The tangential stiffness of the moving pulley halve of a CVT

The tangential stiffness of the moving pulley halve of a CVT

A. Mauritz
DCT 2005.44

Student number: 0470798
Eindhoven, 14-04-2005

Table of contents

Introduction

Every car has a gearbox. The function of a gearbox is to transmit the torque of the engine to the torque that is asked, at the wheels.
The gearbox is available in several types. The most common one, in the Netherlands, is a manual gearbox, which has a number of fixed gear ratios. There are a number of other gearboxes, one of them is transmission with continuously variable ratios, this is a CVT.
A continuously variable transmission (CVT) is a system that makes it possible to slowly vary the transmission ratio. This can be achieved through two shafts, perpendicular placed, with two pulley halves on them, which carry the push belt. The first shaft (primary shaft) is driven by the motor; the torque is passed on to the secondary shaft by the push belt. The secondary shaft, in its turn, gives its torque to the wheels. The ratio change of the transmission between the shafts is accomplished by moving the pulley halves closer to each other or further from each other. To achieve this, one pulley halve must be able to move in axial direction on the shaft. This is done by means of balls. On the primary shaft, three rows, with each four balls, are placed. On the secondary shaft there are four rows with each three balls.
There is a lot of research being done on the CVT to improve it, but the tangential stiffness of the moving pulley halve with respect to the shaft is not yet researched.

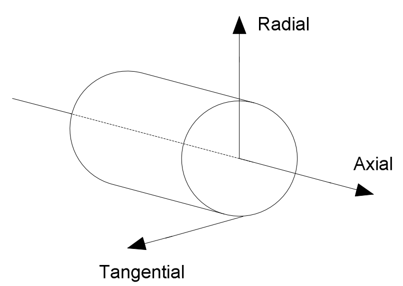

Figure 0.1: Used terminology with respect to the shaft

The push belt is moving between the two pulley halves, in such a way that it causes a tangential force on both the pulley halves. The fixed pulley halve doesn’t move, but the moving pulley halve has a limited tangential stiffness. If the tangential force is too big, then the moving pulley halve will rotate with respect to the fixed pulley halve. At that moment, the force transmitted by the belt will be put mainly on the fixed pulley halve instead of being evenly divided between both pulleys. Also, the push belt can only transmit the force on one side, on this side there is more grip than on the other side. Due to this, one side of the push belt will want to go faster then the other side, which gives strain on the elements in the belt.
This will occur intermittently, each time giving a blow to the bullets and the elements in the push belt, this causes wear and reduces the life of a CVT.

The questions which will be researched here are:

- How high the tangential stiffness is of the moving pulley halve?
- What is the influence of the diameter of the balls on the stiffness?

The researched shaft is the secondary shaft.

Chapter 1: The secondary shaft

The secondary shaft consist of a shaft with a fixed pulley halve on it. Furthermore there's a moving pulley halve slid over the shaft. This pulley halve is attached to a clamp ring. On the inside of this ring there are grooves, this is mirrored in the shaft, in here the balls lay enclosed. This setup acts as a bearing. Because of this an axial movement of the moving pulley halve is possible.

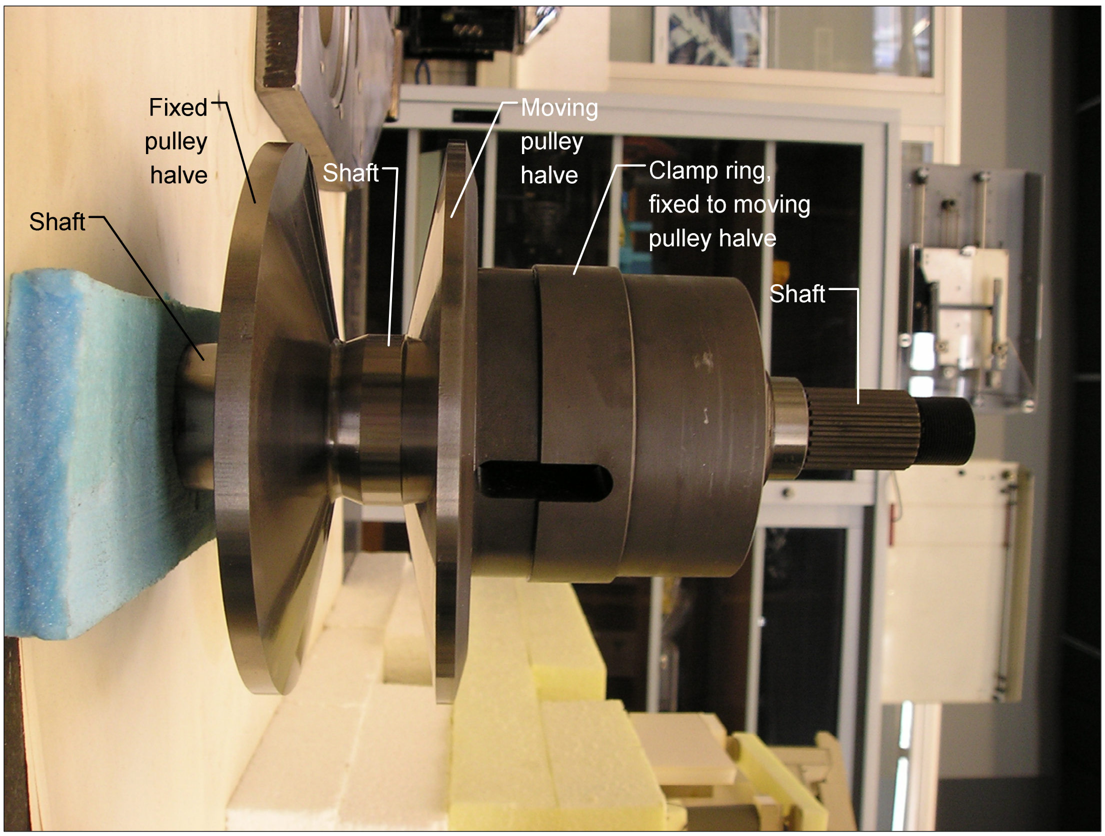

Figure 1.1: Picture of the secondary shaft

Chapter 2: Numerical approach

To estimate some dimensions of the experimental setup it is necessary to first make a number of calculations.

§ 2.1 Applied load

In a CVT the maximum incoming torque from the engine is 220 Nm. At that point, the part of the push belt on the primary shaft has a minimal radius (33.941 mm), on the secondary shaft the radius is maximal (78.948 mm).

The secondary shaft gets a torque of $220 * \frac{78.946}{33.941} = 512$ Nm.

The torque is evenly divided by the two pulley halves, they both receive $\frac{512}{2} = 256$ Nm.

With a weight of about 50 kg, the applied load is $50 * 9.81 = 490.5$ N.

To get the torque needed, the load must be placed at a distance of $\frac{256}{490.5} = 0.522\,\text{m}$.

The distance between the edge of the moving pulley halve and the centre line of the shaft is 84.5 mm. This means that an addition of 438 mm from the edge of the pulley halve is needed.

§ 2.2 Calculated stiffness

To get more insight in the forces needed to get a movement that can be measured the stiffness has to be estimated through calculations.
The calculated stiffness of a ball with a diameter of 5.96 mm is being calculated here. This will be the minimal stiffness, because this is the smallest ball being used in the experiments.
The stiffness will be calculated with the surface contact theory of Hertz [see appendix 6]. In this problem we have a circular contact, in which the contact at the shaft is hollow, with a radius of 3 mm, and the ball is round, with a radius of 2.98 mm.

The force, for one ball, at the radius of the position of the balls, with respect to the centre line of the shaft, is: $F_b = F_a \frac{r_a}{r_b} \frac{1}{n}$

With: F_a = applied force (490.5 N)
r_a = radius at which the force is applied (= 522 mm)
r_b = radius of the position of the balls (= 26.3 mm)
n = number of balls (= 12)

This gives a force of F_b = 811 N.
This force has a vertical direction. When a force is applied, the clamp ring makes a rotation. There is a little bit of room, so the clamping ring rotates until it comes in contact

with a ball. The ball and the clamping ring rotate until they hit the shaft. The force line goes through both contact points, this line has a rotation of 45° with respect to the calculated force.
So $F = F_b / \cos(45°) = 1147$ N.

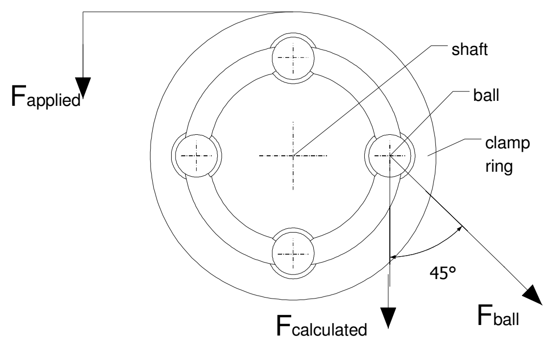

Figure 2.1: The angle between F_{ball} and $F_{calculated}$

The calculated stiffness (through the contact surface tension theory of Hertz) is $1.5*10^9$ N/m. [see Appendix 1]

The displacement u is given by: $c(r)*u = c(r)*\left(\frac{R}{r}\right)^2 *u = F$

With: r = distance at where the displacement is measured (0.3 m)
R = radius at which the balls are situated ($26.3*10^{-3}$ m)
c(r) = stiffness ($1.5*10^9$ N/m)
F = applied force at R (811 N)

This gives a displacement of $u = 7.0*10^{-5}$ m.
This can be measured with the available ‘millitron probe 1300’. It has a minimal measuring distance of $1*10^{-6}$ m.

Chapter 3: Experimental set-up

To research the stiffness of the moving pulley an experimental set-up has to be made. There is one important requirement that has to be met, the shaft and its pulleys may not be damaged.
To measure the tangential stiffness accurately, the shaft must be rigidly placed on a table and the rotation of the shaft must be prevented. Further more, a momentum must be placed on the moving pulley of 2500 N and the rotation of the moving pulley halve must be measured relative to the fixed pulley.

§3.1 The moving pulley halve

The moving pulley halve has to have a load placed on it, this must be on a distance of about 0.5 meters, because then the load is manageable (the load is in that case around the 500 N). A plate will be fixed on the pulley halve, with the help of clamps.
In appendix 2 it is proven that six socket-head screws of 8 mm provide a sufficient force to keep the pulley halve from slipping with respect to the attached plate.
During assembly it became obvious that the clamps, which were placed between the pulley halves, were almost impossible to tighten. It was decided to put the screws, which are used to clamp the moving pulley halve, in from the side of the plate. The hole in the clamps was already made; these holes had a smooth surface. Because the screws are now coming from the other way, the hole must have a thread, instead of a smooth surface to give the screws grip. With the thread, the hole is too big for a screw of 8 mm. The used screws are now 10 mm.
Attached to the plate is an arm, were the weights can be put on, of approximately 0.5 m. The arm is also made out of a plate. This plate is welded to the plate clamped to the pulley halve. The main requirement for this plate is that it is not very heavy, but to keep the arm from kipping (this is a movement around the axial axis due to a radial force), it must be relatively broad. To achieve this, the plate is bended on the sides.

$$F_{kip} = 4\sqrt{\frac{E * I_b * G * I_t}{l^2}}$$

With $I_b = \frac{1}{2} * h * t^3$, $I_t = \frac{1}{3} * h * t^3$ and $G = 0.4E$ (for metals). (1)
E = Young's modulus (400 N/mm^2)
t = thickness (3 mm)
h = height (200 mm)
l = length (300 mm)
G = shear modulus
I_b = moment of inertia around the axis in the direction of the length
I_t = moment of inertia around the axis in the direction of the thickness

This simplifies the formula to:

$F_{kip} = 0.42 \frac{h * t^3 * E}{l^2} = 10.08N$ This is too little because there will be a load of 500 N. With the sides bended, t becomes 18 mm and F_{kip} = 2.18 kN, this is sufficient.

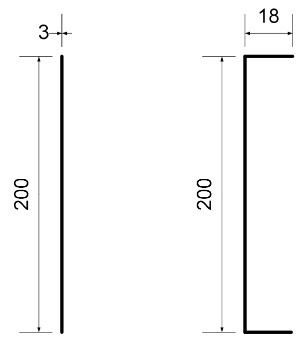

Figure 3.1: Dimensions of the flat plate and the bended plate

At the end of the arm a tube is fixed. A stud end can be attached to this tube by a nut. At the end of the stud a ring is fixed. With the help of a shackle the weight arm can be attached.

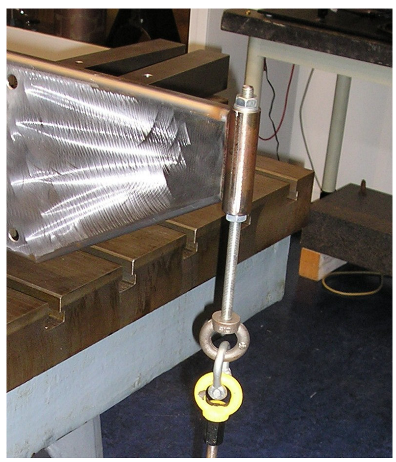

Figure 3.2: The tube with the stud end

To keep the pulley halve from being damaged, the parts that touch the pulley are made of aluminium. This is because aluminium is weaker than steel, so if the force it so high that there will be plastic deformations, the aluminium will deform and not the steel.
There are two areas where aluminium is used. A thin plate is glued on the clamps and a ring is glued into the plate.

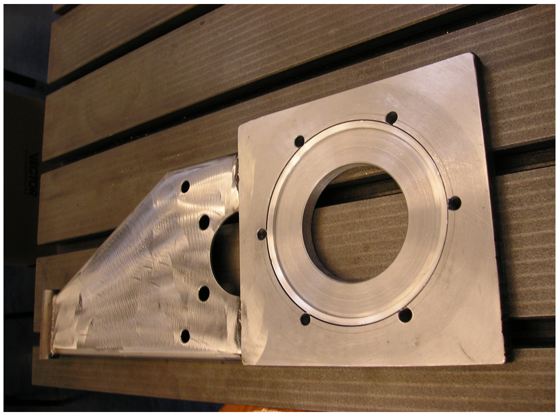

Figure 3.3: The plate with the arm, which is attached to the moving pulley halve

Figure 3.4: On the left side is the clamp used to fix the plate on de moving pulley halve, on the right side, the one for the fixed pulley halve

§3.2 Fixing the shaft

The shaft must be fixed in order to prevent it from rotating and moving horizontally and vertically. Prevention of the rotation is done by fixing the fixed pulley halve, prevention of the translation is done by fixing the shaft itself.

§ 3.2.1 Rotation

A plate with an arm is mounted on the fixed pulley halve. This is a mirror of the plate fixed on the moving pulley halve. Here the original assemble method, with the 8 mm screws, can be used. The end of the arm is fixed on the table, with a stud and a number of nuts. This prevents a rotation.

Figure 3.5: The fixation of the arm to the table

§ 3.2.2 Translation

The translation of the shaft on the side of the fixed pulley halve is prevented by fixing a plate, with a round hole in it, around the shaft and mounting it on the table. The plate is divided in two parts for easy assembly and studs are used to fix the two parts together. There is a block placed under the two plates which has two holes with screw thread in it. The stud is going through all this and a block in the grooves in the table makes sure the plate and the block on the table are fixed rigidly to the table.
The plate has to be mounted on a smooth part of the shaft. On the side of the fixed pulley halve, the shaft sticks out very little. Also, the plate attached to the fixed pulley halve is attached on the side of the shaft. The room that was left was too little to fit the two parts on it. It was decided to make a cylinder and slide it over the shaft and attach the cylinder on the table.

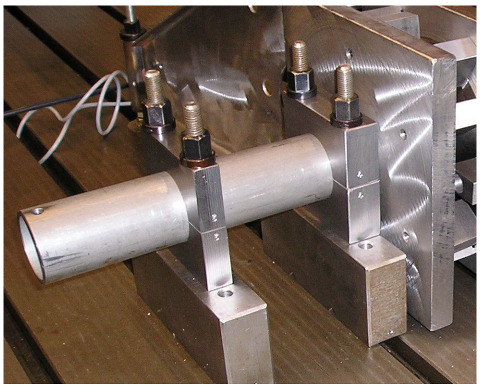

Figure 3.6: The fixation of the shaft on the side of the fixed pulley halve

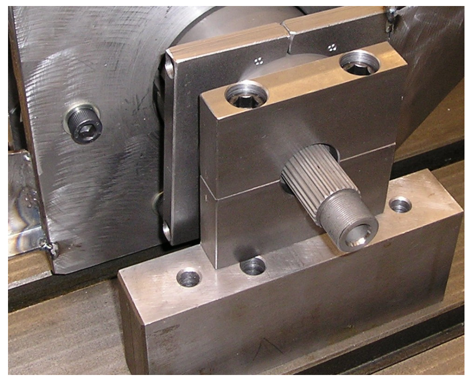

Figure 3.7: The fixation of the shaft on the side of the moving pulley halve

On the other side about 2 cm of the shaft was smooth. This was enough to place the two plates. There two socket-head screws were used. But the distance between the clamping areas and the grooves in the table don't match. Therefore this side cannot be fixed to the table with a block in the groove. On the block, situated under the plate, two heavy beams are placed. On a distance of about 50 cm another block is place to support the beams on the other side. The weight of the beams will prevent translation of the shaft on this side.

The use of the stud on the one side and the socket-head screws on the other side are due to the fact that it was made from materials that were available. There was no constructional reason for this.

§ 3.3 The measuring arm

The object is to measure the rotation of the moving pulley with respect to the fixed pulley. This has to be done on a distance of more than 20 cm with respect to the centerline of the shaft. It is decided to attach the arm with a plate with a round hole in it, this plate is divided in two and it is fixed together again by two socket-head screws.

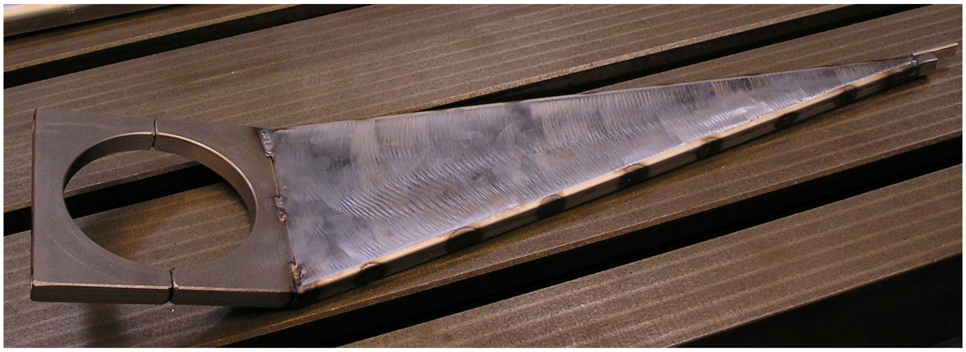

Figure 3.8: The measuring arm

This can be attached very easily on the clamp ring. The measurement of the relative rotation with respect to the fixed pulley halve is desired. So it is decided to attach the probe to the arm which fixates the fixed pulley halve and let the measuring arm rest on the probe. When the moving pulley halve rotates the measuring arm moves and the probe can measure the displacement of the moving pulley halve with respect to the fixed pulley halve.

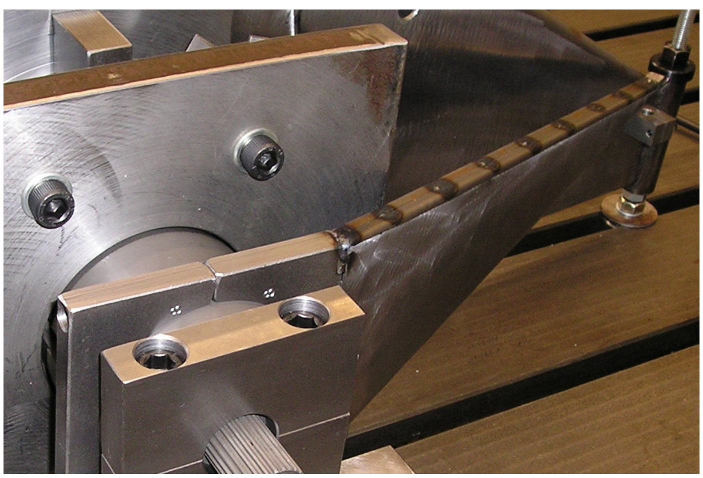

Figure 3.9: The measuring arm attached to the clamp ring

Figure 3.10: The end of the measuring arm and the probe, fixed on the arm, attached to the fixed pulley halve and the table

Chapter 4: The results

The used balls have a diameter between 5.96 mm and 5.98 mm. These balls are from the firm Van Doorne's Transmissie BV who also fabricates the CVT. The balls with the diameter of 5.96 mm where the smallest balls available and though there were balls up to 6.02 mm, the maximum used here was 5.98 mm. All the balls have been measured, they varied with a maximum of 3 µm in diameter.

It was given that the axial movement of the moving pulley halve must be achievable by hand. With a ball diameter of 5.98 mm the axial movement was almost impossible to achieve by hand and there was a great fear for plastic deformation of the grooves in the clamp ring and the shaft. Therefore it was decided to stop at a diameter of 5.98 mm.

With the experiment two different measuring series were made. One was when the weights were going on, the other one was when the weights were taking off. The first series is described as up, the second one as down.

The graphical results are shown in this chapter, the numerical results are found in appendix 3, 4 and 5 for respectively a ball diameter of 5.96, 5.97 and 5.98 mm.

§ 4.1 Ball diameter 5.96 mm

It is seen in the charts that the lines of the different measurements do not overlap, as is expected, because every time you put up a weight the same displacement should be measured. But after 11 kg the lines become nearly perpendicular. So the stiffness (after about 11 kg) for every measurement is virtually the same. This indicates that there is some tangential movement of the moving pulley halve when there is no load placed. After about 11 kg all the balls have a firm contact with the grooves and after that the actual stiffness is measured.

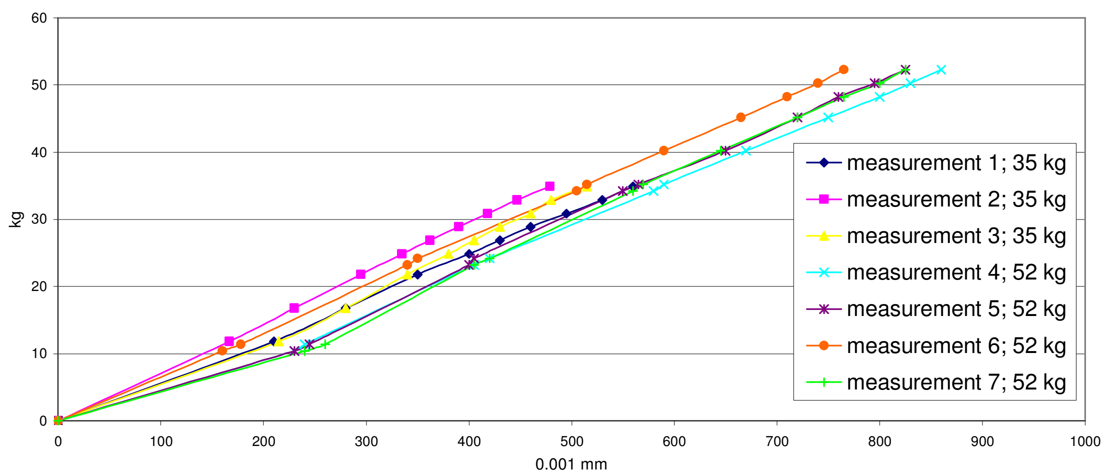

Figure 4.1: The measuring results for a ball diameter of 5.96 mm, placing the weights on

The displacement u is given by: $c*u = c*\left(\frac{R}{r}\right)^2 *u = F$

With: r = distance at where the displacement is measured ($405*10^{-3}$ m)

R = radius at which the balls are situated ($26.3*10^{-3}$ m)

c = stiffness

F = force at R

Here: $F = \frac{F_{applied} * r_{applied}}{R}$

With: $r_{applied}$ = radius at which the load is placed ($448.25*10^{-3}$ m)

$F_{applied}$ = the applied load

This gives: $c = 4040.8 * \frac{F_{applied}}{u}$

Because here the results of the measurements are reliable from a load of about 11 kg, the total load isn't used here in the formula, but the load difference between the load just higher than 11 kg and the maximal load. This is also done for the displacement.
This gives a stiffness between $2.62*10^9$ N/m and $3.05*10^9$ N/m. [appendix 3]
It can be seen from the results in appendix 3 that in general the stiffness is greater when only 35 kg is placed. This is caused by the fact that the line representing the stiffness is not linear, this is due to the virtual play (this will be explained later).
The load of approximately 52 kg represents the biggest force that is placed on the moving pulley halve by the push belt. So the stiffness that corresponds with this load should be taken as the stiffness of the balls. This means that the stiffness of the moving pulley halve for balls with a diameter of 5.96 mm is between $2.62*10^9$ N/m and $2.86*10^9$ N/m.

The virtual play is the difference between the displacement of putting the weights on and taking it off. The virtual play is caused by hysteresis. (1)
A force is placed on the construction. At first there is only movement due to elastic deformation. If the force exceeds the friction coefficient, there will be slip, which means that two parts of the construction move with respect to each other. When the force is taken of, the parts do not move back, because the friction coefficient is not exceeded. This process is called hysteresis.
To get all the parts back on the place where they started, an equally high force has to be applied, only this time in the opposite direction. Then the friction coefficient is exceeded again, but the movement is now in the opposite direction.
It is impossible to say what part of the virtual play takes place in the CVT or the construction built around it.

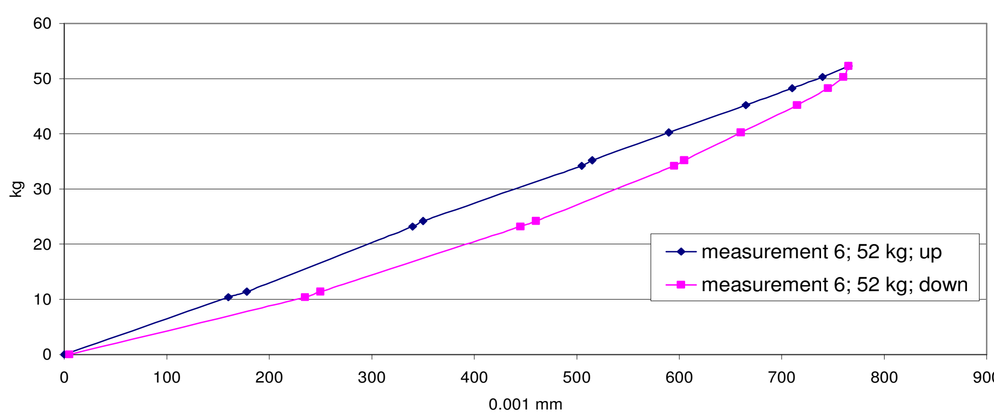

Figure 4.2: Measurement 6, difference between placing the weights on and taking them off

The virtual play here is a so called 'vlinderdasje'. Here the friction is a function of the applied load. It can be calculated through:

$$S_v = \frac{2W}{c} = \frac{2F * \mu}{c}$$

With: W = friction (N)
S_v = virtual play (m)
c = stiffness (N/m)
F = load (N)
μ = friction coefficient (0.20) (2)

For the virtual play in measurement 6 [see appendix 3] the virtual play is:

$$S_v = \frac{2F * \mu}{c} = \frac{2 * (52.297 * 9.81) * 0.20}{2.77 * 10^9} = 7.41 * 10^{-8} m$$

The virtual play is an insecurity in the place. So the measured movement has an insecurity. Because the smallest unit measured is $1*10^{-6}$ m and the virtual play is $7*10^{-8}$ m, the virtual play doesn't have a great effect on the measurement.

§ 4.2 Ball diameter 5.97 mm

With a ball diameter of 5.97 mm all the measurement have nearly the same displacement. This indicates that there is very little play. This doesn't mean that there is no virtual play. The axial movement of the moving pulley halve can be achieved relatively easy. The stiffness of the balls is computed by taking the difference between zero and the maximum load, this is also the case for the displacement. The stiffness that is given as the stiffness of the balls is the stiffness that corresponds with a maximum load of about 52 kg.
The stiffness of the balls is between $3.44*10^9$ N/m and $3.51*10^9$ N/m.

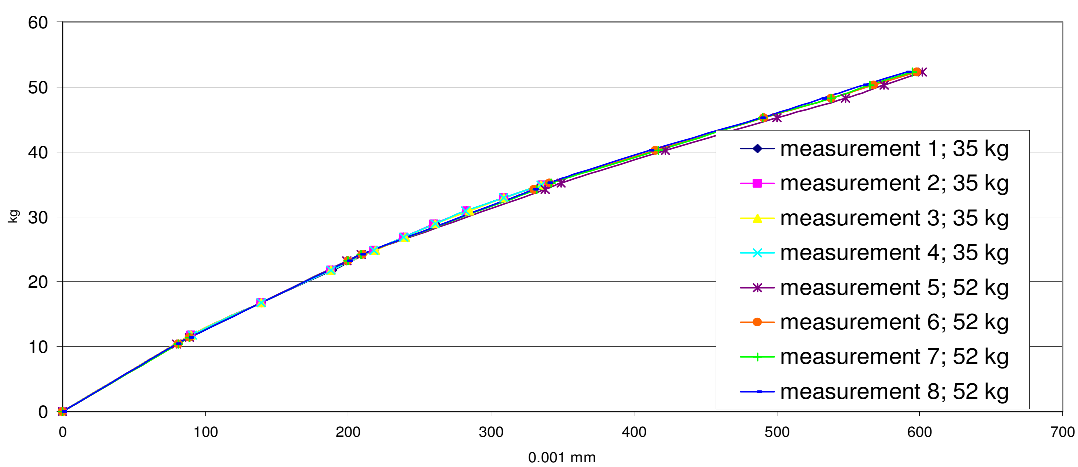

Figure 4.3: The measuring results for a ball diameter of 5.97 mm, placing the weights on

The virtual play with this ball diameter is: $S_v = 6*10^{-8}$ m.

§ 4.3 Ball diameter 5.98 mm

Just like with the balls with the diameter of 5.97 mm, all the measurements overlap. The stiffness of the balls, with a load of about 52 kg, is between $3.80*10^9$ N/m and $3.85*10^9$ N/m.

The problem with a ball diameter of 5.98 mm is that the axial movement of the moving pulley halve is very hard to achieve.

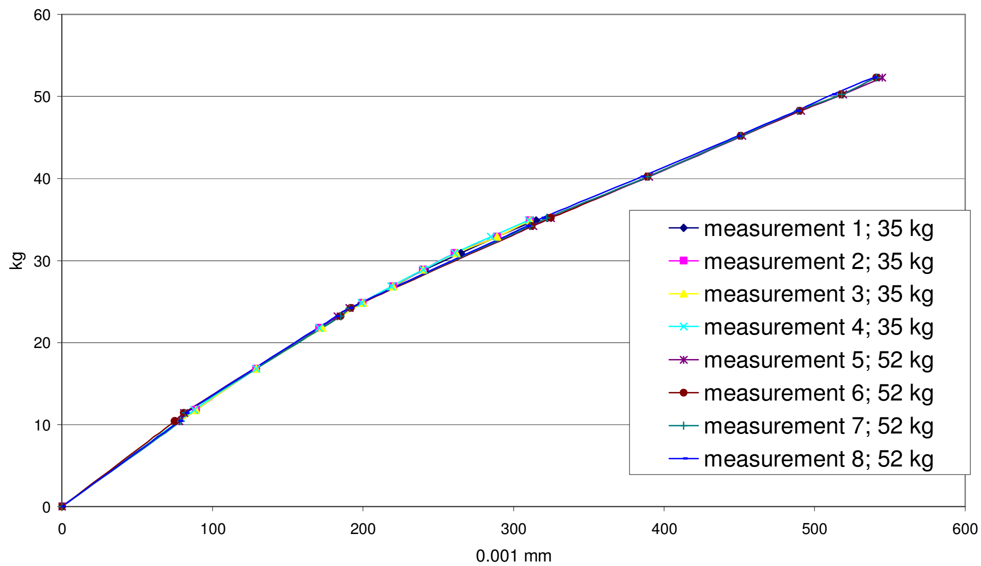

Figure 4.4: The measuring results for a ball diameter of 5.98 mm, placing the weights on

For this diameter the virtual play is: $S_v = 5*10^{-8}$ m.

§ 4.4 Error analysis

The errors that are present in this analysis are the reading errors (about 1 µm), the errors in the measuring equipment (accurate up to 0.1 µm) and unwanted movements of the whole set-up (around 1 µm). Also the virtual play gives an error (which is maximal 0.1 µm).

For all the balls the measuring procedure is the same, so when the stiffness of the different ball diameters is compared, it gives a good view on the variation of the stiffness with different ball diameters.

And these errors combined have a magnitude of about 2 µm. the results are given in the order of 100 µm, so only the last digit is not accurate.

Chapter 5: Conclusions and recommendations

The stiffness must be as great as possible, but an axial movement of the moving pulley halve on the shaft must be attainable with a force that is relatively low.
This gives the following table:

Ball diameter (mm)	Stiffness (10^9 N/m)	Axial movement	Displacement of the moving pulley halve at the edge, with the maximum load (μm)
5.96	2.7	Achievable with one finger	54
5.97	3.5	Achievable with two hands	39
5.98	3.8	Not achievable by hand	35

Table 5.1: Conclusions

It can be seen in the table that the stiffness increases with a bigger ball diameter.
With the given demands a ball diameter of 5.97 mm is the best option for this configuration.

In this report a very specific case is being researched. It is advisable to research other cases. In particular, the primary shaft (because of the different configuration, namely three rows of four balls each) and another secondary shaft (the used secondary shaft is not made perfectly, is the shaft is rotated by 180° even the smallest balls do not all fit).
Also it is advisable for a correct result that the hysteresis, and the corresponding virtual play, is further investigated.
The force at which the grooves get indents from the balls should be investigated as well. These indents cause play locally, which is undesired.
To find an optimal ball diameter there should be balls with diameters between 5.96 and 5.97 mm and between 5.97 and 5.98 mm.

Literature list

1) Rosielle, C.P.J.N., Reker, E.A.G., (2003), *Constructie Principes 1*. Eindhoven: Tue

2) Dubbel, H., Sas, von F. (Ed.), Bouché, Ch. (Ed.), Leitner, A. (Ed.), (1974), *Taschenbuch für de Maschinenbau,* auflage 13. Berlin: Springer-Verlag.

3) Jeveka bv Catalogus, page K-382

Appendix 1: The calculated stiffness

The stiffness is being calculated with the theory of Hertz. [Appendix 6]

The formula for the stiffness is: $\frac{\partial F}{\partial \delta} = R^{1/2} E_r \delta^{1/2}$

With: $\frac{1}{R} = \frac{1}{R_{11}} + \frac{1}{R_{21}}$ and $\delta = \sqrt[3]{\frac{9F^2}{4RE_r^2}}$

R_{11} = radius of the ball ($2.98*10^{-3}$ m)
R_{21} = - radius of the hollow part of the shaft ($26.3*10^{-3}$ m)
E_r = E-modulus of Steel ($2.07*10^{11}$ N/m)
F = applied force (1147 N)
δ = flattened surface (in m)
$\frac{\partial F}{\partial \delta}$ = c = stiffness (in N/m)

This gives: R = 0.45 m and δ = $5.35*10^{-6}$ m

This gives a totally reliable result for $a = \sqrt[3]{\frac{FR}{E_r}}$ in which $a \leq 0.10 * R_{11}$

In this case, $a = 1.4*10^{-3}$ en $0.10 * R_{11} = 2.98*10^{-4}$
So the result obtained is not exact, but can give an indication of the stiffness and the movement. There is research being done to find an exact calculation, but there is no other theory that can provide a better result than the theory of Hertz.
Thus the stiffness per ball, with an angle of 45° with respect to the vertical axis, is:

$c = 3.2*10^8 \, N/m$

There are twelve balls with two contact surfaces, situated under a corner. The wanted stiffness is the tangential stiffness, this is obtained by: c_{wanted} = c * cos 45º = $2.3*10^8$ N/m

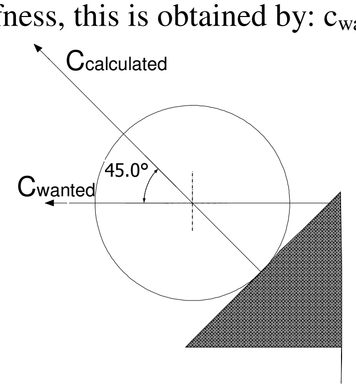

Figure A1.1: The angle between $C_{calculated}$ and C_{wanted}

Each ball has two contact surfaces, which can be represented as a ball clamped between two springs. Due to the configuration, the ball 'feels' a stiffness of $0.5*c_{spring}$.

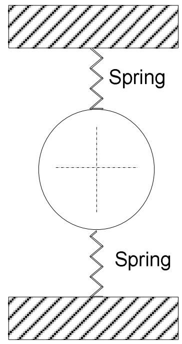

Figure A1.2: The ball clamped between two springs

The twelve balls together have a stiffness of $c_{wanted} * 6 = 1.5 * 10^9 N/m$.

Appendix 2: Calculation of the prestressing force

The moving pulley halve is rotated through a load. But the load cannot be put directly on the pulley halve, it has to be applied to a plate, that is connected to the pulley halve. If six socket-head screws with a diameter of 8 mm are used is prestressing force enough to keep the pulley halve from moving with respect to the plate?

Every screw can be prestressed with a maximum force of N=17 kN. [(3)]
F_{max} = 500 N on a distance of 0.522 m. At the edge of the pulley halve, at 0.0845 m the force is: $F_{\max} = \frac{500 * 0,5}{0,0845} = 2959N$

This is equally divided by the six screws, which get a maximum force of: $\frac{F_{\max}}{6} = 493N$

The pulley is being clamped by a prestressing force of the screw on a distance of about 8 mm from the centre line of the screw to the point where the clamping engages on the pulley halve.

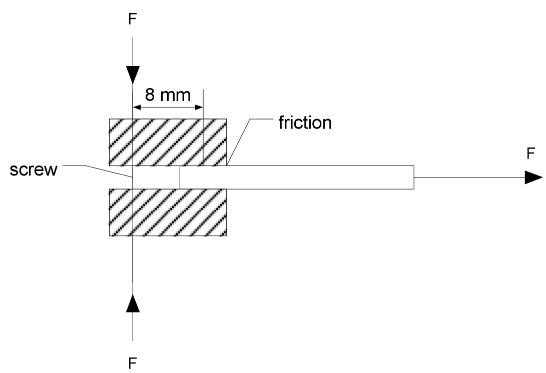

Figure A2.1: Representation of the clamping force

The pulley halve is being clamped due to friction between the pulley halve and two planes.
In this case: $F_{\max} = 2 * \mu * N$
With μ the friction coefficient. This is, for an adhesive, unlubricated contact with steel on steel: μ=0.20. [(2)]

$$N = \frac{F_{\max}}{2 * \mu} = 1233N$$

The screw is on a distance of about 8 mm, so the force (N) in the screw is:

$$N_{bolt} = \frac{N}{0,08} = 15049N$$

This can be achieved by these screws.

What is the tightening torque required to achieve this?
This can be calculated by the following equation: [(2)]

$$M_t = F_v\left[0{,}16 * P + 0{,}5 * \mu' * d_2 + 0{,}25 * \mu'' * (D_a + D_i)\right]$$

With: M_t = tightening torque (Nm)
F_v = prestressing force (N)
P = pitch (1.25 mm)
d_2 = core centre line (7.19 mm)
D_a = diameter socket head (12.8 mm)
D_i = outer centre line screw (8 mm)
μ' = 0.16
μ'' = 0.15

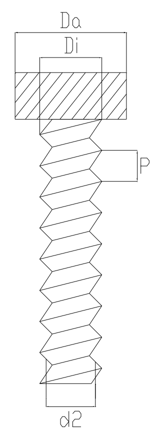

Figure A2.2: A drawing of the dimensions

$M_t = 1.45 * F_v$
So the tightening torque is: M_t = 22 kNmm = 22 Nm
The screw can have a maximum tightening torque of 24 Nm, so it can be used. [3]

Appendix 3: Measuring results; ball diameter 5.96 mm

measurement 1; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $2.62*10^9$ N/m
0	0	0	
11.8	210	275	
16.793	280	355	
21.794	350	420	
24.83	400	460	
26.856	430	490	
28.883	460	510	
30.887	495	530	
32.9	530	550	
34.915	560	560	

measurement 2; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $2.93*10^9$ N/m
0	0	5	
11.8	167	211	
16.793	230	275	
21.794	295	342	
24.83	335	379	
26.856	362	401	
28.883	390	422	
30.887	418	445	
32.9	447	468	
34.915	479	479	

measurement 3; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $3.05*10^9$ N/m
0	0	0	
11.8	215	250	
16.793	280	315	
21.794	340	375	
24.83	380	415	
26.856	405	440	
28.883	430	460	
30.887	460	480	
32.9	480	500	
34.915	515	515	

measurement 4; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: $2.62*10^9$ N/m
0	0	10	
10.403	230	310	
11.402	240	330	
23.202	405	535	
24.207	420	550	
34.207	580	685	
35.214	590	695	
40.207	670	755	
45.208	750	810	

48.244	800	840	
50.271	830	850	
52.297	860	860	

measurement 5; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 2.80*10^9 N/m
0	0	10	
10.403	230	310	
11.402	245	320	
23.202	400	515	
24.207	405	520	
34.207	550	650	
35.214	565	660	
40.215	650	720	
45.208	720	765	
48.244	760	800	
50.271	795	820	
52.297	825	825	

measurement 6; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 2.77*10^9 N/m
0	0	5	
10.403	160	235	
11.402	178	250	
23.202	340	445	
24.207	350	460	
34.207	505	595	
35.214	515	605	
40.207	590	660	
45.208	665	715	
48.244	710	745	
50.271	740	760	
52.297	765	765	

measurement 7; 52 kg			
kg	0.001 mm heen	0.001 mmm terug	Stiffness: 2.86*10^9 N/m
0	0	10	
10.403	240	320	
11.402	260	340	
23.202	405	520	
24.207	420	530	
34.207	560	650	
35.214	570	660	
40.207	645	720	
45.208	720	775	
48.244	765	800	
50.271	800	820	
52.297	825	825	

Appendix 4: Measuring results; ball diameter 5.97 mm

measurement 1; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.13*10^9$ N/m
0	0	0	
11.8	90	110	
16.793	139	161	
21.794	189	211	
24.83	218	248	
26.856	239	270	
28.883	261	291	
30.887	283	311	
32.9	309	325	
34.915	335	335	

measurement 2; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.12*10^9$ N/m
0	0	0	
11.8	90	111	
16.793	139	160	
21.794	188	211	
24.83	218	248	
26.856	239	270	
28.883	260	291	
30.887	283	311	
32.9	309	323	
34.915	336	336	

measurement 3; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.13*10^9$ N/m
0	0	0	
11.8	90	111	
16.793	139	158	
21.794	188	211	
24.83	219	248	
26.856	240	270	
28.883	261	291	
30.887	285	311	
32.9	309	324	
34.915	335	335	

measurement 4; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.13*10^9$ N/m
0	0	0	
11.8	91	111	
16.793	139	158	
21.794	188	211	
24.83	218	248	
26.856	239	269	
28.883	261	291	
30.887	282	310	
32.9	309	322	

34.915	335	335	

measurement 5; 52 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $3.44*10^9$ N/m
0	0	2	
10.403	80	134	
11.402	89	145	
23.202	199	295	
24.207	209	302	
34.207	338	430	
35.214	349	438	
40.207	422	498	
45.208	500	555	
48.244	548	582	
50.271	575	595	
52.297	602	602	

measurement 6; 52 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $3.47*10^9$ N/m
0	0	1	
10.403	81	139	
11.402	89	147	
23.202	200	291	
24.207	210	300	
34.207	330	422	
35.214	341	432	
40.207	415	488	
45.208	491	550	
48.244	538	575	
50.271	568	588	
52.297	598	598	

measurement 7; 52 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $3.48*10^9$ N/m
0	0	2	
10.403	81	139	
11.402	89	145	
23.202	200	290	
24.207	209	299	
34.207	332	421	
35.214	342	431	
40.207	417	488	
45.208	490	545	
48.244	538	571	
50.271	565	585	
52.297	595	595	

measurement 8; 52 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $3.51*10^9$ N/m
0	0	2	
10.403	80	139	
11.402	89	143	
23.202	200	289	

24.207	209	299	
34.207	330	418	
35.214	340	428	
40.207	411	486	
45.208	489	542	
48.244	532	571	
50.271	561	582	
52.297	591	591	

Appendix 5: Measuring results; ball diameter 5.98 mm

measurement 1; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.39*10^9$ N/m
0	0	1	
11.8	89	120	
16.793	129	161	
21.794	171	209	
24.83	199	235	
26.856	220	251	
28.883	241	270	
30.887	265	289	
32.9	289	302	
34.915	315	315	

measurment 2; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.45*10^9$ N/m
0	0	0	
11.8	89	119	
16.793	129	161	
21.794	171	209	
24.83	200	235	
26.856	220	251	
28.883	240	270	
30.887	261	288	
32.9	289	301	
34.915	311	311	

measurement 3; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.45*10^9$ N/m
0	0	2	
11.8	89	119	
16.793	129	161	
21.794	173	201	
24.83	200	231	
26.856	220	249	
28.883	240	268	
30.887	262	283	
32.9	289	300	
34.915	311	311	

measurement 4; 35 kg			
kg	0.001 mm up	0.001 mm down	Stiffness: $4.46*10^9$ N/m
0	0	1	
11.8	88	118	
16.793	129	159	
21.794	171	201	
24.83	199	230	
26.856	219	249	
28.883	240	268	
30.887	261	281	
32.9	285	299	

34.915	310	310	

measurement 5; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 3.80*10^9 N/m
0	0	1	
10.403	78	122	
11.402	81	132	
23.202	183	272	
24.207	191	284	
34.207	313	391	
35.214	325	400	
40.207	390	449	
45.208	452	491	
48.244	491	520	
50.271	519	537	
52.297	545	545	

measurement 6; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 3.83*10^9 N/m
0	0	2	
10.403	75	125	
11.402	81	136	
23.202	185	275	
24.207	192	288	
34.207	312	391	
35.214	325	399	
40.207	389	448	
45.208	451	491	
48.244	490	519	
50.271	518	532	
52.297	541	541	

measurement 7; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 3.83*10^9 N/m
0	0	2	
10.403	79	122	
11.402	82	135	
23.202	185	272	
24.207	191	285	
34.207	311	389	
35.214	322	399	
40.207	389	441	
45.208	451	489	
48.244	489	515	
50.271	518	531	
52.297	541	541	

measurement 8; 52 kg			
kg	0.001 mm up	0.001 mmm down	Stiffness: 3.85*10^9 N/m
0	0	1	
10.403	78	122	
11.402	82	132	
23.202	182	271	

24.207	191	282	
34.207	309	385	
35.214	319	391	
40.207	385	439	
45.208	449	485	
48.244	488	511	
50.271	512	530	
52.297	539	539	

Appendix 6: Contact surface tension theory by Hertz

College Tribologie 4B690

Cursusjaar 2001 - 2002

Uitreikbundel 6:

Samenvatting van de theorie van Hertz

(contraforme kontakten)

aan de hand van kopieën van de overhead sheets

Deze bundel bevat de kopieën van een aantal overhead sheets. die worden gebruikt in het college van 18 december 2002. Met de daarin gepresenteerde formules is het mogelijk om de normaalspanningen σ_{Hz}. de toenadering δ. en de afmetingen van het contactoppervlak (a. b) te bepalen. De waarden voor de Hertze hulpvariabelen μ_ω en ν_ω worden hierin bepaald met benaderingsformules voor kort en lang elliptisch contact. Dat kan ook via andere wegen. zoals interpolatie in een tabel of aflezing uit een grafiek. Het gebruik van de formules geeft echter minder snel aanleiding tot foutieve uitkomsten. en verdient daarom de voorkeur.

Eindhoven. 3 december 2001

Harry van Leeuwen

Samenvatting van de theorie van Hertz

Karakterisering van gekromde lichamen in punt P

Beschrijving van lichamen

uit de differentiaalmeetkunde volgt:

- er is één raakvlak (index 0) in punt P
- er zijn twee hoofd(krommings-)vlakken (indices 1 en 2) in punt P. die leggen de geometrie middels hoofdkromtestralen vast

Lichaam 1

hoofdkromtestralen van lichaam 1: R_{11} en R_{12}

(waarbij eerste index voor lichaam. tweede voor hoofdvlak)

afstand van raakvlak tot lichaam 1 ter plaatse (x_1. x_2):

$$d_1 = \frac{x_1^2}{2R_{11}} + \frac{x_2^2}{2R_{12}}$$

Lichaam 2

hoofdkromtestralen van lichaam 2: R_{21} en R_{22}

(waarbij eerste index voor lichaam. tweede voor hoofdvlak)

afstand van raakvlak tot lichaam 2 ter plaatse (x_1. x_2):

$$d_2 = \frac{x_1^2}{2R_{21}} + \frac{x_2^2}{2R_{22}}$$

Totale afstand

Zonder vervorming is de totale afstand nu:

$$d = d_1 + d_2 = \frac{x_1^2}{2R_1} + \frac{x_2^2}{2R_2}$$

waarin

$$\frac{1}{R_1} = \frac{1}{R_{11}} + \frac{1}{R_{21}}$$

en

$$\frac{1}{R_2} = \frac{1}{R_{12}} + \frac{1}{R_{22}}$$

de gereduceerde kromtestralen

Voorbeelden

Te behandelen:

- plat vlak
- bol (kogel)
- cilinder
- kegel
- torus

Hulpparameters t.b.v. karakterisering van een contact

Bij samenvallende hoofdvlakken:

- de gereduceerde kromtestraal in hoofdvlak 1:

$$\frac{1}{R_1} = \frac{1}{R_{11}} + \frac{1}{R_{21}}$$

- de gereduceerde kromtestraal in hoofdvlak 2:

$$\frac{1}{R_2} = \frac{1}{R_{12}} + \frac{1}{R_{22}}$$

- (gereduceerde) rolstraal

$$r = \min(R_1, R_2)$$

- (gereduceerde) welvingsstraal

$$R = \max(R_1, R_2)$$

- welvingsverhouding

$$\omega = \frac{\max(R_1, R_2)}{\min(R_1, R_2)} = \frac{R}{r} \geq 1$$

onderscheid (niet principieel maar handig):

$\omega = 1$	cirkelcontact
$1 < \omega < \infty$	elliptisch contact
$1 < \omega < 30$:	kort elliptisch contact

$20 < \omega < \infty$: lang elliptisch contact

$\omega \to \infty$ lijncontact

Bij niet-samenvallende hoofdvlakken:

$\Rightarrow$ …..

kunstgreep nodig; deze theorie blijft dan onverkort geldig

Bij holle/bolle loopvlakken

indien het lichaam bol is. dan is $R_{ij} > 0$

indien het lichaam hol is. dan is $R_{ij} < 0$

zodat ook de afstand tussen een bol in een hol lichaam (bijv. as in lager) kan worden gemodelleerd

Bepaling van Hertze grootheden σ_{Hz}. a. b

- cirkelcontact (exact)

$$\sigma_{Hz,cirkel} = \frac{3F}{2\pi a^2} = \sqrt[3]{\frac{3F E_r^2}{2\pi^3 r^2}}, \quad a = b = \sqrt[3]{\frac{3Fr}{2E_r}}, \quad \delta_{cirkel} = \sqrt[3]{\frac{9F^2}{4RE_r^2}}$$

- elliptisch contact (exact)

$$\sigma_{Hz} = \frac{3F}{2\pi a b} \quad a = \mu_\omega \sqrt[3]{\frac{3Fr}{E_r}} \quad b = \nu_\omega \sqrt[3]{\frac{3Fr}{E_r}}, \quad \delta_{ell} = \frac{a^2}{2R} + \frac{b^2}{2r}$$

met benaderingen:

- voor kort elliptisch contact ($1 \le \omega < 30$):

$$\mu_\omega \approx 0.794\,\omega^{11/24} \qquad \nu_\omega \approx 0.794\,\omega^{-4/21}$$

- voor lang elliptisch contact ($20 < \omega < 10^5$):

$$\mu_\omega \approx 1.015\,\omega^{8/21} \qquad \nu_\omega \approx 0.794\,\omega^{-4/21}$$

- lijncontact (exact)

$$\sigma_{Hz} = \sqrt{\frac{F E_r}{2\pi l r}} = \frac{2F}{\pi b l}, \quad b = \sqrt{\frac{8Fr}{\pi E_r l}} = \frac{4\sigma_{Hz} r}{E_r}, \quad \delta_{lijn,theor} \to \infty$$

en bij benadering. voor $b \ll l$:

$$\delta_{lijn} \approx \frac{2\,F}{\pi\,E_r\,l}\left(1+\ln\left\{\frac{\pi}{2}\left(\frac{l}{r}\right)^2\right\}-\ln\left(\frac{F}{E_r\,l\,r}\right)\right)$$

waarmee voor staal/staal contact bij benadering geldt:

$$\delta_{lijn,\,St/St} \approx \frac{2{,}58\,F^{0{,}9}}{E_r^{0{,}9}\,l^{0{,}8}}$$

De maximale belasting in wentellagers volgens Stribeck (1901)

Vooronderstelling: lagers zonder speling

- kogellagers

$$F_{\max} \approx \frac{4{,}37\,F_{rad}}{z}$$

- rollenlagers

$$F_{\max} \approx \frac{4{,}06\,F_{rad}}{z}$$

waarin

F_{rad} = de totale radiale belasting op het lager

F_{max} = de belasting in het zwaarst belaste contact

z = het totale aantal wentellichamen in het lager