

De berekening van $\int \int_{\Delta x} \int_{\Delta y} \mathbf{d}x \mathbf{d}y$

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DE BEREKENING VAN $\iint_{\Delta} x^p y^q dx dy$

W.A.M. Brekelmans

najaar 1971

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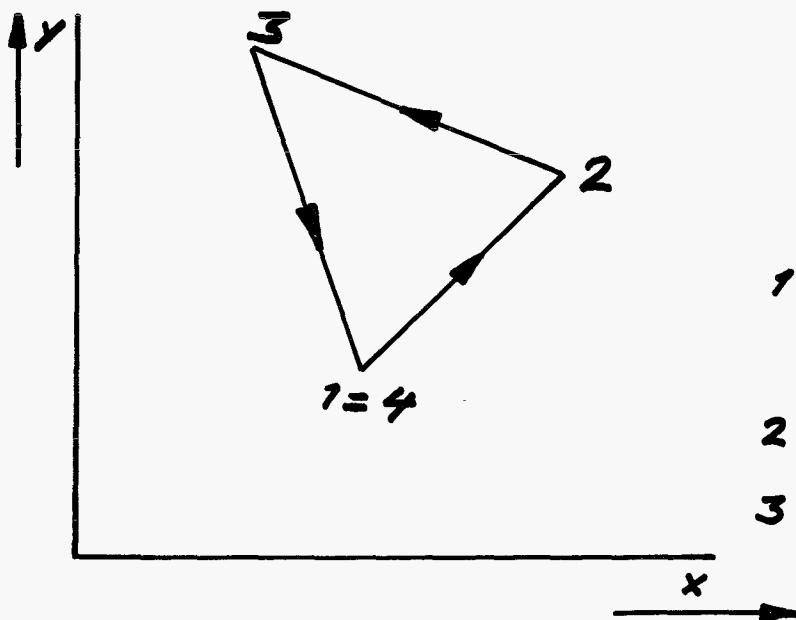
De berekening van $\iint_{\Delta} x^p y^q dx dy$

Voor een beperkt aantal combinaties van p en q zullen we eenvoudig - programmeerbare relaties afleiden voor:

$$\chi_q^p = \iint_{\Delta} x^p y^q dx dy$$

waarbij het integratiegebied wordt gevormd door een driehoek.

Deze driehoek ligt vast door middel van de coördinaten van de hoekpunten. De volgorde van nummering is aangegeven in onderstaande figuur:



- 1 : (x_1, y_1)
of (x_4, y_4)
- 2 : (x_2, y_2)
- 3 : (x_3, y_3)

Voor positieve waarden van p en q ,
de waarde nul niet uitgezonderd,
geldt:

$$\chi_q^p = \frac{2p!q!}{(p+q+2)!} \sum_{\ell=1}^3 A_\ell \sum_{i=0}^p \sum_{j=0}^q \binom{i+j}{i} \binom{p+q-i-j}{p-i} x_\ell^{p-i} x_{\ell+1}^i y_\ell^{q-j} y_{\ell+1}^j$$

met $A_\ell = \frac{1}{2}(x_\ell y_{\ell+1} - x_{\ell+1} y_\ell)$

Wanneer we definiëren:

$$\alpha(p, q) = \frac{2p!q!}{(p+q+2)!}$$

$$\beta(p, q, i, j) = \binom{i+j}{i} \binom{p+q-i-j}{p-i}$$

geldt:

$$\chi_q^p = \alpha(p, q) \sum_{\ell=1}^3 A_\ell \sum_{i=0}^p \sum_{j=0}^q \beta(p, q, i, j) x_\ell^{p-i} x_{\ell+1}^i y_\ell^{q-j} y_{\ell+1}^j$$

Voor slechts die waarden van p
en q , die voldoen aan de onder-
staande relaties, zal de formule
voor χ_q^p verder worden uitgewerkt.

$$p \geq 0$$

$$q \geq 0$$

$$p+q \leq 4$$

α

$q \backslash p$	0	1	2	3	4
0	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{15}$
1	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{30}$	$\frac{1}{60}$	
2	$\frac{1}{6}$	$\frac{1}{30}$	$\frac{1}{90}$		
3	$\frac{1}{10}$	$\frac{1}{60}$			
4	$\frac{1}{15}$				

β

$q \backslash p$	0	1	2	3	4
0	$\begin{array}{c c} \sqrt{0} & 0 \\ \hline 0 & 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \\ \hline 0 & 1 \ 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \\ \hline 0 & 1 \ 1 \ 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \ 3 \\ \hline 0 & 1 \ 1 \ 1 \ 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \ 3 \ 4 \\ \hline 0 & 1 \ 1 \ 1 \ 1 \ 1 \end{array}$
1	$\begin{array}{c c} \sqrt{0} & 0 \\ \hline 0 & 1 \\ \hline 1 & 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \\ \hline 0 & 2 \ 1 \\ \hline 1 & 1 \ 2 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \\ \hline 0 & 3 \ 2 \ 1 \\ \hline 1 & 1 \ 2 \ 3 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \ 3 \\ \hline 0 & 4 \ 3 \ 2 \ 1 \\ \hline 1 & 1 \ 2 \ 3 \ 4 \end{array}$	
	$\begin{array}{c c} \sqrt{0} & 0 \\ \hline 0 & 1 \\ \hline 1 & 1 \\ \hline 2 & 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \\ \hline 0 & 3 \ 1 \\ \hline 1 & 2 \ 2 \\ \hline 2 & 1 \ 3 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \ 2 \\ \hline 0 & 6 \ 3 \ 1 \\ \hline 1 & 3 \ 4 \ 3 \\ \hline 2 & 1 \ 3 \ 6 \end{array}$		
	$\begin{array}{c c} \sqrt{0} & 0 \\ \hline 0 & 1 \\ \hline 1 & 1 \\ \hline 2 & 1 \\ \hline 3 & 1 \end{array}$	$\begin{array}{c c} \sqrt{0} & 0 \ 1 \\ \hline 0 & 4 \ 1 \\ \hline 1 & 3 \ 2 \\ \hline 2 & 2 \ 3 \\ \hline 3 & 1 \ 4 \end{array}$			
	$\begin{array}{c c} \sqrt{0} & 0 \\ \hline 0 & 1 \\ \hline 1 & 1 \\ \hline 2 & 1 \\ \hline 3 & 1 \\ \hline 4 & 1 \end{array}$				

Boovenstaande tabellen geven de waarden van α en β in afhankelijkheid van de bepalende variabelen.

$$\chi_0^0 = \sum_{l=1}^3 A_l$$

$$\chi_1^0 = \frac{1}{3} \sum_{l=1}^3 A_l [y_l + y_{l+1}]$$

$$\chi_2^0 = \frac{1}{6} \sum_{l=1}^3 A_l [y_l^2 + y_l y_{l+1} + y_{l+1}^2]$$

$$\chi_3^0 = \frac{1}{10} \sum_{l=1}^3 A_l [y_l^3 + y_l^2 y_{l+1} + y_l y_{l+1}^2 + y_{l+1}^3]$$

$$\chi_4^0 = \frac{1}{15} \sum_{l=1}^3 A_l [y_l^4 + y_l^3 y_{l+1} + y_l^2 y_{l+1}^2 + y_l y_{l+1}^3 + y_{l+1}^4]$$

$$\chi_0^1 = \frac{1}{3} \sum_{l=1}^3 A_l [x_l + x_{l+1}]$$

$$\chi_1^1 = \frac{1}{12} \sum_{l=1}^3 A_l [x_l y_{l+1} + 2x_l y_l + 2x_{l+1} y_{l+1} + x_{l+1} y_l]$$

$$\chi_2^1 = \frac{1}{30} \sum_{l=1}^3 A_l [3x_l y_l^2 + x_{l+1} y_l^2 + 2x_l y_l y_{l+1} + 2x_{l+1} y_l y_{l+1} + x_l y_{l+1}^2 + 3x_{l+1} y_{l+1}^2]$$

$$\chi_3^1 = \frac{1}{60} \sum_{l=1}^3 A_l [4x_l y_l^3 + x_{l+1} y_l^3 + 3x_l y_l^2 y_{l+1} + 2x_{l+1} y_l^2 y_{l+1} + 2x_l y_l y_{l+1}^2 + 3x_{l+1} y_l y_{l+1}^2 + x_l y_{l+1}^3 + 4x_{l+1} y_{l+1}^3]$$

$$\chi_0^2 = \frac{1}{6} \sum_{l=1}^3 A_l [x_l^2 + x_l x_{l+1} + x_{l+1}^2]$$

$$\chi_1^2 = \frac{1}{30} \sum_{l=1}^3 A_l [3y_l x_l^2 + y_{l+1} x_l^2 + 2y_l x_l x_{l+1} + 2y_{l+1} x_l x_{l+1} + y_l x_{l+1}^2 + 3y_{l+1} x_{l+1}^2]$$

$$\chi_2^2 = \frac{1}{90} \sum_{e=1}^3 A_e \left[6x_e^2 y_e^2 + 3x_e^2 y_e y_{e+1} + x_e^2 y_{e+1}^2 + 3x_e x_{e+1} y_e^2 + \right. \\ \left. + 4x_e x_{e+1} y_e y_{e+1} + 3x_e x_{e+1} y_{e+1}^2 + \right. \\ \left. + x_{e+1}^2 y_e^2 + 3x_{e+1}^2 y_e y_{e+1} + 6x_{e+1}^2 y_{e+1}^2 \right]$$

$$\chi_0^3 = \frac{1}{10} \sum_{e=1}^3 A_e \left[x_e^3 + x_e^2 x_{e+1} + x_e x_{e+1}^2 + x_{e+1}^3 \right]$$

$$\chi_1^3 = \frac{1}{60} \sum_{e=1}^3 A_e \left[4y_e x_e^3 + y_{e+1} x_e^3 + 3y_e x_e^2 x_{e+1} + 2y_{e+1} x_e^2 x_{e+1} + \right. \\ \left. + 2y_e x_e x_{e+1}^2 + 3y_{e+1} x_e x_{e+1}^2 + y_e x_{e+1}^3 + 4y_{e+1} x_{e+1}^3 \right]$$

$$\chi_0^4 = \frac{1}{15} \sum_{e=1}^3 A_e \left[x_e^4 + x_e^3 x_{e+1} + x_e^2 x_{e+1}^2 + x_e x_{e+1}^3 + x_{e+1}^4 \right]$$

Door middel van de volgende afkortingen kunnen de gegeven formules enigszins vereenvoudigd worden.

$$a = x_e + x_{e+1}$$

$$aa = y_e + y_{e+1}$$

$$b = x_e^2 + x_{e+1}^2$$

$$bb = y_e^2 + y_{e+1}^2$$

$$c = x_e x_{e+1}$$

$$cc = y_e y_{e+1}$$

$$d = x_e y_e + x_{e+1} y_{e+1}$$

De resultaten zijn neergeschreven in een direct programmeerbare vorm

$$\chi_0^0 = \sum_{\ell=1}^3 A_{\ell}$$

$$\chi_1^0 = \frac{1}{3} \sum_{\ell=1}^3 A_{\ell} [aa]$$

$$\chi_2^0 = \frac{1}{6} \sum_{\ell=1}^3 A_{\ell} [bb+cc]$$

$$\chi_3^0 = \frac{1}{10} \sum_{\ell=1}^3 A_{\ell} [aa * bb]$$

$$\chi_4^0 = \frac{1}{15} \sum_{\ell=1}^3 A_{\ell} [bb * (bb+cc) - cc * cc]$$

$$\chi_0^1 = \frac{1}{3} \sum_{\ell=1}^3 A_{\ell} [a]$$

$$\chi_1^1 = \frac{1}{12} \sum_{\ell=1}^3 A_{\ell} [a * aa + a]$$

$$\chi_2^1 = \frac{1}{30} \sum_{\ell=1}^3 A_{\ell} [2 * aa * d + a * bb]$$

$$\chi_3^1 = \frac{1}{60} \sum_{\ell=1}^3 A_{\ell} [a * aa * (bb - 2 * cc) + d * (3 * bb + 4 * cc)]$$

$$\chi_0^2 = \frac{1}{6} \sum_{\ell=1}^3 A_{\ell} [b+c]$$

$$\chi_1^2 = \frac{1}{30} \sum_{\ell=1}^3 A_{\ell} [2 * a * d + aa * b]$$

$$\chi_2^2 = \frac{1}{90} \sum_{\ell=1}^3 A_{\ell} [b * (bb + 3 * cc) + c * (3 * bb - 6 * cc) + 5 * d * d]$$

$$\chi_0^3 = \frac{1}{10} \sum_{\ell=1}^3 A_{\ell} [a * b]$$

$$\chi_1^3 = \frac{1}{60} \sum_{\ell=1}^3 A_{\ell} [a * aa * (b - 2 * c) + d * (3 * b + 4 * c)]$$

$$\chi_0^4 = \frac{1}{15} \sum_{\ell=1}^3 A_{\ell} [b * (b+c) - c * c]$$