

Practical convergence of an underactuated H-drive system

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Practical convergence of an under-actuated H-drive system

division DSD
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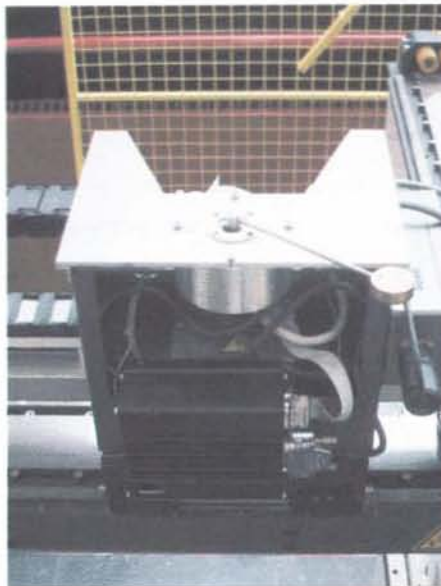
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Practical convergence of an underactuated H-drive system.



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1 Introduction

Control of underactuated systems has attracted a lot attention in the recent past. An underactuated system is defined as a system that has more degrees of freedom than actuators. Some examples of such systems are surface vessels, spacecraft, mobile robots and helicopters. But underactuated systems are not limited to these examples. Underactuated systems generate control problems that require fundamental non-linear control methods.

Many of these underactuated systems are subject to nonholonomic constraints. A nonholonomic constraint is a constraint that is not integrable; i.e. the constraint cannot be written as time derivatives of some function of the generalised coordinates. The nonholonomic constraints can be divided into two classes; the first order nonholonomic constraints and the second order nonholonomic constraints. The first order nonholonomic constraints are defined as constraints on the generalised coordinates and the velocities that are not integrable. Second order nonholonomic constraints are defined as constraints on the generalised coordinates, the velocities and the accelerations that are non-integrable. Linear control methods cannot be used, not even locally because the linearization of the system around the equilibrium may not be controllable.

The underactuated mechanical system used is the H-drive with an additional link and it is subject to a second order nonholonomic constraint. Underactuated systems with second order nonholonomic constraints cannot be controlled with smooth time invariant stabilising feedback laws because these systems have a structural obstruction for this.

A method to design a controller for such a system is first transforming the system into the chained form. This transformation makes it possible to design a controller for the system, while in the mechanical coordinates a controller will not be easy to design.

After the coordinate transformation is complete the feedbacks that stabilise the system can be derived. The model of the H-drive with the additional link, the co-ordinate transformation and the continuous time-varying feedbacks to stabilise the system is described in section 2, The model.

In the designed controller there are several parameters that need to be chosen.

The parameters of this controller are optimised with respect to the deviation of the desired point and the rate of convergence in the first part of section 3, Simulations.

But there are unmodelled dynamics in the system like for instance stick slip and cogging, which are not compensated for in the controller. Due to this the system will never reach the desired point for which the feedback stabilisation is designed. It will oscillate around this point.

To make sure that the system will converge to the desired point the controller is adjusted.

This means that when the state of the system approximates the required state the controller will be switched off. This is described in the second part of section 3, simulations.

The results of the simulations are verified with experiments, which are described in section 4, Experiments.

Finally in section 5, recommendations some recommendations are given to continue the research in this area.

2 The Model

The underactuated system used is a mechanical system known as the H-drive with an additional non-actuated rotational link. This H-drive is an XY-table with three linear motors and is used for pick and place operations on printed circuit boards.

It consists of two parallel y-axes that are connected by a beam, the x-axis. This x-axis is connected to the y-axes with a joint. Because of this the positions of the y_1 - and y_2 -axes are not necessary equal. But servomotors actuate the axes and because servomotors are position steered they thus attempt to follow the desired trajectory. They compensate the friction in the links.

The y-axes can be considered as one axis because the dynamics of each y-axis is assumed to be identical to the other.

The H-drive is illustrated below.

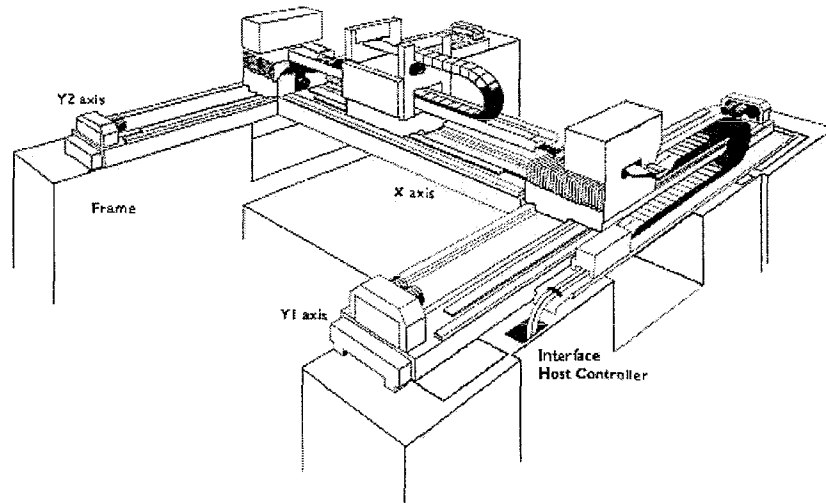


Figure 1: The H-drive

The orientation of the additional link is measured with an encoder. The rotational link is mounted on the linear motor of the x-axis and can rotate freely.

The x-position and the y-position as well as the orientation of the rotational link are controlled by the x and y motors.

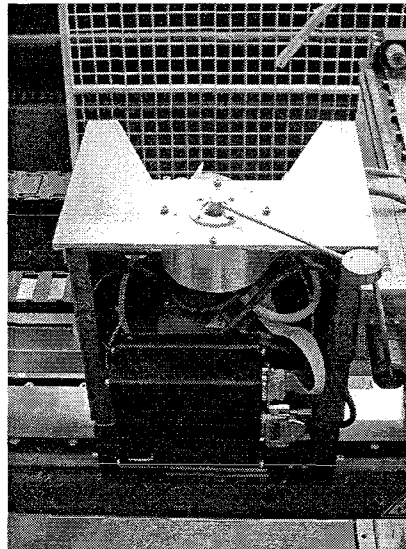


Figure 2: The additional unactuated link

2.1 The dynamical system

The H-Drive with the additional rotational link is an underactuated mechanical system with two inputs, i.e. the currents i_x and i_y to the servomotors, and three positions, i.e. the positions X, Y and the

orientation θ of the rotational link. As mentioned earlier, the H-Drive is powered with servomotors. And the positions $Y_1(t)$ and $Y_2(t)$ will be assumed to be equal, i.e. $Y_1(t) = Y_2(t) \forall t$.

The mass of the x motor and the two y motors is denoted as m_x and m_y respectively. And the mass and inertia of the rotational link and the beam are denoted by m_3, I_3 and m_b, I_b respectively. The longitudinal forces from the Y-axis are denoted by F_y , and the transversal force from the X-axis by F_x . The distance from the rotational joint at the position $[r_x, r_y]$ to the centre of mass of the rotational link is denoted by l . The system moves in a horizontal plane and is not influenced by gravity.

By using the Lagrange-Euler formulation it is straightforward to calculate the dynamic model of the H-Drive. By assumption, the origin O of the global co-ordinate system is located at $(X, Y) = (-0.3, 0.5)$ (near the centre of the H-Drive set-up). The generalised coordinates are given by the joint coordinates and orientation of the link, i.e. $q = [r_x(t), r_y(t), \theta(t)]$. The joint positions r_x and r_y can be written in terms of the encoder measurements $[X, Y]$ as follows

$$\begin{aligned} r_x &= Y - 0.5 \\ r_y &= -X - 0.3 \end{aligned} \quad (1)$$

If we assume true linear dynamics of the motors, with motor constant k_m , then the dynamic model is given by

$$\begin{aligned} m_x \ddot{r}_x(t) - \frac{m_3 l}{2} \sin(\theta(t)) \ddot{\theta}(t) - \frac{m_3 l}{2} \cos(\theta(t)) \dot{\theta}^2(t) &= k_m i_y \\ m_y \ddot{r}_y(t) + m_3 l \cos(\theta(t)) \ddot{\theta}(t) - m_3 l \sin(\theta(t)) \dot{\theta}^2(t) &= k_m i_x \\ (I_3 + m_3 l^2) \ddot{\theta}(t) - m_3 l \sin(\theta(t)) \ddot{r}_x(t) + m_3 l \cos(\theta(t)) \ddot{r}_y(t) &= 0 \end{aligned} \quad (2)$$

The masses along the x and y direction are given by

$$\begin{aligned} m_x &= m_y + \frac{m_b}{2} + \frac{(m_x + m_3)}{2} \\ m_y &= m_x + m_3 \\ I &= I_3 + m_3 l^2 \end{aligned} \quad (3)$$

This model can be transformed into the second-order chained form system, as will be shown in the following section.

2.2 The chained form

In the control of underactuated systems a possible method is transforming the system into the chained form. This simplifies the dynamical equations and is more suitable to design a controller, [1].

The second-order chained form can be used to design controllers for systems with second order nonholonomic constraints.

The second order chained form for the system (2) is given by:

$$\begin{aligned} \ddot{\xi}_1 &= u_1 \\ \ddot{\xi}_2 &= u_2 \\ \ddot{\xi}_3 &= \xi_2 u_1 \end{aligned} \quad (4)$$

The co-ordinate transformation corresponds to the position of the centre of percussion (C.P.) of the rotational link. The C.P. of a link can be interpreted as follows; if one would apply a force perpendicular to the link and at a certain point below or above the C.P., then a rotation of the link will occur. If however, a force perpendicular to the link is applied exactly at the C.P., then no rotation of the link occurs. The C.P. is therefore useful in order to generate pure rotational motions of the link, in

which the C.P. stays at rest. By performing repeated translational and rotational motions of the link, it is possible to move the unactuated and free rotating link from any initial configuration to any final configuration.

Define the configuration variable $q = [r_x, r_y, \theta]$. Any (equilibrium) point, with zero velocity can be mapped to the origin $\xi = 0$ of the chained form. In this chapter the equilibrium $(q, \dot{q}) = (0, 0)$ is mapped to the origin $(\xi, \dot{\xi}) = (0, 0)$ of the extended chained form. The feedback transformation is given by

$$\begin{bmatrix} i_x \\ i_y \end{bmatrix} = \begin{bmatrix} \left(-\frac{m_3 l}{2} \cos(\theta) \dot{\theta}^2 + \left(m_x - \frac{m_3 l}{2\lambda} \sin^2(\theta) \right) v_x + \left(\frac{m_3 l}{2\lambda} \sin(\theta) \cos(\theta) \right) v_y \right) \\ \left(-m_3 l \sin(\theta) \dot{\theta}^2 + \left(\frac{m_3 l}{\lambda} \sin(\theta) \cos(\theta) \right) v_x + \left(m_y - \frac{m_3 l}{\lambda} \cos^2(\theta) \right) v_y \right) \end{bmatrix} \quad (5)$$

where v_x and v_y are new inputs and $\lambda = \frac{I}{m_3 l}$.

The parameter λ equals the effective pendulum length of the rotational link, when treated as a rigid-body pendulum suspended from the passive joint. This length also equals the distance from the joint to the so-called 'centre of percussion' of the link.

This feedback transformation result in the following partially feedback linearized system:

$$\begin{aligned} \ddot{r}_x &= v_x \\ \ddot{r}_y &= v_y \\ \ddot{\theta} &= \frac{1}{\lambda} (\sin(\theta) v_x - \cos(\theta) v_y) \end{aligned} \quad (6)$$

The state transformation follows from the relations

$$\begin{aligned} \xi_1 &= r_x + \lambda(\cos(\theta) - 1) \\ \xi_2 &= \tan(\theta) \\ \xi_3 &= r_y + \lambda \sin(\theta) \end{aligned} \quad (7)$$

By taking the new inputs v_x and v_y as follows, the system is transformed into the extended chained form

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \begin{bmatrix} \frac{u_1}{\cos(\theta)} + \lambda \dot{\theta}^2 \\ \lambda (u_2 \cos^2(\theta) - 2\dot{\theta}^2 \tan(\theta)) \end{bmatrix} \quad (8)$$

The co-ordinate transformation is only valid for $\theta \in (-\pi/2 + k\pi, \pi/2 + k\pi)$, for $\theta = \pi/2 \pm k\pi$, $k \in \mathbb{N}$ the co-ordinate transformation is not well defined.

The nonholonomic constraint is preserved under the co-ordinate and feedback transformation. It is transformed into the last equation of (4).

This system is stabilised with the following continuous time-varying feedbacks. For the derivation of these equations see [1].

$$\begin{aligned} u_1 &= -k_1 \xi_1 - k_2 \dot{\xi}_2 + h(\xi_1, \dot{\xi}_1, \xi_3, \dot{\xi}_3) \sin(t/\varepsilon) \\ u_2 &= -k_3 k_4 \xi_2 - k_4 \dot{\xi}_2 - k_3 k_4 \frac{(k_5 \xi_3 + k_6 \dot{\xi}_3)}{\sigma h(\xi_1, \dot{\xi}_1, \xi_3, \dot{\xi}_3)} \sin(t/\varepsilon) \end{aligned} \quad (9)$$

with $\sigma=1/2$ and ξ denotes the state of the second order chained form and the homogeneous norm is given by

$$h(\xi_1, \dot{\xi}_1, \xi_3, \dot{\xi}_3) = \sqrt{\xi_1^2 + \dot{\xi}_1^2 + \|\xi_3\| + \|\dot{\xi}_3\|} \quad (10)$$

2.3 Friction

If friction and cogging forces are included in the model, the transformed system will not be equal to the second-order chained form system. Consider the underactuated system with friction given by

$$\begin{aligned} \ddot{m}_x r_x - \frac{m_3 l}{2} \sin(\theta) \ddot{\theta} - \frac{m_3 l}{2} \cos(\theta) \dot{\theta}^2 &= k_m i_y + \tau_{f,y} \\ \ddot{m}_y r_y + m_3 l \cos(\theta) \ddot{\theta} - m_3 l \sin(\theta) \dot{\theta}^2 &= -k_m i_y + \tau_{f,y} \\ I \ddot{\theta} - m_3 l \sin(\theta) \dot{r}_x + m_3 l \cos(\theta) \dot{r}_y &= \tau_{f,\theta} \end{aligned} \quad (11)$$

where $\tau_{f,i}$ $i = \{X, Y, \theta\}$ denote the friction forces of the motors and the friction torque of the rotational link. By recalculating the transformation, it can be shown that the system (11) is transformed into

$$\begin{aligned} \ddot{\xi}_1 &= u_1 - \left(\lambda - \frac{m_3}{m_x} \right) \sin(\arctan(\xi_2)) \Delta(\xi, \tau_{f,x}, \tau_{f,y}, \tau_{f,\theta}) + \frac{\tau_{f,y}}{m_y} \\ \ddot{\xi}_2 &= u_2 + (1 + \xi_2^2) \Delta(\xi, \tau_{f,x}, \tau_{f,y}, \tau_{f,\theta}) \\ \ddot{\xi}_3 &= u_3 - \left(\lambda - \frac{m_3}{m_y} \right) \cos(\arctan(\xi_2)) \Delta(\xi, \tau_{f,x}, \tau_{f,y}, \tau_{f,\theta}) + \frac{\tau_{f,x}}{m_x} \end{aligned} \quad (12)$$

where the perturbation Δ of the extended chained form system is given by

$$\Delta(\xi, \tau_{f,x}, \tau_{f,y}, \tau_{f,\theta}) = \frac{\left(\frac{m_3 l}{m_x} \sin(\theta) \right) \tau_{f,y} - \left(\frac{m_3 l}{m_y} \cos(\theta) \right) \tau_{f,x} + \tau_{f,\theta}}{I - \frac{(m_3 l)^2}{2m_x} \sin^2(\arctan(\xi_2)) - \frac{(m_3 l)^2}{m_y} \cos^2(\arctan(\xi_2))} \quad (13)$$

The third equation of (12) shows that any residual perturbation in the X-axis, such as friction or cogging forces $\tau_{f,x}$ that are not compensated, will directly act as an additive perturbation in the dynamics of the chained form variable ξ_3 . This makes it even more difficult to control the ξ_3 dynamics, since the second-order chained form being uncontrollable for $\xi_2=0$ or $u_1=0$ and, as a result, the perturbations can not be fully compensated. Therefore it is essential to use a low-level servo system to compensate friction, cogging forces and additional perturbations in both the X and Y axis.

Note that designing the system such that the perturbation $\Delta(\xi, \tau_{f,x}, \tau_{f,y}, \tau_{f,\theta})$ is not present in the

first and third equation of (12), *i.e.* $\lambda - \frac{m_3 l}{m_y} = 0$, is not possible. The term $\lambda - \frac{m_3 l}{m_y}$ can be written as

$$\frac{I}{m_3 l} - \frac{m_3 l}{m_y} = \frac{I_3}{m_3 l} + \left(1 + \frac{m_3}{m_y} \right) l \quad (14)$$

In the case $m_3 \leq m_y$, considered in this thesis, this term is always positive, and can not be equal to zero. Moreover, if $m_3 \geq m_y$ and $\lambda - \frac{m_3 l}{m_y} = 0$ holds, then the denominator of the perturbation (13)

would become zero for small $\xi_2=0$ and the perturbation would become infinitely large, *i.e.* as $\xi_2 \rightarrow 0$ we have that $\Delta \rightarrow \infty$.

The values of m_3 and m_y are 0.04 kg and 9.16 kg respectively.

As mentioned earlier, the X and Y axis are controlled directly by servo controllers. This means that friction and cogging forces that are present in the motors are compensated by the servo-loop.

Therefore, the friction forces $\tau_{f,x}$ and $\tau_{f,y}$ can be neglected, and we focus on the friction torque that is present in the rotational joint of the link. Additionally, the servo controllers compensate the influence of the link on the dynamics of the motors. Therefore, it suffices to consider the partially feedback linearized system given by (6) and the terms with $m_3 l$ are assumed to be negligible.

The transformed mechanical system then reduces to

$$\begin{aligned}\ddot{\xi}_1 &= u_1 + \Delta_1(\xi_2, \dot{\xi}_2) \\ \ddot{\xi}_2 &= u_2 + \Delta_2(\xi_2, \dot{\xi}_2) \\ \ddot{\xi}_3 &= \xi_2 u_1 + \Delta_3(\xi_2, \dot{\xi}_2)\end{aligned}\tag{15}$$

where the perturbation terms are given by,

$$\begin{aligned}\Delta_1 &= -\frac{\xi_2}{\sqrt{1+\xi_2^2}} \frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{m_3 l} \\ \Delta_2 &= (1+\xi_2^2) \frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{I} \\ \Delta_3 &= \frac{1}{\sqrt{1+\xi_2^2}} \frac{\tau_{f,\theta}(\xi_2, \dot{\xi}_2)}{m_3 l}\end{aligned}\tag{16}$$

In the previous equation the inverse coordinate transformation, *i.e.* $\dot{\theta}(t) = \frac{\dot{\xi}_2(t)}{(1+\xi_2(t)^2)}$, has been

used to write the friction term $\tau_{f,\theta}(\dot{\theta})$ in terms of $(\xi_2, \dot{\xi}_2)$ and the terms $\sin(\arctan(\xi_2))$ and $\cos(\arctan(\xi_2))$ have been simplified. We conclude that the additive perturbations, such as friction and cogging, present in the rotational joint of the mechanical system result in additive perturbations in the resulting second-order chained form system.

Compensating all the perturbations with the chained form inputs u_1 and u_2 is not possible. The perturbations Δ_1 and Δ_2 can be compensated directly by the chained form inputs u_1 and u_2 . By defining $u_1 = \bar{u}_1 - \Delta_1$ and $u_2 = \bar{u}_2 - \Delta_2$, the perturbed chained form system becomes

$$\begin{aligned}\ddot{\xi}_1 &= \bar{u}_1 \\ \ddot{\xi}_2 &= \bar{u}_2 \\ \ddot{\xi}_3 &= \xi_2 \bar{u}_1 + \Delta_3(\xi_2, \dot{\xi}_2) - \xi_2 \Delta_1(\xi_2, \dot{\xi}_2)\end{aligned}\tag{17}$$

This shows that compensating the perturbations Δ_1 and Δ_2 actually increase the perturbation in the dynamics of ξ_3 . Since this equation cannot be controlled directly this is not a good idea and it will not be used in the simulations.

3 Simulations

This chapter is divided into two parts.

In the first part of the chapter the parameters are optimised. Because the system will oscillate about the desired position the criteria used in this optimisation are the deviation of the required point and the rate of convergence, in other words how fast does the system converge.

In the second part of this chapter the controller is adjusted in a way that assures that the system will converge.

3.1 Parameter optimisation

All the simulations include friction in all directions.

The parameters to be tuned are $k_1, k_2, k_3, k_4, k_5, k_6$ and ε .

The parameters k_1 and k_2 are the gains of the stabilising part of the controller u_1 . The parameters k_3 and k_4 are the gains of the backstepping; they determine the convergence of the link orientation. The parameters k_5 and k_6 determine the convergence of the y -position of the link. The last parameter ε determines the frequency.

After several simulations the following can be concluded

The maximum current allowed in the system is 5 A. If k_1 and k_2 are chosen too large this will be exceeded. The parameter k_2 has to be smaller than k_1 to prevent instability. If the gains are chosen as $k_1=c^2$ and $k_2=2c$, with c a positive constant, the poles are real and have damping. If the poles are chosen as $k_1=c^2$ and $k_2=c\sqrt{2}$ the poles become complex and have considerable less damping. And because the system is controlled by changing the orientation of the link, the damping should not be too large.

The parameters k_3 and k_4 have to be large to prevent the system from becoming unstable. The best results are achieved when k_3 and k_4 are about 4 times larger than k_1 . If k_3 and k_4 are chosen to be smaller than k_1 or less than 4 times as large as k_1 the system is unstable. k_3 and k_4 have to be approximately equal.

If k_5 and k_6 are chosen too large the control effort u_2 will be very large and the system will become unstable.

With these restrictions k_1 can be maximal 4 or the maximum current is exceeded.

The best results are achieved with

$$k_1=4, k_2=2\sqrt{2}, k_3=15, k_4=15, k_5=9, k_6=6, \varepsilon=0.25$$

These settings are used for the rest of the simulations and for the experiments

3.2 Practical convergence.

These simulations are also with friction. The simulations start at $t_{begin} = 1$, to prevent numerical problems.

The controller designed to control this system is a non-linear time varying controller. The desired position will never be exactly reached as a result of friction in the rotational link. The system will oscillate around the desired position (see figure 3). The desired state is

$$\begin{aligned}x &= -0.3 & \dot{x} &= 0 \\y &= 0.5 & \dot{y} &= 0 \\ \theta &= 0 & \dot{\theta} &= 0\end{aligned}$$

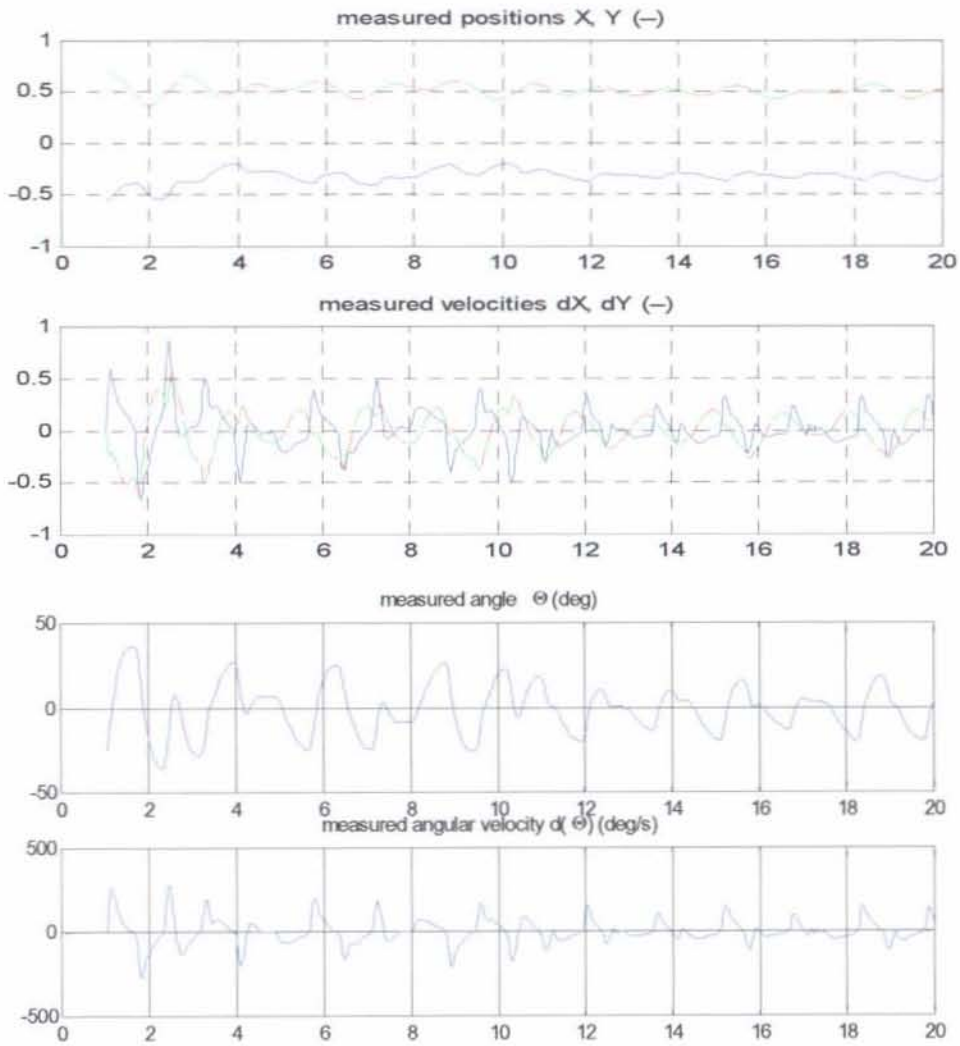


Figure 3: The original controlled system

To ensure convergence the controller is adjusted.
The power-supply to the motors is ended when

$$|x(t_u) - x_d| \leq \alpha \quad (18)$$

t_u is the time when this is true for the first time and α is the maximum allowed error.

Written in the different components this becomes;

$$\begin{aligned} |x_e(t_u)| \leq \alpha_x, \quad |y_e(t_u)| \leq \alpha_y, \quad |\theta_e(t_u)| \leq \alpha_\theta \\ |\dot{x}_e(t_u)| \leq \alpha_{\dot{x}}, \quad |\dot{y}_e(t_u)| \leq \alpha_{\dot{y}}, \quad |\dot{\theta}_e(t_u)| \leq \alpha_{\dot{\theta}} \end{aligned} \quad (19)$$

This means that when the difference between the state and the desired state is smaller than a given threshold the controller is switched off.

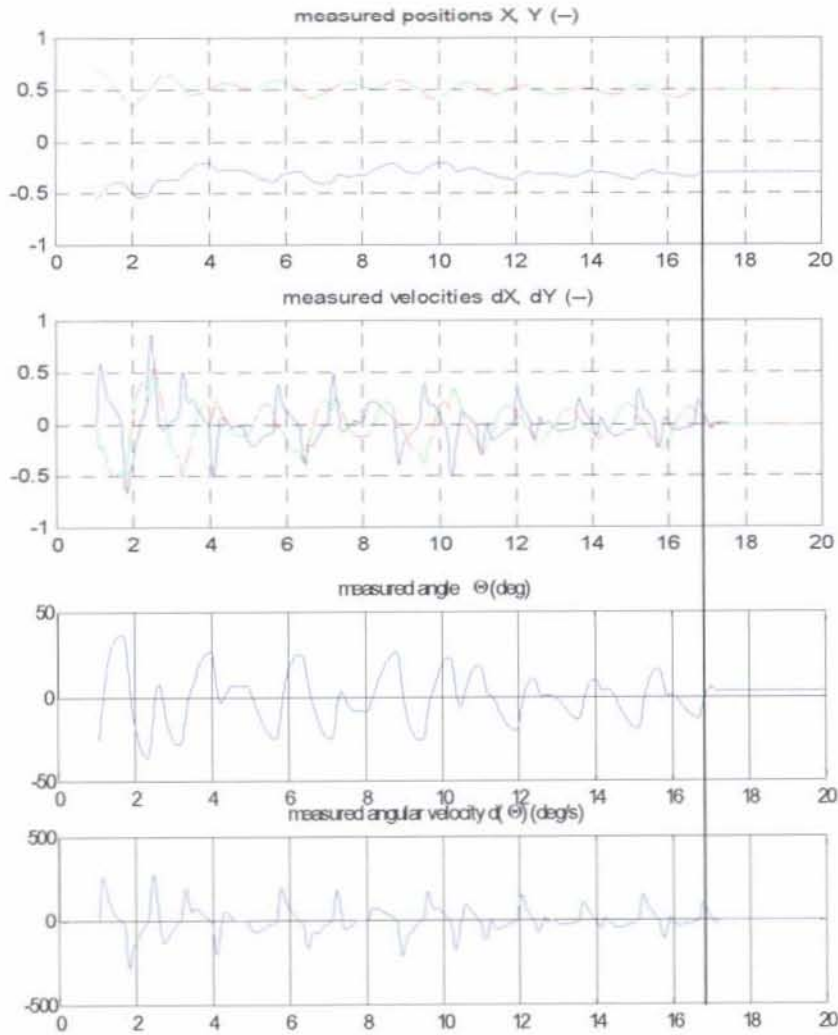


Figure 4: The system with the adapted controller

In the simulations very good results are achieved, see figure 5.

There are two controllers, the one designed in simulink and the controller that actually controls the servomotors.

On $t = t_1$ the controller that controls the servomotors is switched off but the controller in simulink on the other hand isn't switched off.

When the controller of the servomotors is switched off there will remain some velocity in the system and sometimes, say at t_2 , the system exceeds the given threshold again. Then the controller of the servomotors is switched on again. But the controller in simulink has tried all the time to correct the error in the system. So when the controller that controls the servomotors is switched on again the control-effort is high and creates large accelerations in the system.

To prevent this an integrator is inserted and when the power-supply is disabled it will not be enabled again.

Some of the results are shown below.

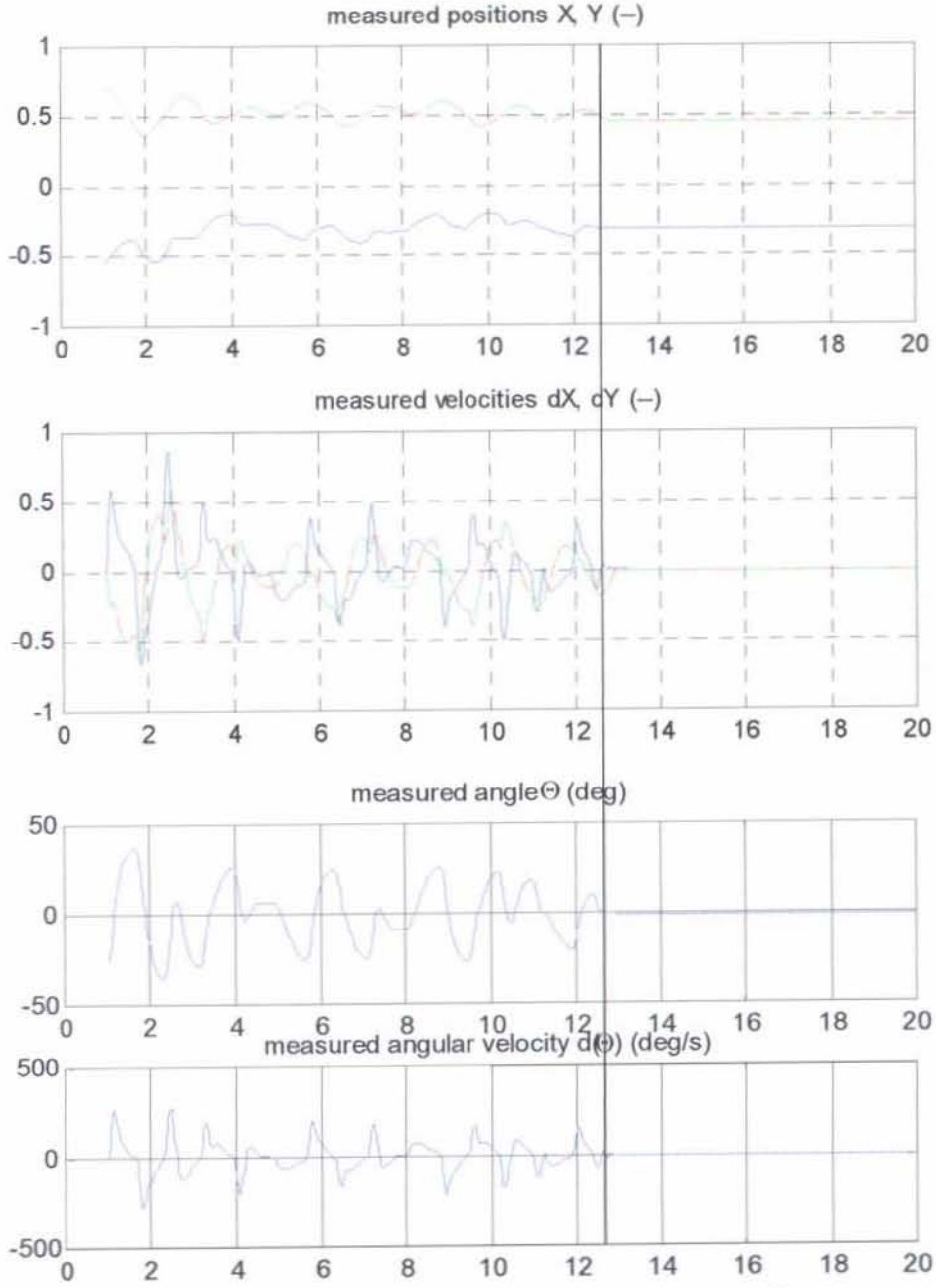


Figure 5: The results for the interval $|x_e| \leq 0.05$, $|y_e| \leq 0.05$, $|\theta_e| \leq \frac{5\pi}{180}$.

$$|\dot{x}_e| \leq 0.1, |\dot{y}_e| \leq 0.1, |\dot{\theta}_e| \leq 0.05$$

With the interval set on

$$\begin{aligned} |x_e| \leq 0.05, |y_e| \leq 0.05, |\theta_e| \leq \frac{5\pi}{180} \\ |\dot{x}_e| \leq 0.1, |\dot{y}_e| \leq 0.1, |\dot{\theta}_e| \leq 0.05 \end{aligned} \quad (20)$$

the (final) error in the x, y and θ direction are respectively

$$error_{x_f} = -0.01 \quad error_{y_f} = -0.04 \quad error_{\theta_f} = -0.8$$

When the thresholds are higher than the ones above the system converges faster but there remains a large error.

But on the other hand when the thresholds are lower it will take longer before the system is converged. In the simulations the system eventually will always converge. So the thresholds can be chosen very low in the simulations.

The thresholds for the x and y direction can be chosen lower than the threshold for the θ direction because the first two can be controlled directly. The threshold for the velocity of the θ direction is chosen low so that when the desired state is reached and the controller is switched off the orientation of the link will not change much. In the simulations the best results with respect to the absolute error is achieved when the thresholds of the x and y velocities are not too low.

Figure 5 shows clearly that when the controller is switched off after about 12 seconds there still remains some velocity in the system. The sign of the velocity changes after switching off the controller. This is due to the fact that the velocity is still increasing (decreasing) when the controller is switched off.

4 Experiments

The real system on the other hand has some unmodelled dynamics, like cogging and stick-slip. Because of this the thresholds of the switch-off interval cannot be chosen too low. If the thresholds are chosen below a certain value ε_{low} , the system sometimes isn't able to get close enough to the desired state and the system will not converge to the desired point but will oscillate around it.

This ε_{low} is a different value for every state variable.

Every experiment is done at least 6 times to make sure that the variance is small.

The most emphasis is on the absolute error of the orientation of the link. This is done because the link is unactuated. The x and the y position can be corrected afterwards by allowing the system to move with low velocities to the desired x and y values since, because of the friction in the link (sticking), it will not move.

The best result, with respect to the absolute error, of the experiment with the switch- off interval

$$\begin{aligned} |x_e| \leq 0.1, |y_e| \leq 0.1, |\theta_e| \leq \frac{5\delta}{180} \\ |\dot{x}_e| \leq 0.1, |\dot{y}_e| \leq 0.1, |\dot{\theta}_e| \leq 0.05 \end{aligned} \quad (21)$$

is in figure 6.

It is shown here that the final errors, in other words the errors that remains after the system has stopped, in the x, y and θ direction are respectively

$$error_{1,xf} = 0.0 \quad error_{1,yf} = 0.07 \quad error_{1,\theta f} = 0.07$$

Note that the figures show the r_x and the r_y positions. These are derived from the X and the Y coordinates through equation (1).

It can be seen that in the r_x direction there still remains a velocity after switching off the controller.

As said earlier the experiments are conducted at least 6 times and this is the best result achieved with this switch-off interval.

For the rest of the experiments with this switch-off interval the errors are

$$error_{2f} = [error_{xf}; error_{yf}; error_{\theta f}] = [0.01; 0.06; 0.12]$$

$$error_{3f} = [0.01; 0.06; 2.12]$$

$$error_{4f} = [0.01; 0.09; 3.09]$$

$$error_{5f} = [0.0; 0.06; -2.06]$$

$$error_{6f} = [0.01; 0.07; 1.56]$$

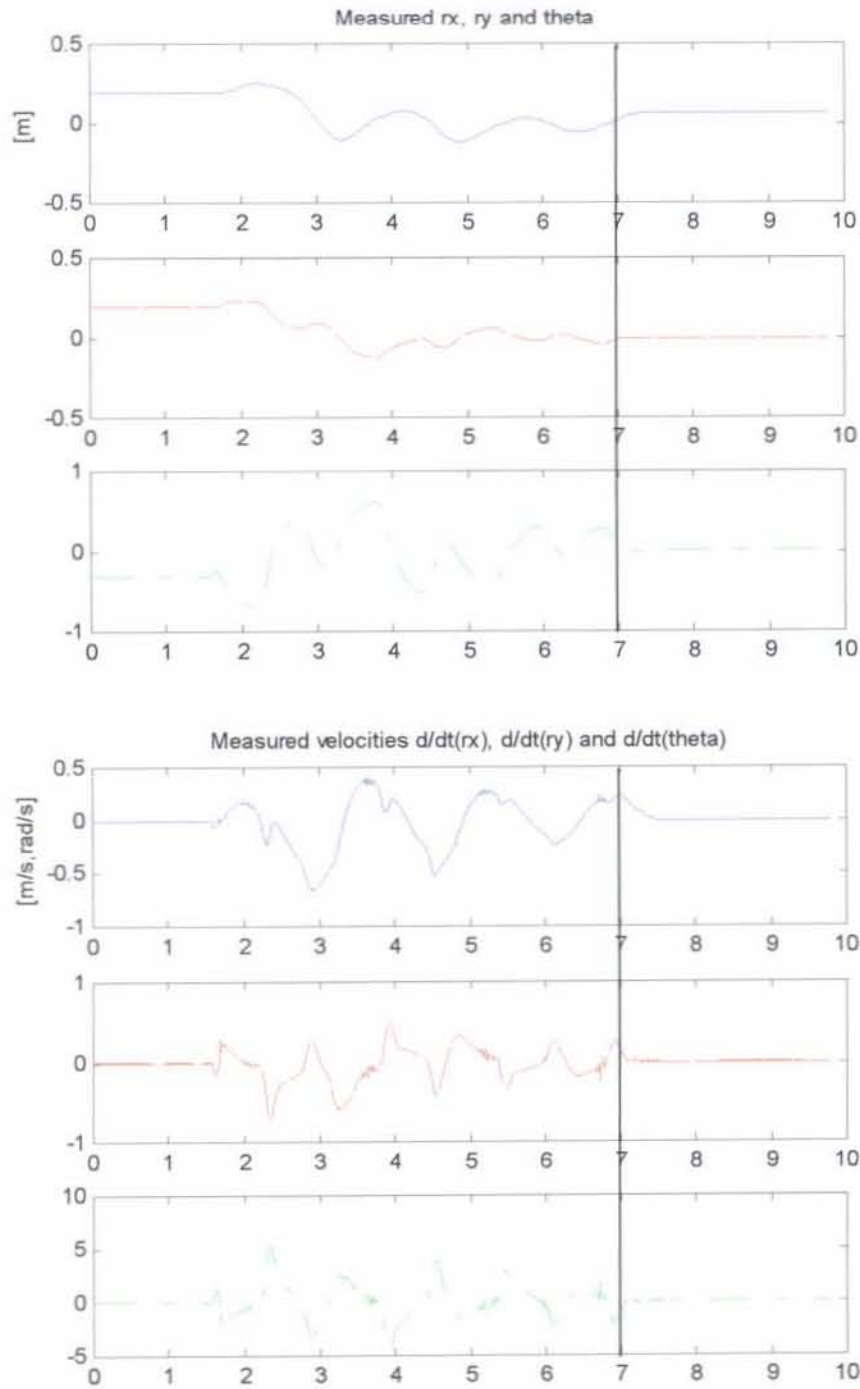


Figure 6: experiment with the Interval set on $|x_e| \leq 0.1$, $|y_e| \leq 0.1$, $|\theta_e| \leq \frac{5\pi}{180}$
 $|\dot{x}_e| \leq 0.1$, $|\dot{y}_e| \leq 0.1$, $|\dot{\theta}_e| \leq 0.05$

The best results were achieved with the switch-off interval

$$\begin{aligned} |x_e| \leq 0.1, |y_e| \leq 0.05, |\theta_e| \leq \frac{5\delta}{180} \\ |\dot{x}_e| \leq 0.1, |\dot{y}_e| \leq 0.1, |\dot{\theta}_e| \leq 0.05 \end{aligned} \quad (22)$$

(see figure 7).

After several experiments it can be concluded that with this switch-off interval the system will only converge if the initial angle is larger than a certain value otherwise the system will not converge. The angle has to be in the range of $-21 \leq \theta \leq 0$ degrees.

The initial angle cannot be positive because of the initial x and y position of the system. The initial position is close to the physical boundary of the system and if the angle would be positive initially the control effort will try to move the system out of its physical boundaries.

This experiment was repeated 16 times of which 8 times the initial angle was chosen larger than -21 degrees. If the angle $\theta \leq -21$ degrees the system will not converge. It appears to get stuck in a sort of periodic behaviour. This is because of unmodelled dynamics like for instance stick slip and friction.

But in the rest of the experiments, with the angle $0 \geq \theta \geq -21$ degrees, the following results are achieved.

$$error_{1f} = [error_{xf}; error_{yf}; error_{\theta f}] = [-0.02; -0.05; 5.0]$$

$$error_{2f} = [0.0; 0.0; 0.42]$$

$$error_{3f} = [0.0; 0.0; 0.36]$$

$$error_{4f} = [0.0; 0.02; 0.21]$$

$$error_{5f} = [0.0; 0.04; 0.15]$$

$$error_{6f} = [0.0; 0.0; 0.46]$$

$$error_{7f} = [0.01; 0.04; 3.51]$$

$$error_{8f} = [0.0; 0.06; -0.75]$$

The final errors are in a range of

$$-0.02 \leq error_{x,f} \leq 0$$

$$-0.05 \leq error_{y,f} \leq 0.06$$

$$-0.75 \leq error_{\theta,f} \leq 5.0$$

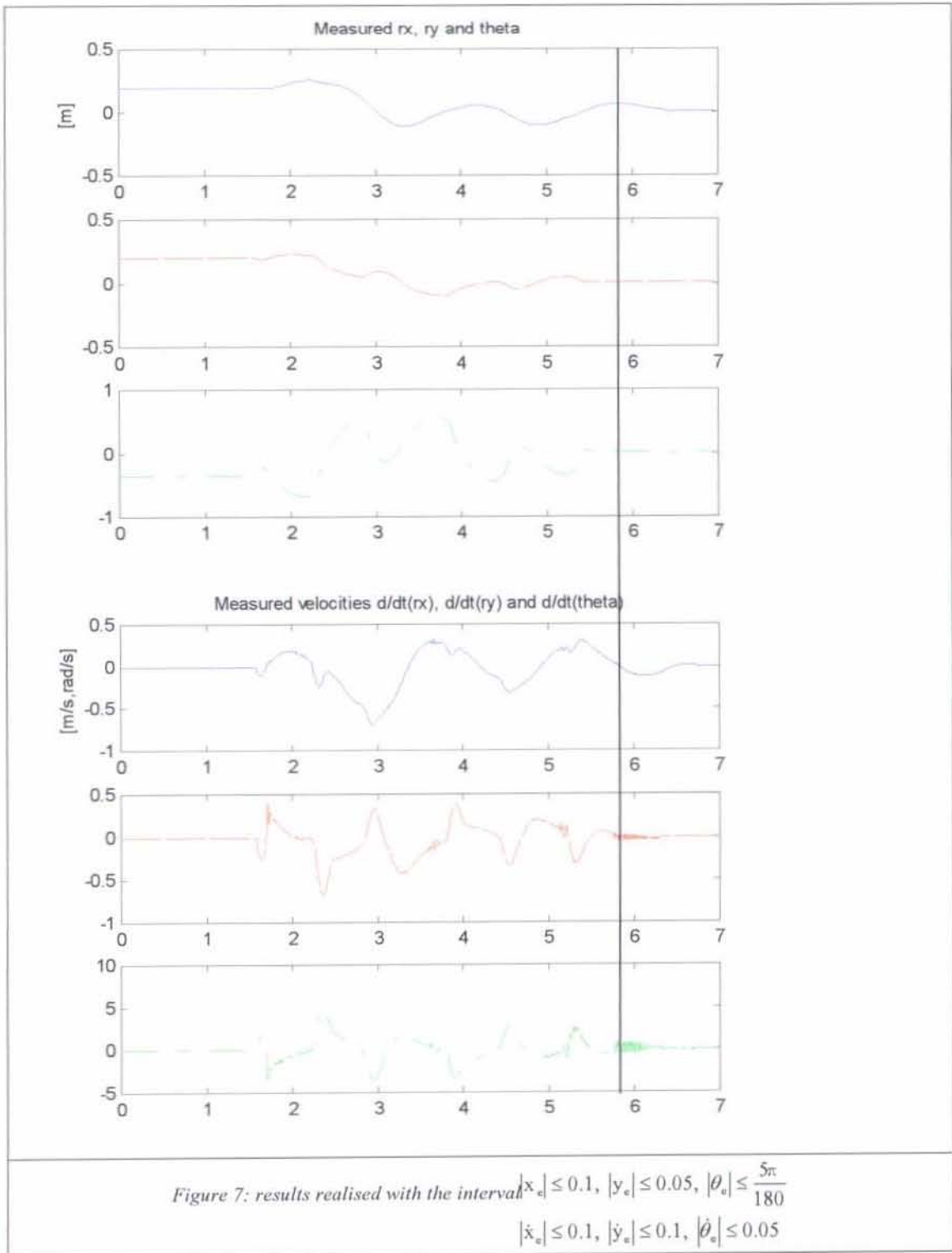


Figure 7: results realised with the interval $|x_e| \leq 0.1$, $|y_e| \leq 0.05$, $|\theta_e| \leq \frac{5\pi}{180}$
 $|\dot{x}_e| \leq 0.1$, $|\dot{y}_e| \leq 0.1$, $|\dot{\theta}_e| \leq 0.05$

5 Recommendations

In the current research the controller only switches off the power supply to the motors. Because of the problems that occur when the system exceeds the given threshold again, it is not allowed to switch on again. It has to be investigated if better results can be achieved when the controller is deactivated totally when a certain threshold is crossed. Then it probably would not be a problem if the given threshold is exceeded again and the controller is re-activated.

It should be investigated whether it is possible to transform the system with the friction into the second order chained form. Then it is possible to choose the feedback laws so that the friction is compensated. When this is possible the system should converge to the desired value. This will be hard and might be impossible.

At this moment the friction is dependent on the position the link is in. One of the reasons for this phenomenon is tilting of the link. With some simple adjustments to the link it should be possible to level the link better. This should give better results.

The designed controller (without the practical convergence) should be able to converge to the desired point if there was no friction in the system. The friction in the x and y direction are compensated for by the servomotors. So when it would be possible to eliminate the friction in the link the achievements of the controller can be evaluated better. The friction in the link can perhaps be eliminated with an additional motor. This motor should only compensate the friction in the link, nothing more. This makes it possible to evaluate the controller more carefully. Because if the friction is eliminated the controller should be able to converge the system to the desired point.

Summary

The control of an underactuated system with a second order nonholonomic constraints is not straightforward. Because of the second order nonholonomic constraint the system cannot be controlled by a linear controller or by a smooth time invariant stabilising feedback law. The linearization is not controllable.

The system used consists of two parallel y-axes that are connected by a beam, the x-axis. This system is known as the H-drive. A rotational link is mounted on the linear motor of the x-axis and is non-actuated. Because the axes are steered by servomotors and their dynamics are assumed to be identical the two y-axes are considered as one.

To make it possible to design a controller for this system a coordinate transformation is applied. This transforms the system into the chained form. The system can be stabilised with continuous time varying feedbacks, which contain parameters that need to be tuned, k_1 , k_2 , k_3 , k_4 , k_5 , k_6 and ϵ .

After several simulations the optimal parameters with respect to the deviation of the required point and the rate of convergence are $k_1=4$ $k_2=2\sqrt{2}$ $k_3=15$ $k_4=15$ $k_5=9$ $k_6=6$ $\epsilon=0.25$.

Because of unmodelled dynamics in the system and the friction in the rotational link the system does not converge to the desired point. The system will oscillate around the desired position. In order to ensure that the system will converge to the desired point the controller is adjusted.

The power supply to the motors is ended as soon as the state, i.e. the x, y and θ direction and the x, y and θ velocities, is all below a certain threshold.

After the power supply is switched off there will remain some velocity in the system because the velocity is not necessarily zero and it is possible that the one or more of the state variables will exceed the threshold again. The system will not be switched on again when this happens. This is because the controller exists of two parts the controller that controls the servomotors and the controller in simulink and when the state is below the threshold the first controller is switched off but the controller in simulink is still working. When the system would be switched on again the controller in simulink will have calculated a very large control-effort to correct the remaining error and will create very large control errors.

The rate of convergence and the remaining error are dependent on the chosen thresholds.

Very good results are achieved with respect to the absolute (final) error.

References

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Appendix 1 practical convergence (simulink)

