p, p+k) ; [m^3/sec]$$

$\hat{T}_e$  is expressed in eV.

At a constant temperature  $c$  is only a weak function of the principal quantumnumber  $p$ . It is obvious that  $C(p,p+1) \gg \sum_{q>p+1} C(p,q)$ .

Thus the step  $\Delta p = 1$  is the most probable one. For ionization we use [2] to obtain

$$S(p) = sp^2 \quad \text{where}$$

$$s = 2.14 \cdot 10^{-13} \hat{T}_e^{-1/2} \psi(u_p) [m^3/sec]$$

is again a weak function of  $p$ ;  $u_p = E_p / (kT_e)^{-1}$  and  $E_p = Ry/p^2$ .

Detailed balancing (D.B.) gives us the expression for the recombination :

$$Q(p) = 2 q sp^4 \quad \text{with} \\ q = 1,66 \cdot 10^{-28} g(+)^{-1} \hat{T}_e^{3/2} \exp u_p / kT_e u_p \quad (3)$$

Note, that  $g(p)q n_e^2$  is the Sahapopulation. Now we define the "stepflow"  $J$  in the excitationspace at the "location"  $p$  as the excitation processes leading from  $p$  to  $p+1$  minus the number of deexcitation processes from  $p+1$  back to  $p$ .

$$J = n_e n(p) C(p,p+1) - n_e n(p+1) C(p+1,p)$$

Introducing  $\eta(p) = n(p)/g(p)$  and using D.B. we obtain for large enough  $T_e$  and  $p$  :

$$J(p) = n_e g(p) C(p,p+1)(n(p) - n(p+1))$$

If we use a continuous description and infinitesimalization of the energy step with  $\eta = dn/dp$ , we can write :

$$J(p) = -2c n_e p^6 \eta' \quad (4)$$

We define the ionization flow I as the number of ionization processes  $m^{-3}s^{-1}$  from "location" p. We obtain :

$$I(p) = 2s n_e p^4 \eta(p) \quad (5)$$

The recombination flow in the excitation space at location p will be :

$$R = 2q s n_e^3 p^4 \quad (6)$$

Now we write down the continuity equation (1) in the form :

$$\frac{dJ}{dp} = R - I \quad (7)$$

With the analytical expressions (2)-(4) we get the simple second-order differential equation :

$$c p^2 \eta'' + 6cp\eta' = s\eta - sq n_e^2$$

with the solution :

$$\eta(p) = b p^{-5.5} + q n_e^2 \quad (8)$$

The last term at the r.h.s. is the Sahapopulation. It is the boundary condition for  $p = \infty$ . The constant b comes from the boundary condition for  $p=1$  and is equal to the neutral density. In general, the exponent x will be :  $x = -2.5 - 0.5 \sqrt{25+4.s/c}$  and is dependent of the quotient  $s/c$  i.e. of the function  $\psi$ . The exponent -5.5 is obtained by putting  $s/c = 2.75$  according to the tables of [2] for  $p=6$  and  $T_e=3.5$  eV. If we neglect the ionization we find  $\eta \propto p^{-5}$ ; if we neglect the deexcitation from high to low p-values the solution will be  $\eta \propto p^{-6}$ . This is what Fujimoto found.

In fig. 1 we show experimental results of ArI [3], HeII [4] and Na [5]. The slope of  $p^{-5.5}$  is also indicated. The agreement is good. There is no difference between the various systems. They all behave H-like and the S.E. approximation turns out to be a good description. As mentioned in [3] we succeeded in keeping the argon neutral density constant in the experiment. So the values of b are independent of  $n_e$ . At  $n_e = 1.5 \cdot 10^{19} m^{-3}$  the  $2p_2$  level ( $p=2.3$ ) is not yet in the saturation phase. At higher densities this happens as shown.

It is interesting to compare the stepflow with the

ionization- and recombinationflow. For this purpose we substitute (8) in (4), (5) and (6) and we find :

$$\begin{aligned} J &= 11 bc n_e p^{-0.5} \\ I &= 2 bs n_e p^{-1.5} + 2 qs n_e^3 p^4 \\ R &= 2 qs n_e^3 p^4 \end{aligned}$$

These curves are plotted in fig. 2.

It is important to remark that even when the population density has reached the P.L.T.E. value within e.g. 1% the stepflow remains nearly a slowly decaying value. The recombination is very fast growing function of p as well as the second term of the ionization. But I-R is never greater than the stepflow. The small surplus over the Sahapopulation has to be removed by stepwise excitation even far in P.L.T.E. The conclusion from the analysis and the experiments is, that even for complicated atomic systems a relatively simple excitation population exists which can be characterized by a simple power law, provided that the excitation saturation phase is reached.

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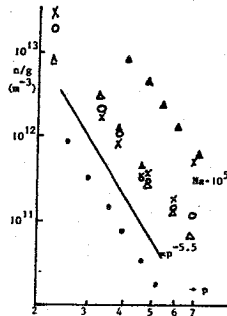


Fig. 1 Level densities as functions of the effective quantum p;  $\Delta$  Na, ref. 5;  $\bullet$  He II, ref. 4;  $\times$   $n_e = 1.5 \cdot 10^{19} m^{-3}$ ;  $\circ$   $n_e = 6.8 \cdot 10^{18} m^{-3}$ ;  $\times$   $n_e = 16 \cdot 10^{18} m^{-3}$ , Ar I, ref. 3

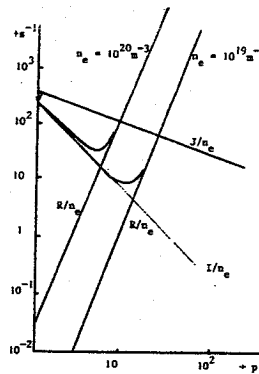


Fig. 2 The various streams  $J/n_e$  (ionization),  $I/n_e$  (ionization) and  $R/n_e$  (recombination) as functions of the effective quantum number p.