

## Note on a fundamental problem in the theory of cutting : the shear angle solution

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NOTE ON A FUNDAMENTAL PROBLEM IN THE THEORY  
OF CUTTING: THE SHEAR ANGLE SOLUTION

H.J.J. KALS

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In spite of the numerous attempts to determine the law(s) which govern the shear angle  $\phi$  in cutting, the lack of adequate criteria leading to predictable chip thickness, load characteristics and contact temperatures is still an important problem in the cutting theory. Roughly, the existing shear angle solutions are either based on the theory of plasticity involving slipline fields or on the principle of minimum energy using bulk forces. Most incorporate uniform stress fields and a constant coefficient of friction across the chip-tool contact area. It is presently accepted that the contact between chip and tool is partly adhesive, so that the assumption of a constant coefficient of friction is very doubtful. As these theories emphasize the relevancy of the friction force, the absence of a proper description of the interaction between chip and rake face may explain the unsatisfactory results. It has been shown<sup>1)</sup> that both normal and tangential stress distributions at the chip-tool interface are far from uniform, while the existence of a secondary shear zone has also become evident now. Furthermore, when using the principle of minimum energy, the interaction between primary and secondary shear zone should be taken into account: the chip-tool contact length  $KB_0$  is a function of the shear angle  $\phi$ .

Using a uniform tangential stress distribution and incorporating both the dependence of the contact length on the shear angle and an assumed influence of temperature and strain rate on the shear stress in the secondary zone, Spaans<sup>2)</sup> came to a solution based on the principle of minimum energy. However, using the analysis for qualitative purposes only, he did not give an experimental verification of the results.

At present it is not clear whether or not the cutting process obeys the condition of minimum energy. There is however evidence that the cutting process is a temperature-stabilised system. To clarify matters, a cutting model containing certain assumptions regarding the stress distribution on the rake face of the tool is needed.

Concerning the normal stress distribution, the analysis used by Yellowley and Barrow<sup>3)4)</sup> has been adopted. Referring to experimental results of Primus<sup>1)</sup>, they concluded that the distribution of the normal stress across the majority of the chip-tool contact length can be expressed as

$$\sigma_{\gamma d} = \sigma_o \left(1 - \frac{x}{KB_o}\right)^m \quad (1)$$

where  $\sigma_o$  is the maximum normal stress (in the vicinity of the cutting edge):

$$\sigma_o = \tau(1 + \sin 2(\phi - \gamma_o)) \quad (2)$$

and

- $x$  = the distance to the cutting edge
- $\tau$  = the shear yield strength in the shear plane
- $KB_o$  = contact length
- $m$  = a constant not depending on cutting conditions, type of tool and work material,

while a constant stress  $\sigma_o$  covers the feed region over the length  $h/2\cos\gamma_o$  (see fig. 1;  $h$  = uncut chip thickness,  $\gamma_o$  = rake angle). By considering force equilibrium and energy distribution over primary and secondary deformation zone, it is possible to express the apparent coefficient of friction in the mode as suggested by Zorev<sup>5</sup>):

$$\mu_\gamma = \frac{\lambda_c \cos\gamma_o}{1 + a_e - \lambda_c \sin\gamma_o} \quad (3)$$

The natural contact length between chip and tool can then be expressed as

$$KB_o = \frac{2h\gamma_c (1 + a_e - \lambda_c \sin\gamma_o)}{a_e \cos\gamma_o (1 + \sin 2(\phi - \gamma_o))} - \frac{1}{2\cos\gamma_o} \quad (4)$$

where

- $\gamma_c$  = shear strain
- $\lambda_c$  = the cutting ratio
- $a_e$  = ratio of dissipated energies in primary and secondary zone respectively.

The analysis holds for cutting speeds beyond the region of the BUE where  $a_e$  is known to be fairly independent of cutting speed and feed (Zorev<sup>5</sup>) ( $a_e$  depends on the combination work-/toolmaterial as well as on the rake angle).

Proceeding in the same way, it is also possible to find an expression for the sticking length  $l_s$ , i.e. that part of the contact length which is controlled by adhesive phenomena (fig. 1). The adhesive region is characterised by a constant shear yield strength  $\tau_0$ , while over the remaining part of the contact length the tangential force is caused by friction, i.e.:

$$\tau_{\gamma d} = \tau_0 \left( \frac{x}{KB_0 - l_s} \right)^m \quad (5)$$

$$\tau_0 = \tau \cos 2(\phi - \gamma_0) \quad (6)$$

From equilibrium considerations it follows for the sticking length:

$$l_s = \frac{m+1}{m} \cdot \frac{h\gamma_c}{a_e} \left\{ \frac{\lambda_c}{\cos 2(\phi - \gamma_0)} - \frac{1}{\cos \gamma_0} \left( \frac{1+a_e - \lambda_c \sin \gamma_0}{1 + \sin 2(\phi - \gamma_0)} \right) \right\} + \frac{h}{2 \cos \gamma_0} \quad (7)$$

Both contact length and sticking length are functions of the shear angle; the functions are depicted in figs. 2a, 2b and 2c.

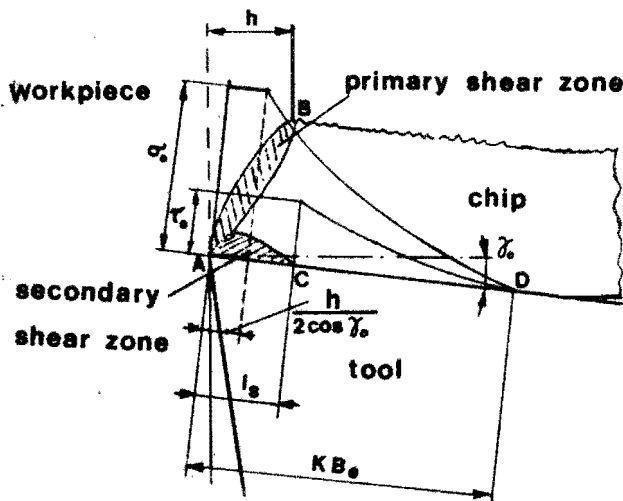


Figure 1

Characterizingly the sticking length curves show a minimum, while the contact length curves do not. Another fact is that  $l_s$  is subjected to a substantial higher sensitivity to variations in the energy ratio  $a_e$  than  $KB_0$  does. Cutting experiments carried out for a number of different work materials and carbide grades show that in the cutting process adjustment to a certain shear angle occurs, the value of which being to a degree

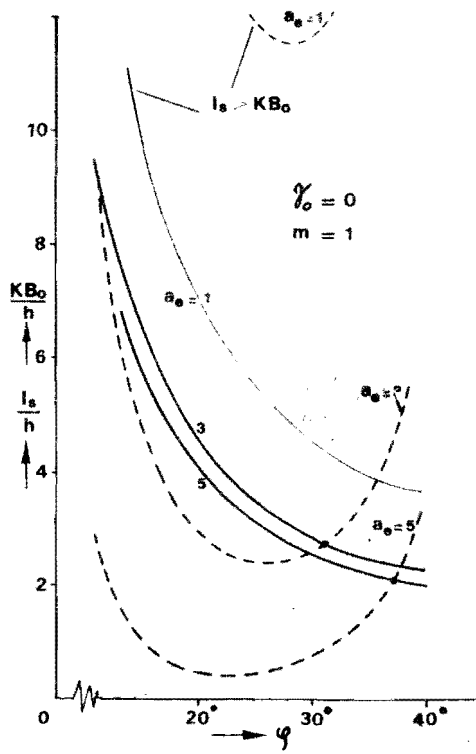


Figure 2a

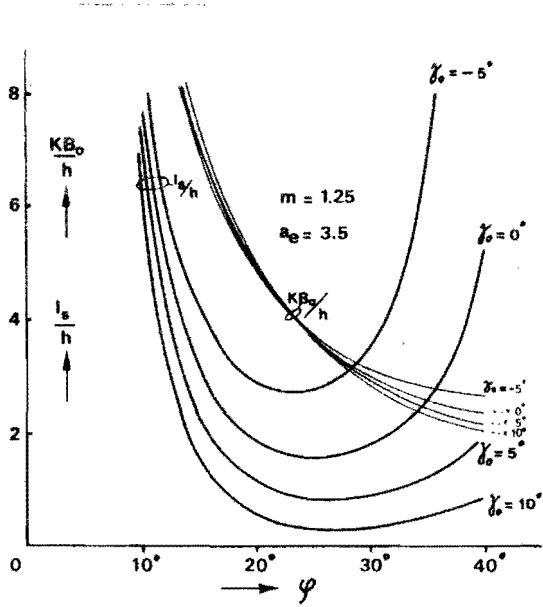


Figure 2b.

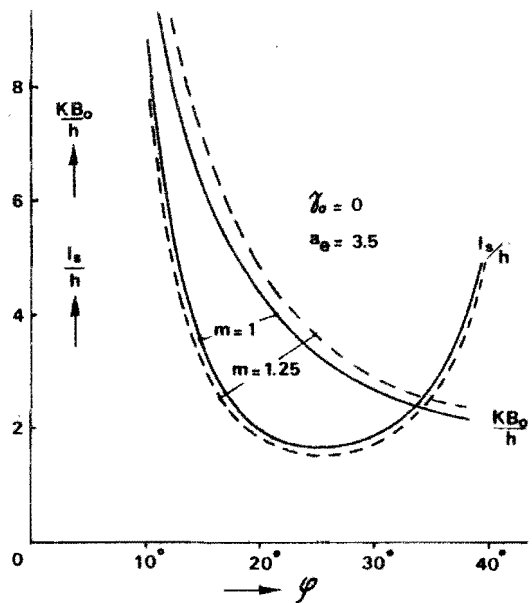


Figure 2c

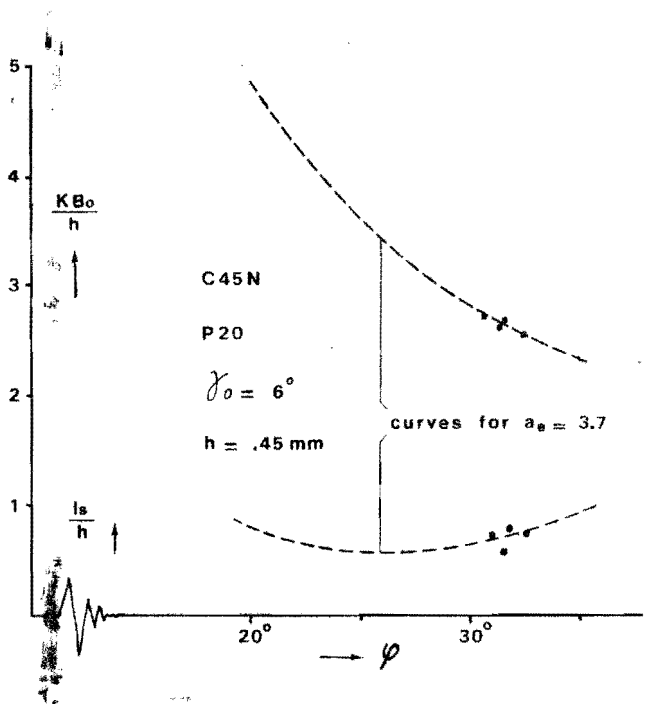


Figure 3a

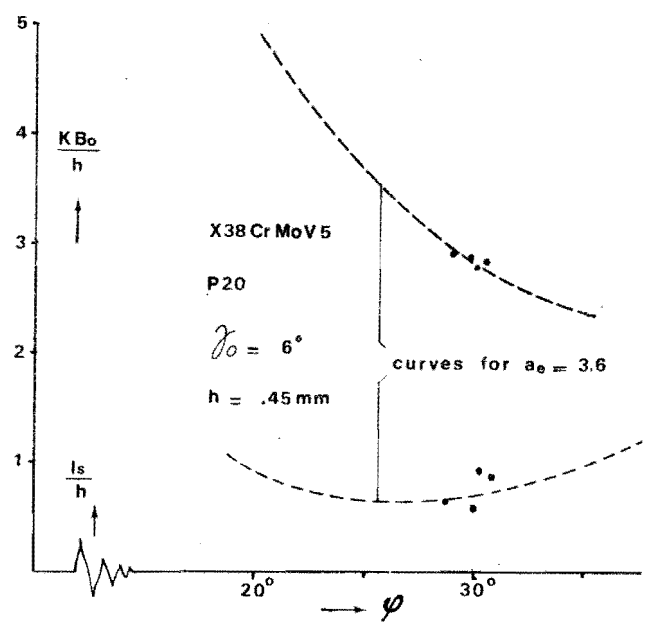


Figure 3b

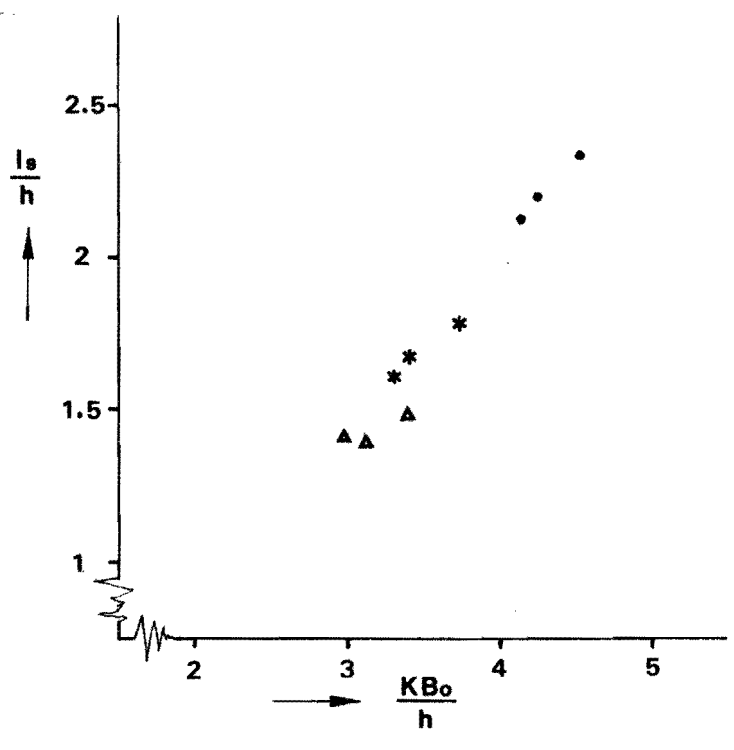


Figure 4

somewhat higher than the value coinciding with the minimum value of the sticking length (fig. 3). These facts together with the occurrence of a -more or less- triangular shaped secondary shear zone suggest that the cutting process is a temperature-stabilised system (see further on). Fig. 4. shows that the ratio  $l_s/KB_0$  is fairly constant regardless of feed and speed; this being a result of the shear angle changing only slightly with feed and speed, while  $a_e$  is not affected by these variables. Results from wear-scar measurements on the rake face (initially only the frictional contact area is subjected to wear) show that the scatter in  $KB_0$ -values substantially exceeds that of the  $l_s$ -values. It is the author's believe that - as far as continuous chips are concerned and no BUE is present - the sticking length is the prime cutting parameter; the contact length results from it, but is also subjected to secondary (such as natural chip curl) and fortuitous processes.

The idea of the cutting process being a temperature-stabilized system is based on two phenomena; the first phenomenon is related to the magnitude of the influenced zone when metals are subjected to plastic deformation, the second phenomenon concerns the dependence on temperature of the strain hardening ability. An example of the first phenomenon is shown in the hardness test; the influenced area increases with increasing value of the strain hardening coefficient  $n$  ( $\sigma = C\delta^n$ ). For steels the strain-hardening ability increases with temperature till about 300 °C. A further increase in temperature has a decreasing ability of strain hardening as result. The temperature at which strain hardening is no longer possible increases with the strain rate. In the cutting process the material passing through the secondary shear zone will gradually increase in temperature and loose its strain hardening ability as a result of which the thickness of the secondary zone gradually decreases to a minimum value at which sticking changes into friction. The maximum temperature in the sticking region is reached in the vicinity of C. A temperature fluctuation  $\Delta\theta_C$  is believed to start the following train of events:

- increase  $\Delta\theta_{C1}$ ,
- decrease of  $l_s$ ,
- decrease of tangential force on rake face,
- increase of shear angle,
- decrease of temperature rise in primary zone,
- - (together with the decrease of  $l_s$ )- decrease  $\Delta\theta_{C2}$ .



Results of Blankenstein<sup>6)</sup> show that for 35NC6G,  $v = 200$  m/min and  $h = 0.235$  mm, the frequency of the dynamic component of the cutting force is about 5 kHz. This means that during one period the chip travels over a distance of  $v/\lambda_c \times 1/f = 3.33/(1.75 \times 5000) \text{ m} \approx 0.38$  mm (The value of the cutting ratio  $\lambda_c = 1.75$  results from cutting experiments using the work-piece material X38CrMo5 and a P 10 carbide tool).

The latter value being quite common for the sticking length under comparable conditions, it seems that the sticking region is closely related to the segmentation frequency; chip speed and sticking length determine the time constant of the oscillatory system with the average temperature  $\theta_c$  taking a minimum value. In this respect it is interesting to consider the findings of

- Mac Manus<sup>10)</sup>: Oscillations of the shear plane and temperature variations play an important role in the formation of segmented chips; the effective shear angle responds sensibly in phase with temperature variations inducted by an alternating current through the chip tool contact zone.
- Trigger et al.<sup>11)</sup>: The occurrence of smooth chips with soft ductile materials when the chip tool contact length is restricted to about 1/3 of the natural contact length; i.e. a non-oscillatory behaviour when the contact length becomes smaller than the natural sticking length.

The average temperature can be calculated with the aid of the equation

$$\theta_c = \frac{1}{\rho c} \left\{ \tau \gamma_c (1 - \beta) + \alpha \frac{\tau_o l_s}{\lambda h} \right\} \quad (8)$$

where  $\rho c$  = specific volumetric heat (assumed to be constant for the moment),  
 $\beta$  = that part of the heat-generated in primary zone- which flows into the workpiece,  
 $\alpha$  =  $(\theta_c - \theta_s)/(\theta_a - \theta_s)$ ;  $\theta_s$  being the chip temperature at the end of the primary zone and  $\theta_a$  being the average chip temperature at C. ( $\alpha$  settles the nonuniformity of heat generation and heat distribution in the chip).

The value of  $\beta$  can be calculated with the formula

$$\beta = 0.57 (R \tan \phi)^{-0.45} \quad (9)$$

$$(R = \frac{c}{k} v h \text{ ("thermal number")})$$

which covers experimental results of Nakayama and Boothroyd<sup>7)9)</sup>. A difficulty arises when the determination of  $\alpha$  is concerned. Assuming a plane heat source and neglecting the flow of heat into the tool, Rapier<sup>8)9)</sup> was able to calculate an expression for the temperature distribution over the chip-tool contact length; the maximum temperature occurs at the end of the contact between chip and tool where

$$\alpha = 1.13 \sqrt{\frac{Rh}{KB_o} \frac{c}{K}} \quad (10)$$

However, all the experimental evidence indicates that the maximum temperature occurs within the chip-tool contact length while the actual values are substantial lower than those calculated with the aid of equation (10). Boothroyd suggest that the discrepancy is due to the assumption of a plane heat source and shows theoretically that a triangular shaped secondary shear zone compares well with the experimental evidence. He also proves that the thickness of the heat source is of substantial interest. The introduction of a heat source of finite thickness, however, requires the use of numerical methods to determine the temperature distribution which in our case leads to time consuming computer programmes. Therefore it was decided to carry out some preliminary calculations adopting Rapier's approach (eq. (10)). Additionally, since we are interested in the temperature at the end of the sticking length, the quantity  $KB_o$  has to be replaced by  $l_s$ . Finally we arrive at the equation

$$\alpha = 1.13 \sqrt{\frac{Rh}{l_s} \frac{c}{K}} \quad (11)$$

and thus we are able to determine the shear angle solution from

$$\frac{d\theta_C}{d\phi} = 0 \quad (12)$$

The results are depicted in the orthodox way in fig. 5, together with the well-known solutions of both Lee & Schaffer and Merchant.

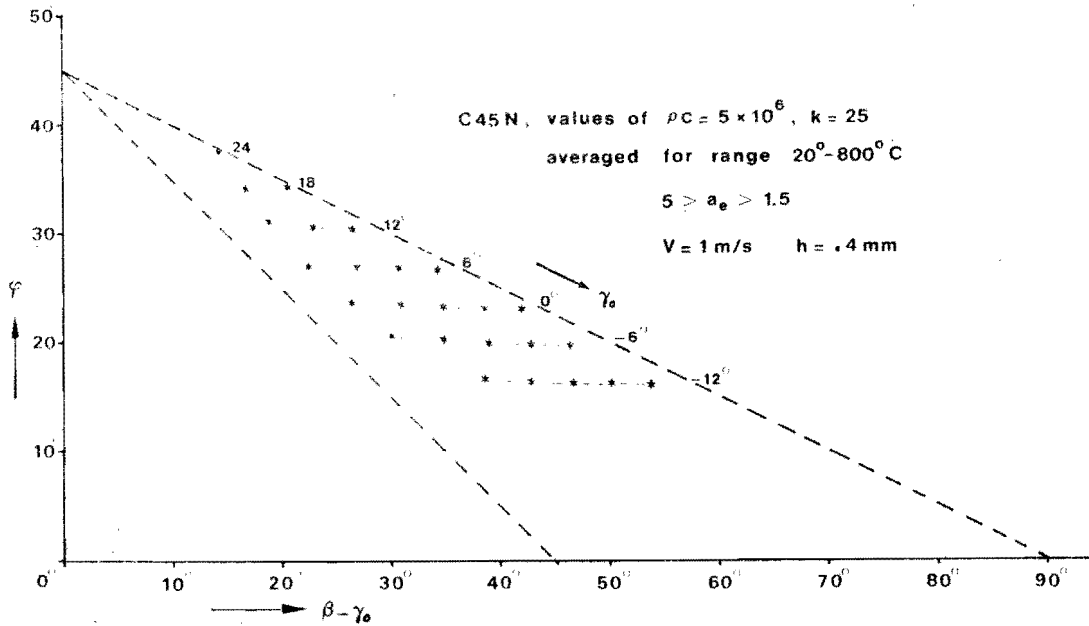


Figure 5

The solutions which cover actual cutting conditions (i.e. beyond the BUE for  $1.5 < a_e < 5$ ;  $0.45 < \mu_\gamma < 1.00$ ) are situated in the area limited by the Lee & Schaffer and Merchant solutions. It is noticed, however, that the results do not satisfy the general tendency of the shear angle to increase monotonously with decreasing value of the friction angle  $\beta$  whilst keeping the rake angle  $\alpha_0$  constant. It is believed that this behaviour is due to the adoption of a plane heat source on the rake face with a resulting over-estimation of the maximum temperature rise in the chip. It is noticed that for  $\beta \rightarrow 0$  and  $\alpha \rightarrow 1$  the solution would meet the criterion of least energy.

Our present work is directed to the introduction of a triangular shaped heat source. In doing so one has to overcome the problem of making proper assumptions regarding the distribution of stress and strain as well as the initial thickness of the triangular heat source.

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