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Abstract

In this paper an optimal deterministic identification problem is solved in which a new measure for the misfit between data and system is minimized. It is shown that the misfit can be expressed as the Hankel norm of a specific operator. Optimal autonomous models are obtained by factorizing an optimal Hankel norm approximant of the Laplace transformed data matrix. An upperbound on the misfit between model and data is derived for a class of non-autonomous models of prescribed complexity. The identified autonomous systems are viewed as closed-loop behaviors of a feedback interconnection of two systems. Stability of these feedback interconnections is discussed.

Key Words

optimal system identification, stabilization, linear systems, behaviors

1 Introduction

In the usual methods of system identification uncertainty of models is expressed as uncertainty in the parameters defining the model. In traditional axiomatic frameworks this uncertainty is given a stochastic interpretation in the sense that deviations of nominal parameter values are modeled by prescribed probability distributions. The recent interest in deterministic techniques to quantify model uncertainty stems mainly from the present inability of robust control theory to cope with probabilistic assumptions on estimated model parameters.

A mathematical description of model uncertainty requires a quantification of a distance measure between models. In the context of deterministic system identification a distance measure needs to be specified between observed data and models belonging to an a priori specified set of candidate models. It is important that this misfit criterion is chosen independent of the parametrization of the model class and independent of the way individual models are represented. As such, parametrization and representation issues should be clearly separated from the model identification problem.

In the present paper we consider a deterministic identification problem using model sets of prescribed complexity. We restrict attention to the class of linear time-invariant systems and introduce a misfit criteria for the distance between data and elements in this model class. The proposed misfit function is independent of model representations and is characterized as the Hankel norm of an operator which is determined by the data and a specific (co-inner) kernel representation of the model. Optimal approximate models with prescribed complexity are characterized in this way.

In the last section of this paper we will view these optimal models as the closed-loop behavior of a plant-controller feedback interconnection. Internal stability of these interconnections is characterized in an input-output independent context and it is shown how the class of all stabilizing controllers for a given plant (or the class of all plants stabilized by a given controller) can be parametrized.

2 Models and data

Consider a finite set of observed time series

$$\tilde{w}_i : \mathbb{Z}_+ \longrightarrow W, \quad i = 1, \ldots, n \tag{2.1}$$

where W denotes the *signal space* which is assumed to be a q dimensional real vector space, i.e., $W = \mathbb{R}^q$. We address the problem to identify linear time-invariant systems that model these time series.

Let $l_2(\mathbb{Z}_+, \mathbb{R}^q)$ (or l_2^+ for short) denote the set of time series $w : \mathbb{Z}_+ \to \mathbb{R}^q$ for which $\|w\|_2^2 := \sum_{t \in \mathbb{Z}_+} w^T(t) w(t) < \infty$. It is assumed throughout that $\tilde{w}_i \in l_2^+$ for all $i = 1, \ldots, n$.

Definition 2.1 An l_2 *system* is a system $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B})$ with time set $\mathbb{Z}_+$, signal space $\mathbb{R}^q$ and behavior $\mathcal{B} \subseteq l_2(\mathbb{Z}_+, \mathbb{R}^q)$.

Denote by $\mathcal{S}_2^q$ the class of all l_2 systems $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B})$ defined on the time set $\mathbb{Z}_+$ whose behavior $\mathcal{B} \subseteq l_2^+$ is a linear shift-invariant and closed subset of $l_2(\mathbb{Z}_+, \mathbb{R}^q)$. This class of linear time-invariant systems has been extensively studied in the work of Willems [8] and Heij [5] and corresponds to the class of systems whose behavior can be represented as the (l_2^+-) kernel of a finite number of polynomial difference equations in the system variables or, alternatively, as the (l_2-) solution space of a linear time-invariant state space system with finite dimensional state space.

Definition 2.2 A system $\Sigma \in \mathcal{S}_2^q$ is said to be *autonomous* if there exists $t \in \mathbb{Z}_+$ such that the mapping $\pi : \mathcal{B} \to \mathcal{B}|_{[1,t]}$ defined by the restriction $\pi_t(w) := w|_{[1,t]}$ is injective.

Definition 2.3 A system $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B})$ is *unfalsified* by the data (2.1) if $\tilde{w}_i \in \mathcal{B}$ for $i = 1, \ldots, n$. We call a system $\Sigma_1 = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_1)$ *more powerful* than $\Sigma_2 = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_2)$ if $\mathcal{B}_1 \subseteq \mathcal{B}_2$.

This defines a partial order $\subseteq$ on systems in $\mathcal{S}_2^q$ by defining $\Sigma_1 \subseteq \Sigma_2$ if Σ_1 is more powerful than Σ_2. Equivalently, Σ_1 will be called a *subsystem* of Σ_2 if $\Sigma_1 \subseteq \Sigma_2$. The *most powerful unfalsified system* (for the data (2.1)) is that system $\Sigma_{\text{MPUM}} \in \mathcal{S}_2^q$ which is unfalsified by (2.1) and which is more powerful than any other $\Sigma \in \mathcal{S}_2^q$ which is unfalsified by (2.1). In the following we use the fact that $\Sigma_{\text{MPUM}} \in \mathcal{S}_2^q$ exists and is unique for any set of time series (2.1) which belong to l_2^+. See [5, 8, 9].

3 Models for data

In this section we address the question to characterize all systems $\Sigma \in \mathcal{S}_2^q$ which are unfalsified by the data (2.1).

Define the Laplace transform of the data sequences $\tilde{w}_i$ by putting

$$W(z) := \tilde{W}(1) z^{-1} + \tilde{W}(2) z^{-2} + \ldots \tag{3.1}$$

where $z \in \mathbb{C}$ and $\tilde{W}(t) := [\tilde{w}_1(t) \ldots \tilde{w}_n(t)]$, $t \in \mathbb{Z}_+$ denotes the data matrix. We will assume the following.

Assumption 3.1 $W(z)$ is rational and analytic for all $z \in \mathbb{C}$ with $|z| < 1$.

Specific examples of data sets that satisfy assumption 3.1 include finite sets of frequency response measurements, spectral data, or polynomial-exponential data series. See [6] for a methodology to approximate data sets by polynomial-exponential time series which satisfy assumption 3.1 using risk minimization techniques.

Let $\Sigma_{\text{MPUM}} = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_{\text{MPUM}})$ denote the most powerful unfalsified model for this data and let σ denote the shift $(\sigma w)(t) = w(t+1)$.

Proposition 3.2 *Σ_{MPUM} is well defined and autonomous. Its behavior*

$$\mathcal{B}_{MPUM} = span\ \{\sigma^t(\tilde{w}_i);\ i = 1, \ldots, n,\ t \in \mathbb{Z}_+\}$$

is the smallest linear shift invariant finite dimensional and closed subspace of l_2^+ that contains $\tilde{w}_i$ for all $i = 1, \ldots, n$.

Proof. See e.g. [2, 8]. ∎

To represent $\mathcal{B}_{\text{MPUM}}$ introduce the power series

$$\Theta(z) := \Theta_0 + \Theta_1 z + \cdots + \Theta_k z^k + \cdots$$

where $z \in \mathbb{C}$ and Θ_i are constant real matrices of dimension $g \times q$. Assume that $\Theta \in H_\infty^-$ (all entries of $\Theta(z)$ are analytic and bounded functions for $z \in \mathbb{C}$ with $|z| < 1$). Then Θ represents a linear time-invariant system through the difference equations

$$\Theta(\sigma)w = 0. \qquad (3.2)$$

Formally, (3.2) defines the l_2 system $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}(\Theta)) \in \mathcal{S}_2^q$ with

$$\mathcal{B}(\Theta) := \{w \in l_2^+ \mid \Theta(\sigma)w = 0\}. \qquad (3.3)$$

Let $\hat{\mathcal{B}}$ denote the Laplace transform of $\mathcal{B}$. Then Σ is in the frequency domain equivalently described by

$$\hat{\mathcal{B}} = \{\hat{w} \in \mathcal{H}_2^+ \mid (\Pi_+ \Theta \hat{w})(z) = 0,\ z \in \mathbb{C}\}.$$

Here, $\mathcal{H}_2^+$ denotes the image of l_2^+ under the Laplace transform and Π_+ denotes the canonical projection $\Pi_+ : \mathcal{L}_2 \to \mathcal{H}_2^+$.

The most powerful unfalsified system Σ_{MPUM} admits an $\mathcal{H}_\infty^-$ kernel representation which is obtained as follows

Theorem 3.3 *Let $W = \Theta_{mpum}^{-1}\Psi$ be a left coprime factorization over H_∞^- of W. Then $\Sigma_{MPUM} = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_{MPUM})$ where $\mathcal{B}_{MPUM} = \mathcal{B}(\Theta_{mpum})$ or, equivalently, $\hat{\mathcal{B}}_{MPUM} = \ker \Pi_+ \Theta_{MPUM}$.*

Once Σ_{MPUM} is known, a system Σ is unfalsified by the data (2.1) if and only if $\Sigma_{\text{MPUM}} \subseteq \Sigma$.

Theorem 3.4 *Let Σ_{MPUM} be represented by Θ_{MPUM} and let $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}(\Theta))$. Then the following statements are equivalent.*

1. $\Sigma_{MPUM} \subseteq \Sigma$
2. $\Theta = \Lambda \Theta_{MPUM}$ *for some rational* $\Lambda \in H_\infty^-$.

All unfalsified models $\Sigma \in \mathcal{S}_2^q$ are therefore generated by Σ_{MPUM} by premultiplying Θ_{MPUM} with rational elements in H_∞^-. We will use this result in section 4 below.

4 Optimal approximate models

4.1 Model complexity

In this section $\Sigma = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B})$ belongs to $\mathcal{S}_2^q$ and it is supposed that $\mathcal{B} = \mathcal{B}(\Theta)$ with Θ a rational element in H_∞^-.

Definition 4.1 The *complexity* $c(\Sigma)$ of Σ is a pair $c(\Sigma) = (m(\Sigma), n(\Sigma))$ where $m(\Sigma) = q - \text{rank}\,(\Theta)$ and $n(\Sigma)$ is the McMillan degree of Θ.

In this definition $m(\Sigma) + n(\Sigma)$ is to be interpreted as the total degree of freedom to uniquely determine a trajectory in Σ. This consists of the dimension $m(\Sigma)$ of the input space in a (and hence any) input-output representation of Σ and the dimension $n(\Sigma)$ of the space of initial conditions (or the state space dimension in any minimal state space representation of Σ). Introduce a lexicographic ordering on system complexities as follows. Define $c(\Sigma_1) \preceq c(\Sigma_2)$ if

$$\begin{cases} m(\Sigma_1) = m(\Sigma_2),\ n(\Sigma_1) = n(\Sigma_2) & \text{or} \\ m(\Sigma_1) < m(\Sigma_2),\ n(\Sigma_1) = n(\Sigma_2) & \text{or} \\ m(\Sigma_1) = m(\Sigma_2),\ n(\Sigma_1) < n(\Sigma_2) \end{cases}$$

Since autonomous systems $\Sigma \in \mathcal{S}_2^q$ have finite dimensional behavior, it follows that their corresponding kernel representations have full rank or, equivalently, $m(\Sigma) = 0$. Therefore, the least complex systems are the autonomous ones.

4.2 Misfits

The discrepancy between model and data is formalized by the definition of a misfit function between the data (2.1) and models in $\mathcal{S}_2^q$. We assume that the data is represented by the matrix W defined in (3.1). The misfit is defined as follows

Definition 4.2 The *misfit* between $\mathcal{B}$ and W is defined as

$$d(\mathcal{B}, W) := \sup \left\{ \frac{< Wx, v >}{\|v\|\ \|x\|} \mid v \in \hat{\mathcal{B}}^\perp, x \in \mathcal{H}_2^- \right\}$$

Here,

$$\hat{\mathcal{B}}^\perp = \{v \in \mathcal{H}_2^+ \mid < v, w >= 0 \text{ for all } w \in \mathcal{B}\}$$

is the orhogonal complement of $\hat{\mathcal{B}}$ in $\mathcal{H}_2^+$ and W is viewed as a multiplicative operator $W : \mathcal{H}_2^- \to L_2$, $W : x \mapsto Wx$.

Clearly $d(\mathcal{B}, W) \geq 0$ and $d(\mathcal{B}, W) = 0$ if, and only if Σ is unfalsified by the data. Note that the misfit is independent of representations of $\Sigma \in \mathcal{S}_2^q$. In other words, $d(\mathcal{B}, W)$ is a non-parametric criterion.

The following theorem relates the misfit to the Hankel norm of a specific operator. See also [7] for other characterizations of the misfit $d(\mathcal{B}, W)$.

Theorem 4.3 *Let $\Sigma \in \mathcal{S}_2^q$ and let its behavior $\mathcal{B} = \mathcal{B}(\Theta)$ where Θ is* co-inner, *i.e. $\Theta\Theta^* = I$. Then*

$$d(\mathcal{B}, W) = \| \Pi_+ \Theta W \|_H$$

where $\| \cdot \|_H$ denotes the induced operator (or Hankel) norm of the composite function $\Pi_+\Theta W$ viewed as a mapping from $\mathcal{H}_2^-$ to $\mathcal{H}_2^+$.

Co-inner kernel representations of systems in $\mathcal{S}_2^q$ indeed exist.

Proposition 4.4 *The behavior $\mathcal{B}$ of every system $\Sigma \in \mathcal{S}_2^q$ admits a representation $\mathcal{B} = \mathcal{B}(\Theta)$ with Θ co-inner.*

4.3 Optimal identification

The approximate modeling problem consists of finding low complexity models which minimize the misfit between model and data. Precisely, given the data (2.1) together with prescribed complexity (m, n), find systems $\Sigma \in \mathcal{S}_2^q$ with $c(\Sigma) \preceq (m, n)$ such that the misfit $d(\mathcal{B}, W)$ is minimal. A complete solution for the case where $m = 0$ is given in the following result.

Theorem 4.5 *Let W be given by (3.1) and let Σ_{MPUM} be the most powerful unfalsified model for W. Suppose that $c(\Sigma_{MPUM}) = (0, N)$ and let*

$$\sigma_1 \geq \sigma_2 \ldots \geq \sigma_N > 0$$

denote the singular values of W. Denote by $W_n \in H_\infty$ an optimal Hankel norm approximant of W of McMillan degree $\leq n$. Let $\Sigma_n = (\mathbb{Z}_+, W, \mathcal{B}_n)$ be the most powerful unfalsified model associated with W_n. Then

1. *Σ_n is autonomous.*
2. *$d(\mathcal{B}_n, W) = \sigma_{n+1}$.*
3. *$d(\mathcal{B}'_n, W) \geq d(\mathcal{B}_n, W)\}$ for all $\Sigma'_n \in \mathcal{S}_2^q$ with $c(\Sigma'_n) \preceq c(\Sigma_n)$.*
4. *if $\Sigma_n \subseteq \Sigma'_n$ then $d(\mathcal{B}'_n, W) \leq \sigma_{n+1}$.*

Proof. See [7]. ■

Note that $c(\Sigma_n) = (0, n^*)$ with $n^* \leq n$ the McMillan degree of W_n. Conclude from Theorem 4.5 that

$$\Sigma_n = \arg\min\{d(\mathcal{B}, W) \mid \Sigma \in \mathcal{S}_2^q, c(\Sigma) \leq (0, n)\}$$

For given data (2.1), Theorem 4.5 leads to the following constructive method for the computation of a kernel representation of Σ_n.

1. Compute an optimal Hankel norm approximant $W_n \in H_\infty$ of W with McMillan degree n [4].
2. Let $W_n = \Theta_{\text{MPUM},n}^{-1} \Psi_n$ be a left coprime factorization over H_∞^- of W_n.
3. Put $\mathcal{B}_n = \mathcal{B}(\Theta) := \ker \Theta_{\text{MPUM},n}(\sigma)$.

The optimal approximate model is then given by $\Sigma_n = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_n)$ whereas all models which have Σ_n as subsystem have misfit $\leq \sigma_{n+1}$.

5 Stabilization of systems

In this section kernel representations of autonomous systems are used as descriptions of closed-loop behaviors. A "closed-loop[1]" behavior consists of the interconnection of two l_2 systems Σ_p and Σ_c which will be referred to as the *plant* and the *controller*, respectively.

Definition 5.1 Let Σ_p and Σ_c be elements of $\mathcal{S}_2^q$. Their *interconnection* is defined as the system $\Sigma_{\text{int}} := \Sigma_p \wedge \Sigma_c = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_{\text{int}})$ where $\mathcal{B}_{\text{int}} = \mathcal{B}_p \cap \mathcal{B}_c$.

Hence, interconnection and intersection are synonymous. It is easily seen that Σ_{int} again belongs to $\mathcal{S}_2^q$.

Definition 5.2 The interconnection $\Sigma_{\text{int}} = \Sigma_p \wedge \Sigma_c$ is called a *feedback interconnection* if Σ_{int} is autonomous.

[1] in this input-output independent context the traditional looping configuration of input and output signals is not implied.

Definition 5.3 A feedback interconnection $\Sigma_{\rm int} = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B})$ is said to be *internally stable* if

$$\mathcal{B} = \{w \in l_2^{\rm loc}(\mathbb{Z}_+, \mathbb{R}^q) \mid \Theta(\sigma)w = 0\}$$

for all Θ for which $\mathcal{B} = \mathcal{B}(\Theta)$. Here, $l_2^{\rm loc}(\mathbb{Z}_+, \mathbb{R}^q)$ denotes the set of locally square summable time series $w : \mathbb{Z} \to \mathbb{R}^q$. In that case, Σ_c is called a *stabilizing* system for Σ_p and Σ_p is said to be *stabilized* by Σ_c.

Stated otherwise, in an internally stable feedback interconnection the set of locally square summable solutions of (3.2) coincides with the set of square summable solutions of (3.2). In particular this implies that the locally square summable solutions $w(t)$ of a feedback interconnected system converge to zero as $t \to \infty$.

Let Θ be a non-singular $q \times q$ rational matrix with entries in H_∞^-. Suppose that Θ represents a feedback interconnection $\Sigma_{\rm int} = (\mathbb{Z}_+, \mathbb{R}^q, \mathcal{B}_{\rm int})$, i.e. $\mathcal{B}_{\rm int} = \mathcal{B}(\Theta)$. Since the behavior of both plant and controller contain $\mathcal{B}_{\rm int}$ as a subset we derive the following property as an immediate consequence of Theorem 3.4.

Proposition 5.4 *Let Σ_{int} be a feedback interconnection of Σ_p and Σ_c and let its behavior $\mathcal{B}_{int} = \mathcal{B}(\Theta)$ with $\Theta \in \mathcal{H}_\infty^-$. Then there exist $\Lambda_p, \Lambda_c \in \mathcal{H}_\infty^-$ such that*

$$\mathcal{B}_p = \mathcal{B}(\Lambda_p\Theta) \qquad (5.1)$$
$$\mathcal{B}_c = \mathcal{B}(\Lambda_c\Theta). \qquad (5.2)$$

Conversely, if $\Lambda = (\Lambda_p^T\ \Lambda_c^T)^T$ is a unit in H_∞^- then the interconnection of (5.1) and (5.2) yields the feedback interconnection Σ_{int} with behavior $\mathcal{B}_{int} = \mathcal{B}(\Theta)$.

In order to investigate internal stability of interconnected systems it is common to introduce fictitious signals in a plant-controller configuration. Let $\Theta \in \mathcal{H}_\infty^-$ be as in Proposition 5.4 and consider the equation

$$\bar{w} = \Theta(\sigma)w \qquad (5.3)$$

Introduce the system $\Sigma_{full} = (\mathbb{Z}_+, \mathbb{R}^{2q}, \mathcal{B}_{\rm full})$ with *full behavior*

$$\mathcal{B}_{\rm full} = \{(w, \bar{w}) \in l_2^+ \mid (5.3) \text{ is satisfied}\}.$$

Note that $\mathcal{B}_{\rm full} = \mathcal{B}([\Theta\ \ -I])$. Let π_w and $\pi_{\bar{w}}$ denote the canonical projections $\pi_w(w, \bar{w}) := w$ and $\pi_{\bar{w}}(w, \bar{w}) := \bar{w}$. Internal stability is then characterized as follows.

Theorem 5.5 *Let Σ_{int} be a feedback interconnection and let Σ_{full} be its associated full behavior. Then the following statements are equivalent.*

1. *Σ_{int} is internally stable.*
2. *$\pi_{\bar{w}}\mathcal{B}_{full} = l_2^+$.*
3. *there exists a non-singular $\Theta \in \mathcal{H}_\infty^-$ such that $\Theta^{-1} \in \mathcal{H}_\infty$ and $\mathcal{B}_{int} = \mathcal{B}(\Theta)$.*
4. *for all non-singular $\Theta \in \mathcal{H}_\infty^-$ for which $\mathcal{B}_{int} = \mathcal{B}(\Theta)$ there holds $\Theta^{-1} \in \mathcal{H}_\infty$.*

Theorem 5.5 has the interpretation that internal stability is equivalent to the property that the fictitious signal $\bar{w}$ in $\mathcal{B}_{\rm full}$ can be considered as a free variable in l_2^+. Since for all $\bar{w} \in l_2^+$, $\pi_w\mathcal{B}_{\rm full}$ is autonomous, the variables $(w, \bar{w})$ of $\Sigma_{\rm full}$ can be partitioned into a q-dimensional *input variable* $\bar{w}$ and a q-dimensional *output variable* $w = \Theta^{-1}\bar{w}$ which belongs to l_2^+ whenever $\Sigma_{\rm int}$ is internally stable. The *poles* of the feedback interconnection are the invariant zeros of Θ or, equivalently, the poles of Θ^{-1}.

Suppose that $\Lambda_p \in \mathcal{H}_\infty^-$ has rank $p < q$ and consider the systems Σ_p and $\bar{\Sigma}_p$ with behavior $\mathcal{B}_p := \mathcal{B}(\Lambda_p\Theta)$ and $\bar{\mathcal{B}}_p := \mathcal{B}(\Lambda_p)$, respectively. Let Σ_c and $\bar{\Sigma}_c$ be defined analogously. We address the question to characterize the class of controllers $\Sigma_c \in \mathcal{S}_2^q$ which stabilize Σ_p. First observe that Σ_p can be viewed as the interconnection $\Sigma_{\rm full} \wedge \bar{\Sigma}_{{\rm full},p}$ where

$$\bar{\mathcal{B}}_{{\rm full},p} := \{(w, \bar{w}) \in l_2^+ \mid \bar{w} \in \bar{\mathcal{B}}_p\}$$

That is,

$$\mathcal{B}_p = \pi_w(\mathcal{B}_{\rm full} \cap \bar{\mathcal{B}}_{{\rm full},p})$$

We remark that the number $m(\Sigma_p) = q - p > 0$ corresponds to the dimension of the input space in any input-output representation of Σ_p.

Theorem 5.6 *Suppose that Σ_{int} is an internally stable feedback interconnection with behavior $\mathcal{B}_{int} = \mathcal{B}(\Theta)$. Then the following statements are equivalent.*

1. *$\Sigma_p \wedge \Sigma_c$ is an internally stable feedback interconnection.*
2. *$\bar{\Sigma}_p \wedge \bar{\Sigma}_c$ is an internally stable feedback interconnection.*
3. *$\Lambda := [\Lambda_p^T\ \Lambda_c^T]^T$ is non-singular and $\Lambda^{-1} \in \mathcal{H}_\infty$*

The Laplace transform $\hat{\mathcal{B}}_p$ of $\mathcal{B}_p$ is given by

$$\begin{aligned}\hat{\mathcal{B}}_p &= \{\hat{w} \in \mathcal{H}_2^+ \mid \Pi_+ \Lambda_p \Theta \hat{w} = 0\} \\ &= \{\hat{w} \in \mathcal{H}_2^+ \mid < \Lambda_p \Theta \hat{w}, \hat{v} >= 0 \ \forall \hat{v} \in \mathcal{H}_2^+\} \\ &= \{\hat{w} \in \mathcal{H}_2^+ \mid < \hat{w}, \Theta^* \Lambda_p^* \hat{v} >= 0 \ \forall \hat{v} \in \mathcal{H}_2^+\} \\ &= \{\hat{w} \in \mathcal{H}_2^+ \mid \hat{w} \in [\Theta^* \Lambda_p^* \mathcal{H}_2^+]^\perp\} \\ &= [\text{im } \Theta^* \Lambda_p^*]^\perp\end{aligned}$$

where $\Theta^* \in H_\infty$ denotes the dual of $\Theta \in H_\infty^-$. Therefore, the behavior of Σ_p can equivalently be represented as the orthogonal complement (in $\mathcal{H}_2^+$) of the $\mathcal{H}_2^+$ image of $\Theta^* \Lambda_p^*$.

The following result provides a characterization of stabilizing systems for Σ_p.

Theorem 5.7 *The following are equivalent*

1. Σ_c *stabilizes* Σ_p.
2. $\hat{\mathcal{B}}_p^\perp + \hat{\mathcal{B}}_c^\perp = \mathcal{H}_2^+$
3. $\mathcal{B}_c = \mathcal{B}(\Lambda_c \Theta)$ *where* $\Lambda := \begin{pmatrix} \Lambda_c \\ \Lambda_p \end{pmatrix}$ *is a non-singular* $q \times q$ *matrix and* $(\Lambda\Theta)^{-1} \in \mathcal{H}_\infty$.

The above Theorem 5.7 provides a characterization of all stabilizing controllers of the linear time-invariant system Σ_p. Note that input/output representations of Σ_p and/or Σ_c are not necessary to provide such a characterization.

References

[1] A.C. Antoulas, J.C. Willems, "A Basis Free Approach to Linear Exact Modeling," Technical Report #91–05, Dept. ECE, Rice University, 1991.

[2] A.C. Antoulas, J.C. Willems, "A Behavioral Approach to Linear Exact Modeling," IEEE Transactions on Automatic Control, Vol. 36, pp. 1776-1802, 1993.

[3] A.C. Antoulas, "A New Approach to Modeling for Control," *to appear in:* Linear Algebra and its Applications, Special issue on Systems and Control, 1994.

[4] K. Glover, "All Optimal Hankel Norm Approximations of Linear Multivariable Systems and their L_∞ Bound," Int. Journal of Control, Vol. 39, pp. 1115–1193, 1984.

[5] C. Heij, "Deterministic Identification of Dynamical Systems," Lecture Notes in Control and Information Sciences, Vol. 127, Springer-Verlag, 1989.

[6] E. Weyer, "System Identification in the Behaviuoral Framework,", Ph.D. Thesis, Department of Engineering Cybernetics, The Norwegian Institute of Technology, Trondheim, Norway, 1992.

[7] S. Weiland and A.A. Stoorvogel, "Optimal Approximate Model Identification in the Hankel Norm," Proc. 10th IFAC SYSID Symposium, Kopenhagen, Denmark, Vol. 3, pp. 197-201, 1994.

[8] J.C. Willems, "From time Series to Linear System; Part I: Finite Dimensional Linear Time Invariant Systems"; Part II: "Exact Modeling," Automatica, Vol. 22, no. 5, pp. 561–580, resp. Vol. 22, no. 6, pp. 675–694, 1986.

[9] J.C. Willems, "Paradigms and Puzzles in the Theory of Dynamical Systems," IEEE Trans. Automatic Control, Vol. 36, No. 3, pp. 259–294, 1991.