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European Journal of Ultrasound 1 (1994) 171–182

EUROPEAN JOURNAL OF ULTRASOUND

Scientific paper

Processing scheme for velocity estimation using ultrasound RF cross correlation techniques

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(Received 2 November 1993; revision received 23 December 1993; accepted 29 December 1993)

Abstract

Objective: A modeled cross correlation function for (radio frequency) RF ultrasound signals can be used to estimate the velocity of moving scatterers. Since the RF signal can be regarded as the sum of random phase, relatively narrow, band signals the correlation product will vary only slowly over the observation window, suggesting that the number of correlation products to calculate the cross correlation coefficients may be reduced without affecting the root-mean-square (rms) error of the estimate. *Methods:* Computer generated RF signals with a Gaussian spectral power distribution, simulating a signal returned by blood with a given velocity range, are subjected to software evaluation. The quality factors of the signal Q_s and the noise Q_n, the center frequency, the signal-to-noise ratio, and the mean velocity and velocity range are varied over realistic ranges. The observation window is split up in subsegments, each containing 4 RF sample points. For a fixed length of the observation window and number of RF lines the rms error of the velocity estimate is obtained as function of the factor by which the number of subsegments was reduced for the calculation of the correlation coefficients. *Results:* It is observed that for the signal conditions considered for an RF signal with quality factor Q_s, the standard deviation of the velocity estimate hardly increases for a reduction factor up to $Q_s/2$. *Conclusions:* Reduction of the number of subsegments used for velocity estimation in accordance to the quality factor of the RF signal will reduce computational complexity without notably affecting the standard deviation of the velocity estimate.

Key words: Cross correlation; RF processing; RF sampling; Ultrasound; Velocity estimation

1. Introduction

In conventional pulsed Doppler systems the received radio frequency (RF) signal is demodulated in quadrature and averaged over a short segment in depth to yield a point of the complex Doppler signal sampled at the pulse repetition frequency (PRF). For color coded Doppler systems only short segments of the sampled Doppler signal (in time) are used to get an estimate for the mean

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0929-8266/94/$07.00
SSDI 0929-8266(94)00029-D

velocity of the scatterers contributing to the RF signal segment of interest. The autocorrelation technique as introduced by Kasai et al. (1985) is now commonly employed to get a Doppler velocity estimate. It has been shown that the variance of the velocity estimate is independent of the length of the RF signal segment (Hoeks et al. 1993). This is due to the averaging process following demodulation whereby phase variation over depth is discarded. A reduction in variance is only obtained by evaluating longer segments in time, i.e. more samples of the Doppler signal (package length). A large package length reduces temporal resolution and, eventually, image size or frame rate. Moreover, a fractional deviation of the center frequency of the RF signal from the demodulation frequency, possibly due to frequency-dependent attenuation, will result in a corresponding fractional error in the velocity estimate. This fractional error increases for an increasing fractional bandwidth of the RF signal and will be more prominent for Doppler systems with a high resolution along the ultrasound beam (large fractional bandwidth).

Recently, other methods have been suggested to estimate the mean velocity of scatterers within an observation window at a given depth. They are all based on estimating the location of the peak of the cross correlation function of corresponding segments of subsequent echo-signals (Weinstein and Kletter 1983; Bonnefous and Pesque 1986; Hein and O'Brien 1993). For an accurate estimate of the shift in depth the RF signal has to be digitized at a high sampling frequency in combination with an adequate interpolation scheme for the cross correlation function around its suspected peak (Bonnefous and Pesque 1986; Embree and O'Brien 1990; Foster et al. 1990). For larger shifts, i.e. higher velocities, the peak value of the cross correlation function will gradually vanish due to transit time effects (Ferrara and Algazi 1991). An alternative approach is based on the approximately known shape of the spectral power distribution of the RF signal. Then a model can be developed for the cross correlation function (CCM) having only 4 unknown parameters, i.e. the signal-to-noise ratio (SNR), the center frequency and bandwidth of the RF signal, and the velocity (de Jong et al. 1990; Hoeks et al. 1993). Only 5 coefficients around the origin of the two-dimensional cross correlation function suffice to solve the unknown velocity:

$$\hat{v} = \frac{c * PRF}{2f_s} \frac{\arctan2\left[R' R(0,T) \sin\left\{\arccos\left(\frac{R(\tau,0)}{R(0,0)}\right)\right\}\right]}{\arccos\left(\frac{R(\tau,0)}{R(0,0)}\right)} \quad \text{[m/s]} \tag{1}$$

with

$$R' = \frac{R(\tau,T) - R(-\tau,T)}{2}$$

where τ denotes the sample index along the beam, f_s the sampling frequency, and c the velocity of sound in the medium. The arctan2 function is a double argument arctangent preserving the sign information. The ratio of arctangent and arccos gives the displacement as a real number in units of the sampling interval. The factor in front of it converts the estimate to m/s. Generally, the transmitted ultrasonic pulse consists of one or several cycles of a periodic carrier signal. The center frequency of the received RF signal is often practically the same as the carrier frequency. Eq. 1 does not contain the carrier frequency, demonstrating that the approach, as in all cross correlation methods, is insensitive to it. Without using additional knowledge on the velocity (de Jong 1991) aliasing will occur if the shift in phase in between observations exceeds half a period of the carrier frequency. For processing, the RF signal should be sampled according to the Nyquist bound, i.e. twice the anticipated maximal RF frequency. Since the bandwidth of the RF signal is generally less than the center frequency a sampling frequency of about 4 times the carrier frequency will be adequate.

The correlation coefficients in Eq. 1 are estimated using:

$$\hat{R}(i\tau, jT) = \frac{1}{(PL-j)\,(NS-i)} \sum_{k=0}^{PL-1-j} \sum_{m=0}^{NS-i-1} y_k(m)\,y_{k+j}(m+i) \quad i,j = 0,1 \tag{2}$$

where PL (package length) is the number of echolines considered and NS the number of sample points within the RF signal segment. Since only correlation coefficients are used with minimal shifts in depth (i) and time (j), there is no need for tracking of the signal window as is advised for the cross correlation method based on the location of the peak (Ferrara et al. 1992). The variance of the velocity estimate will decrease for an increasing product of PL and NS, demonstrating that for the cross correlation method axial resolution may be exchanged for temporal resolution (Hoeks et al. 1993).

Eq. 2 employs all possible correlation products within the data segment of interest. Because of the limited bandwidth of the RF signal the cross correlation function will only gradually decay for an increasing depth lag. Therefore, it may be inferred that the effective degree of freedom is less than the considered number of correlation products over depth. For a given length of the signal segment in sample points, the degree of freedom will increase with the bandwidth of the received RF signal. This explains why, for a larger fractional bandwidth and a given segment length, the variance of the velocity estimate will be less (Hoeks et al. 1993). On the other hand the CCM method is based on an approximation of the cross correlation function following from the assumed shape of the spectral power distribution which is only valid for relative low fractional signal bandwidths. It has been shown, however, that the CCM method gives good results in terms of bias and variance for fractional bandwidths equal to or less than 0.25 times the center frequency (Hoeks et al. 1993).

The observation that the degree of freedom is less than the number of correlation products over depth leads to the development of the following processing scheme. Let us assume that the RF signal is sampled about 4 times its carrier frequency; then 4 consecutive samples will cover approximately one period at the carrier frequency. Restricting basic processing to a period (subsegment) will make the anticipated result independent of the instantaneous phase of the RF signal with respect to the sampling grid. Processing may be confined to a number of subsegments evenly distributed over the signal segment (decimation). The ratio M of the observation length in sample points (NP) and the number of sample points considered will represent a corresponding increase in processing speed (Fig. 1). For $M = 1$ the subsegments are bordering on each other and processing will be almost identical to the original contiguous approach except for skipping cross correlation over the boundaries of the subsegments. In the following, $M = 0$ denotes the case that all possible correlation products are considered (contiguous segment).

In this paper we address the bias and standard

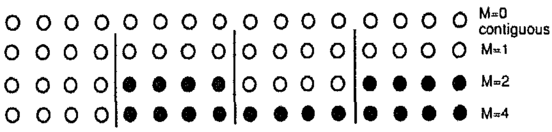

Fig. 1. Schematic presentation of the data points considered for processing (O). For the top row the data are treated as a contiguous series.

deviation of the velocity estimate based on the CCM method using a reduced number of sample points over depth (RCCM). For this purpose computer generated RF signals with a given center frequency and bandwidth and simulating moving scatterers over time with a given velocity range around an imposed mean velocity will be subjected to processing thereby varying SNR, package length *PL*, signal segment length *NP*, and reduction factor *M*. The results will be compared with the results obtained for the contiguous case for the same signal and processing conditions to arrive at a good choice for *M* in terms of gain in processing speed and increase in standard deviation and bias of the velocity estimate.

2. Signal generation and processing

The method to generate and evaluate simulated RF signals is described elsewhere (Hoeks et al. 1993). In short it is assumed that scatterers have a random spatial distribution, and therefore a random spectral phase distribution, and move along the ultrasound beam. Then, displacement in between observations corresponds to a shift in phase of the signal, i.e. sliding an observation window over a data series having the prescribed spectral composition. Generally the imposed shift will be a fraction of the sampling distance of the RF signal, requiring resampling of the signal to restore the phase relationship between the sampling signal and the observation window. If only a single signal source is considered then all scatterers are assumed to move at the same speed. A velocity distribution with a given width v_w around a mean velocity v_m is generated by considering the sum of a large number of independent signals (set at 21). The imposed shift per observation interval ($1/PRF$) for each of the signals is evenly distributed over the specified displacement range. A further decorrelation is achieved by performing to each signal and during each observation a random additional shift from a zero mean rectangular probability distribution with a width of v_w divided by the number of signals employed.

The generation of each of the signals is similar. Two independent long series of zero-mean Gaussian distributed random samples are windowed in the frequency domain by a Gaussian function with a given width and centered at a given frequency, and subsequently transferred to the time domain. The starting position of an observation window with a given length in sample points is randomly selected. Due to the imposed displacement during each observation interval the starting position changes linearly from observation to observation. The phase relationship between the beginning of the observation window and the sampling grid is restored using linear interpolation between adjacent sample points. As has been demonstrated (Hoeks et al. 1993), the error introduced by the interpolation scheme is marginal if the original sampling frequency is about 16 times the center frequency. After resampling, the sample frequency is reduced by a factor of 4 by decimation resulting in a ratio of sample frequency and center frequency comparable with practical applications.

It is also assumed that the additive noise has a Gaussian shaped spectral power distribution, centered at the same frequency as the RF signal but with a spectral width equal to or greater than the RF bandwidth, implying some form of band pass filtering. Since the noise contribution in subsequent observations is independent there is no need for resampling and decimation. It is sufficient to add to the RF signal, at the specified signal-to-noise ratio (SNR), random selections from a long series containing the additive noise.

The parameters for signal generation are the ratio of carrier frequency f_c, and sample frequency f_s, the quality factor of the signal (Q_s) and of the additive noise (Q_n), the signal-to-noise ratio (SNR), the mean velocity (or displacement during each observation interval) v_m and the width v_w of the velocity distribution. For these simulations the

Table 1
Summary of the signal and processing parameters used

Parameter		Value	Unit
f_c	RF center frequency	0.30, 0.25, 0.20	f_s
Q_s	Quality factor signal	4, 8	f_c/B_s
Q_n	Quality factor noise	4, 8	f_c/B_n
SNR	Signal-to-noise ratio	5, 10, 15, 20, 25	dB
v_m	Mean velocity	0.0, 0.4, 0.8, 1.2, 1.6	Sample points/T
v_w	Width velocity distribution	0.0, 0.4, 0.8	Sample points/T
NP	Length observation window	64	Sample points
PL	Package length	16	Lines
M	Reduction factor	0, 1, 2, 4	

sample frequency is used as the reference. The velocities are expressed in units of sample distance per time interval (T). To convert the velocity unit used to an actual one the value should be multiplied with c/f_s according to Eq. 1. The processing parameters are the length (NP) of the observation window expressed in sample points, the number (PL, package length) of composed RF-signal observations used to obtain the velocity estimate upon, and the reduction factor M. The parameters and selected values are summarized in Table 1. For each parameter combination 100 independent packages are analysed to arrive at estimates for the bias and standard deviation of the velocity estimate.

3. Results

In the following figures the rms error in the velocity estimates, expressed as a fraction of the sample distance, are given for various settings of the signal parameters (Table 1) as function of the reduction factor M. It should be noted that $M = 0$ is used as a code to indicate that the signal segment is treated as a contiguous signal, while for higher M the boundaries between the subsegments are taken into account. The effect of the processing parameters NP and PL on the standard deviation and bias of the velocity estimate is discussed elsewhere (Hoeks et al. 1993). For all of the results reported, they are kept at $NP = 64$ and $PL = 16$. To keep the presentation compact the rms error for $f_c = 0.3$, $f_c = 0.25$ and $f_c = 0.2$ are given alongside each other.

Fig. 2 depicts the observed rms error as function of the imposed signal-to-noise ratio for signals with a relatively narrow bandwidth ($Q_s = Q_n = 8$), a mean velocity $v_m = 0.8$ and a velocity width $v_w = 0$. The rms error decreases sharply for in-

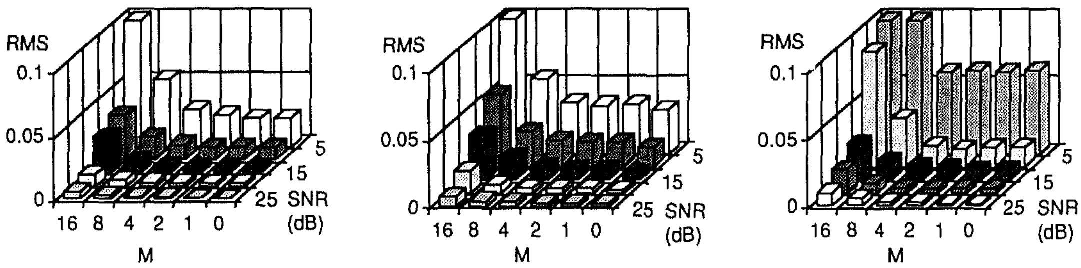

Fig. 2. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 8$, $v_m = 0.8$, $v_w = 0$ (for symbols, see Table 1).

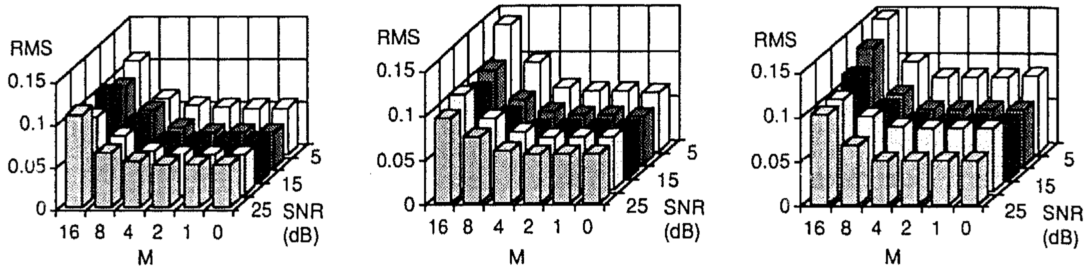

Fig. 3. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 8$, $\nu_m = 0.8$, $\nu_w = 0.8$.

creasing SNR and gradually for higher carrier frequencies, f_c, but remains rather constant for a reduction factor up to 4. There is no notable difference between $M = 0$ and $M = 1$. Increasing the velocity width to $\nu_w = 0.8$ (Fig. 3) results in the same observation for f_c but now the drop in the rms error for higher SNRs is considerably less, indicating that the velocity width is a large source of error. The latter observation is confirmed in Fig. 4 where the rms error is given for three velocity widths ($Q_s = Q_n = 8$, $\nu_m = 0.8$, SNR = 15 dB). Only for zero velocity width is the rms error negligible according to earlier observations (Hoeks et al. 1993; Embree and O'Brien 1990). Varying the mean velocity over the range from 0 to 1.6 sample points per time interval hardly affects the rms error (Fig. 5, $Q_s = Q_n = 8$, $\nu_w = 0.8$, SNR = 15 dB). Only for $f_c = 0.3$ and $\nu_m = 1.6$ is a sharp increase in the rms error to above 0.1 noted (it should be noted that the displayed error range is limited to 0.1) which can be explained by aliasing. At this carrier frequency a mean velocity of 1.6 sample points per time interval corresponds to a shift over half a period of the carrier frequency which is the limit for aliasing. The velocity width of $\nu_w = 0.8$ further increases the probability of aliasing.

By comparison with Fig. 2 ($Q_s = Q_n = 8$) it can be seen that enlarging the noise bandwidth by a factor of 4 (Fig. 6, $Q_s = 8$, $Q_n = 2$, $\nu_m = 0.8$, $\nu_w = 0$) reduces the rms error of the velocity estimate considerably. Apparently, the averaging involved in the computation of the correlation coefficients is now more effective. This is in concordance with the expectation that the ratio of bandwidth and the number of points (in depth and in time) governs the efficacy of the averaging process. This is confirmed in Fig. 7 where both Q_s

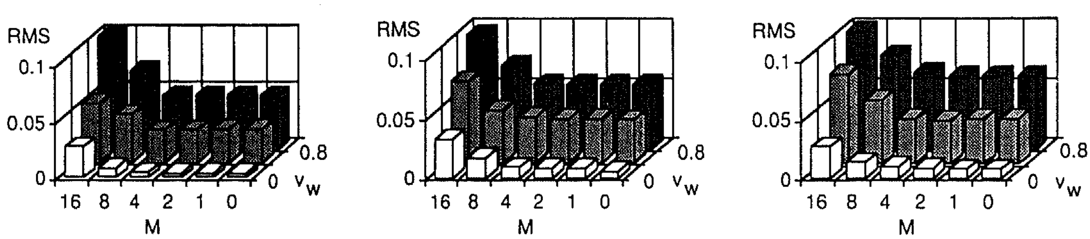

Fig. 4. Observed rms error in velocity estimate as function of the velocity width ν_w and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 8$, $\nu_m = 0.8$, $SNR = 15$ dB.

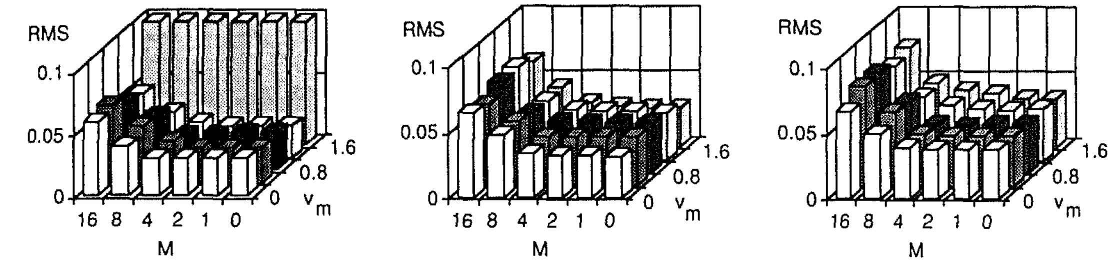

Fig. 5. Observed rms error in velocity estimate as function of the mean velocity width ν_m and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 8$, $\nu_w = 0.8$, $SNR = 15$ dB.

and Q_n have been reduced to 4. The other parameters are the same as in Fig. 2. Enlarging the fractional bandwidths by a factor of 2 gives a reduction in the rms error. The rms error now remains fairly constant up to $M = 2$ and increases for higher reduction factors. The same tendency is observed in Fig. 8 where the rms error is given for exactly the same parameter settings as in Fig. 2 except for the quality factors of the signal and noise components. Changing the velocity width from 0 to 0.8 (Fig. 9) or the mean velocity from 0 to 1.6 (Fig. 10) does not affect the observation that the reduction factor may be safely set to $Q_s/2$ without significantly affecting the rms error. It should be noted that Figs. 9 and 10 may be directly compared with Figs. 4 and 5, respectively. The only difference between both sets of figures is the quality factor of the composed RF signal. Only if the quality factor, Q_n, of the additive noise is considerably lower than Q_s (compare Fig. 11 with Fig. 6) and the SNR is low, does the relation between Q_s and reduction factor not apply. However, for these conditions the fractional bandwidth of the composed signal is also governed by the spectral distribution of the noise. So, for a low SNR the quality factor of the noise should be taken into account (resulting in a lower reduction factor) while for a high SNR, Q_s should be considered.

Up to now little attention has been paid to the bias in the estimate. This is mainly because for visual inspection of the time-dependent velocity distribution in a plane, as is done with color coded Doppler systems, the appearance of the image is governed by random error rather than bias. Nevertheless, bias may become important when convert-

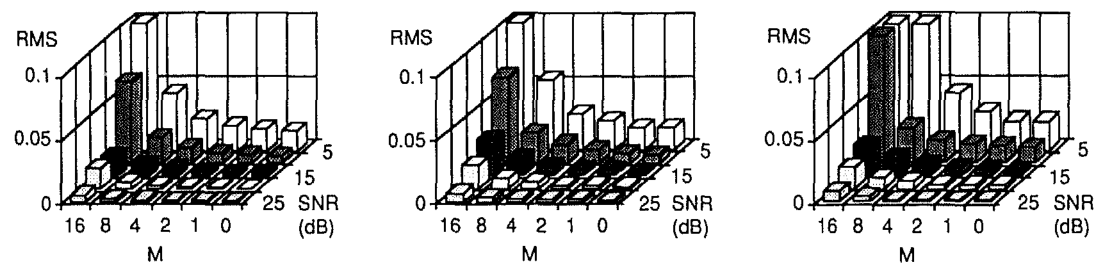

Fig. 6. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = 8$, $Q_n = 2$, $\nu_m = 0.8$, $\nu_w = 0$.

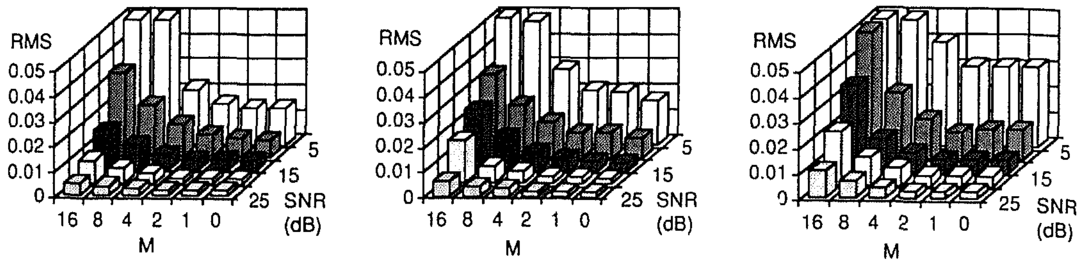

Fig. 7. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 4$, $\nu_m = 0.8$, $\nu_w = 0$.

ing the observed colors to absolute velocity values. Table 2 summarizes numerically the bias, and within parentheses the rms error, observed for various signal parameter settings (for all values listed $\nu_m = 0.8$, $\nu_w = 0.4$). It can be concluded that the bias in the estimate is far smaller than the rms error, independent of the reduction factor applied. As stated before the carrier frequency does not influence the bias.

4. Discussion

Estimation of the velocity of scatterers averaged over a depth and time window of the RF signal based on the location of the peak of the cross correlation function of corresponding RF segments involves a large number of operations. The computational complexity can be reduced considerably by assuming that the shape of the spectral power distribution of the RF signal is rectangular (de Jong et al. 1990) or Gaussian (Hoeks et al. 1993). Using a model for the cross correlation function (CCM) based on the assumed spectral distribution estimation of the velocity requires only 5 coefficients of the cross correlation function. The CCM approach has the advantage that the sample frequency of the RF signal does not have to be larger than the bound given by the Nyquist limit. Even if only 5 correlation coefficients are evaluated, the required number of operations is still considerably larger than for the Doppler approach. A further reduction in mathematical operations can be achieved by noting that the statistical properties of the RF signal gradually change over depth due to the limited bandwidth of the RF signal. Because of the high correlation, nearby RF data points will

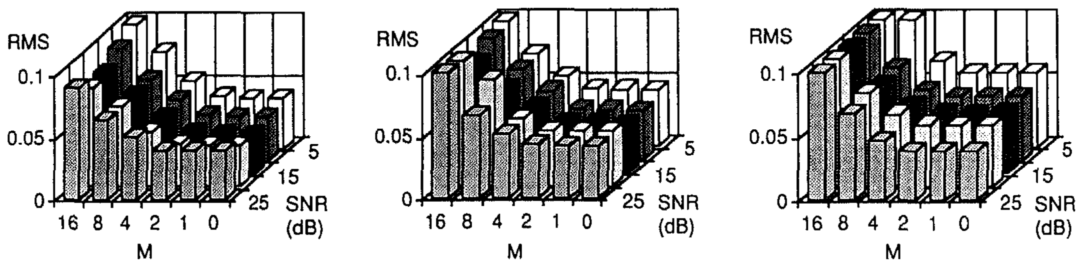

Fig. 8. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 4$, $\nu_m = 0.8$, $\nu_w = 0.8$.

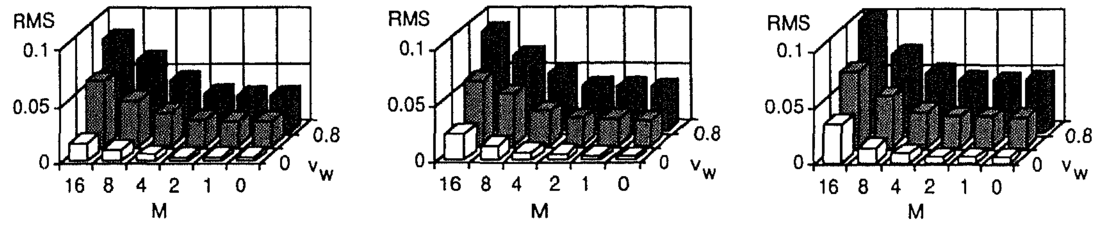

Fig. 9. Observed rms error in velocity estimate as function of the velocity width v_w and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 4$, $vm = 0.8$, $SNR = 15$ dB.

hardly contribute to a reduction in the standard deviation of the velocity estimate. In the present study it is suggested that the observation window be partitioned over depth in subsegments, each with a length of 4 sample points. Depending on the bandwidth of the RF signal one may decimate the subsegments contributing to the correlation coefficients. To validate this approach, computer generated RF signals containing velocity information were subjected to statistical evaluation. Reducing the number of subsegments for analysis by a factor M up to about $Q_s/2$ did not notably affect the standard deviation of the velocity estimate. For moderate ($Q_s = 4$) and narrow band RF signals this will result in a considerable improvement in processing speed.

The signal simulation as used in the present study assumes only movement along the ultrasound beam. Under practical conditions this is quite a rare situation. For peripheral vessels, running parallel to the skin surface, the angle of observation will generally be about 60°. Then the velocity component perpendicular to the beam will be larger than the component along the beam. The spatial resolution for measuring the velocity, i.e. the path length of a particle through the measurement volume, is a combination of axial resolution (RF band width) and the local beam width. There is a physical limitation to beam width. When focusing the beam of ultrasound to less than approximately 4 wavelengths, the inherent inhomogeneities in sound intensity cause unwanted strong amplitude modulations of the signal returned by the particles crossing the beam. For a regular observation angle of 60° reduction of axial resolution below beam width will not further enhance spatial resolution of velocity, but will reduce the transit time for a particle to cross the

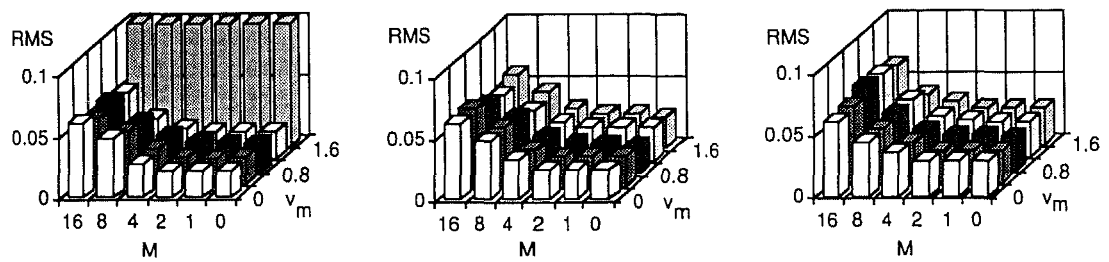

Fig. 10. Observed rms error in velocity estimate as function of the mean velocity width v_m and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = Q_n = 4$, $v_w = 0.8$, $SNR = 15$ dB.

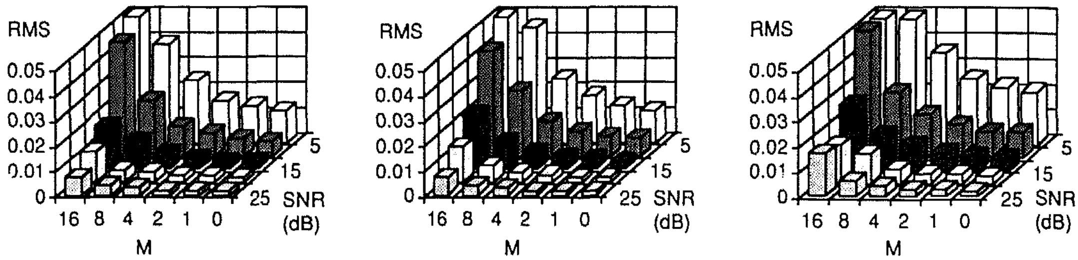

Fig. 11. Observed rms error in velocity estimate as function of the signal-to-noise ratio (SNR) and reduction factor (M) for $f_c = 0.3$ (left), $f_c = 0.25$ (middle), and $f_c = 0.2$ (right). Signal conditions: $Q_s = 4$, $Q_n = 1$, $v_m = 0.8$, $v_w = 0$.

observation window, thus causing an increase of the standard deviation of the velocity estimate. It is, therefore, advised to set the axial resolution of the ultrasound system to about half a beam width, which is achieved by emission of bursts containing 4 periods ($Q_s = 4$) for a beam width of 4 wavelengths. Thus a lower bound is set to the Q of the ultrasound system. As a consequence the data size may be reduced by at least a factor of 2, resulting in an improvement of processing speed with the same factor.

As long as the carrier frequency, f_c, may be assumed to be constant the reduction factor, M, can be fully expressed by the quality factor. However, if the carrier frequency is changed while the fractional bandwidth remains the same then the bandwidth of the RF signal varies accordingly: a higher f_c gives a wider bandwidth. For the same length of the observation window in sample points, averaging of the correlation products is more effective in explaining the lower rms error for higher carrier frequencies. On the other hand, when the frequency components in the signal approach the sample frequency limit the risk of aliasing increases, as demonstrated in Figs. 5 and 10.

If the quality factor, Q_s, of the signal conveying the velocity information exceeds that of the additive noise, (Q_n), which quality factor will dominate depends on the SNR (Fig. 6 and 11). For a low SNR the spectral noise distribution will dominate the averaging process while for increasing SNR the quality factor of the composed signal gradually shifts up to Q_s. It should be noted that a large difference between Q_s and Q_n cannot be used to ascertain the quality of a velocity estimator. Noise outside the signal band can be reduced anyway by suitable bandpass filtering with a corresponding improvement of the SNR. What really matters is the SNR for the coinciding spectral distribution.

To keep the simulations simple the package length and the length of the observation window was kept constant. As reported elsewhere (Hoeks et al. 1993) the standard deviation of the velocity estimate will go down for increasing package and observation lengths. On the other hand, one may consider reducing the observation length NP to improve processing speed or axial resolution. However, if NP becomes smaller than Q_s times the sample frequency, a decimation of subsegments with $Q_s/2$ may result in only a single subsegment, giving a larger standard deviation. The number of subsegments used for velocity estimation should be at least 2.

For the present evaluation the length of a subsegment was set at approximately a period of the carrier frequency or 4 points for a sample frequency of 4 times the carrier frequency. It looks attractive to reduce the length of a subsegment further to only 2 sample points but then the correlation coefficients vary with the (change in) instantaneous phase of the received signal. This will

Table 2
Bias (and the rms error) observed for various signal parameter settings

Q_s	Q_n	f_c	M (reduction factor)				
			1	2	4	8	16
8	8	0.3	−0.001 (0.030)	−0.001 (0.030)	−0.001 (0.030)	−0.001 (0.047)	−0.005 (0.061)
		0.25	−0.007 (0.036)	−0.006 (0.036)	−0.005 (0.039)	−0.007 (0.052)	0.002 (0.074)
		0.2	−0.009 (0.041)	−0.009 (0.041)	−0.010 (0.042)	−0.011 (0.052)	−0.014 (0.071)
8	2	0.3	−0.001 (0.028)	0.000 (0.029)	0.000 (0.033)	0.001 (0.049)	0.003 (0.068)
		0.25	−0.004 (0.035)	−0.004 (0.036)	−0.004 (0.040)	−0.008 (0.050)	−0.004 (0.076)
		0.2	−0.003 (0.037)	−0.004 (0.038)	−0.003 (0.045)	−0.008 (0.061)	−0.010 (0.072)
4	4	0.3	0.002 (0.021)	0.003 (0.021)	0.002 (0.032)	0.004 (0.043)	0.006 (0.069)
		0.25	−0.001 (0.025)	−0.001 (0.024)	0.001 (0.029)	0.003 (0.045)	−0.010 (0.063)
		0.2	−0.001 (0.030)	−0.001 (0.030)	0.002 (0.033)	0.010 (0.046)	0.001 (0.114)
4	1	0.3	0.008 (0.026)	0.007 (0.028)	0.011 (0.032)	0.011 (0.042)	0.011 (0.068)
		0.25	−0.003 (0.028)	−0.004 (0.029)	−0.001 (0.033)	−0.003 (0.043)	0.000 (0.062)
		0.2	−0.014 (0.029)	−0.013 (0.031)	−0.010 (0.034)	−0.010 (0.046)	−0.009 (0.090)

The bias is the estimate minus the expected value. It and the standard deviation of the velocity estimate are given as a function of signal (Q_s) and noise (Q_n) bandwidth, and carrier frequency (f_c) for a mean velocity $v_m = 0.8$, velocity range $v_w = 0.4$, SNR = 10dB, package length of 16, and length observation window of 64 sample points. All velocities are given in sample points per time interval.

introduce a larger error in the velocity estimate, especially for a poor to moderate SNR (SNR < 15 dB) or a short observation window in depth.

5. Conclusion

Evaluation of the cross correlation function over possible shifts in depth and in time is a time consuming procedure. The number of calculations can be reduced considerably if the two-dimensional cross correlation function can be modeled using a priori information about the shape of the spectral power distribution of the received RF signal. A further improvement in processing speed can be obtained by discarding segments of the received signal sampled at 4 times the assumed carrier frequency of the transmitted pulse. In a first approach the observation window over depth was partitioned in subsegments, each with a length of 4 sample points. The number of subsegments used for analysis can be reduced by $Q_s/2$ without a significant loss of accuracy. For higher quality factors (lower axial resolution) the gain in processing speed will increase accordingly while the bias and standard deviation of the velocity estimate are hardly affected.

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