Control of 2D turbulence

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M. Lauret

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Traineeship report

Coach(es): Dr. H.J.H. Clercx, Drs. D. Molenaar

Supervisor: Prof. dr. H. Nijmeijer

Technische Universiteit Eindhoven
Department Mechanical Engineering
Dynamics and Control Technology Group

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Abstract

Pattern formation is often observed in two-dimensional turbulence because here vortices tend to merge rather than to break up and decay like they do in three dimensions (like cigarette smoke does). With an experimental setup in mind, theoretical and numerical research has been done on the possibility of controlling this sort of pattern formation with open- and closed-loop control. The setup is a square container containing a shallow layer of fluid and magnets beneath it to influence the flow behavior. Simulations show that with controlling the strength of an active magnet under this setup the patterns can be conserved longer than without active control.

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1 Introduction

At the Department of Applied Physics a lot of research has been done on quasi-two-dimensional (Q2D) turbulence. This form of turbulence is observed in flows of fluids or gasses in which the thickness of the fluid or gas is negligible in comparison to the length and width. The behavior of these flows is totally different from the three dimensional case. In the two dimensional case pattern formation or coherent structures are often observed. Typical examples of this kind of flows are the flow of air in the atmosphere and the flow of water in the oceans.

I want to find out if it is possible to control the behavior of this kind of flows in an experimental setup. This setup is a square container with a shallow fluid layer in which an array of vortices can be started. Theory, experiments and simulations show that these vortices merge and form different structures, see for example Clercx *et al.* [4] and Clercx *et al.* [5]. The research question is: Is it possible to control or influence this pattern formation with open- or closed-loop control of the strength of magnets beneath the experimental setup? The Department of Applied Physics is especially interested in the question if one can stabilize the vortices at their starting position. They should not merge with each other or move from their initial position. This is baptised 'pattern conservation'. If this can be achieved, research can be done on the transport properties of the setup at higher Reynolds numbers.

I start with an investigation of results obtained in an earlier stage from experiments and simulations of shallow flows in square containers. Then a literature survey on flow control and partial differential control has been carried out. This theory is more difficult then classical control theory because the flow of fluids is described by the Navier-Stokes equation which is a difficult nonlinear partial differential equation with possible chaotic behavior. It is not clear how the pattern formation of the fluid flow can be described by low-order models. For this reason the investigation has been started by considering simple vortex configurations. Simulations are done with a simplified configuration and periodic perturbations of an active magnet beneath the setup. The results of these simulations could hint at the structure of the controller needed to control the pattern formation. Later on the effect of a proportional controller has been simulated. With this controller, the patterns can be stabilized for a longer time. Because the research on flow control is new in the section of Dynamics and Control, care has been taken to paint a broader picture and to show some more advanced techniques that are not used yet but may be used in the future.

The report is organised as follows.The working of the experimental setup is explained in chapter 2. In chapter 3 a short overview of the theory of 2D flow is given. Also a short description of the used numerical code is given. The results of the simulations and the analysis of these results are given in chapter 4. Chapter 5 contains our conclusions and recommendations. The report finishes with an appendix on different recent theories on control that can be useful to apply on our control problem.

2 Experimental setup

2.1 Introduction

The experimental setup is located at the Cascade building. It is a horizontal square container with a shallow fluid layer. Below the container is an array of permanent magnets. The flow that appears in the container is caused by applying an electrical current (between two opposite side walls, see figure 1) to a electrolyte solution with permanent magnets below the surface of the tank. The resulting flow can be analysed in two ways. One can inject a colored liquid in the water or one can put a lot of tracer particles in the water. These particles are recorded by a high-resolution CCD camera that is attached above the container. The velocity field can be computed from the recorded data stored on disk. Note that computing the vorticity or velocity fields of a short experiment takes about half an hour. Most information found in this chapter is based on the reports of Leblond [2] and van Bokhoven [22].

2.2 Description of the setup

The setup, figure 1, is a container with dimensions 52 x 52 x 4 cm (length x width x hight). Straight above the container there is a camera and below the

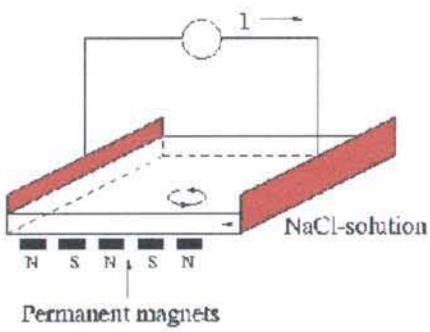

Figure 1: Experimental setup, note the alternating magnets under the square container

container there is a checkerboard like array of 10 x 10 magnets. The polarity of the magnets changes like the colors of a checkerboard. The magnets have a thickness of 5 mm, a diameter of 2,5 cm and a magnetic field of maximal 1,09 Tesla.

The water is a solution of $NaCl$ in water. An electrical current flows from one electrode to the opposite electrode. As a result of this, Na^+ and Cl^- ions go to respectively the negative and the positive electrode. If the ions flow through the magnetic field of a magnet there is a Lorentz force perpendicular on the electrical current. Thus dipoles are formed. These dipoles are swiftly turned into monopole vortices so there is an array of alternating vortices. Depending on a special kind of Reynolds number there are different forms of pattern formation of the vortices. The vortices can merge into one big vortex (see figure 2) or they can show a 'chaotic' distribution of vorticity, Leblond [2].

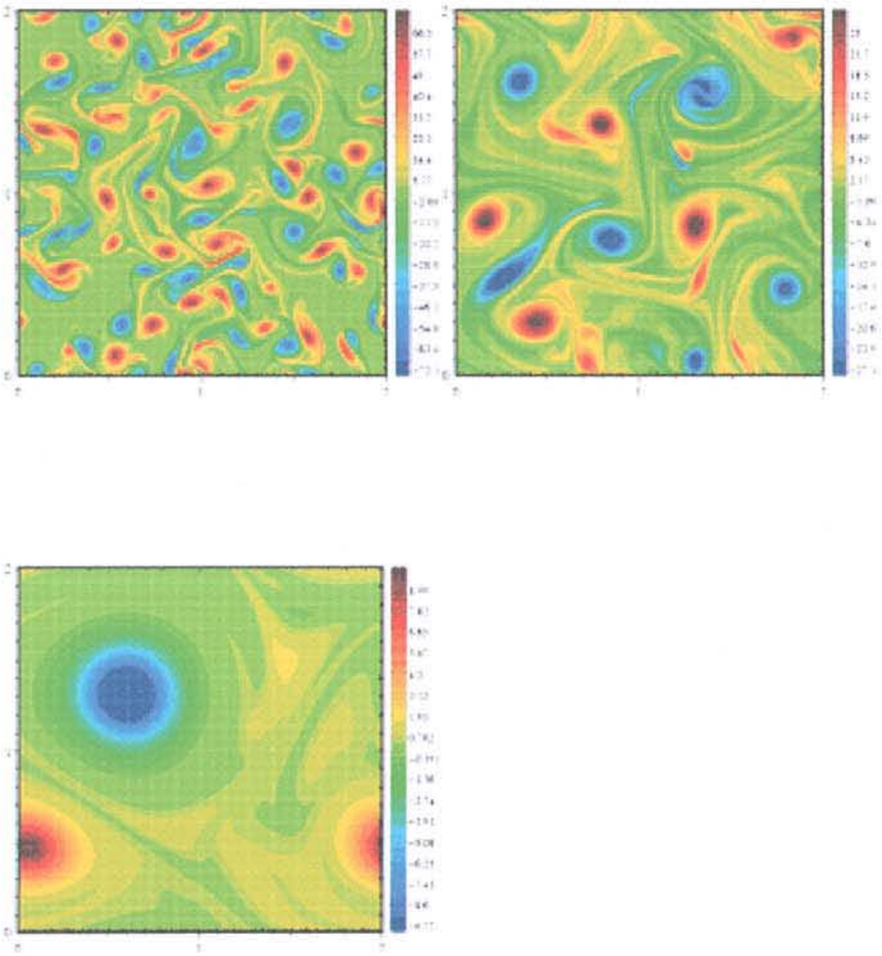

Figure 2: A typical example of vortex merging in a 2D flow (Source: dr. H.J.H. Clercx). From left to right we see a simulation in which the vorticity changes during the time. Red colour indicates counter clockwise rotation. Blue indicates clockwise rotation.

3 Two dimensional turbulence

3.1 Theory

Most basic theory and equations on 2D flow presented in this section comes from van Bokhoven [22], Leblond [2], Schepers [20] and van Heijst [23]. More general theory on turbulence can be found in Berkooz *et al.* [12] and Nieuwstadt [17]. Specific literature on pattern formation of fluids can be found in McWilliams [16] and Cross *et al.* [8]. The first one is specific on Q2D turbulence and its decay properties and the second one is about the mechanisms of pattern formation in general. The section on the numerical code is entirely based on information from dr. H.J.H. Clercx.

Without thermal effects the flow of an incompressible Newtonian fluid on a domain $\mathcal{D}$ is determined by two equations. The continuity equation (conservation of mass)

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

and the Navier-Stokes equation (conservation of momentum):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} - g\mathbf{e}_z + f\mathbf{e}_x, \tag{2}$$

in which t is time, $\mathbf{v} = \mathbf{v}(x, y, z) = (u, v, w)$ the velocity in three-dimensional Cartesian coordinates, ν the kinematic viscosity, ρ the fluid density, f the forcing term and g the gravitational acceleration. $p(\rho)$ is the pressure.

$$\nu \nabla^2 \mathbf{v} = \nu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\mathbf{v} + \nu\frac{\partial^2}{\partial z^2}\mathbf{v}. \tag{3}$$

If the vertical velocity is small compared to the horizontal ones, the vertical term can be modelled as Rayleigh damping

$$\nu\frac{\partial^2}{\partial z^2}\mathbf{v} = \lambda \mathbf{v} \tag{4}$$

With λ giving the amount of dissipation due to bottom friction. This estimation can be done if the flow is quasi-two-dimensional turbulent with a shallow fluid layer (from now on called Q2D turbulence), see Clercx *et al.* [4]. We now assume that the velocity is 2D and in the (x,y) plane, $\mathbf{v} = (u, v)$. If we define vorticity as

$$\omega \equiv \nabla \times \mathbf{v} \tag{5}$$

which in two-dimensions becomes

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{6}$$

We can think about this vorticity as the local rotation of the fluid. It is convenient to think about vorticity instead of velocity in turbulence problems (especially in two dimensions). We can rewrite the Navier-Stokes equation into a two-dimensional vorticity equation

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} \cdot \nabla)\omega = \nu \nabla^2 \omega - \lambda \omega + f. \tag{7}$$

The stream function ψ is defined as

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \tag{8}$$

$$\omega = -\nabla^2 \psi. \tag{9}$$

In the experimental setup we have a fixed boundary ($\partial \mathcal{D}$) which is the boundary of the square container. We assume a no-slip boundary condition to complete the mathematical formulation

$$\mathbf{v} = 0, \ at \ \partial \mathcal{D}. \tag{10}$$

With these basic equations and variables we can introduce some quantities and behaviors that are relevant to our specific problem. The kinetic energy of the flow on the domain is

$$E = \frac{1}{2} \int_{\mathcal{D}} \mathbf{v}^2 dA. \tag{11}$$

We can write the change of energy as:

$$\frac{dE}{dt} = -2\nu\Omega - 2\lambda E + \int_{\mathcal{D}} \psi f dA, \tag{12}$$

where the enstrophy Ω is defined as

$$\Omega = \frac{1}{2} \int_{\mathcal{D}} \omega^2 dA. \tag{13}$$

According to equation (12) dissipation of energy is caused by bottom friction due to λ and viscous dissipation due to Ω. The injection of energy is an interaction between the forcing function f and the stream function ψ, see the integral term in equation (12). As the streamfunction is *a priori* unknown the energy change is not completely determinable by the forcing. There is also the palinstrophy, a measure for the vorticity gradients in the flow,

$$P = \frac{1}{2} \int_{\mathcal{D}} (\nabla \omega)^2 dA \tag{14}$$

and the very important Reynold number which is defined here as:

$$Re = \frac{vL}{\nu}$$

in which L is a typical length scale (here always one), v is the velocity and ν the viscosity.

Energy has a remarkable property in nearly inviscid Q2D turbulence. If the velocity field is decomposed in complex Fourier series one can show that due to nonlinear interaction the energy goes to greater length scales in the flow thus there is a tendency to form larger and larger vortices, see figure 2. This is called the inverse energy cascade. Inverse because in three-dimensional turbulence the term energy cascade is used for the flow of energy to smaller and smaller length scales. Cigarette smoke, for example, forms smaller and smaller vortices until they are dissipated. Because dissipation takes place at small length scales two-dimensional vortices, which have greater length scales, decay much slower.

Two other important quantities are the eddy-turnover time and the angular momentum. First, the eddy-turnover time is the inverse of the maximum vorticity of a vortex, it is the time in which the center of the vortex makes one complete turn. Logically, the eddy-turnover frequency is the maximum vorticity. Second, the angular momentum is defined as

$$L = \int_{\mathcal{D}} r \times \mathbf{v} dA. \tag{15}$$

In Clercx *et al.* [6], [5] research has been done on the spin-up during the decay of 2D turbulence. We are interested in non-decaying turbulence but some important lessons can be taken from these papers. First, the angular momentum seems to be an important variable to describe the motion of the flow and it seems to be a better constant of motion than the kinetic energy. This means that the angular momentum is constant if the pattern stays the same. Second, the wall plays an essential role in the pattern formation. Angular momentum of unbounded viscous flows is conserved when the total circulation is zero (as it is in bounded domains with no-slip walls). So if the angular momentum spins up in a bounded decaying flow, this must be because of wall effects. Our experimental setup has no initial vorticity and forcing, so it is not exactly clear what this effect will do in our case. A formula can be introduced which gives the approximate position of the center (x_c, y_c) of a vortex given a certain vorticity field:

$$x_c = \frac{\int_{\mathcal{D}} x\omega dA}{\Gamma}, y_c = \frac{\int_{\mathcal{D}} y\omega dA}{\Gamma}, \tag{16}$$

with

$$\Gamma = \int_{\mathcal{D}} \omega dA. \tag{17}$$

If one knows the domain of each vortex one can compute its center.

Turbulence, being strongly nonlinear and unpredictable, has an analogy with chaos theory. However, since turbulence is a spatio-temporal phenomenon it can not be explained with chaos theory alone because this theory considers low-dimensional dynamical systems. Infinite dimensionality and nonlinearity of the describing equations does not only make the analysis more difficult but also the theory of controlling this kind of systems, see for example Fursikov and Imanuvilov [10] and Fursikov [9].

3.2 Control goals

It is important to realize what our main goal is. Certain small vortices exist at begin time t_0 and start to move through the domain or become bigger sized vortices after some time Δt. Can we keep the original vortices from merging or moving for a time substantially greater than Δt? Note that we do not necessary have to keep the vorticity field the same for this time as different vorticity fields can have the same pattern of vortexes. We will call this goal 'pattern conservation'.

This control goal is thus somewhat weaker than stabilizing the system around a certain vorticity field, see the appendix on flow control, section A.5, for an exact definition of stabilizability in this case. We also do not try to really control the system as this would imply that we would try to steer the system to certain vorticity fields totally different than the original one. However, it is good to note that recently it has been proven that the 2D Navier-Stokes system is stabilizable and controllable under certain conditions, Imanuvilov and Fursikov [10] and Fursikov [9]. See the appendix on flow control, section A.5, for exact definitions.

3.3 Assumptions and estimations

Because of the complexity of the problem we start with a simpler configuration than the experimental setup with 10x10 magnets. To estimate if we can conserve the patterns of this system we start with doing simulations with an imaginary system with 3x3 alternating magnets, figure 3.

This simplified system has dimensions $2L \times 2L$ and a typical dimensionless length $L = 1$ (thus $A = 4$). The amplitude and sign of the forcing in the center of the container can be controlled as a function of time. In the end, it is desired that the vortex above this central forcing behaves approximately the same as the other vortices. It is assumed that the amplitude of the passive forcings has a Gaussian profile,

$$f = c \cdot e^{-\frac{r^2}{R^2}}, \tag{18}$$

with r the radius measured from the middle of the forcing and somewhat larger than $R = 0,1$.

The supplied energy is an interaction between stream function and forcing, see 12. So this input is not completely controlled. However, it is possible to make an estimation of the energy input. The forcing is known and we can assume that the the vorticity field is the same as the forcing field. This estimation will give us the order of the vorticity. With this estimation and some trial-and-error simulations, forcing parameter c can be found for each different Reynolds

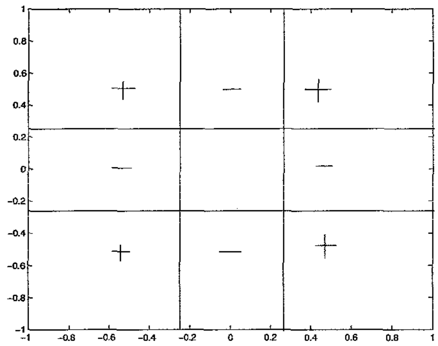

Figure 3: array of alternating magnets, the center forcing on (0,0) can be controlled as a function of time

number so that the energy of the system is normalised 2 (When $E = 2$ it simplifies the work with the dimensionless variables as explained below).

Because the energy $E = \frac{1}{2}\int_D v^2 dA$ we know that the root mean square velocity $v_{rms} = \sqrt{\frac{1}{2}E}$ (remind that $A = 4$). Real time (t) is scaled via

$$t^* = \frac{L}{v}t \tag{19}$$

with L=1. If $v_{rms} = 1$ than the dimensionless time has the same numerical value as real time. Also the Reynolds number in the simulation code is scaled as $1/\nu$. If $v_{rms} = 1$ and L=1 than the real Reynolds number, $Re = \frac{v_{rms}L}{\nu}$, becomes $\frac{1}{\nu}$.

3.4 Numerical code

In a Direct Numerical Simulation (DNS) of 2D turbulence in bounded square or rectangular domains, all relevant length scales (including boundary layers near no-slip walls and the enstrophy dissipation scale) should be well resolved. For this purpose a 2D Chebyshev pseudospectral code has been developed [3], in which the spatial discretization of the relevant flow variables, such as velocity u, vorticity and stream function, are expanded in a series of Chebyshev polynomials. Some advantages of this method are:

- absence of numerical dissipation and dispersion,
- exponential convergence,
- high resolution near no-slip walls,
- possibility to use Fast Fourier Transforms (FFT).

The time integration scheme is based on the second-order accurate Adams-Bashforth (advection terms) Crank-Nicolson (viscous terms) scheme.

The numerical code has been implemented in FORTRAN. Parameters that are input are the time step (maximal time step depends on the grid size), the grid size (33x33 or 129x129), the initial Reynolds number (10-100) and parameters for the time behaviour of the external forcing of the flow.

4 Simulation results

4.1 Introduction

Most simulations are done on the simplified configuration with 3x3 alternating forcings. The amplitude of the middle forcing can be a function of time and also of properties of the system like energy or angular momentum. Grid-size, Reynolds number and the time function of the middle forcing are used as input. With the parameters used, it was clear that the grid-size (129x129 Chebyshev modes was the highest resolution used) was of no major influence on the observed pattern formation. So the simulations shown here are done with a small grid-size of 33x33 Chebyshev modes. Notice that because of the low resolution the values of the vorticity are very coarse. The dimensionless Reynolds number is very low (in a range from 10 to 100) to avoid too difficult flow behavior in the beginning.

4.2 Test simulations

In the beginning simulations are done with 10x10 magnets and a Reynolds number of $Re = 200$ to understand the behavior of the experimental setup and to get accustomed with the simulation code. See figure 4. Clearly, merging of vortices can be shown. In the left picture we see the vorticity field some short time after the beginning is shown. In the right picture the vorticity field after longer time is shown. Just like in picture 2 the creation of a large positive (counter clockwise rotating) vorticity patch (in red) that slowly evolves to a vortex can be seen.

4.3 Simulations with open-loop control

In all the open-loop simulations, the center forcing is periodically excited around its static dimensionless value 1. The forcing (F) is given by:

$$F(t) = asin(ft) + 1 \tag{20}$$

The frequency f, amplitude a and Reynolds number Re are varied. As has been said before, the Reynolds number is maximally chosen to be 100. For the amplitude values are used with the same magnitude as the amplitudes of the passive forcings. The frequencies used are maximally a little higher than the eddy turnover frequency. Higher frequencies will probably have very small effects on the system as it has second order characteristics. The simulations were typically done for times of an order of 100 seconds. Longer simulation times never contained any new information if time was larger than a few hundred seconds.

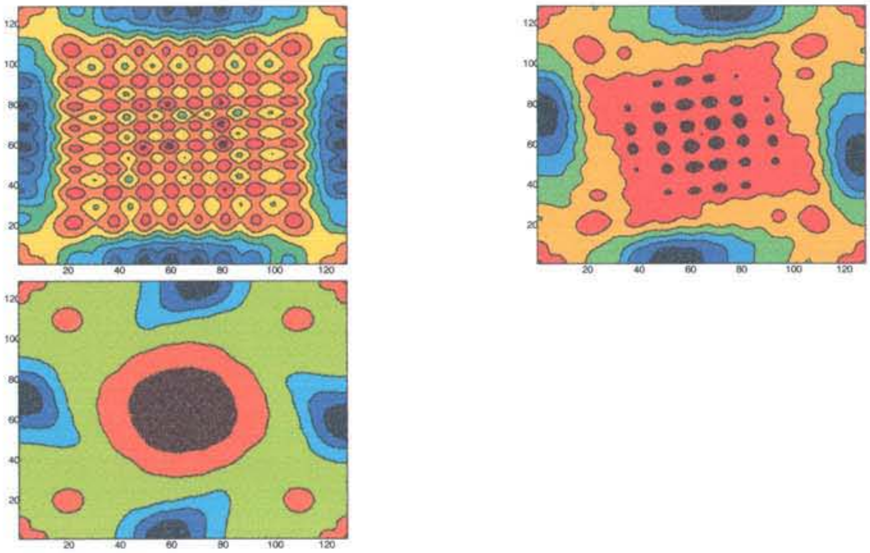

Figure 4: Vorticity field at Re=200 with an array of 10x10 magnets. Time increases from the left to the right. The gridsize is indicated by the x- and y-axis. Red indicates counter-clockwise rotation and blue indicates clockwise rotation.

4.3.1 Simulations with Re = 10

After the configuration is adjusted to the simplified 3x3 magnet array, simulations are done at a Reynolds number of ten. With an amplitude of the periodic disturbance, see (20), of $a = 1.0$, frequencies of 0 to 30 Hz (with steps of five Hertz) are used to influence the system. Some simulations are also done on a 129x129 grid to check the precision of the simulations. A finer grid gives no significant differences with the coarse grid solutions.

The maximal vorticity is around 20 in most of the simulations done at $Re = 10$ (in figure 5 it is a little lower). So this is the typical eddy turnover frequency. If the center magnet is excited with a frequency of 25 Hz there seems to be a sort of mode-locking (the frequency of the forcing can be seen in the frequency of the energy, enstrophy, palinstrophy and angular momentum), see figure 5. In figure 5 can be seen that the angular momentum periodically vibrates around zero and the vortices stay stable in their 3x3 configuration.

4.3.2 Simulations with Re = 50

Simulations have also been conducted at a Reynolds number of 50 and with different amplitudes and frequencies of the periodic forcing. All simulations at $Re = 50$ give vorticity plots as displayed in figure 6. It is seen that the vortices have a small tendency to start moving away from their original positions. A little

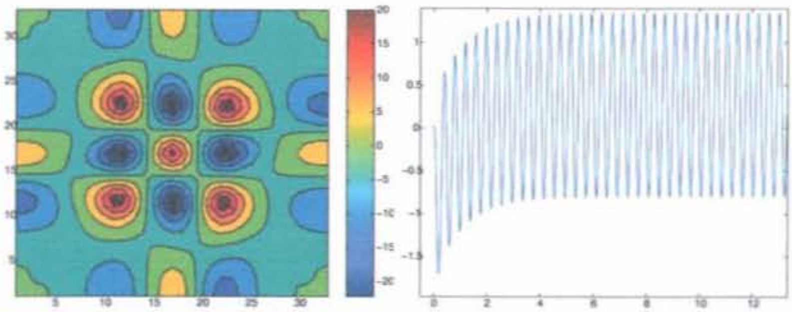

Figure 5: Vorticity plot and angular momentum-time (s) plot, Re=10 a=3 and f=25 Hz. Vorticity stays this way during the whole time of the simulation

counter clockwise rotation can be observed. However, after this little rotation, the vortices stay at their place for the rest of the simulation time. Therefore, I consider this pattern as the same pattern as in figure 5. As noticed in the section on simulations with $Re = 10$, this seems to indicate some sort of mode-locking.

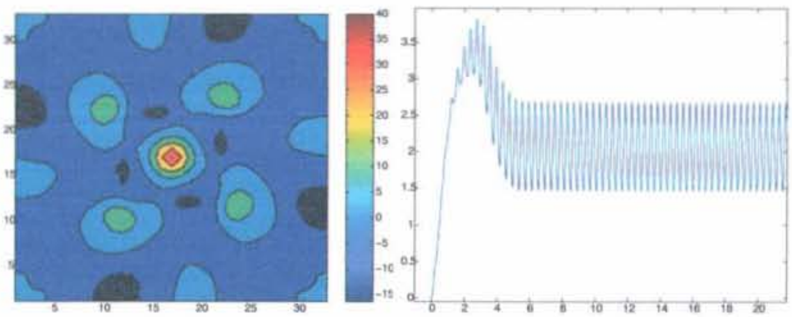

Figure 6: Vorticity plot and Energy-time (s) plot, Re=50, a=3, f=10 and t=10

4.3.3 Simulations with Re = 100

Simulations at $Re = 100$ showed that there was typical pattern formation. The vortices merged into a tripolar vortex, see figure 7. In figure 8 even some sort of pentapole (five somehow connected vortices) can be seen. It has to be noticed that this can be a result of the gridsize (33x33 Chebyshev modes), this has not been checked.

At $Re = 100$, $a = 6$ and $f = 25$ a tripole is formed after some time, in figure 9 the results show what happens if the vortices merge.

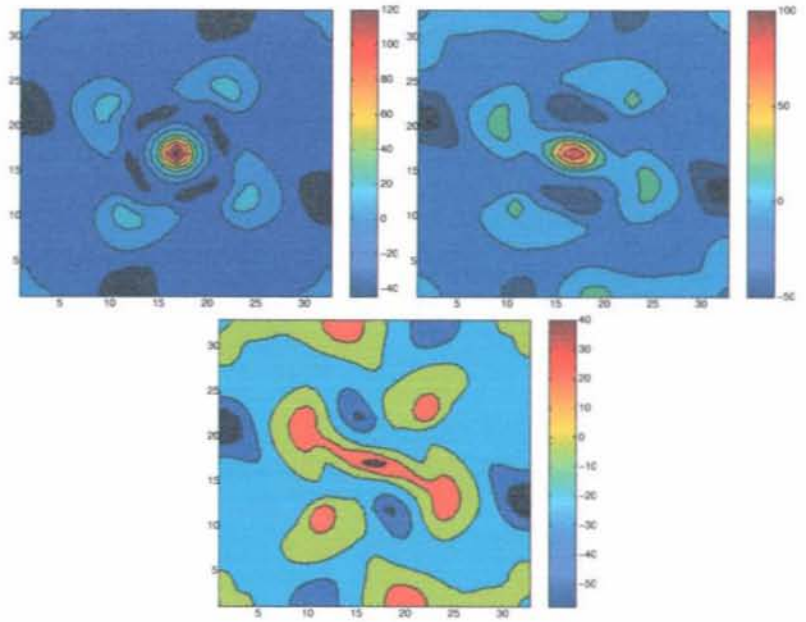

Figure 7: Vorticity plot, Re=100, a=6, f=25 resp. t=15, 25, 40

4.3.4 Observations on open-loop control

After observing these simulations some particular behavior can be noted. Angular momentum and energy signals have the same qualitative behavior. The mode-locking phenomenon can be observed in all these signals. Also, the behavior of the vortices can be more or less observed in the angular momentum and energy. Around the time that the patterns of vorticity change one can also observe a decrease of the angular momentum and energy, see figure 9 and 10. The theory of the inverse energy cascade for 2D turbulence says that energy is less dissipated if the vortices are merged. If the patterns change one can always see a decrease of energy.

That the patterns have a tendency to rotate counter clockwise can be explained from the angular momentum. There are four vortices with positive (counter clockwise) vorticity out of the center and four negative vortices. Because the positive vortices are in the corners their distance to the center is greater. From the definition of angular momentum, (15), it is concluded that the total angular momentum of these eight vortices is already positive. Combined with the positive angular momentum of the center vortex this means that the structure will tend to turn counter clockwise. Knowing this, the question arises if the pattern could be stabilized by introducing a negative center magnet that generates just as much negative angular momentum as there is positive angular momentum because of the other eight magnets. The problem that arises

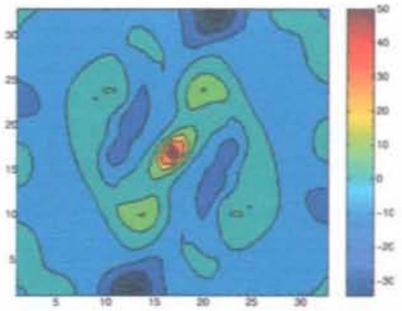

Figure 8: Vorticity plot Re=100, f=0 and t=10

then is that the goal was to have positive vorticity in the center (alternating vortices) and this would then probably turn into a negative center vortex.

The most convincing formations of some special sort of tripoles is seen in figure 7. In figure 7 it is clear that the three poles are not segregated by negative vorticity anymore. This seems to be a good criterium to decide that the pattern has changed. There also seems to be a total rotation of vorticity in that system. The positive vorticity that connects these three main vortices seems to turn around and connects to the other two positive vortices. This can be hard to observe from the vorticity plots that are always taken at a fixed time. It would be clearer if the vorticity would be observed as a movie. Sometimes even some kind of fivepoles are seen. In figure 8 all positive vortices are connected by (a small) positive vorticity. This is very interesting but it goes beyond our control goal, keeping the original pattern like in figure 6.

Note that the open-loop control used (with periodical excitation) does not conserve the pattern. It seems that the pattern merging behavior is qualitatively independent of the frequencies and amplitudes of this excitations.

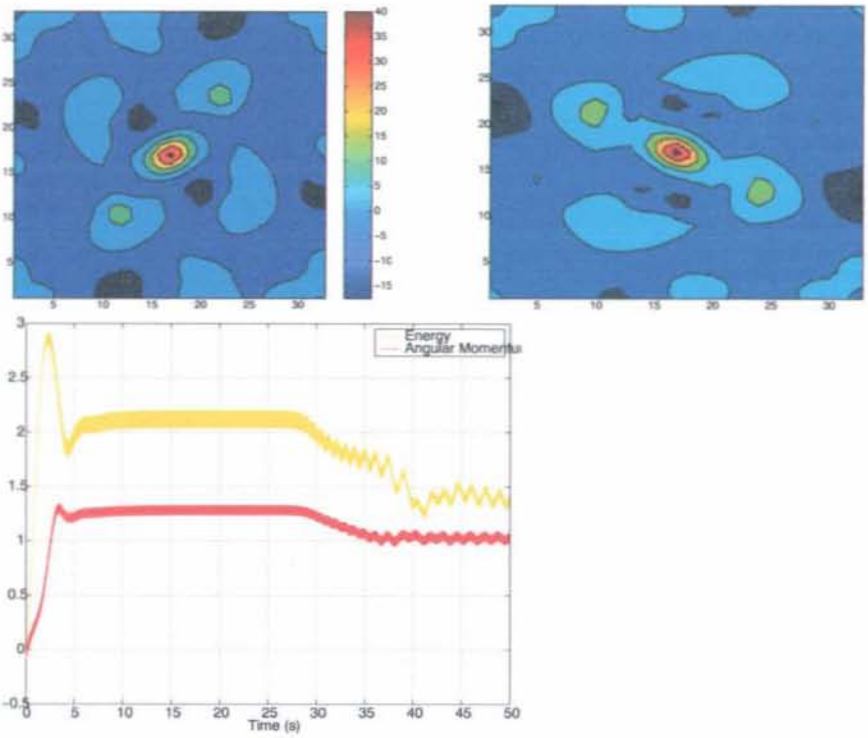

Figure 9: Vorticity Re=100, a=6, f=25 at resp. t=30 and t=31 sec. Below, angular momentum and energy plot

4.4 Feedback controller

The periodic excitation of the center magnet did not succeed in keeping the vortices from merging. So feedback control is proposed to compute the time dependent amplitude of the center forcing. What variable can be used for feedback? The vorticity field would be a complex measure (it is a 33x33 matrix if a small grid size is used) and the position of the centers of the vortices are difficult to compute. Other possibilities are energy or angular momentum. These variables are already outputs of the numerical code and as observed in the last section, their behavior seems to be closely related to the pattern behavior. The angular momentum is used as feedback variable because of research done on its relation with the pattern behavior, see Clercx *et al.* [5], [6].

The error e of the angular momentum is defined as

$$e(t) = L(t) - L_0, \tag{21}$$

L_0 is the reference value of the angular momentum in which the vortices should stay at their start position. The control law is then given by

$$f(t) = P \cdot e(t - \Delta t). \tag{22}$$

In which Δt is the time step used in the simulation. Note that we can not use the error at the actual time in the simulation because this is a function of the forcing $f(t)$. The time step is rather small (depending on the grid size) so it is not assumed that this time delay will create problems. This reference value is estimated from uncontrolled simulations at a certain Reynolds number where the pattern is stable for some time and the angular momentum is constant in that time interval. For example, in the energy plot of figure 9 this can be observed from 10 to 25 seconds and the value of the reference angular momentum is $L_0 \approx 1.3$. Note that this is a rather heuristic approach, it is well possible that for other reference values the controlled system will behave the same. Also note that, according to (22), P should be negative if L_0 is positive. I have chosen $P = -10$ for our simulations. This choice is rather arbitrarly. Significant different values did the job as well.

In figure 10 the energy and angular momentum can be seen in the controlled and uncontrolled case. The vorticity plots of these cases are respectively plotted in figure 11 and 12.

4.4.1 Observations on closed-loop control

The closed-loop simulations at $Re = 50$ do not show anything unlike figure 6. So at $Re = 50$ this figure describes the pattern that can be seen by uncontrolled, open-loop controlled and closed-loop controlled simulations.

At $Re = 100$ one observes that this original pattern becomes unstable. With the closed-loop control it stays stable and the results are not unlike figure 6. The angular momentum, which is actually controlled, has a very small error with its reference value $L_0 = 1$. So the control seems to work for a period of 50 seconds. After this time the energy begins pulsating, indicating that the controller does work. The pattern stays more or less stable but the middle vortex disappears. This is explainable with the control law. If the error is zero than the forcing is also zero. So the center magnets does not work anymore, see figure 11. If controlled behavior is compared with the uncontrolled behavior the difference is striking.

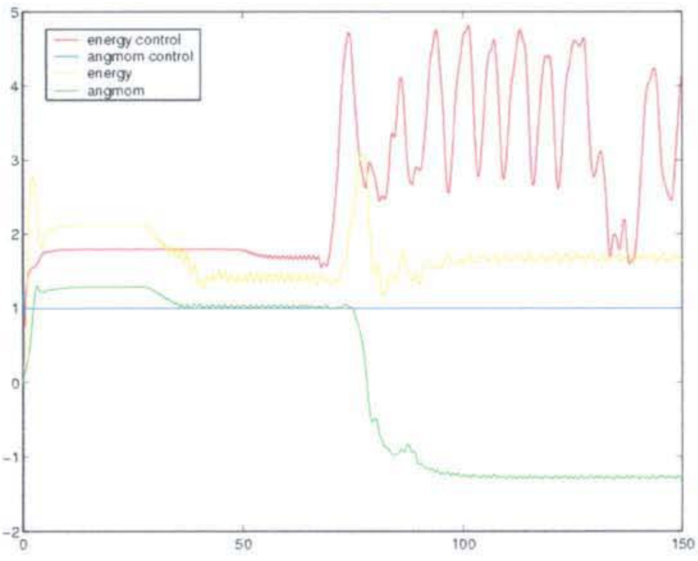

Figure 10: Energy-time (s) and angular momentum-time plot, feedback controlled and uncontrolled at $Re = 100$. Red en blue lines are resp. the energy and angular momentum of the controlled system as a function of time. The yellow and green lines are resp. the energy and angular momentum of the uncontrolled system as a function of time. Note that around 25 s the uncontrolled systems behavior changes. The large peaks in the energy of the controlled system are a result of the controller.

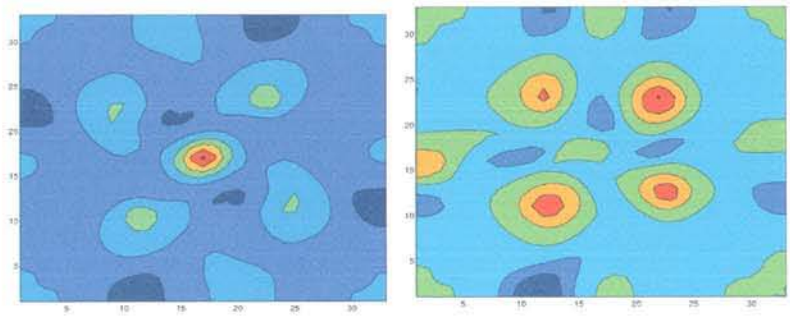

Figure 11: Vorticity plot for Re=100 with feedback control, after 50 (left picture) and 140 (right picture) sec.

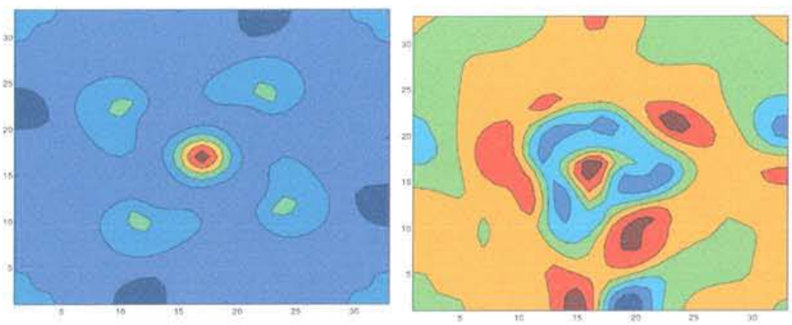

Figure 12: Vorticity at Re=100 without control, after 15 (left picture) and 100 (right picture) sec.

5 Conclusions and recommendations

5.1 Conclusions

In the short time available for the research done more questions than answers have risen. Despite this, we can already take some serious conclusions.

So far, the goal of the investigations was pattern conservation in a simplified configuration of 9 magnets. Simulations with closed-loop control show that this control did have some success. It did not work perfectly but it has been shown that in certain situations the pattern behavior can be influenced. This gives hope for the future.

The closed-loop controller just mentioned is designed on the knowledge of some basic physical features of the system. So far, we have no real theoretical understanding of the control problem. Given the Navier-Stokes equations the normal tools of control theory are not really usable any more. It is even difficult to state exact our goal. In the Appendix different directions have been mentioned and based on the knowledge so far, most of them, except the algebraic approach, could be useful for practical investigations in the future. There will be a big gap to close between these theories and the reality of our experimental setup.

We also did many simulations with open-loop control. Only periodic controls were used. The results did not give any hints how to control the patterns with this method. However, this method is very important as closed-loop control may have been succesful in simulations but it will be very difficult to implement it in the experimental setup. This is because of the large time needed to compute the vorticity field from the measurements. A time-delay of half an hour (or even minutes) will have disastrous effects on the use of closed-loop control.

Further, some interesting observations have been made. We have seen a form of mode-locking between the frequency of the forcing and different physical macro variables like energy, enstrophy and angular momentum. Also a sort of three and five pole has been observed. It is different of the three poles normally observed in Q2D flows because these normally have alternating rotating vortices while our three pole has only counter clockwise rotating vortices. The physical relevance of this phenomenon must be analysed. It could be a rather trivial result but it is an interesting idea to use active control to create new vortex patterns so far unobserved.

As has just been mentioned, the measurement of the vorticity field will be a problem for control. Not only does it take a lot of time to compute but it contains also a lot of redundant information for us. We are only interested in the place of the vortices and the merging behavior on macro scale.

5.2 Recommendations

After giving the conclusions about the research done we can give some recommendations for further investigation and pose some open and interesting questions.

Our first recommendations are practical ones. It would be more efficient to modify the existing code, as it has been developed for physical research, to fit better the needs for designing controllers. Also, it would be interesting to do some very easy experiments on the experimental setup with open-loop control to notice if there are some hidden real-world problems not noticed yet.

Before we can do this experiment, better knowledge of open-loop control should be obtained. This would probably ask for a better understanding of the behavior of the system as an open-loop controlled system can become quickly unstable because of external noise or unmodelled dynamics.

What would be important features of the system behavior from a control perspective? One thing that certainly is necessary is a better understanding of the pattern formation. What exactly do we mean with stabilizing a pattern ('pattern conservation') and what happens if we say that the pattern changes? This should be understood on a more fundamental level and the analogy with pattern formation in the Rayleigh-Bénard flow (on which more research has been done) could be useful. The behavior of some properties as energy, enstrophy, angular momentum, dissipation and the influence of the boundary and Reynolds number should be understood better in relation to our control problem and an attempt could be made to connect this physical understanding with the theory of energy based control and the influence of dissipation in control. By doing more detailed simulations and if possible a linear stability analysis we could see why the pattern becomes unstable, it seems that the boundary is an important factor in this. Studying the active forcing in simulations with closed-loop control can hint at a way open-loop control could work. If the closed closed-loop forcing goes to a steady value (zero in our former simulations) we could use this value for open-loop control. When the system is controllable this way, we actually use feedback control as some sort of adaptive algorithm. We come back on this view on the end of the recommendations.

If some progress is made in this global understanding one could investigate with simulations the relation between the forcing (input) and variables as energy, enstrophy, angular momentum and vortex locations. What is the transfer function between these variables and is there some local control possible? Also measuring the (co-)relation between energy or angular momentum and as output the vortex locations could clarify a lot about the causal relationship between these variables.

As we have seen, feedback control seems to be able to stabilize the pattern. The only disadvantage why we can not use closed-loop control is that every output considered so far (vorticity, place of vortices, energy, angular momentum) depends on the velocity field. So it would take too much time to compute this. If we could estimate one of these outputs in another and faster way, feedback control would be back in the picture. However, no fruitful ideas on this subject have emerged yet.

After some better understanding of the system has been achieved it would be a good idea to have a look at the theory of control of flow, especially Coron [7].

As a last recommendation we have a practical idea that could be very useful for flow simulations in general. A great and general problem is fixing the Reynolds number as it is function of the velocity. So if the Reynolds number is given before the simulation it still is possible that the result of the simulation gives another Reynolds number. This is why there is a trial-and-error approach to find the right forcing that will gives the desired Reynolds number. We have seen that the feedback control of angular momentum works very good in the model, there is only a very small error. If we can control the energy as a function of the complete magnetic field (so we control the force of all nine magnets) just as good then we could find the right passive forcing in one run. After fixing the passive magnets at this forcing value we can do simulations with one active magnet. Again simulations with feedback control are used as some sort of adaptive algorithm.

A Flow control

A.1 Introduction

Before we start simulating and analysing the behavior of the 3x3 magnets configuration, we study the research already done on flow control. Our goal is to control the pattern formation of the vortices but at first it would be nice if we could maintain the nine starting vortices. Without control these vortices have the tendency to merge and with higher Reynolds number this tendency grows. It would be nice if we could use the tools and concepts of classical control theory. Natural questions rise for instance on the controllability, stabilizability, observability of the system and the existence and construction of optimal/robust controllers. That the used model of the system is a system of nonlinear partial differential equations and thus of infinite dimension does not mean that it can not be controlled by classical control methods. Just like the vibration of a flexible beam can be modelled by a mass-spring model for some applications it is possible that also this system can be modelled by a low-dimensional model locally around the vortices starting positions. However, this is not obvious. Therefore we have studied literature on control of partial differential systems and on flow control.

There are a lot of different views on the problem. Hereunder an effort is made to summarize these views.

A.2 Problems

As is said in the previous chapter, stability in flows is a difficult subject. But also controllability and observability are difficult because the classical definitions are not suited for infinite dimensions and the classical techniques to check these properties often use the rank of certain finite dimensional matrices. It is not clear how to check these properties for a system described by the Navier-Stokes equation. Actually, it is not even completely clear what would be the best way to define these properties for our problem.

A.3 Complex Ginzburg-Landau equation

The Complex Ginzburg-Landau equation (CGLE) is a much investigated equation with a very rich behavior. It is used for studying pattern formation and turbulence. In Xiao *et al* [24] it is shown that they quite easily can control the behavior and can control pattern formation by boundary control. Because this equation has one space dimension and it is much easier to analyse than the two dimensional Navier-Stokes equation it could be interesting to investigate it. Especially because there is also an experimental setup in the Cascade building that can quite well be described by the CGLE. This setup could be investigated if the given setup is too difficult to control.

A.4 Finite dimensional control

It is natural to try to reduce the system to a finite dimensional system of order 1 or 10. Standard, finite dimensional control can then be used.

In fluid science there is a standard method, proper orthogonal decomposition (POD) [12]. With this you can reduce fluid flows to a low-dimensional model (in the order of 10 dimensions). A similar approach has been used in Park/Sung [11]. They succeeded in controlling patterns in a two-dimensional Rayleigh-Bénard flow with optimal control. This kind of flow arises if the temperature at the bottom of a two-dimensional fluid layer is higher than the temperature at the surface of the fluid layer. Under certain conditions the system becomes unstable and pattern formation is observed. By controlling the temperature distribution at the bottom of the flow, the flow is stabilized.

In Abarbanel [1] on identification of (the phase space of) chaotic systems there is a small but interesting example considering turbulence. Data is used to identify the behavior of the boundary layer with its coherent structures. They use identification tools specifically made for chaotic systems identification. They use the mutual information (the same as in Shanon's information theory) in the data signal to know the best sample time and a concept called false neighbourhood to prove that the behavior will be described by a 8 dimensional model. This is interesting because in almost all literature on turbulence linear identification techniques are used. If the systems becomes chaotic it can be hard to get real information out of the broadband spectrum, it seems that those new concepts can sometimes give more understanding.

A.5 Partial differential control

Two recent mathematical papers from Fursikov [10, 9] use theory of partial differential equations (like distributions) and functional analysis to investigate controllability and stabilizability in the 2D Navier-Stokes equations with boundary control (Dirichlet boundaries). His analysis of the complete Navier-Stokes equation is very difficult. However, his analysis of simplified equations like the Stokes equation (in which the nonlinear term is neglected) is understandable with some knowledge of functional analysis. His results are that the 2D Navier-Stokes equation and simplified equations like the Stokes equation are controllable and stabilizable.

It can be instructive to give the definitions he uses to define controllability and stabilizability in this problem. The definitions are not unlike the standard definitions for finite dimensions but the fact that they are applied to partial differential equations makes them somewhat less intuitive to understand. It also makes it much more difficult to solve questions about controllability and stabilizibility in this type of systems.

Given a 2D Navier-Stokes equation defined in a bounded domain Ω which is controlled by Dirichlet boundary conditions for velocity vector field.

$$(\hat{v}(x), \nabla\hat{p}(x)), x\epsilon\Omega \tag{23}$$

is a steady-state solution and let the initial condition be $v(t,x)|_{t=0} = v_0(x)$ and $v|_{\partial\Omega} = 0$. Suppose $\hat{v}|_{\partial\Omega}$ and $\hat{v}$ is an unstable singular point for the 2D Navier-Stokes equation. This implies that the solution $(v(t,x), \nabla p(t,x))$ will generally diverge from $(\hat{v}(x), \nabla\hat{p}(x))$. If we introduce boundary control

$$v|_{\partial\Omega} = u \tag{24}$$

Assume

$$\|\hat{v} - v_0\|_{H^1} < \varepsilon \tag{25}$$

for small ε. With H^k a Sobolev space which is a space with all functions f with $f \epsilon L_2$ and all their derivatives up to k. Given $\sigma > 0$, if we can find $u(t,x), t > 0, x\epsilon\partial\Omega$ such that

$$\|v(t,\cdot) - \hat{v}(\cdot)\|_{H^1(\Omega)} \leq ce^{-\sigma t} \tag{26}$$

we call the problem stabilizable.

The most basic way to define controllability in this problem is to ask for the existence of a control function $u(t)$ that can bring $v(t)$ close to $\hat{v}(t)$ in arbitrary time T as $t \to \infty$.

A very recent unpublished paper from Coron [7] places the results of Fursikov and J.-L. Lions (see Lions [14] for a very good introduction on flow control) in the vocabulaire of nonlinear control theory. He also remarks that geometric tools (like Lie brackets) seem to be less effective in infinite dimensions than in finite dimensions. Because he uses his theories (in an accessible way) on flow control, this article seems to be usable in further research.

A.6 Differential algebra for control

There also exists research on partial differential control in an algebraic setting. In the introduction of the recent book of Pommaret [18] some claims are made about the proof of controllability in infinite dimensions. He claims proving controllability with operator theory (in which the operator represents the system in the same way a matrix can represent a finite dimensional dynamical system) is equivalent with proving that a certain D-module is torsion free. It takes a lot of differential algebra to explain the details but in essence a module has nearly the same structure as a vector space [13] and if you can prove that a certain type of element (torsion element) does not occur in the D-module (which represents the system) than the system is controllable. If there exists good algorithms to check this non-torsion property and it is not too difficult to construct the D-module given a certain system this is a very interesting theorem.

The theory given in this book also gives methods for optimal control of nonlinear partial differential systems but the mathematical theory needed to understand this is very difficult. Cohomological algebra, differential algebra and differential module theory are used on an advanced level. In [19] a short but not easy summary of this theory is given.

A.7 Information-theoretic approach

Sometimes one can hear people speak about control problems in terms of information. Statements like:'The quality of the control depends on the information in the output of the system' are made. This way of thinking about control can be made exact by using the concepts of Shanon's information theory. In this theory concepts like information-entropy are used to describe communication processes. It has been used for decades in fields like coding, cryptography and (electronic) communication. However, if one has a control theoretic background communication channels seem to have a close relationship with controllers. It is no surprise that it is possible to use information theory to analyse control issues. Only a very recent paper of Lloyd and Touchette [15] has made some exact statements in terms of information entropy about controllability, observability, optimality and the gain of closed-loop control over open-loop control. The origins of their approach lie in their attempts to control quantum states and they want to eventually use this in quantum cryptography and quantum computing.

It seems that there are some possible profits of using this theory. There is no essential difference in this theory between linear and nonlinear systems nor between deterministic and stochastic systems. However, making the theory work for a continues system will take some work. Also the theory provides no way of actually constructing a controller yet.

By using the concept of entropy also stability and optimality can be defined. A necessary condition to be able to stabilise a system is to decrease its entropy or immunize it from sources of entropy because of uncertainties.

A.8 Energy and passivity based control

For global control of underactuated robots there exists an approach called energy and passivity based control [21]. The concepts of energy and passivity (no energy production in the system) are used as an alternative way of approaching stability and constructing stabilizing feedback control. Due to a lack of time this theory has not been deeply investigated but it is interesting for future research.

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