

Designing a hydraulic actuator for the tyre measurement tower

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Designing a Hydraulic Actuator for the Tyre Measurement Tower

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DCT 2006.26

Bachelor End Project

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Abstract

English

When developing modern control systems like anti-lock braking systems (ABS), traction control (TC), brake assistant (BAS) and electronic stability programs (ESP) to increase safety and comfort in vehicles, accurate tyre models are very important. Numerical simulation models are needed to predict the vehicle's behaviour for various driving conditions. Testing facilities like the tyre measurement tower are needed to provide the necessary experimental data to validate these tyre models. The tyre measurement tower of the Dynamics and Control section of the TU Eindhoven, is already capable of simulating different situations, but it needs hydraulic actuators to subject the wheel to vertical vibrations and alternating steer angles. This report discusses the design of a vertical hydraulic actuator for the tyre measurement tower. A theoretical model of the hydraulic actuator and the measurement tower is developed, to determine what size of hydraulic cylinder is needed to fulfill the demands for good tyre measurement. This is done for a-symmetrical and symmetrical cylinder types. The size of these hydraulic cylinders limit the choice between servo valves, and this choice has a substantial effect on the dynamics of the total system. Some available hydraulic cylinders and servo valves are implemented in the model, and the performances are analyzed. From this analysis the most suitable hydraulic cylinder and servo valve is determined to complete the measurement tower. Finally, recommendations are given on the usage of other hydraulic cylinders, and on additional components that are required for a functioning hydraulic system.

Dutch

Bij het ontwerpen van moderne regelsystemen zoals "antiblokkeersystemen" (ABS), "wegligging regelaar" (TC), "rem assistent" (BAS) en "elektronische stabiliteit programma's" (EPS) voor het verhogen van de veiligheid en comfort in voertuigen, zijn nauwkeurige autoband modellen heel belangrijk. Numerieke simulatiemodellen zijn nodig om het voertuiggedrag voor verschillende rij-situaties te voorspellen. Test opstellingen zoals de bandenmeettoren zijn nodig om de nodige experimentele data te leveren waarop deze modellen gevalideerd kunnen worden. De bandenmeettoren van de sectie 'Dynamics and Control' op de TU Eindhoven, kan al verschillende situaties simuleren, maar het ontbreekt nog aan hydraulische actuators die het wiel verticaal kunnen laten trillen en de stuurhoek kunnen veranderen. Dit verslag bespreekt het ontwerp van een verticale hydraulische actuator voor de bandenmeettoren. Een theoretisch model van de hydraulische actuator en de meettoren is ontwikkeld om te bepalen hoe groot een hydraulische cilinder moet zijn die voldoet aan de eisen voor een goede meting. Dit is gedaan voor zowel a-symmetrische als symmetrische cilinder types. De grootte van deze hydraulische cilinders beperkt de keuze voor een servo klep, en deze keuze heeft een substantieel effect op de dynamica van het gehele systeem. Verschillende beschikbare cilinders zijn geïmplementeerd in het model, en de prestaties zijn geanalyseerd. Uit deze analyse is de meest geschikte hydraulische cilinder en servo klep bepaald om de bandenmeettoren mee te vervolledigen. Uiteindelijk worden er nog aanbevelingen gegeven over het toepassen van andere hydraulische cilinders, en over welke bijkomende componenten nog nodig zijn voor een functionerend hydraulisch systeem.

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List of symbols

Variables

A	piston area	$[m^2]$
C_0	oil spring stiffness	$[N/m]$
$C_{0,min}$	minimum spring stiffness	$[N/m]$
C_{01}, C_{01}	spring stiffnesses for symmetrical cylinder	$[N/m]$
C_h	hydraulic stiffness	$[N/m]$
E	bulk modulus of oil	$[N/m^2]$
F_{ext}	external force	$[N]$
$\sum F$	sum of the loads	$[N]$
H_{ol}	open loop transfer function	$[-]$
$ H_{cl} $	magnitude of closed loop transfer function	$[-]$
K_{tot}	total stiffness	$[N/m]$
P_s	system pressure	$[N/m^2]$
P_1	pressure above the piston	$[Pa]$
P_L	pressure of loads	$[N/m^2]$
S	stroke of the piston	$[m]$
V_1	volume above the piston	$[m^3]$
a	amplitude sinusoidal	$[m]$
d	damping constant	$[Ns/m]$
f	frequency	$[Hz]$
f_b	bandwidth frequency	$[Hz]$
$f_{0,a-symmetrical}$	natural frequency a-symmetrical cylinder	$[Hz]$
$f_{0,cyl}$	natural frequency cylinder	$[Hz]$
$f_{0,symmetrical}$	natural frequency symmetrical cylinder	$[Hz]$
$f_{0,sys}$	natural frequency system	$[Hz]$
k	spring constant	$[N/m]$
k_a	constant acceleration feedback	$[-]$
k_m	velocity gain	$[1/s]$
k_p	constant pressure feedback	$[-]$
k_v	constant for servo valve	$[1/s]$
m	mass	$[kg]$
q	flow	$[l/min]$
q_1, q_2	flow between port openings in the servo valve	$[l/min]$
q_n	normalized flow	$[l/min]$
s	integrator laplace domain	$[1/s]$
t	time	$[s]$
x	spool displacement in servo valve	$[m]$
$ x $	absolute value of spool displacement	$[m]$
x_{max}	maximum spool displacement	$[m]$
y	position (displacement) of the piston	$[m]$
\dot{y}	velocity of the piston	$[m/s]$

β	damping in the cylinder	$[-]$
$\omega_{0,a-symmetrical}$	natural frequency a-symmetrical cylinder	$[rad/s]$
$\omega_{0,cyl}$	natural frequency cylinder	$[rad/s]$
$\omega_{0,symmetrical}$	natural frequency symmetrical cylinder	$[rad/s]$
$\omega_{0,sys}$	natural frequency system	$[rad/s]$

Chapter 1

Introduction

1.1 Background

In the world of tyre, wheel and automotive industries testing equipment is very important when designing a new and better product. Almost every vehicle manufacturer uses the "Magic Formula", a tyre model developed by Professor Hans Pacejka, in their vehicle dynamics simulations. This tyre model is the subject of continuous development. When developing modern control systems like anti-lock braking systems (ABS), traction control (TC), brake assistant (BAS) and electronic stability programs (ESP) to increase safety and comfort, tyre modelling is crucial [1].

The research field of vehicle dynamics, of the Dynamics and Control section of the TU/e, is strongly focused on the development of simulation models to predict vehicle behaviour for various driving conditions (e.g. different road surfaces). In the Automotive Laboratory, testing facilities are available for experimental research to validate these numerical simulation models. In this report the measurement tower is discussed, which is an experimental setup of the Dynamics and Control section of the TU Eindhoven (figure 1.1).

The measurement tower consists of a construction on which the wheel carrier is mounted. The wheel is in contact with a large drum (3) that drives the wheel at different speeds.

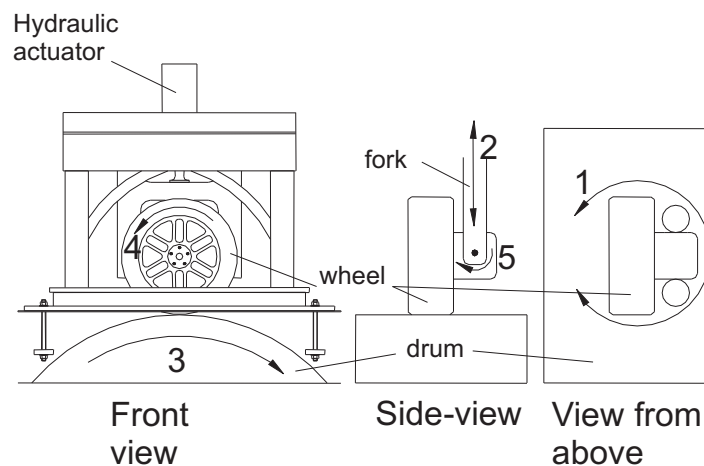


Figure 1.1: The measurement tower with a hydraulic actuator [2]

The measurement tower is able to simulate a few different situations: the steer angle (1) can be altered, and the drum (3) can drive the wheel (4) at different speeds. Experimental setups like the measurement tower at the TU/e mostly include a few hydraulic actuators. With hydraulic actuators, the wheel can easily be subjected to (dynamic) displacements (2) and (alternating) steer angles (1). Adding these hydraulic actuators makes it possible to determine the dynamic response of a tyre.

1.2 Aim of this report

The aim of this report is to complete the measurement tower with a hydraulic actuator. With a hydraulic actuator, the measurement tower will be able to measure the dynamic response of different kind of tyres. This hydraulic actuator (see figure 1.1) must be capable of delivering dynamic loads on the wheel. In this report, only a vertical actuator is considered.

The first goal of this report is to design a hydraulic actuator that fulfills the demands for good tyre measurement. Some of these demands are given in an earlier report [2], and some new demands are added. From a developed model, the size of the cylinder can be determined. This is done for both a-symmetrical and symmetrical cylinders. The size of these hydraulic cylinders limit the choice on servo valves. From the size of the cylinders the needed flow is determined, which is a guideline for determining the servo type of valve.

Secondly, the data of available hydraulic cylinders and servo valves is implemented in the model. The results on the dynamic response are discussed for different types of servo valves and hydraulic cylinders. From this analysis, the most suitable hydraulic cylinder and servo valve is determined for the measurement tower.

1.3 Overview

This report consists of two parts. The first part deals with modelling and designing a hydraulic actuator with the right size. First a model of the hydraulic cylinder with measurement tower is developed in chapter 2. This physical model will be implemented in SIMULINK, to be able to simulate and develop the hydraulic actuator.

In chapter 3, the model of the hydraulic actuator is adapted to meet the design demands. A controller is designed and some considerations are presented to increase the bandwidth of the system. After tuning the system, the size of the theoretically developed hydraulic actuator is known.

To validate the developed model, the simulated transfer functions from the SIMULINK program are compared with measurements on an existing hydraulic actuator in chapter 4.

The second part, chapter 5, deals with available hydraulic cylinders and servo valves. These cylinders and servo valves are simulated, and the results are analyzed.

Finally, chapter 6 concludes which of the hydraulic cylinders and servo valves are most suitable for the measurement tower. Further, some recommendations are given.

Chapter 2

Modelling

In this first chapter the measurement tower and the hydraulic actuator will be analyzed. The model of the measurement tower is known from an earlier report [2], and so are some of the demands for the hydraulic actuator. The purpose of this chapter is to develop a method for determining the size of the hydraulic cylinder so that it meets the demands set in the earlier report. Therefore, the measurement tower and the hydraulic actuator are modelled in MATLAB and SIMULINK.

2.1 The measurement tower

As mentioned earlier, the model of the measurement tower is already known and so are the values of the variables. The model is represented by a mass-spring-damper system shown in fig 2.1, where m represents the total mass of the wheel carrier, rim and tire. The constants k and d are typical stiffness and damping values for a passenger car tyre.

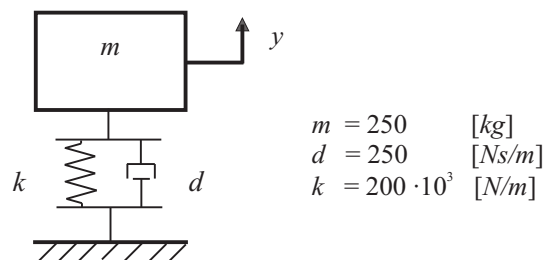


Figure 2.1: Model of the measurement tower

2.2 The actuating system

The hydraulic actuator will first be modelled as a cylinder with a flow supply delivered by an ideal servo valve. After analyzing the model and the outcoming results, the servo type of valve that is needed to fulfill the required demands on pressure and flow can be determined. This will be done in chapter 5.

First the a-symmetrical cylinder will be discussed. This cylinder consists of a rod that is attached to the piston. This rod connects the piston with a mechanical system, in this case the mass-spring-damper system of the measurement tower (see figure 2.2). The piston, connecting rod and mass of the measurement tower are modelled as a rigid body with a total mass m . Note that this changes the definition

of the total mass m presented in the previous paragraph.

Consequently, the displacement y of the mass m in figure 2.1 now equals the position of the rod of the cylinder which is positive defined downwards. To realize a positive piston displacement, the servo valve should be able to deliver a positive flow q to the hydraulic cylinder. In figure 2.2, the hydraulic cylinder is controlled by a valve. The spool displacement x in this valve, changes the port opening that results in a change in the flow q from servo valve to hydraulic cylinder. This flow results in a positive or negative piston motion. Part of the flow q is also needed to compensate for the leakages that are present in the cylinder.

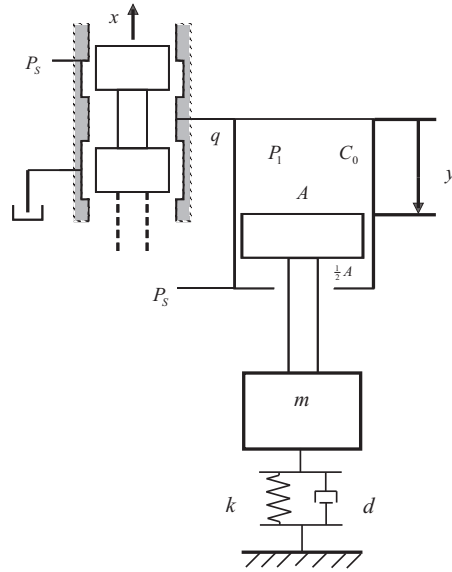


Figure 2.2: A-symmetrical cylinder connected to the measurement tower

In the hydraulic cylinder shown above there are a few properties that are of interest for modelling. An ideal servo valve is assumed, meaning that the valve has no dynamics and it completely controls the motion of the piston. When the flow q exceeds the compensation flow needed for leakages, the piston will be forced to move downwards ($y > 0$). This creates an oil volume (column) above the piston with a certain stiffness C_0 (see equation 2.1 and figure 2.2). This (column) stiffness depends on the stroke y , piston area A and the bulk modulus of oil E , and can be modelled as a spring between the piston and cylinder end.

$$C_0 = \frac{AE}{y} \quad (2.1)$$

This (spring) stiffness C_0 in the a-symmetrical cylinder is in parallel to the spring k of the tyre in the measurement tower. The model can be simplified by combining these springs:

$$K_{tot} = C_0 + k \quad (2.2)$$

The spring stiffness in the hydraulic actuator can now be replaced with the total stiffness K_{tot} . The damper d in the measurement tower model can be included in the external force F_{ext} , which already contains the gravity force, see figure 2.3:

$$F_{ext} = mg - d\dot{y} \quad (2.3)$$

Replacing the stiffness in the hydraulic actuator with the total stiffness K_{tot} and including the damper d in the external force F_{ext} reduces the model of figure 2.2 to the model of figure 2.3. Here it is chosen

to integrate the spring and damper of the measurement tower in the model of the hydraulic cylinder. This reduced model approaches the theoretical models in the applied literature of the course *Hydraulic Servo Systems* [3], what simplifies the determination of the equations needed to describe this model.

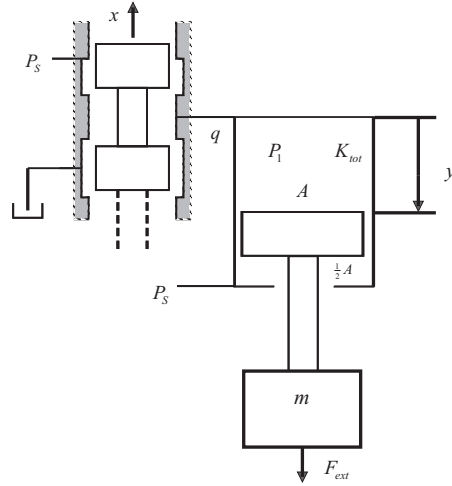


Figure 2.3: Reduced model of an asymmetrical cylinder on measurement tower

2.3 The hydraulic model

To find the equations needed for designing the hydraulic actuator, the hydraulic model must be further examined. The valve will still be regarded ideal and the pressure supply will be assumed constant. In chapter 5, the effect of available valves shall be introduced, analyzed and modelled. For the purpose of better understanding, the derivation of equation 2.4 is given in appendix B. Equation 2.4 applies to an a-symmetrical cylinder controlled by a critical center servo valve, like the one in figure 2.3.

$$\dot{y} = k_m \left(x - \frac{\sum F}{C_h} \right) - \frac{\sum \dot{F}}{C_0} \quad (2.4)$$

As in figure 2.2 and equation 2.1, C_0 refers to the spring stiffness of fluid inside the a-symmetric cylinder. An equation similar to equation 2.4 can be derived for symmetric cylinders (see appendix A). In this equation, x denotes the spool displacement in the servo valve and C_h refers to the hydraulic stiffness of the cylinder. This hydraulic stiffness reacts as a spring (i.e. stiffness) to the mechanical system, but has the unit of damping. The value of the hydraulic stiffness strongly depends on the type of servo valve that is used (see appendix B). The velocity gain k_m gives the linear relation between the spool displacement of the valve x and velocity \dot{y} of the piston. This velocity gain is a constant that strongly depends on the valve geometry (see Appendix B). The sum of the forces $\sum F$ in equation 2.4 consists of the external forces acting on the rigid body with mass m in figure 2.3. Gravity and the damping of the tyre in measurement tower model were already considered external forces in equation 2.3, which yields equation 2.5:

$$\sum F = m\ddot{y} - F_{ext} = m\ddot{y} + d\dot{y} - mg \quad (2.5)$$

According to paragraph 2.2, spring stiffness C_0 can be replaced by the total spring stiffness K_{tot} , which includes the spring in the measurement tower model:

$$\dot{y} = k_m \left(x - \frac{\sum F}{C_h} \right) - \frac{\sum \dot{F}}{K_{tot}} \quad (2.6)$$

The model introduced in the previous paragraph is now totally described by equations (2.6) and (2.5). Before implementing this in a MATLAB and SIMULINK program, the model will be further completed to meet our needs.

2.4 Model of the system in SIMULINK

Before the model can be implemented in SIMULINK, equations (2.6) and (2.5) are transferred to the Laplace domain. First, equation 2.6 is rewritten to another form:

$$\frac{\sum \dot{F}}{K_{tot}} = k_m \left(x - \frac{\sum F}{C_h} \right) - \dot{y} \quad (2.7)$$

Transferring (2.7) and (2.5) to the Laplace domain yields:

$$\frac{s \sum F(s)}{K_{tot}} = k_m \left(X(s) - \frac{\sum F(s)}{C_h} \right) - sY(s) \quad (2.8)$$

$$\sum F(s) = ms^2Y(s) + dsY(s) - F_{ext} \quad (2.9)$$

In equation 2.9, the gravity force mg is now replaced with F_{ext} , because it is independent on the displacement of the rigid body y :

$$\sum F(s) = ms^2Y(s) + dsY(s) - mg \quad (2.10)$$

In this form, the equation approximates the one used in literature [3]. After transferring the equations to the Laplace domain, the model can be put in a block representation 2.4.

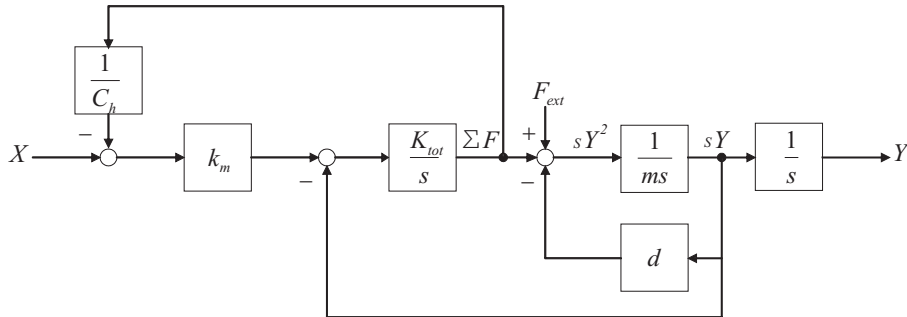


Figure 2.4: Block representation of model in Laplace domain

In figure 2.4, both equations (2.8) and (2.10) can be found starting in the middle of the block representation, namely with the sum of the forces $\sum F_{ext}$. The loop right of this unit corresponds to equation (2.8), and the loop on the left side to equation (2.10).

In general, the hydraulic actuator with measurement tower can also be completely described by the block representation of figure 2.4. This representation can directly be transferred to a computer program in SIMULINK.

In figure 2.5, the model in SIMULINK is shown. Clearly, on the right side the block representation of figure 2.4 can be found, including position feedback control. This feedback includes a controller that has to be designed and a servo valve with no dynamics. As mentioned earlier, the effects of a real servo valve will be determined once the size of the hydraulic cylinder is known (chapter 5). In this model the dynamic behaviour will be modelled with only sinusoidal input signals. Note that the gravity force

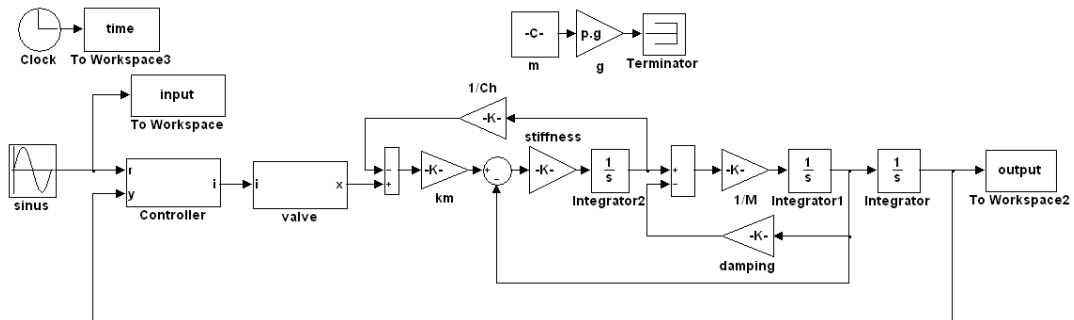


Figure 2.5: The model implemented in SIMULINK (see also appendix C)

is connected to a terminator and not included in the model. The gravity force mg , also denoted as external force F_{ext} , is independent of the piston displacement y . This force can be used as a static load on the tyre, next to a fixed load of the hydraulic actuator. Because the model will only examine dynamic responses, these static (steady-state) loads will not be included in the model and can be neglected. In the next chapter the model from figure 2.5 will be further adapted to the actual demands on dynamic behaviour.

Chapter 3

Design Of The Actuating System

In this chapter, the model of the hydraulic actuator and measurement tower will be adapted to the demands of the hydraulic actuator. From the earlier *HTS graduation report* [2], the demands are known on which the hydraulic actuator will be sized. First these demands will be recalled and updated, and then the system will be optimized for this situation. After analyzing the results, the sizes of the different types of hydraulic cylinders are known.

3.1 Demands on the hydraulic actuator

The demands, in the earlier *HTS graduation report* referred to as 'wishes', are shown in table 3.1. For this report, some of the demands on the hydraulic actuator were altered. These new demands are depicted next to the demands of the HTS-report.

	Demands HTS-report	New Demands
Maximum static force [N]	9000	9000
Stroke [mm]	160	160
Sinusoid frequency [Hz]	8	20
Sinusoid amplitude [mm]	20	10

Table 3.1: Demands on the hydraulic actuator

The demands on maximum force and stroke are restraints on the measurement tower itself. The static force of the actuator may not exceed 9 kN, because when it reaches 10 kN the measuring instrument on the wheel will be damaged. In the HTS-report a limit of 9 kN is chosen, including a safety factor. The stroke is limited too, because the maximum displacement of the wheel carrier in the measurement tower is restricted to 160 mm. The last two demands are variable, and only here the demands of the HTS-report differ from the new demands on the behaviour of the hydraulic actuator. The new demands, on which the size of the vertical hydraulic actuator will be designed, require an increased sinusoidal frequency and have a reduced demand on amplitude. Now the demands are known, the size of the hydraulic actuator can be determined.

3.2 Design of a hydraulic actuator

Now the system parameters are known and the demands are set, the model can be further developed to determine the size of the hydraulic cylinder. A way to start is to determine the minimum natural frequency needed to perform under the severest demands. From the natural frequency, the piston

area of the cylinder can be calculated. Equation 3.1 applies to a-symmetrical cylinders, but the design process of symmetrical cylinders is similar (see appendix A).

$$\omega_{0,cyl} = \sqrt{\frac{C_{0,min}}{m}} = \sqrt{\frac{AE}{mS}} \quad (3.1)$$

In this equation, $C_{0,min}$ denotes the minimum oil spring stiffness and S refers to the stroke (in contrast with equation 2.1, where the displacement y was considered). To determine the minimum natural frequency of the cylinder $\omega_{0,cyl}$, equation 3.1 must be used. The natural frequency, and oil stiffness, strongly depend on the displacement of the piston in the hydraulic cylinder. The other variables in the right part of equation 3.1 are known and assumed constant. The dimension of the hydraulic cylinder will be determined on basis of the minimum natural frequency, and thus on the stroke S (see appendix A). In this report f_0 and ω_0 both refer to natural frequency (see list of symbols), using equation 3.2:

$$f_0 = \frac{\omega_0}{2\pi} \quad (3.2)$$

First a natural frequency will be selected, to be able to implement the model in the simulation program MATLAB. From this natural frequency, other system parameters (e.g. the piston area A) can be determined. Note that in (3.1), only the spring stiffness of the cylinder $C_{0,min}$ is used and not the total stiffness K_{tot} mentioned in paragraph 2.2. The spring stiffness of the cylinder is variable, and so is the natural frequency of the hydraulic cylinder. Now only the hydraulic cylinder is considered, and not the total system that includes the measurement tower. To calculate the natural frequency of the total system, the spring stiffness $C_{0,min}$ should be replaced by the total spring stiffness K_{tot} .

There is a standard routine to determine if the natural frequency is chosen right, i.e. meets the demands on frequency and amplitude. First a value for natural frequency must be chosen or guessed. This natural frequency limits the bandwidth of the closed loop feedback system. So, if the natural frequency is high enough, the system will follow the input frequency on phase and amplitude closely, otherwise the output signal will have a phase delay and amplitude decrease.

In general, the bandwidth f_b is defined to be the maximum frequency at which the output of the system will track an input sinusoid in satisfactory manner. In this report, the bandwidth is defined to be the frequency where the sinusoidal input signal r on the control system is attenuated to the output y . In a Bode diagram, this refers to the frequency where an open loop transfer function intersects with the 0 dB line in the magnitude plot.

First the transfer function must be determined, before something can be stated about the bandwidth of the system. Equation 3.3 shows the general open loop transfer function of a hydraulic actuator. This equation can be derived from the standard equation 2.4 (see appendix B). In this equation, k_v is a constant that equals the velocity gain k_m , when assuming an ideal servo valve. β denotes the damping of the system, and a realistic values can be found in the literature [3].

$$H_{ol} = \frac{k_v}{s \left(\frac{s^2}{\omega_{0,cyl}^2} + s \frac{2\beta}{\omega_{0,cyl}} + 1 \right)} \quad (3.3)$$

Now, a good estimate can be made on how the hydraulic system will react on different input frequencies. When a realistic value for damping ($\beta = 0.1$) is chosen and an ideal servo valve ($k_v = k_m = 1$) is assumed, the transfer function can be calculated. If the natural frequency of the hydraulic servo cylinder is chosen to be 20 Hz, MATLAB returns the magnitude plot of figure 3.1.

Note that the transfer functions in figure 3.1 show a -1 slope for frequencies lower than the natural frequency, and a -3 slope for frequencies higher than the natural frequency. The -1 slope comes from the hydraulic cylinder, where the spool displacement x in the servo valve yields the needed flow

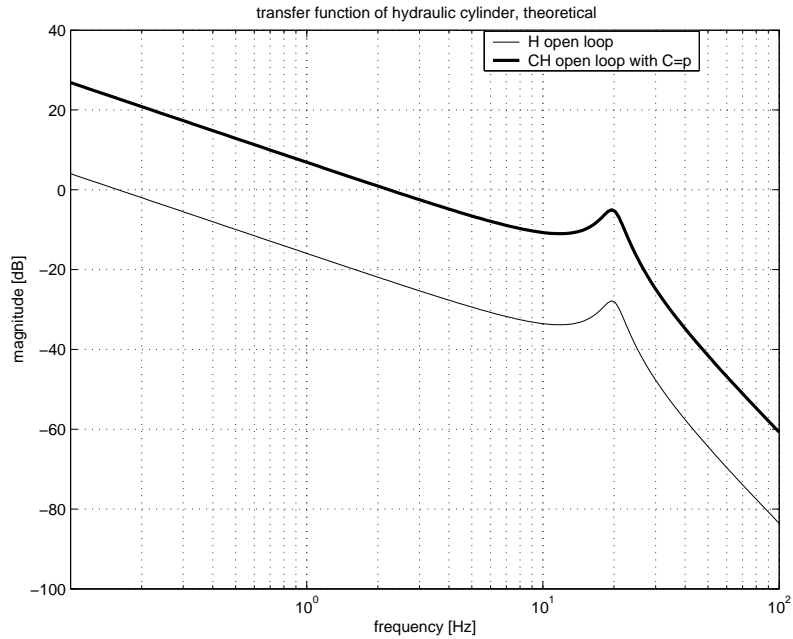


Figure 3.1: A-symmetric hydraulic cylinder with $f_0 = 20$ [Hz].

to move the piston in the hydraulic cylinder. The actuator's behaviour on low frequencies is considered to be an integrator (see appendix B). The other physical properties of a hydraulic cylinder result in a second order system, which is in series with the integrator. A second order system has a typical -2 slope in a frequency response diagram. Because these systems are in series, the -3 corresponds to the total of the first and second order system. This can also be derived from the transfer function of a hydraulic cylinder with servo valve, presented in equation 3.3.

In figure 3.1, the lowest graph refers to the open loop transfer function with no controller. For this transfer function of the a-symmetrical cylinder, also described by equation 3.3, a controller can be designed.

For reasons of stability, there is a simple guideline that restricts the gain of a proportional controller: the top of the resonance peak should be 5 dB under the zero dB line. This follows from the stability criterion for the magnitude of the closed loop transfer function $|H_{cl}| = m = 1.3$ in a Nyquist diagram, stated in literature [3]. This makes that the open loop characteristic in figure 3.1 can be increased in gain, which results in a larger bandwidth of the system. The open loop transfer function with controller has a bandwidth of approximately 2.2 Hz.

Because this example above does not meet the demands, the natural frequency has to be increased. For this a-symmetric cylinder, an sinusoid input signal r with a frequency of 20 Hz would be attenuated to the output y with a factor of 0.56.

Note that in case of a symmetric cylinder, the design process is identical but with different values for the surface area of the piston A and the minimum (spring) stiffness $C_{0,min}$ (see appendix A).

3.3 Increasing bandwidth

A possible solution to increase the bandwidth of the system is choosing a larger cylinder, with a larger piston area, which results in a higher natural frequency (see equation 3.1). From the previous paragraph can be concluded that the natural frequency of the cylinder should be at least well above the mentioned 20 Hz, to create a bandwidth of 20 Hz. Increasing the natural frequency automatically leads to a larger cylinder.

Another way to increase the bandwidth is making use of pressure or acceleration feedback. Both the pressure and the acceleration feedback will give the same outcome by increasing the damping of the hydraulic cylinder, which reduces the resonance peak at the natural frequency. In case of acceleration feedback, the piston acceleration in the hydraulic cylinder is measured by a transducer. The output of this signal is used as a feedback signal on the spool displacement x of the servo valve (see figure 3.2).

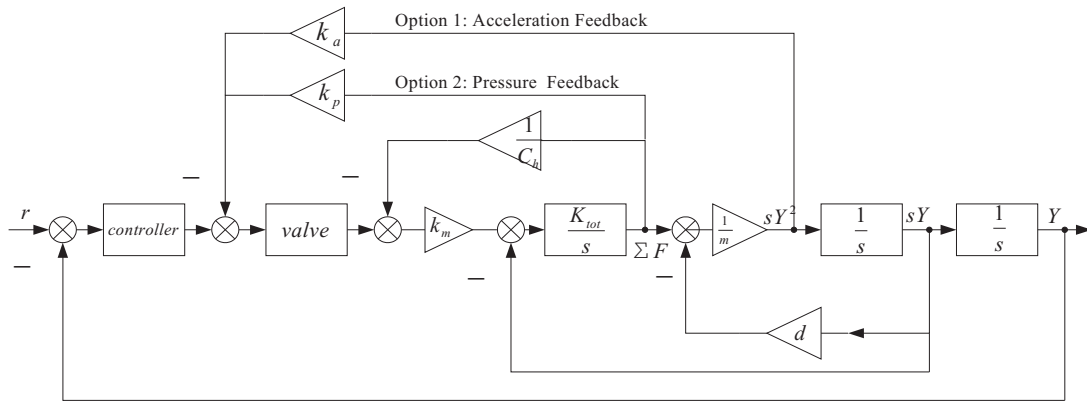


Figure 3.2: Pressure and acceleration feedback.

An alternative for measuring acceleration is using a pressure transducer which is mounted on the standing cylinder. The pressure inside the cylinder is measured, and the output of the transducer is again used as feedback on the spool displacement x of the servo valve.

Figure 3.3 shows a Bode diagram of two closed loop transfer functions. First, the closed loop transfer function of hydraulic cylinder is plotted with a natural frequency of 40 Hz (upper graph). The only difference with figure 3.1 is the choice of a larger hydraulic cylinder. The proportional controller is designed, according to the guideline in the previous paragraph, on a magnitude of -5 dB at the natural frequency of the open loop transfer function for reasons of stability.

Secondly, the transfer function of the system is shown with pressure feedback, as modelled in the block diagram of figure 3.2. The transfer function of this system is determined by putting a white noise signal on the input of the system, and then measuring the output.

For the considered pressure or acceleration feedback, the constants k_a and k_p should be determined by trial and error. The intended result is to increase damping and reducing the resonance peak, as figure 3.3 shows. From the phase plot can be seen that the phase has shifted 180 degrees at the natural frequency. This is in accordance with the literature, but is of minor importance when developing a vertical hydraulic actuator for the measurement tower.

Now the pressure feedback is applied and the resonance peak is damped, the proportional controller

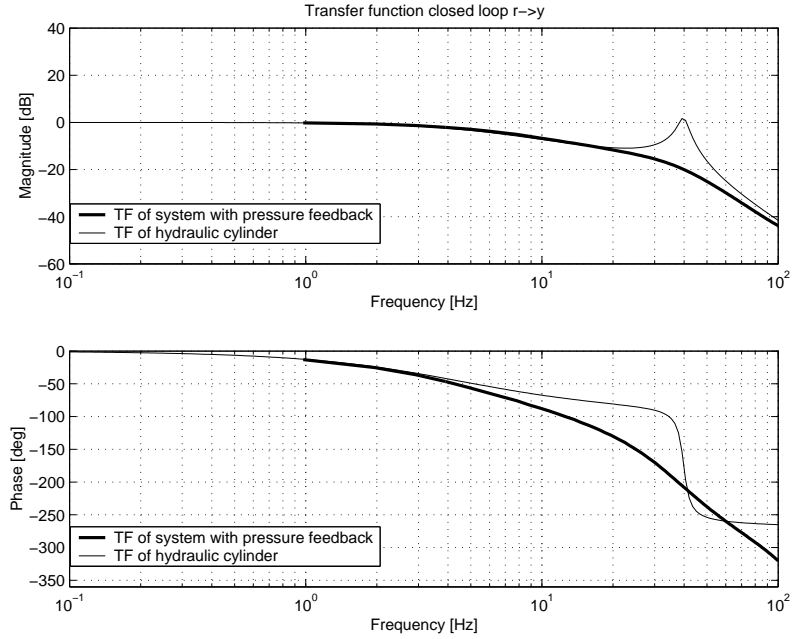


Figure 3.3: Transfer function closed loop with and without pressure feedback.

can be tuned to increase the bandwidth of the system (see figure 3.4). In the literature [3] is also mentioned how far the gain of the p-action can be increased before the system becomes unstable. The controller has to be tuned by taking into account the resonance peak in the Bode diagram. When the controller is tuned too high, the system will be on the edge of stability. On the other hand, if the controller is tuned too weak the performance of the system will be poor for its size and costs.

The system in its current form, including pressure feedback and completely tuned, has a bandwidth at about 23 Hz. On that frequency the system shows a phase shift of 108 degrees. This means that a sinusoidal input signal with frequency of 23 Hz, would not attenuate the system's output signal on amplitude and would have a phase shift of 108 degrees.

The hydraulic actuator is only designed on the magnitude response, because phase response is not relevant for this application. Most importantly should the actuator be able to put a sinusoidal signal (with frequencies up to 20 Hz) on the measurement tower. This is a constant signal, where a phase shift between the input signal r and output signal y is of minor importance as long as the system stays stable.

3.4 Results of simulations

In the previous paragraph a hydraulic actuator is developed on the demands of table 3.1. Note that the hydraulic cylinder is designed on natural frequency, and that this does not affect the choice of a symmetric or a-symmetric cylinder. If a symmetric and a-symmetric cylinder have a equal natural frequency, they will have a different size due to the different piston area.

The natural frequency of both types of hydraulic systems is designed on a frequency of 40 Hz. With equation 3.1, the minimum spring stiffness and piston area for an a-symmetrical can be calculated: $C_{0,min} = 15,8 \cdot 10^6 [N/m]$ and $A_{a-sym} = 2567 [mm^2]$.

The symmetrical cylinder has a equal spring stiffness: $C_{0,min,a-sym} = 15,8 \cdot 10^6 [N/m]$, and the

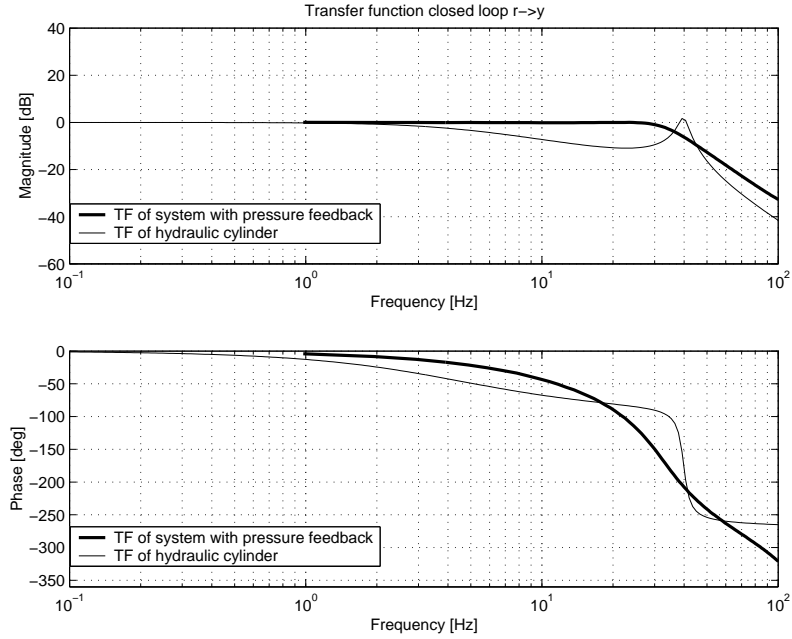


Figure 3.4: Transfer function of tuned system with pressure feedback.

piston area can be calculated with equation A.6: $A_{sym} = 631,6 [mm^2]$.

More precisely, the piston area of an a-symmetrical cylinder has to be four times larger than that of a symmetrical cylinder. This can be easily derived from appendix A. In general, a symmetrical type of cylinder is preferred above an a-symmetrical, but this also shows in price difference between the two.

These hydraulic cylinders are designed on a natural frequency of 40 Hz, and in accordance with equation 3.1 the spring stiffness is calculated. As mentioned, the total spring stiffness K_{tot} of the system does not only consist of the spring stiffness of the cylinder $C_{0,min}$, but also of a spring k from the tyre in the measurement tower. This means that the natural frequency of the system $\omega_{0,sys}$ is somewhat higher than the natural frequency of the cylinder $\omega_{0,cyl}$.

$$\omega_{0,sys} = \sqrt{\frac{K_{tot}}{m}} = \sqrt{\frac{C_{0,min} + k}{m}} \quad (3.4)$$

Summing both stiffnesses yields the total stiffness of the system ($K_{tot} = 16 \cdot 10^6 [N/m]$) which is not so much different from the cylinder's spring stiffness, because the spring in the measurement tower is very small compared with the oil's spring stiffness. According to equation 3.4, the system's natural frequency can be calculated and yields $f_{0,sys} = 40,25 [Hz]$.

Note that this also applies to figures 3.3 and 3.4, where the plot with resonance peak refers to the hydraulic cylinder and the other to the whole system. Because of the little difference, this does not affect the results on size for the designed cylinder.

Chapter 4

Model Validation

The general approach that was chosen to model the hydraulic actuator is based on theory presented in appendix B. To verify that the equations and assumption in this approach are valid, the model will be compared with measurements executed on a hydraulic actuator system.

The measurements were executed on an a-symmetrical cylinder that was connected to a load with a mass of 130 kg. The piston was positioned on a stationary position y , to be able to consider a constant natural frequency. The input signal r working on the spool displacement in the servo valve was a sinusoidal with low frequency and amplitude. A random noise signal with a frequency contents up to 200 Hz was used to actuate the system, while the system was controlled by a weak proportional controller to keep the cilinder in the desired position.

Figure 4.1 shows the measured frequency response of this hydraulic cylinder with servo valve. Here, the piston was positioned on half of cilinder's stroke, which corresponds to a position of $y = 100$ [mm]. The characteristic data of this hydraulic cylinder was inserted into the theoretical model, which yields the simulated frequency response.

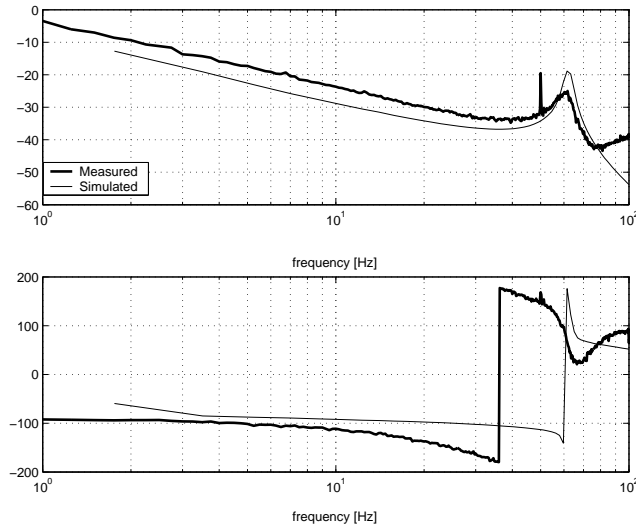


Figure 4.1: Simulated and measured transfer function open loop, $y = 100$ [mm]

First, and most importantly, it can be noticed that the natural frequencies in this figure are equal; $f_0 = 62$ [Hz]. Secondly, both transfer functions show a -1 slope in the frequency range under the natural frequency, and a -3 slope at higher frequencies than the natural frequency, in accordance with the explanation on page 10.

In the measured transfer function from figure 4.1, there is a small part of the -3 slope visible, but at frequencies above 70 Hz the slope of the transfer function seems to rise. This can be explained by the method of processing the data in the measuring device. Around a frequency of 70 Hz, the measured output signal has a magnitude of less than -40 dB, which means that the output signal is decreased to an amplitude that is 100 times smaller than the input signal. Because the big difference between signal level, and the processing of time signals in the frequency domain the measurement will only consist out of noise. Around a frequency of 50 Hz and 150 Hz, a small sharp peak can be found that can be explained by the frequency of the main voltage that lies at 50 Hz.

When comparing the graphs in figure 4.1 more closely, it strikes that the resonance peaks at the natural frequency are not equal in size. The size of a resonance peak is determined by the damping in the system. In the simulated transfer function the damping is mainly determined by the hydraulic stiffness C_h in the hydraulic model (see appendix B). The used hydraulic cylinder has hydrostatic bearings, what raises the leakage in total hydraulic system and automatically leads to a smaller hydraulic stiffness C_h . Next to this extra leakage, no additional damping from the servo valve is considered. In the real situation, the servo valve does have an effect on the system's damping, which is very dependent from valve to valve. This explains the size differences between the resonance peaks in the transfer functions of the simulated model and the real hydraulic cylinder with servo valve.

A servo valve has its own frequency response, and this not only effects the resonance peak. The measured transfer function in figure 4.1, consists of the separate transfer functions of the hydraulic cylinder and the servo valve. The effects a servo valve has on the frequency response of the system will be further examined in chapter 5. Already can be said that the difference in height between the transfer functions in figure 4.1 is a result of the non ideal servo valve.

Because the piston was positioned at a fixed position, the natural frequency could be considered to be constant. When the position (or displacement y) of the piston is changed, this automatically leads to a different natural frequency. In the transfer functions of figure 4.1, the piston was positioned to half of the stroke S , which equals a position of $y = 100$ [mm]. In figures 4.2 and 4.3, other piston positions are considered.

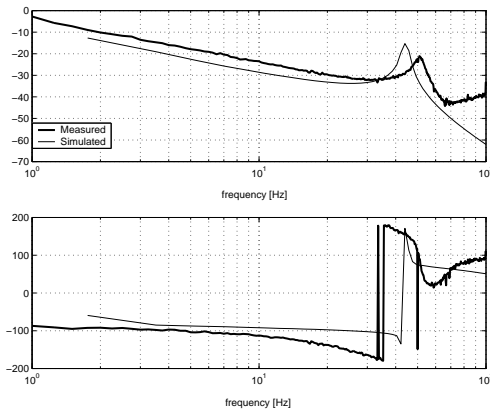


Figure 4.2: Measured and simulated, $y = 200$ [mm]

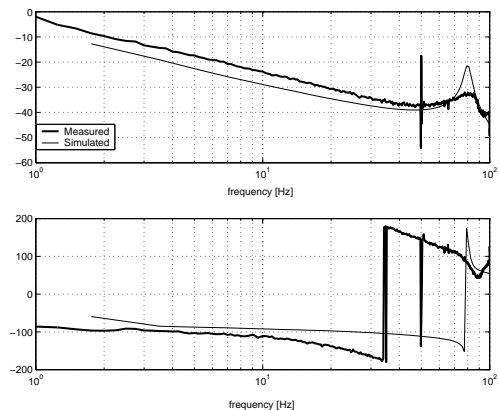


Figure 4.3: Measured and simulated, $y = 60$ [mm]

Figure 4.2 shows the measured and simulated transfer function of the hydraulic actuator with a position of $y = 200$ [mm], which equals the stroke S . In this Bode diagram a natural frequency around $f_0 = 50$ [Hz] can be found for both transfer functions. Figure 4.3 refers to a position around $y = 60$ [mm], which is considered the minimum fixed displacement for this hydraulic cylinder.

Chapter 5

Simulating Servo Systems

In the previous part of the report, the aim was to design a hydraulic cylinder on basis of the given demands. Now that the sizes of the developed symmetrical and a-symmetrical cylinders are known, the model can be extended. Out of the size of the hydraulic cylinders, the needed flow and pressure can be determined and a servo valve can be chosen and implemented in the model. The dynamics of a servo valve model may be approximated by a second order approximation of the frequency response data from available servo valves. In combination with these servo valves, some available hydraulic cylinders will be implemented and analyzed.

5.1 Including the servo valve in the model

So far, the servo valve in our model was assumed ideal. To be able to assume that a servo valve acts ideal, the natural frequency of the servo valve should be about twice as high as the natural frequency of the hydraulic system (see course *Hydraulic Servo Systems* [3]). Although this assumption is a good approximation of reality, in this chapter the servo valve will assumed non ideal.

The sizes of the developed hydraulic cylinder types are determined in paragraph 3.4. From the sizes and the demands on frequency and amplitude from table (3.1), the flow q can be determined. First the maximum velocity of the piston is determined.

Assume that the piston's motion follows a harmonic input signal, described by equation 5.1:

$$y = a \cdot \sin(2\pi ft) \quad (5.1)$$

y describes the piston displacement, f the frequency of sinusoidal signal and a , t respectively relate to sinusoid amplitude and time. The velocity of the piston \dot{y} can be obtained by differentiating this function:

$$\dot{y} = -2\pi f \cdot a \cdot \cos(2\pi ft) \quad (5.2)$$

Hence, the maximum velocity:

$$v_{max} = 2\pi f \cdot a \quad (5.3)$$

Assuming a sinusoidal signal with frequency of $f = 20$ [Hz] and amplitude of $a = 10$ [mm], yields a maximum velocity of $v_{max} = 1.26$ [m/s]. From this maximum velocity, the maximum flow can be determined:

$$q = v_{max} \cdot A \quad (5.4)$$

The flow demand for the theoretical determined a-symmetrical and symmetrical cylinders are given in table 5.1.

	Piston area A [mm ²]	Flow q [l/min]
a-symmetrical cylinder	2567	194
symmetrical cylinder	632	48.6

Table 5.1: Theoretical flow demands for servo valves

The servo valve model will be approximated with data from available servo valves to describe its dynamic behaviour. This data consist of a frequency response and is plotted in a Bode diagram. This data is fitted with a second order approximation in a MATLAB program, see figure 5.1. The data in this figure refers to a standard two stage servo valve (Moog model 76-104) with a pressure range of 15 to 210 bar, and can deliver a nominal flow of $q = 57$ [l/min] at $\Delta P = 70$ [bar].

This flow restricts the choice between the cylinder types in table 5.1. Only the symmetrical cylinder has a flow requirement that can be provided by this servo valve. Figure 5.2 shows the transfer function of the developed hydraulic actuator with this servo valve. Also depicted in this figure is the transfer function of the hydraulic actuator with an ideal assumed servo valve. It is shown that this servo valve does have its influences on the frequency response of the hydraulic actuator. Although this model includes pressure feedback, determined according to the method used in paragraph 3.3, two natural frequencies can be found. These refer to the natural frequencies of the hydraulic system and the servo valve. Implementing a real servo valve also changes the damping of the system, which means that the controller has to be tuned for this specific situation.

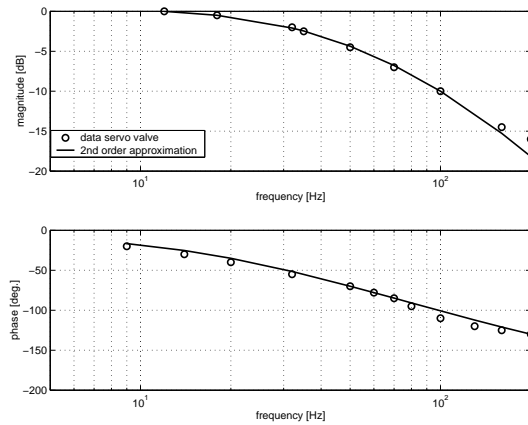


Figure 5.1: Servo valve approximation

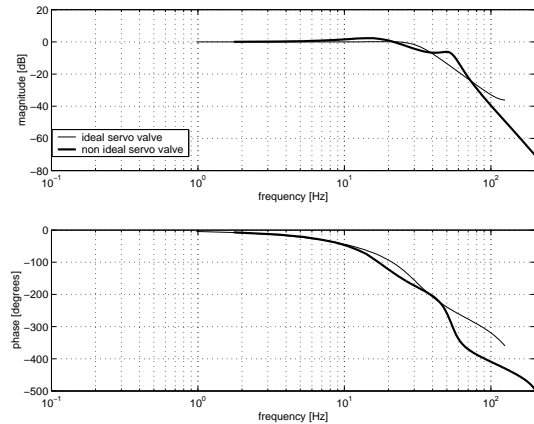


Figure 5.2: Model with non ideal servo valve

Already can be concluded that the developed symmetrical cylinder combined with this servo valve complies with the requirements on which the system was designed. According to the definition on page 9, a bandwidth of $f_b = 22$ [Hz] can be found in figure 5.2. Note that the transfer function in the magnitude plot of the Bode diagram rises above the 0 dB line around a frequency of 15 Hz. This means that the system's input sinusoid signal is amplified around this frequency. This dynamic behaviour can be adjusted by reducing the p-action of the controller, or expanding the model with e.g. a notch filter.

5.2 Available cylinders

Now that the model of the hydraulic actuator is expanded with a non ideal servo valve, some different situations can be regarded. In this paragraph, the sizes of available cylinders will be implemented in

the model in combination with available servo valve data.

In table 5.2 the sizes of a few available hydraulic cylinders are given. The measurement cylinder was used to validate the model in chapter 4, and the other cylinders are part of the *Series 248 Hydraulic Actuators* from *MTS Systems Corporation*.

The piston area A is denoted for each cylinder and with equation 5.4 the flow q is determined. From the piston area of these hydraulic cylinders, the natural frequency ω_0 can be determined with equation 2.1 (or equation A.6 for symmetrical cylinders). In the equations for the natural frequency, the mass m and position y are assumed to be equal to the constants used in the model of the hydraulic actuator and measurement tower in figure 2.3).

	Piston area A [mm ²]	Flow q [l/min]	Natural frequency f_0 [Hz]
Measurement cylinder (a-sym)	1900	143	35
MTS model 248.01 (sym)	523	39,4	36
MTS model 248.02 (sym)	832	62,7	46
MTS model 248.03 (sym)	1452	110	61

Table 5.2: Available cylinders

In table 5.2, the MTS models 248.01 and 248.02 seem to be the most applicable hydraulic cylinders to use for the measurement tower. This can be concluded out of the natural frequencies and the required flow. Like in the theoretical developed hydraulic actuator, natural frequencies around 40 Hz are needed to meet the demands from table 3.1.

The maximum required flows q , denoted in table 5.2, that these hydraulic cylinders need to perform in the most extreme circumstances are needed to choose the appropriate servo valve. Standard two stage servo valves, like the one in figure 5.1, are capable of delivering flows up to 65 l/min. When flows above 65 l/min are required, other types of servo valves, e.g. three stage servo valves, must be considered. Three stage servo valves can deliver more flow to the cylinder but have lower natural frequencies. This means that their dynamic behaviour is worse than that of a two stage servo valve. Three stage servo valves deliver more flow, but the spool size restricts their motion dynamics. In this report, only two stage servo valves will therefore be discussed.

5.3 Options

To be able to use a a-symmetrical hydraulic cylinder in combination with a two stage servo valve, the demands in table 3.1 must be reconsidered. One option to reduce the required flow q is to decrease the demands on amplitude and frequency, see equations 5.3 and 5.4. Another option is to choose an a-symmetrical cylinder with a smaller piston area A . In this paragraph a few alternatives will be discussed for the hydraulic cylinders from table 5.2. In all of these options, the mass and the position of the piston are assumed to be equal with the constants used in the theoretical model, thus $m = 250$ [Kg] and $y = 160$ [mm].

5.3.1 A-symmetrical hydraulic cylinder

The a-symmetrical cylinder from 5.2 was already discussed in chapter 4. Here, this hydraulic actuator will be implemented in the measurement tower model with the servo valve from figure 5.1. This servo valve can provide flows up to 57 l/min, which does not fulfill the requirements on flow in table 5.2. According to equation 5.4, the flow q that the hydraulic cylinder needs is determined by the piston area A and the maximum velocity of the piston v_{max} . So, the only way to reduce the flow requirement for this hydraulic cylinder is to restrict the velocity v_{max} . If the flow is limited to $q = 57$ [l/min], then the piston's maximum velocity is limited to $v_{max} = 0.5$ [m/s], which equals a sinusoidal input signal

with a frequency of $f = 10$ [Hz] and an amplitude of $a = 8$ [mm].

When the a-symmetrical cylinder is implemented in our model, the transfer function can be determined as in chapter 3, and a natural frequency $f_0 = 35$ [Hz] can be found. In figure 5.3, the transfer functions of the hydraulic cylinder with the servo valve from figure 5.1 is depicted. Also the transfer function of the hydraulic cylinder without pressure feedback and an ideal valve is shown as a reference. From this figure, a bandwidth can be found around $f_b = 16$ [Hz]. Although the piston is limited on a maximum velocity, still some different sinusoidal piston motions can be regarded. For instance, a input sinusoidal with a frequency of $f = 16$ [Hz] and amplitude of $a = 5$ [mm] would not be attenuated to the system's output, and requires a flow of 57 [l/min].

If the considered servo valve (Moog model 76-104) should be used on the theoretical determined a-symmetrical cylinder from table 5.1, the flow demands should also be reduced to $q = 57$ [l/min]. This would result in a maximum flow $v_{max} = 0.37$ [m/s], which yields for a sinusoidal piston motion with frequency $f = 16$ [Hz] an amplitude of $a = 3.6$ [mm], or for $f = 10$ [Hz] an amplitude of $a = 5.9$ [mm].

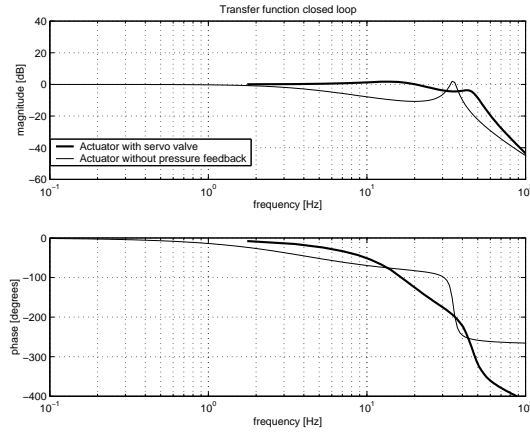


Figure 5.3: LAB cylinder with servo valve on measurement tower

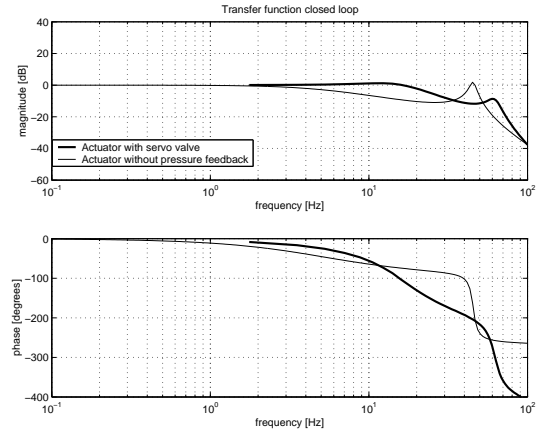


Figure 5.4: Cylinder model MTS 248.02 with servo valve on measurement tower

5.3.2 Symmetrical cylinder MTS model 248.02

Now consider the symmetrical cylinder model 248.02 from table 5.2, with a flow requirement $q = 62.7$ [l/min]. Again, this required flow must be adjusted to be able to combine this cylinder with the Moog 76-104 servo valve. This yields a maximum piston velocity of $v_{max} = 1.14$ [m/s], or sinusoidal piston motion with frequency of 20 Hz and amplitude of 9 mm. These results are very close to the demands from table 3.1.

The transfer function of this symmetrical hydraulic cylinder with servo valve is shown in figure 5.4, next to the transfer function of the hydraulic cylinder without pressure feedback and an ideal assumed servo valve. This Bode diagram, and table 5.2, show the natural frequency of this system: $f_0 = 46$ [Hz]. The bandwidth of the system can be found around $f_b = 16$ [Hz], which equals the bandwidth of a-symmetrical cylinder of figure 5.3, although the natural frequencies of both cylinders are different. This can be explained by the servo valve, that has a big influence on the frequency response of the system.

5.3.3 Symmetrical cylinder MTS model 248.01

Next to the standard Moog 76-104 servo valve, the high response Moog model 76-233 is implemented in our measurement tower model. The maximum flow this servo valve can deliver equals $q = 38$ [l/min]. The hydraulic cylinder most suitable to combine with this servo valve is MTS model 248.01 from table 5.2, with a flow requirement of $q = 39.8$ [l/min]. First, this required flow is reduced what yields a maximum piston velocity of $v_{max} = 1.19$ [m/s], which corresponds to a sinusoidal piston motion with frequency of 20 Hz and amplitude of 9.5 mm. Figure 5.5 shows the second order approximation of the Moog 76-233 servo valve model. The transfer function of the MTS 248.01 symmetrical cylinder with this servo valve is depicted in figure 5.6, together with the transfer function of the hydraulic cylinder without pressure feedback and an ideal assumed servo valve. The natural frequency of this hydraulic actuator can be found at $f_0 = 36$ [Hz], and is also given in table 5.2. The choice for a high response valve, increases the bandwidth of the system to $f_b = 20$ [Hz]. This combination of servo valve and hydraulic actuator comes closest to the demands on sinusoidal frequency and amplitude from table 3.1.

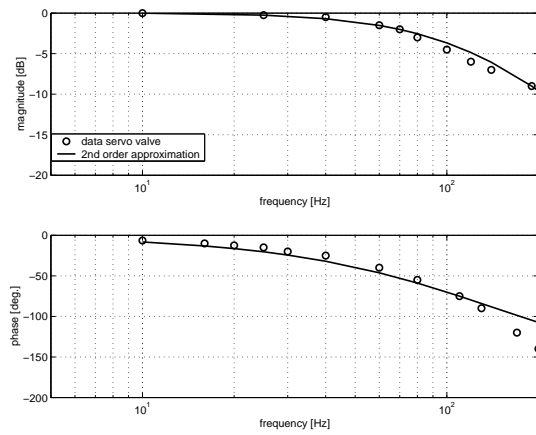


Figure 5.5: Servo valve approximation

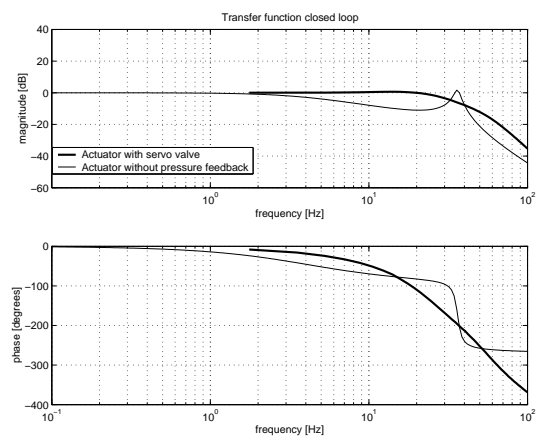


Figure 5.6: Cylinder model MTS 248.01 with servo valve on measurement tower

Chapter 6

Conclusion And Recommendations

In the introduction, the aim of this report has been defined as: to complete the measurement tower with a hydraulic actuator, so that the dynamic response of different kind of tyres can be measured. The first goal is to design a hydraulic actuator on the given demands. To be able to design a hydraulic actuator, a model of the measurement tower and hydraulic actuator has been developed in chapter 2. This model has been implemented in a SIMULINK program, to determine the frequency response.

In chapter 3, the hydraulic actuator has been designed on the given demands and pressure feedback is used to increase the bandwidth of the system. The natural frequency has been set at 40 Hz, which resulted in different piston areas for a-symmetric and symmetric cylinders. The hydraulic cylinder has been designed and the size was determined. From this chapter it is concluded that for equal natural frequencies, symmetric cylinders have smaller piston areas.

To validate the developed model, the simulated transfer functions from the SIMULINK program have been compared with measurements on an available hydraulic actuator in chapter 4.

The second goal of the report is to implement available hydraulic cylinders and servo valves in the model and analyze the results.

First an dynamic servo valve has been implemented in the model of the theoretical hydraulic actuator in chapter 5. The model of the servo valve consist of a second order approximation based on frequency response servo valve data. To find a suitable servo valve, the flow requirements for both of the developed hydraulic actuators (a-symmetric and symmetric) have been determined. For symmetric actuators, the flow requirement can be delivered by a two stage servo valve. For a-symmetric cylinders, the required flow is too high and a three stage servo valve should be used. In case of the measurement tower, two stage servo valves are preferred to three stage servo valves because of their dynamic response. Two stage servo valves have an overall faster dynamic response, and are therefore more suitable for the high response demands for which the hydraulic actuator is developed. The theoretically developed symmetric cylinder has been simulated with an available two stage servo valve, and it is shown that it satisfies the flow requirements ($q < 57$ [l/min]) and the dynamic demands ($f_b = 20$ [Hz]).

Secondly, some available hydraulic cylinders have been implemented in combination with available servo valves. First, an a-symmetric cylinder has been considered. To use this type of cylinder with a two stage servo valve, the flow requirement must be adjusted. It is shown that the required flow is too high. There are two ways to reduce the flow requirement. First, an a-symmetrical cylinder with smaller piston area can be taken, which reduces the natural frequency and thus the dynamics of the system. Secondly, the piston's velocity, in this case the amplitude and frequency of the sinusoidal motion, can be reduced. Both options do not comply with the demands on frequency and amplitude denoted in table 3.1, for which the hydraulic cylinder is developed and it can be concluded that this type of cylinder does not fulfill the requirements.

Next, two symmetric cylinders have been implemented in combination with two different servo valves. In this case, not the cylinders but the related two stage valves resulted in the main difference between dynamic responses. The first symmetric cylinder, MTS model 248.02, in combination with a standard two stage servo valve, Moog model 76-104, results in a bandwidth of $f_b = 16$ [Hz]. Again the required flow has to be reduced, resulting in a limit on amplitude and frequency of the piston's motion ($a = 9$ [mm], $f = 20$ [Hz]). The second symmetric cylinder, MTS model 248.01, has a smaller piston area which results in a lower natural frequency. Because of its smaller piston area, the required flow is reduced, and this makes it possible to use it in combination with a high response servo valve, Moog model 76-233. This results in a bandwidth of $f_b = 20$ [Hz], but also limits the amplitude and frequency of the sinusoidal piston motion ($a = 9.5$ [mm], $f = 20$ [Hz]). The high response servo valve is responsible for a higher bandwidth of the system. This combination of high response servo valve with a smaller a-symmetric cylinder perform best under the requirements.

Finally, it is recommended to use a symmetric hydraulic cylinder in combination with a two stage high response servo valve. The symmetric cylinder is preferred to the a-symmetric cylinder because of its flow requirement. This flow requirement makes it possible to use high response servo valves, and also limits the maximum velocity of the piston movement only in a minor way. This makes it capable of realizing the sinusoidal piston motions that the measurement tower requires.

Before the measurement tower is completed with a hydraulic actuator, the demands on the sinusoidal amplitude and frequency should be reconsidered. Although the symmetrical cylinder gives the best overall performance, the choice for an a-symmetrical cylinder is more convenient. Several a-symmetrical cylinders are already available on the TU/e, and the a-symmetrical cylinder is less expensive than a symmetrical cylinder. If an a-symmetrical cylinder would be applied on the measurement tower, the recommendation is to use it in combination with a two stage servo valve to maintain a fast dynamic response. As explained, the required flow must be decreased, and therefore the demands on sinusoidal piston motion must be reduced. One way to reduce this sinusoidal piston motion is to decrease only the demand on amplitude, to maintain the dynamics of the system. Next to decreasing the demands on sinusoidal amplitude, the demand on maximal stroke can be reduced. This restricts the size of the tyres that can be tested, but it results in a smaller cylinder for an equal natural frequency, and yields a smaller piston area and thus a reduced flow requirement.

After the choice for a hydraulic cylinder and servo valve is made, the size of the needed pump and motor have to be determined to complete the hydraulic system. From the required supply pressure (210 bar) and the needed flow, the necessary pump output can be determined. This result can be used to find the size of motor that will safely yield the necessary output. Additional components for the hydraulic system like the filter, oil tank, pipes and hoses, etc. have more specific demands.

Appendix A

A-symmetric Versus Symmetric Cylinders

In paragraph 2.2 only the a-symmetrical cylinder was discussed. In case of a symmetrical cylinder, equation (2.1) does not apply. In the symmetrical cylinder, the piston is attached on both sides to a rod (see figure A.2). Now the volume left as well as the volume right from the piston contributes to the (column) stiffness.

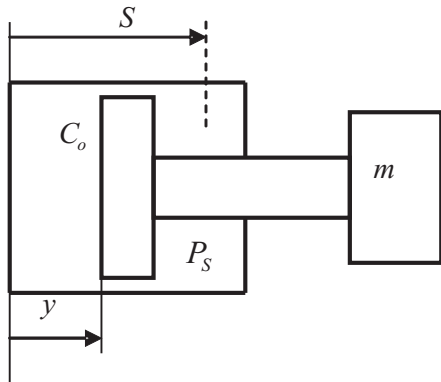


Figure A.1: A-symmetrical cylinder

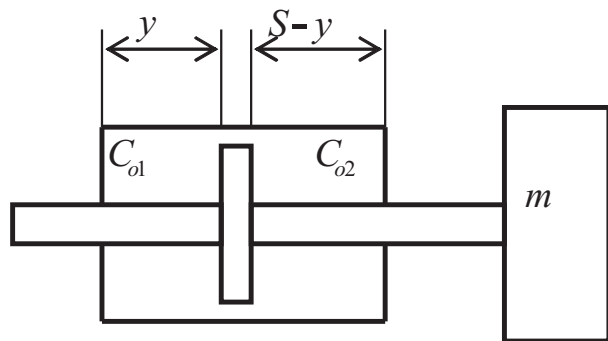


Figure A.2: Symmetrical cylinder

The stroke S is considered to be the maximum distance the piston can move in the cylinder (note that this is different from equation 2.1, where position y was used). Figure A.1 shows that this is in theory the length of the cylinder minus the thickness of the piston. This yields the minimum spring stiffness:

$$C_{0,min} = \frac{AE}{S} \quad (\text{A.1})$$

In case of the symmetrical cylinder in figure A.2, two oil columns contribute to the spring stiffness, so the total stiffness is determined by:

$$C_{0,min} = C_{o1} + C_{o2} \quad (\text{A.2})$$

where,

$$C_{01} = \frac{AE}{y} \quad (\text{A.3})$$

and,

$$C_{02} = \frac{AE}{S-y} \quad (\text{A.4})$$

Suppose that the piston in the symmetrical cylinder is situated in the mid-position. So the volumes on both sides of the piston are equal. In that case y in (A.3) and (A.4) equals half of the stroke S of the cylinder:

$$y = \frac{S}{2} \quad (\text{A.5})$$

The total stiffness can now be described by:

$$C_{0,min} = C_{01} + C_{02} = AE \left(\frac{1}{y} + \frac{1}{S-y} \right) = \frac{4AE}{S} \quad (\text{A.6})$$

So, compared with the a-symmetrical cylinder, the symmetrical cylinder shows a four times larger spring stiffness. This means that there is a lot of difference in the performances of these cylinders, considering that they have the same size. As a result of the spring stiffness, the minimum natural frequencies can be calculated for both types of cylinders:

$$\omega_0 = \sqrt{\frac{C_{0,min}}{m}} \quad (\text{A.7})$$

Consequently, it can be easily derived that the natural frequencies relate as:

$$\omega_{0,a-symmetrical} = 2\omega_{0,symmetrical} \quad (\text{A.8})$$

Appendix B

Equations

In the course *Hydraulic Servo Systems* [3], the standard equation that describes the hydraulic actuator is introduced. In this appendix follows a derivation of this equation.

Consider the a-symmetric actuator in figure B.1, here controlled by a critical center servo valve.

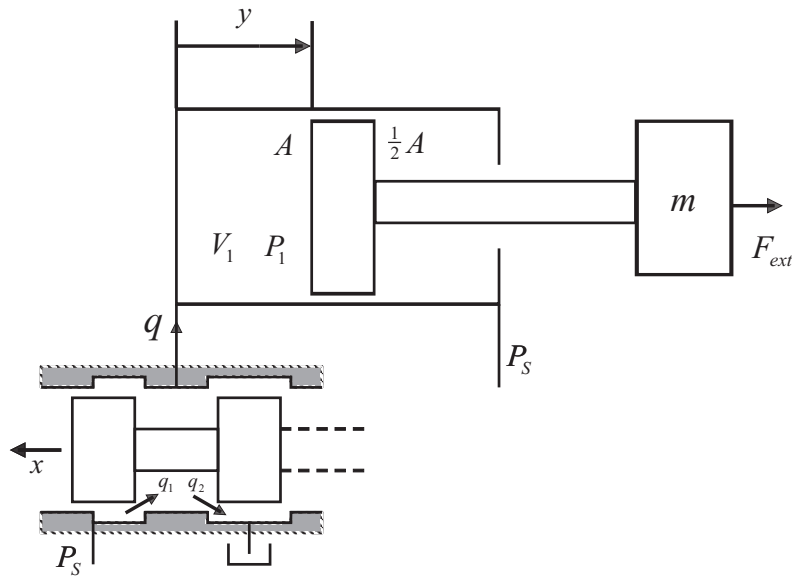


Figure B.1: A-symmetrical actuator controlled by a critical center servo valve.

From figure B.1, the force balance can be written as:

$$m\ddot{y} = AP_1 - \frac{1}{2}AP_s - d\dot{y} + F_{ext} \quad (\text{B.1})$$

Where y denotes the piston displacement, A is the piston area, P_1 is the pressure between piston and cylinder-end and P_s the constant supply pressure. As mentioned in paragraph 2.2, the damping d denotes the damping of the tyre in the measurement tower. F_{ext} consist out any external forces acting on the rigid body with mass m , in this case only the gravity (spring k of the tyre in measurement tower is included with the oil stiffness C_0 in total spring stiffness K_{tot}).

After rearranging (B.1), the force balance is reduced to (B.4):

$$m\ddot{y} + d\dot{y} - F_{ext} = AP_1 - \frac{1}{2}AP_s \quad (\text{B.2})$$

$$AP_1 = \frac{1}{2}AP_s + m\ddot{y} + d\dot{y} - F_{ext} \quad (\text{B.3})$$

$$AP_1 = \frac{1}{2}AP_s + AP_L \quad (\text{B.4})$$

where P_L denotes the pressure caused by the load:

$$P_L = \frac{\sum F}{A} = \frac{m\ddot{y} + d\dot{y} - F_{ext}}{A} \quad (\text{B.5})$$

Elimination of piston area A in (B.4), yields the pressure balance:

$$P_1 = \frac{1}{2}P_s + P_L \quad (\text{B.6})$$

Next, the conservation of mass is applied on the cylinder:

$$q = A\dot{y} + \frac{V_1}{E}\dot{P}_1 \quad (\text{B.7})$$

where V_1 denotes the volume between piston and cylinder-end, E is the bulk modulus of oil, and q is the flow between servo valve and cylinder and can be calculated by:

$$q = q_1 - q_2 \quad (\text{B.8})$$

Combining (B.7) and (B.8) yields:

$$\dot{y} = \frac{q_1 - q_2}{A} - \frac{V_1}{AE}\dot{P}_1 \quad (\text{B.9})$$

Figure B.1 shows that q_1 and q_2 depend of the port opening in the servo valve, and thus on the spool displacement x . From linearisation of the Orifice equations, a more general relation can be found (for complete derivation see literature [3]):

$$q_{1,2} = q_{1,2}(x, P_1, q_n) \quad (\text{B.10})$$

where the normalized flow q_n was introduced. This equation shows that q_1 and q_2 do not only depend on the spool displacement x , but also on the pressure P_1 and the introduced normalized flow q_n . Combining (B.10) and (B.9) yields:

$$\dot{y} = \frac{q_n}{A} \left(\frac{x}{x_{max}} - \frac{|x|}{x_{max}} \frac{P_L}{P_s} \right) - \frac{V_1}{AE}\dot{P}_1 \quad (\text{B.11})$$

where (B.6) is used to substitute pressure P_1 with constant supply pressure P_s and load pressure P_L . In this equation x_{max} denotes the maximum spool displacement, and $|x|$ is the absolute value of the spool displacement. To simplify (B.11), the velocity gain k_m is introduced:

$$\dot{y} = k_m x \quad (\text{B.12})$$

The spool displacement in the servo valve x induces the flow q , and directly relates to the velocity of the piston \dot{y} in the hydraulic cylinder. This linear relation is given by the velocity gain k_m , and after rewriting (B.11):

$$\dot{y} = \frac{q_n}{Ax_{max}} \left(x - \frac{|x|P_L}{P_s} \right) - \frac{V_1}{AE}\dot{P}_1 \quad (\text{B.13})$$

the velocity gain of (B.12) is applied:

$$\dot{y} = k_m \left(x - \frac{|x|P_L}{P_s} \right) - \frac{V_1}{AE} \dot{P}_L \quad (\text{B.14})$$

Next, the hydraulic stiffness C_h is introduced:

$$C_h = \frac{AP_s}{|x|} \quad (\text{B.15})$$

This stiffness can be seen as a damper which depends on the port opening in the servo valve, and differs for each servo valve. The oil's springstiffness for a-symmetric cylinders is defined as:

$$C_0 = \frac{AE}{y} = \frac{A^2E}{V_1} \quad (\text{B.16})$$

where $V_1 = Ay$ is used for the volume between piston and cylinder end. Substitution of C_h and C_0 in (B.14) yields the standard equation:

$$\dot{y} = k_m \left(x - \frac{AP_L}{C_h} \right) - \frac{A\dot{P}_L}{C_0} = k_m \left(x - \frac{\sum F}{C_h} \right) - \frac{\sum \dot{F}}{C_0} \quad (\text{B.17})$$

The hydraulic stiffness C_h (introduced in equation B.15) depends on the actuator type and above all on the spool displacement of the servo valve x . Calculating the hydraulic stiffness is difficult and often very inaccurate, but can easily be determined using the normalized hydraulic stiffness:

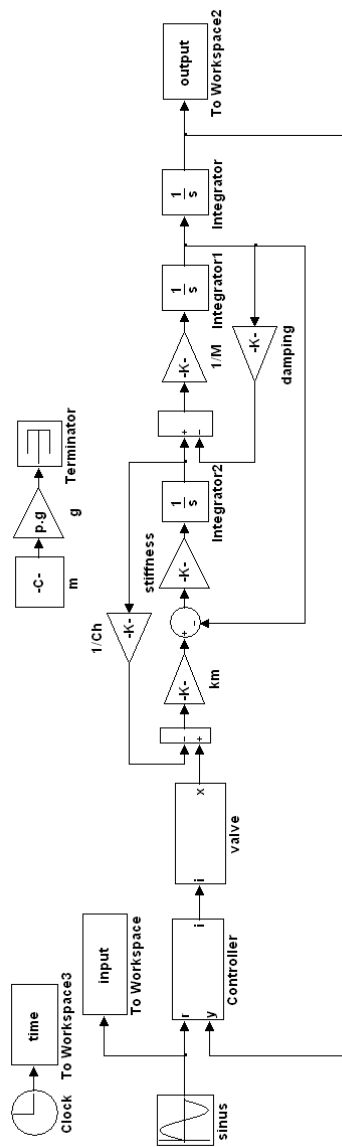
$$C_{h*} = P_L \left(\frac{x}{x_{max}} \right)^{-1} \quad (\text{B.18})$$

In equation B.18, the ratio $\frac{x}{x_{max}}$ can be found by a simple measurement. Practical values are 1 % for a good servo valve, and 2 % for a less good servo valve. Again one is referred to *Hydraulic Servo Systems* [3]. To obtain the hydraulic stiffness, there can be found:

$$C_h = \frac{A}{x_{max}} C_{h*} \quad (\text{B.19})$$

Appendix C

SIMULINK Model



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