Damage processes in solids and structures and their numerical computations

DAMAGE PROCESSES IN SOLIDS AND STRUCTURES AND THEIR NUMERICAL COMPUTATION

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Abstract

A concise overview is given of some recent developments, mainly by the Delft group, of numerical modelling of damage processes. Attention is paid to discrete methods, where all damage is lumped into interface elements, and to enhanced continuum theories, where higher-order terms, either in time or in space, are added to preserve well-posedness of the rate boundary value problem after the onset of softening.

1. Introductory remarks

Standard rate-independent continuum models, i.e. continuum models that do not incorporate an internal length scale, suffer from excessive mesh sensitivity when used in the simulation of damage processes in solids and structures. We observe that the solutions are fully dependent upon the fineness of the discretisation, the direction of the mesh lines [2-5] and that spurious mechanisms may occur also for elements that are normally considered as 'safe' [6]. The underlying reason for this 'pathological' sensitivity of the solution on the discretisation of the body is the change of type of the set of differential equations that describes the rate boundary value problem which governs the mechanical behaviour of the body. When softening type constitutive relations are employed to describe the mechanical behaviour of a solid - as is typically done in damage and degradation processes - the set of governing partial differential equations (equilibrium equations, kinematic equations and constitutive equations) changes character from elliptic to hyperbolic under static loading conditions and from hyperbolic to elliptic under dynamic loading conditions. In either case well-posedness of the rate boundary value problem is lost [13].

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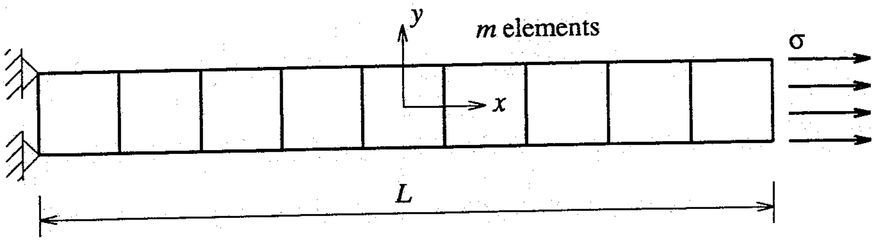

Figure 1. Strain-softening bar subject to uniaxial loading.

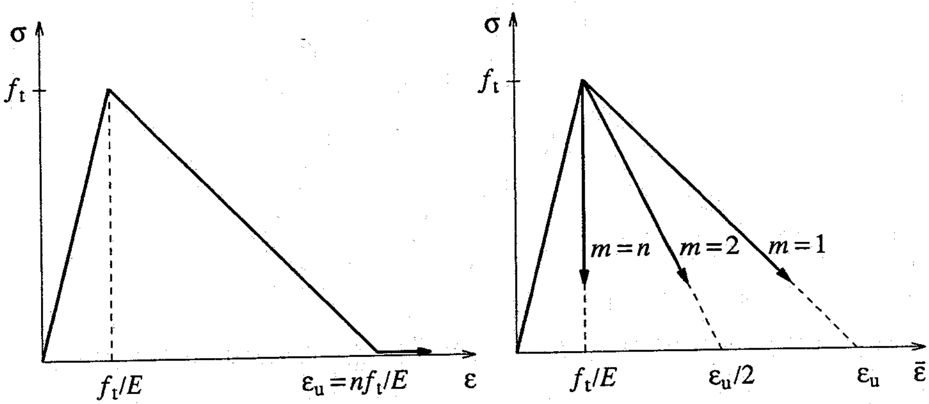

Figure 2. Stress-strain diagram (left) and response of an imperfect bar in terms of an stress-average strain curve (right).

One possible approach to this problem is to recognise that hyperbolic problems under static loading conditions involve discontinuities. Then, interface elements can be employed into which all damage evolution is concentrated. The differential equations in the continuous parts of the structure remain elliptic, and consequently, the boundary value problem remains well-posed. Of course, the success of this method is contingent upon the proper knowledge of the location of these failure zones and upon the correct energy release in these interfaces.

If the location of the failure zones - e.g. cracks, shear bands or rock faults - is not known precisely, enhanced and/or rate-dependent continuum models must be utilised if one is to use softening type stress-strain relations in a meaningful manner. The past few years have seen a variety of such approaches. Below we shall examine a number of these enhanced formulations, including non-local continua, models that are enriched with higher-order spatial derivatives, micro-polar continua and rate-dependent models. Common characteristics of these models are that they can properly capture *size effects* and that under dynamic loading conditions wave propagation becomes *dispersive*.

2. Problem statement and interface formulations

The essential shortcomings of the conventional approach as well as the novel features of enhanced continuum models are best demonstrated by the example of a simple bar loaded in uniaxial tension of Figure 1 [2,3]. Let the bar be divided into m elements. Prior to reaching the tensile strength f_t a linear relation is assumed between the (normal) stress σ and the (normal) strain ε:

$$\sigma = E\varepsilon\,, \tag{1}$$

with E Young's modulus. After reaching the peak strength a descending slope is defined in this diagram through an affine transformation from the measured load-displacement curve. The result is given in the left part of Figure 2, where ε_u marks the point where the load-carrying capacity is totally exhausted. In the post-peak regime the constitutive model can thus be summarized as:

$$\varepsilon = \varepsilon^e + \varepsilon^i\,, \tag{2}$$

which constitutes a decomposition of the strain into an elastic part ε^e:

$$\varepsilon^e = E^{-1}\sigma\,, \tag{3}$$

and a contribution due to inelastic effects (eg, cracking or plastic slip)

$$\varepsilon^i = h^{-1}(\sigma - f_t)\,, \tag{4}$$

where h plays a similar role for the inelastic strain ε^i that E has for the elastic strains. In case of degrading materials $h < 0$ and h may be termed a softening modulus. In the applications discussed in this article h will be assumed to be a constant, so that linear softening is used. Eq. (4) may also be thought of as an integrated form of the evolution equation for the stress rate after failure:

$$\sigma = f_t + h\,\varepsilon^i\,. \tag{5}$$

Now suppose that one element has a tensile strength that is marginally below that of the other $m-1$ elements. Upon reaching the tensile strength of this element failure will occur. In the other, neighbouring elements the tensile strength is not exceeded and they will unload elastically. The result on the *average* strain of the bar $\bar{\varepsilon}$ is plotted in the right part of Figure 2 for different discretisations of the bar. The results show an extreme mesh sensitivity. Convergence to a 'true' post-peak failure curve does not seem to occur. In fact, it does occur, because the governing equations predict the failure mechanism to be a line crack with zero thickness. This is a manifestation of the fact that the rate boundary value problem has changed character from elliptic to hyperbolic. The finite element solution simply tries to capture this line crack, which results in localisation in one element, irrespective of the width of this element. The result on the load-average strain curve is obvious: for an infinite number of elements ($m \rightarrow \infty$) the post-peak curve doubles back on the original loading curve. A major problem is now that, since the constitutive model has been phrased in terms of a *stress-strain* law and *not* as a *force-displacement* relation, the energy that is dissipated tends to zero upon mesh refinement because the area in which the failure process occurs also becomes zero. From a physical point of view this is unacceptable and we must therefore either rephrase our constitutive model in terms of force-displacement relations, which implies the use of special interface elements, or enrich the continuum description such that narrow zones of highly localized deformation can be accommodated, cf. descriptions for boundary layers in fluids. To achieve a correct amount of energy dissipation the fracture energy G_f must be specified as an additional material parameter in the interface. In this fashion Rots [26,27] and Feenstra *et al.* [15,16] have successfully solved problems involving concrete cracking, while Schellekens has simulated problems involving mode-I and mixed-mode delamination in carbon-epoxy composites [28,30].

A complication of a numerical nature that occurs when using interface elements to simulate a discrete fracture surface or a discrete shear band or rock fault, is that the elastic deformations must be suppressed that may occur prior to violation of the tensile strength and/or shear strength. To this end a high 'dummy' value for the elastic modulus in the interface must be specified. When using a traditional Gauß scheme for the numerical evaluation of the stiffness matrix and load vector of the interface element spurious oscillations in the stresses emerge. This phenomenon has been reported first by Rots [26] and has been explained thoroughly by Schellekens [28,29]. In the latter two references also a number of remedies is scrutinised.

3. Non-local continua

A very powerful approach to regularise the standard strain-softening continua has been offered by Pijaudier-Cabot and Bazant [24]. They have suggested to use a non-local continuous damage model in which the local energy release rate is averaged over a spatial domain. While being efficient for total stress-strain relations it seems that for constitutive models which are phrased in a rate format, the approach is numerically less convenient [10,13].

4. Grade-n continua

In an alternative approach Aifantis [1], Schreyer and Chen [31], Coleman and Hodgdon [14] and Lasry and Belytschko [17] have suggested to add higher-order spatial gradients to the constitutive model. With the contributions of Mühlhaus and Aifantis [21] and de Borst and Mühlhaus [12] a formulation for gradient plasticity has been constructed that is amenable to an efficient numerical implementation.

One way to look upon gradient models is to consider them an approximation of fully non-local models. However, the notion transpires that it may be more logical to justify the inclusion of higher-order spatial gradients in the constitutive model directly on micro-mechanical grounds. In this respect much work remains to be done.

5. Micro-polar continua

In a different approach Mühlhaus and Vardoulakis [19] and Mühlhaus *et al.* [22] have suggested to use the Cosserat continuum (a micro-polar continuum) to introduce the internal length scale in the continuum model that is needed to give the localisation band a finite size and to capture the size effect. Numerical simulations with this type of approach were first presented in [7-9,11,20]. Apart from the rate-dependent regularisation to be discussed next, this approach seems to be the most natural extension of the rate-independent, simple solid.

6. Rate-dependent models

Since all solids possess some form of rate-dependence it seems most natural to include rate effects in numerical simulations of dynamic failure. For metals this approach was pioneered by Needleman [23]. Loret and Prévost [18] have made the extension to soils, while Sluys has introduced the methodology for concrete and rocks [32,33,35].

7. A note on wave dispersion

All higher-order continua as well as rate-dependent continuum models share the property that, unlike the standard, rate-independent continuum, wave propagation becomes dispersive, that is, waves with different wave numbers propagate with different velocities and, consequently, the shape of a pulse is altered during propagation. This is of crucial importance when modelling localisation under dynamic loading conditions. For non-local continua discussions involving dispersion and softening were contributed by Pijaudier-Cabot and Benallal [25], for the Cosserat continuum by de Borst and Sluys [9], for rate-dependent continua by Sluys and de Borst [33] and for gradient-dependent continua by Sluys *et al.* [34]. In the latter two publications it was also shown that the combined presence of rate dependence and inertia terms introduces an internal length scale in the solid, thus setting the width of localisation bands. When either of these effects is absent, it seems that a well-posed problem can still be obtained, but that the internal length scale disappears in the continuum description, thus making the width of the band dependent upon the size and the location of the imperfections. A comprehensive treatment of the whole subject is given in Sluys [32].

8. Concluding remarks

Many important aspects concerning damage evolution, softening and localisation of deformation in solids have been left untouched. Micro-mechanical justifications for the introduction of higher-order continua, thermodynamics aspects, element performance, the use of path-following techniques for failure in softening media under quasi-static loading conditions and the issue of accurate time integrators in dynamic failure of quasi-brittle materials, the need for adaptive meshing techniques in case of highly localised failure, the problem of identification of the additional parameters that are needed in the enhanced continuum descriptions - all these issues that are of pivotal importance for the proper simulation of failure in materials and structures have not been addressed. The only, but compelling excuse is the limited space that was available for this contribution. The reader who wishes to familiarise himself a little bit more with this rapidly developing and fascinating field is advised to consult the literature, e.g. the overview article [13].

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