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A.A.F. van de Ven



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Department of Mathematics and Computing Science
Eindhoven University of Technology
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The Netherlands
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A REVIEW OF A VARIATIONAL METHOD APPLIED TO MAGNETOELASTIC BUCKLING PROBLEMS

A.A.F. van de Ven

Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Abstract

Magnetoelastic buckling theory, concerning ferromagnetic and superconducting systems, is reviewed. Special attention is widmed to a variational method, which yields a unified and direct solution for the buckling value.

1. A Historical Survey

The theory of electromagnetic interactions describes the phenomena that can occur when elastic bodies are acted upon by electromagnetic fields. This interaction manifests itself in both the mechanical and electromagnetic balance laws as well as in the constitutive equations. The earlier foundations of this interaction theory are laid by such great names in continuum mechanics as *Toupin, Eringen, W.F. Brown Jr., Tiersten, Parkus, Alblas, K.B. Vlasov, Pao, Hutter, Maugin* and many others (for more complete lists we refer to the books of Hutter and Van de Ven [1], Moon [2], Maugin [3], and Eringen and Maugin [4]). Here, we shall restrict ourselves to a more specific subject, one out of the many varieties in the practice of magnetoelastic interactions, namely magnetoelastic buckling.

Magnetoelastic buckling is a phenomenon in which an elastic structure becomes unstable (buckles) under electromagnetic loading. Such a structure can be, for instance, ferromagnetic or (super)conducting. The first (technical important) investigations on this field are those of *F.C. Moon*. He considered both ferromagnetic as well as, in cooperation with *Chattopadhyay*, conducting systems (confer e.g. Moon and Pao [5], and Chattopadhyay and Moon [6]). A more fundamental theory of magnetoelastic stability was presented by Alblas in [7].

The magnetoelastic buckling of (super)conducting coils seems to be of greater technical importance than that of ferromagnetic systems. Besides the, already mentioned, *Chattopadhyay* and *Moon*, many other authors have worked on this subject of which we only mention *K. Miya, Geiger* and *Jüngst, P. Wolfe* and *J.P. Nowacki*. In this respect, also *S.A. Ambartsumian* and his coworkers must be mentioned (for a review of their earlier work see Ambartsumian [8]) who reported on the (in)stability of magnetic and/or current carrying plates and shells placed in external magnetic fields.

Since, in our view, several of the approaches to magnetoelastic buckling problems thusfar are rather ad hoc, we felt that there is a need for a unified approach. To this end we, that is *Lieshout* and *Van de Ven* together with some coworkers, have constructed a variational principle specially fit for the solution of magnetoelastic buckling problems for ferromagnetic and for superconducting systems. Thus, we have obtained a standard apparatus for this class of problems. In a series of papers we have presented solutions for systems reaching from rather simple ones, such as a set of two parallel rods, to complex ones as helical or spiral coils.

2. Ferromagnetic Systems

In this section we consider the (in)stability of (systems of) ferromagnetic bodies placed in an external magnetic field. Since a stability problem is always in essence a nonlinear

problem, the theory for it must be built upon a nonlinear set of equations for, in this case, a magnetoelastic interaction model. As there exist several of such models one specific model must first be chosen (cf. [1]). After that, the general approach to the problem must be as follows:

Consider a ferromagnetic body \mathcal{B} placed in an external magnetic field \mathbf{B}_0 . Due to the action of magnetic forces \mathcal{B} will deform to an (slightly deformed) intermediate state G_I . It is the stability of this intermediate state we want to investigate; we say that \mathcal{B} buckles when G_I becomes unstable. To investigate the stability of G_I we superpose an extra perturbation on G_I leading to the final state G for \mathcal{B} . By assuming the perturbations small, one can linearize the nonlinear set of equations and boundary conditions referring to state G with respect to the perturbations. This results in a homogeneous, linear set of equations, now referring to G_I . Since the displacements in G_I are always very small (and not essential for the stability problem) we may in practice replace in the linearized perturbed system the intermediate state by the rigid-body state (= undeformed state). The rigid-body magnetic fields have to be calculated first and then the perturbed system can be solved. However, as this is a *homogeneous* system, it will in general only have a trivial zero-solution. Only for a set of (eigen)values for \mathbf{B}_0 this system for the perturbations will have a non-trivial solution. The lowest of these eigenvalues is the buckling value \mathbf{B}_{ocr} .

Moon and Pao, [5], applied this method in the buckling problem of a cantilevered ferromagnetic beam of narrow rectangular cross-section placed in a transverse magnetic field \mathbf{B}_0 . They found that the buckling field \mathbf{B}_{ocr} was proportional to the 3/2 power of the thickness-to-length ratio. This result, however, was in disagreement with their own experimental results. This discrepancy started a discussion in literature between several authors (a.o. *Wallerstein, Peach, Bast, Popelar, Dalrymple, Miya, Hara, Someya, Takagi, Ando*; for a list of references, see [9]). Ultimately, a good explanation was found in a theory accounting for the finite width of the rectangular cross-section (Van de Ven, [9], [10]).

With *K. Miya* ([11],[12]) and *J. Tani* ([13],[14]) only two of the leading Japanese researchers on magnetoelastic buckling are mentioned. Tani presented in Paris 1983 a paper on the magnetoelastic buckling of two nearby ferromagnetic panels ([13]). As a follow up, Tani and Van de Ven cooperated on the magnetoelastic buckling of two parallel rods ([14]); the found correspondence between experiments and theory was satisfactory. The influence of magnetic saturation in soft ferromagnetic bodies was investigated by Van de Ven [15]. It turned out that the soft ferromagnetic model may simply be extrapolated up to the saturation point. Magnetoelastic buckling of plates was, besides the already mentioned Ambartsumian, also studied by *Eringen*, [16], and *Maugin*, [17].

Although the method sketched above could be applied to some simple structures (e.g. a beam or a plate), the analysis became very complex for more complicated systems. Moreover, often a lot of more or less ad hoc approximations were needed to keep the analysis in hand. Therefore, we constructed an alternative approach, in the way of a variational method, in the hope that this would serve us:

- i) a unique method in a clear formulation;
- ii) a straight way (omitting all unnecessary calculations) to an exact solution (if possible);
- iii) a solid basis for numerical calculations if an exact analytical solution is no longer possible.

The set-up of our variational principle, realized mainly by P.H. van Lieshout, was based upon the following Lagrangian (in fact, this is the Lagrangian density; to obtain the total Lagrangian \mathcal{L} this density must be integrated over both the ferromagnetic body \mathcal{B} and the surrounding vacuum)

$$L = -\frac{1}{2} \mu_0(\mathbf{H}, \mathbf{H}) - \rho U + \frac{1}{2\mu_0}(\mathbf{B}_0, \mathbf{B}_0) . \quad (1)$$

What we need here is the second variation of \mathcal{L} , which we shall denote by J . Since the first variation of \mathcal{L} is zero ($\delta\mathcal{L} = 0 \rightarrow G_I$), J must be of the second order in the perturbations and, hence, after the neglect of higher order terms, J becomes a homogeneously quadratic functional in the perturbations. Since the final state G is an equilibrium state the first variation of J must be zero, but since J is homogeneously quadratic this implies that J itself must be zero. Hence,

$$\delta J = 0 \Rightarrow J = 0 . \quad (2)$$

The second relation (2) delivers us directly an explicit value for the buckling field, as can be seen as follows:

split J up in a magnetic (\mathcal{K}) and an elastic part (W); it then turns out that the magnetic part is proportional to B_0^2 , hence

$$J = \mathcal{K} - W = B_0^2 K - W , \quad (3)$$

and then

$$J = 0 \Rightarrow B_{ocr} = \sqrt{\frac{W}{K}} . \quad (4)$$

Hence, if we can calculate W and K (either exact or in a sufficiently accurate approximation) relation (4) yields immediately the buckling load.

Lieshout et. al. employed this method for the calculation of the buckling fields for ([19],[20])

- i) a cantilevered beam of rectangular cross-section (an exact solution was found),
 - ii) a set of two parallel rods,
- both placed in a transverse magnetic field.

3. Superconducting structures

Superconducting systems have found a large field of applications in modern technology. In this aspect such complex structures as spiral, helical or toroidal coils are in use; structures whose complexity makes the study of their stability very complicated. In conducting systems the electromagnetic loading is mainly due to Lorentz forces, originating from an interaction of the electric current with either its own magnetic field or with an external magnetic field. Whenever these Lorentz forces become too large the system buckles. The analysis of this phenomenon can be performed analogously to the perturbation method presented schematically in the begin of the preceding section.

In 1975, *Chattopadhyay* and *Moon*, [6], were the first to give a closed form solution for the buckling problem of an elastic current carrying rod in its own field. Stabilities of conducting strings in a parallel magnetic field were studied by *P. Wolfe*, [21], [22] (bifurcation aspects) and *J.P. Nowacki* [23] (on the basis of a consistent 3 - D-model). *Chattopadhyay*,

[24], proved, by numerical means, that a conducting coil in its own field is always stable, a result confirmed in [25] through a completely analytical solution. In Moon's book, [2] (esp. Ch. 5 and 6) a number of other problems is presented (e.g. circular coils in transverse or toroidal external fields); moreover a fairly complete review of the literature on this subject up to 1984 can be found there.

From the more recent work on the stability of current carrying plates and shells of the school of Ambartsumian we mention [26] (influence of transverse shear) and [27] (nonlinear constitutive behaviour). In this respect, we also refer to the work of *K.B. and R.A. Kazarian*, [28], and *R.N. Ovakimian*, [29].

For realistic problems, however, structural problems of greater complexity must be studied. A first example is [30], in which *Hara and Moon* studied the internal buckling of superconducting solenoid magnets. Secondly, we refer to the work of *K. Miya* and his coworkers (e.g. [31]-[34]) who investigated both theoretically/numerically (finite element analysis) as well as experimentally the stability of superconducting (toroidal and helical) coils in fusion reactors. Buckling calculations and measurements on technologically relevant toroidal magnet systems are also performed by *Geiger and Jüngst*, [35], [36]. They concluded that for the existing TESPE magnet system (*Kernforschungszentrum Karlsruhe*) there was no risk for magnetoelastic buckling.

In the references listed above, due to the great complexities of the structures, a large variety of methods, often using approximations, the degree of accurateness of which could not be indicated, are employed. To overcome this we have adapted our variational method for ferromagnetic bodies of the preceding section to one for superconducting structures, [37]. Evident advantages of this method are:

- i) once a definite form for the variational principle is chosen, the remaining analysis can be performed in an exact way by completely analytical means;
- ii) whenever the principle is used in an approximated sense, the order of the approximations and the conditions under which they are allowable can be clearly indicated.

To make the variational method of Section 2 suitable for systems of superconducting coils, carrying a prescribed current I_0 , the Lagrangian density (1) must be changed into

$$L = \frac{1}{2\mu_0}(\mathbf{B}, \mathbf{B}) - \rho U , \quad (5)$$

(formally, this is a Legendre transformation of (1) in which we pass from the variable \mathbf{H} to \mathbf{B} , cf. [37]). Since the current through the coil is prescribed, \mathbf{B} must satisfy the constraint (Ampère's law)

$$\int_C (\mathbf{B}, d\mathbf{s}) = \mu_0 I_0 , \quad (6)$$

where C is a contour encircling the conductor. The further evaluation is completely analogous to the one in Section 2 and ultimately amounts in (here the electromagnetic term in J is proportional to I_0^2)

$$J = I_0^2 K - W \Rightarrow \delta J = 0 \Rightarrow J = 0 \Rightarrow I_{ocr} = \sqrt{\frac{W}{K}} , \quad (7)$$

an explicit expression for the buckling current I_{ocr} . For not too complicated systems (see [19] and [37]) the values of W and (especially) K can be calculated analytically or numerically (cf. [38],[39]), but both exact, or, when this is no longer possible, in a variational (approximated) way by choosing an appropriate set of admissible fields (see [40]).

4. Some results for superconducting systems

The variational method described in the preceding section has been applied by us to a great class of superconducting systems. These systems always consist of slender beam-like structures. Here, we mention the following systems (in all these examples the total system is slender (for the explicit conditions see the examples) and all the structural (beam-like) members are superconducting, carry a prescribed current I_0 and have a circular cross-section of radius R)

- i) a set of two parallel rods; the rods are infinitely long, periodically supported over distances ℓ , while the distance between the rods is $2a$ (the slenderness condition reads here $R < a \ll \ell$) (cf. [19]);
- ii) a set of two concentric tori (or rings) in one plane; the rings have radii b_1 and b_2 ($b_1 < b_2$) and the distance between them is $2a$ ($= b_2 - b_1$) ($R < a \ll \pi b := \pi(b_1 + b_2)/2$) (cf. [37]);
- iii) a set of two identical coaxial tori (rings); radius b and distance $2a$ ($R < a \ll \pi b$) (cf. [37]);
- iv) sets of n ($n \geq 2$) equidistant parallel rods (as in i) where $n = 2$) (cf. [38]);
- v) an infinite helical conductor, periodically supported over n turns; the radius of the helix is b and the pitch is h , the distance between two turns is $2a = 2\pi h$ and the support length $\ell = 2\pi h n$ ($R < \pi h \ll \pi b$) (cf. [39] or [40]);
- vi) a finite helical conductor (as in v)) of n turns, simply supported in its end points (cf. [40]);
- vii) a flat spiral of n turns; the radius is given as function of the arc φ by $b(\varphi) = b_0 + h\varphi$ ($\varphi \in [0, 2\pi n]$), where h is the constant pitch; the distance $2a = 2\pi h$ and the mean radius $b_1 = b_0 + n\pi h$ ($R < \pi h \ll \pi b_1$); the spiral is simply supported in its end points (cf. [40]).

The problems i), ii) and iii) are solved completely by analytical means (using conformal mapping and complex function theory). These mathematical procedures were necessary to obtain an exact solution for the magnetic fields, both undisturbed (rigid-body fields) as well as disturbed (referring to the deformed coils), which at their turn were needed for the calculation of the electromagnetic interaction integral K , occurring in (7). In this way an exact value for the buckling current is obtained (in practice, this value is only exact within the concept of slender beam theory, i.e. up to $O(a^2/L^2)$, where L is a characteristic length parameter, i.e. $L = \ell$ or πb). The approach to example iv) was also by analytical means, resulting in a set of integral equations for the (perturbed) magnetic field. This set was solved numerically. Nevertheless, the found buckling value was exact (in the same sense as in i)-iii). For the problems v)-vii) we did (up to now) not succeed in finding a completely analytical solution, but we applied our method in a variational sense, resulting in a buckling value which to our conviction is a good approximation. This variational approach was based on: a) a result, proved to be correct for ii) and iii), stating that the value of K for a set of two rings (as in ii) or iii) was equal to the K for a set of two parallel rods ([39],[40]); b) the fact that the

magnetic fields to be found from the law of Biot and Savart are (useful) admissible fields in the sense of our variational principle ([19],[37]; see also furtheron).

As for result a), it could be proved that this also holds for sets of more than two rings. Result a) is a consequence of the slenderness of the systems as can be made plausible by the following reasoning: consider two rings and take a point A_1 on one ring; then the interaction of the second ring with A_1 is concentrated to a point A_2 and its very close neighbourhood, where A_2 is that point on the second ring that is closest to A_1 ; according to the slenderness $b \gg a$ and, hence, in an a -neighbourhood of A_2 the ring may be considered as (locally) straight. This reasoning also applies to a slender helix or spiral and, therefore, the contribution to the integral K of two interacting turns of a helical or spiral coil can be calculated by replacing them locally by a set of two parallel rods. In this way our variational method has yielded results for the examples v)-vii). All these results are schematically presented in the first row ($I_0^{(V)}$) of Table 1.

Table 1. Buckling currents for the examples i)-vii) according to the variational method ($I_0^{(V)}$) and the Biot and Savart method ($I_0^{(B)}$) (E and ν are Young's and Poisson's modulus, respectively, the common factor $\mathcal{J} = (aR^2/L^2)\sqrt{E/\mu_0}$, where $L = \ell$, for i) and iv), $L = \pi b$, for ii), iii), v), vi), and $L = \pi b_1$, for vii); furthermore $a = \pi h$, for v)-vii)).

System	i)	ii)	iii)	iv)	v)	vi)	vii)
$I_0^{(V)}$	$\pi^3 q \mathcal{J}$	$3\pi^3 q \mathcal{J}$	$\frac{6\pi^3 q}{\sqrt{5+\nu}} \mathcal{J}$	$\alpha_n \pi^3 q \mathcal{J}$	$\pi^2 \sqrt{\frac{N(n)}{(2n-1)(1+\nu)\kappa}} \mathcal{J}$	$\pi^3 \sqrt{\frac{2\lambda(n)}{(1+\nu)\kappa}} \mathcal{J}$	$\pi^3 \sqrt{\frac{2\lambda(n,h)}{\kappa}} \mathcal{J}$
$I_0^{(B)}$	$\pi^3 \mathcal{J}$	$3\pi^3 \mathcal{J}$	$\frac{6\pi^3}{\sqrt{5+\nu}} \mathcal{J}$	$\alpha_n \pi^3 \mathcal{J}$	$\kappa = 1$		

$$q = q\left(\frac{R}{a}\right) = \frac{R}{a\sqrt{Q_S}}, \quad Q_S\left(\frac{R}{a}\right) \text{ according to Table 4, [19]}$$

$$\alpha_n : \alpha_3 = \sqrt{2/3}, \quad \alpha_4 = 0.753, \quad \alpha_5 = 0.723, \quad \alpha_\infty = 2/\pi$$

for $N(n)$, $\kappa = \kappa(R/a)$, $\lambda(n)$ and $\lambda(n, h)$: see [40]

$$N(n) = 1 + O(n^{-2}); \quad \kappa = 1 + O(R^2/a^2); \quad \lambda(n) \approx 1/4\pi n$$

We conclude that for all of the problems considered here the buckling current $I_0^{(V)}$ contains the common factor

$$\mathcal{J} = \frac{aR^2}{L^2} \sqrt{\frac{E}{\mu_0}}, \quad (9)$$

whereas the coefficient preceding \mathcal{J} is a function of (R/a) and is different for each problem. For later reference, we note that the factor $q = q(R/a) \rightarrow 1$ for $(R/a) \rightarrow 0$.

Biot and Savart method

A more classical approach to magnetoleastic buckling problems is the so called Biot and Savart method. This method is based on a generalization of the law of Biot and Savart for the external magnetic field $\mathbf{B}(\mathbf{x})$ of a (slender; in fact one-dimensional) conductor and of the associated Lorentz force $\mathcal{F}(\mathbf{x})$ on that conductor, due to the interaction with another circuit (cf. Moon, [2], Section 2.6). For applications to magnetoelastic buckling problems the Lorentz force must be linearized in the perturbations with respect to the rigid-body state.

The linearized part of the force serves as the load parameter in a (curved-) beam equation for the structure. Supplemented by the support conditions, a purely mechanical system is thus obtained (see e.g. [19] or [37]). Whenever this system has a non-trivial (non-zero) solution, the conducting structure becomes unstable. The associated value for I_0 is the buckling value. This is a very direct and mathematically simple method. However, the method is not exact insofar as: i) the conductor is assumed to be one-dimensional (\Rightarrow neglect of current distribution); ii) the boundary conditions for $\mathbf{B}(\mathbf{x})$ are not exactly satisfied; iii) the force due to the self field of the conductor is neglected.

Although it may be expected that the influences of these inaccuracies become smaller if the interacting circuits are farther apart (larger values of a/R), it is not possible to make an a priori guess about the order of the errors. To get an impression, we have applied this method to the examples i)-iv); the results are given in the second row of Table 1 ($I_0^{(B)}$). From a comparison of $I_0^{(V)}$ and $I_0^{(B)}$ we conclude that the $I_0^{(B)}$ -values differ from those of the variational method $I_0^{(V)}$ only by a factor, which is the same for all examples. In fact, all the results would correspond if q would satisfy

$$q = \frac{R}{a\sqrt{Q_S}} = 1 \Rightarrow \frac{1}{\sqrt{Q_S}} = \frac{a}{R} =: m . \quad (10)$$

This is not exactly true (cf. [19], Table 4), but for (a/R) not too close to unity the difference is small and decreases with increasing (a/R) (e.g. for $a/R \geq 4$, the relative difference is less than 5%). Hence, we state that, unless the members of the conducting structural system are very close to each other, the Biot and Savart method yields an acceptable approximation for the buckling current.

Led by this observation we looked for a possibility to use the Biot and Savart fields in our variational method. It was easily established that these fields satisfy the constraints of our principle and, hence, constitute an admissible field. As a check, we first used the Biot and Savart fields in our variational method applied to the example of two parallel rods i)). What we found was a buckling value in between the values $I_0^{(B)}$ and $I_0^{(v)}$ (cf. [40], Fig. 2). Hence, this approach yields an improvement of the Biot and Savart method (towards the variational method) and, therefore, we have used it for the calculation of the buckling currents for helical and spiral conductors (examples v)-vii)). The results are given in Table 1.

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